6. Determinants

Exercise 6A

1. Question

If A is a 2 × 2 matrix such that |A| ≠ 0 and |A| = 5, write the value of |4A|.

Answer

Theorem: If A be k × k matrix then $|pA|=p^k|A|$.

Given, p=4,k=2 and |A|=5.

$|4A|=4^2 \times 5$

$=16 \times 5$

=80

2. Question

If A is a 3 × 3 matrix such that |A| ≠ 0 and |3A| = k|A| then write the value of k.

Answer

Theorem: If Let A be k × k matrix then $|pA|=p^k|A|$.

Given: k=3 and p=3.

$|3A|=3^3 \times |A|$

=27|A|.

Comparing above with k|A| gives k=27.

3. Question

Let A be a square matrix of order 3, write the value of |2A|, where |A| = 4.

Answer

Theorem: If A be k × k matrix then $|pA|=p^k|A|$.

Given: p=2, k=3 and|A|=4

$|2A|=2^3 \times |A|$

$=8 \times 4$

= 32

4. Question

If A_{ij} is the cofactor of the element a_{ij} of $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ then write the value of $(a_{32}A_{32})$.

Answer

Theorem: A_{ij} is found by deleting i^{th} rowand j^{th} column, the determinant of left matrix is called cofactor with multiplied by $(-1)^{(i+j)}$.

Given: i=3 and j=2.

$A_{32}=(-1)^{(3+2)}(2 \times 4-6 \times 5)$

$=-1 \times (-22)$

=22

a_{32}=5

$a_{32}A_{32}$= 5 × 22

=110

5. Question

Evaluate $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$.

Answer

Theorem: This evaluation can be done in two different ways either by taking out the common things and then calculating the determinants or simply take determinant.

I will prefer first method because with that chances of silly mistakes reduces.

Take out x+1 from second row.

(x+1) × $\begin{vmatrix} x^2-x+1 & x-1 \\ 1 & 1 \end{vmatrix}$

⇒(x+1) × (x^2-x+1-(x-1))

⇒ (x+1) × (x^2-2x+2)

⇒ x^3-2x^2+2x+x^2-2x+2

⇒ x^3-x^2+2 .

6. Question

Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$.

Answer

This we can very simply go through directly.

((a+ib)(a-ib))-((-c+id)(c+id)).

⇒ (a^2+b^2)-(-c^2 -d^2).

⇒ a^2+b^2 + c^2+d^2

∵ i × i=-1

7. Question

If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, write the value of x.

Answer

Here the determinant is compared so we need to take determinant both sides then find x.

12x+14=32-42

⇒ 12x=-10-14

⇒ 12x=-24

⇒ x=-2

8. Question

If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x.

Answer

this question is having the same logic as above.

$2x^2$-40=18+14

⇒ $2x^2$=72

⇒ x^2=36

⇒ x=± 6.

9. Question

If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, write the value of x.

Answer

Simply by equating both sides we can get the value of x.

$2x^2+2x-2(x^2+4x+3)=-12$

⇒ -6x-6=-12

⇒ -6x=-6

⇒ $x = 1$

10. Question

If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, find the value of 3|A|.

Answer

Find the determinant of A and then multiply it by 3

|A|=2

3|A|=3 × 2

=6

11. Question

Evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$.

Answer

It is determinant multiplied by a scalar number 2, just find determinant of matrix and multiply it by 2.

2 × (35-20)

2 × 15= 30

12. Question

Evaluate $\begin{vmatrix} \sqrt{6} & \sqrt{5} \\ \sqrt{20} & \sqrt{24} \end{vmatrix}$.

Answer

Find determinant

$\sqrt{6} \times \sqrt{24} - \sqrt{20} \times \sqrt{5}$

$\sqrt{144} - \sqrt{100}$.

=12-10

=2.

13. Question

Evaluate $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$.

Answer

After finding determinant we will get a trigonometric identity.

$2\cos^2\theta + 2\sin^2\theta$

=2

$\because \sin^2\theta + \cos^2\theta = 1$

14. Question

Evaluate $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$.

Answer

After finding determinant we will get a trigonometric identity.

$\cos^2\alpha + \sin^2\alpha$

=1

$\because \sin^2\theta + \cos^2\theta = 1$

15. Question

Evaluate $\begin{vmatrix} \sin 60^\circ & \cos 60^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{vmatrix}$.

Answer

After finding determinant we will get,

Sin60° $= \frac{\sqrt{3}}{2} =$ cos30°

Cos60° $= \frac{1}{2} =$ sin30°

sin 60° × cos30° + sin30° × cos60°

$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

= 1.

16. Question

Evaluate $\begin{vmatrix} \cos 65^\circ & \sin 65^\circ \\ \sin 25^\circ & \cos 25^\circ \end{vmatrix}$.

Answer

By directly opening this determinant

cos65° × cos25° -sin25° × sin65°

= cos(65°+25°) ∵ cosAcosB-sinAsinB=cos(A+B)

= cos90°

= 0

∵ cosAcosB-sinAsinB=cos(A+B)

17. Question

Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$.

Answer

cos15°cos75° - sin75°sin15°

= cos(15°+75°) ∵ cosAcosB-sinAsinB=cos(A+B)

= cos90°

= 0

∵ cosAcosB-sinAsinB=cos(A+B)

18. Question

Evaluate $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$.

Answer

We know that expansion of determinant with respect to first row is $a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$.

0(3 × 6-5 × 4)-2(2 × 6-4 × 4)+0(2 × 5-4 × 3)

= 8.

19. Question

Without expanding the determinant, prove that $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix} = 0$.

SINGULAR MATRIX A square matrix A is said to be singular if |A| = 0.

Also, A is called non singular if |A| ≠ 0.

Answer

We know that $C_1 \Rightarrow C_1 - C_2$, would not change anything for the determinant.

Applying the same in above determinant, we get

$$\begin{bmatrix}40 & 1 & 5\\72 & 7 & 9\\24 & 5 & 3\end{bmatrix}$$ Now it can clearly be seen that $C_1=8 \times C_3$

Applying above equation we get,

$$\begin{bmatrix}0 & 1 & 5\\0 & 7 & 9\\0 & 3 & 3\end{bmatrix}$$

We know that if a row or column of a determinant is 0. Then it is singular determinant.

20. Question

For what value of x, the given matrix $A = \begin{bmatrix}3-2x & x+1\\2 & 4\end{bmatrix}$ is a singular matrix?

Answer

For A to be singular matrix its determinant should be equal to 0.

0= (3-2x) × 4-(x+1) × 2

0= 12-8x-2x-2

0=10-10x

X=1.

21. Question

Evaluate $\begin{vmatrix}14 & 9\\-8 & -7\end{vmatrix}$.

Answer

$\begin{vmatrix}14 & 9\\-8 & -7\end{vmatrix}$.=14 × (-7)-9 × (-8)

= -26

22. Question

Evaluate $\begin{vmatrix}\sqrt{3} & \sqrt{5}\\-\sqrt{5} & 3\sqrt{3}\end{vmatrix}$.

Answer

$\begin{vmatrix}\sqrt{3} & \sqrt{5}\\-\sqrt{5} & 3\sqrt{3}\end{vmatrix}$. = 3√3 × √3 – (-√5 × √5)

= 14.

Exercise 6B

1. Question

Evaluate :

$$\begin{vmatrix}67 & 19 & 21\\39 & 13 & 14\\81 & 24 & 26\end{vmatrix}$$

Answer

$$\begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

$$= \left(\frac{1}{2}\right)\begin{vmatrix} 67 & 19 & 21 \\ 78 & 26 & 28 \\ 81 & 24 & 26 \end{vmatrix}$$ [$R_2' = (1/2)R_2$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} 67 & 19 & 21 \\ -3 & 2 & 2 \\ 81 & 24 & 26 \end{vmatrix}$$ [$R_2' = R_2 - R_3$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} -14 & -5 & -5 \\ -3 & 2 & 2 \\ 81 & 24 & 26 \end{vmatrix}$$ [$R_1' = R_1 - R_3$]

$$= \begin{vmatrix} -14 & -5 & -5 \\ -3 & 2 & 2 \\ 81/2 & 12 & 13 \end{vmatrix}$$ [$R_3' = 2R_3$]

$= (-14)\{(2 \times 13) - (2 \times 12)\} - 5\{(2 \times 81/2) - (-3) \times 13\} - 5\{(-3) \times 12 - 2 \times 81/2\}$

[expanding by the first row]

$= -14 \times (26 - 24) - 5(81 + 39) - 5(-36 - 81)$

$= -14 \times 2 - 5 \times 120 - 5 \times (-117) = -28 - 600 + 585 =$ **-43**

2. Question

Evaluate :

$$\begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$$

Answer

$$\begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & -5 & -5 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$$ [$R_1' = R_1 - R_2$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} 4 & -5 & -5 \\ 50 & 62 & 54 \\ 63 & 54 & 46 \end{vmatrix}$$ [$R_2' = 2R_2$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} 4 & -5 & -5 \\ -13 & 8 & 8 \\ 63 & 54 & 46 \end{vmatrix}$$ [$R_2' = R_2 - R_3$]

$$= \begin{vmatrix} 4 & -5 & -5 \\ -13 & 8 & 8 \\ 63/2 & 27 & 23 \end{vmatrix}$$ [$R_3' = 2R_3$]

$= 4(8 \times 23 - 8 \times 27) - 5\{8 \times 63/2 - (-13) \times 23\} - 5\{(-13) \times 27 - 8 \times 63/2\}$

[expansion by first row]

= 132

3. Question

Evaluate :

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

Answer

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6 \times \begin{vmatrix} 17 & 18 & 6 \\ 1 & 6 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$ [$R_1' = R_1/6$]

Now, for any determinant, if at least two rows are identical, then the value of the determinant becomes zero.

Here, the first and third rows are identical.

So, the value of the above determinant evaluated = **0**

4. Question

Evaluate :

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 2^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$$

Answer

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Expanding by first row, we get,

1(9 × 25 - 16 × 16) + 4(16 × 9 - 4 × 25) + 9(4 × 16 - 9 × 9) = - 31 + 176 - 153 =**- 8**

5. Question

Using properties of determinants prove that:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a).$$

Answer

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ bc-ca & ca-ab & ab \end{vmatrix}$$ [$C_1' = C_1 - C_2$ & $C_2' = C_2 - C_3$]

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ -c(a-b) & -a(b-c) & ab \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -c & -a & ab \end{vmatrix}$$ [$C_1' = C_1/(a - b)$ & $C_2' = C_2/(b - c)$]

= (a - b)(b - c)[0 + 0 + 1{ - a - (- c)}] [expansion by first row]

= **(a - b)(b - c)(c - a)**

6. Question

Using properties of determinants prove that:

$$\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a).$$

Answer

$$\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & b-a & b^2-a^2 \\ 0 & c-b & c^2-b^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix}$$ [$R_1' = R_1 - R_2$ & $R_2' = R_2 - R_3$]

$$= \begin{vmatrix} 0 & b-a & (b-a)(b+a) \\ 0 & c-b & (c-b)(c+b) \\ 1 & a+b & a^2+b^2 \end{vmatrix}$$

$$= (b-a)(c-b)\begin{vmatrix} 0 & 1 & b+a \\ 0 & 1 & c+b \\ 1 & a+b & a^2+b^2 \end{vmatrix}$$ [$R_1' = R_1/(b - a)$ & $R_2' = R_2/(c - b)$]

= (b - a)(c - b)[0 + 0 + 1{(c + b) - (b + a)}][expansion by first column]

= **(a - b)(b - c)(c - a)**

7. Question

Using properties of determinants prove that:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1.$$

Answer

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -2-p & -2p-q \\ -1 & -3-p & -3p-q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$$ [$R_1' = R_1 - R_2$ & $R_2' = R_2 - R_3$]

$$= \begin{vmatrix} 0 & 1 & p \\ -1 & -3-p & -3p-q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$$ [$R_1' = R_1 - R_2$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} 0 & 1 & p \\ -2 & -6-2p & -6p-2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$$ [$R_2' = R_2*2$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} 0 & 1 & p \\ 1 & p & 1+q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$$ [$R_2' = R_2 + R_3$]

= (1/2)[0 + 3(1 + q) - (1 + 6p + 3q) + p(6 + 3p - 3p)] [expansion by first row]

= (1/2)(3 + 3q - 1 - 6p - 3q + 6p) = **1**

8. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z).$$

Answer

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

$= \begin{vmatrix} a & -a & 0 \\ 0 & a & -a \\ x & y & a+z \end{vmatrix}$ [$R_1' = R_1 - R_2$ & $R_2' = R_2 - R_3$]

$= a^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ x & y & a+z \end{vmatrix}$ [$R_1' = R_1/a$ & $R_2' = R_2/a$]

= a^2[a + z - (- y) - (- x)] [expansion by first row]

= **a^2(a + x + y + z)**

9. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x+2a)(x-a)^2.$$

Answer

$$\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix}$$

$= \begin{vmatrix} x+2a & x+2a & x+2a \\ a & x & a \\ a & a & x \end{vmatrix}$ [$R_1' = R_1 + R_2 + R_3$]

$= (x+2a)\begin{vmatrix} 1 & 1 & 1 \\ a & x & a \\ a & a & x \end{vmatrix}$ [$R_1' = R_1/(x + 2a)$]

$= (x+2a)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-a & a-x \\ a & a & x \end{vmatrix}$ [$R_2' = R_2 - R_3$]

$= (x+2a)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-a & -(x-a) \\ a & a & x \end{vmatrix}$

$= (x+2a)(x-a)\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ a & a & x \end{vmatrix}$ [$R_2' = R_2/(x - a)$]

= (x + 2a)(x - a)[x - (- a) + (- a - 0) + (- a)] [expansion by first row]

= (x + 2a)(x - a)(x + a - a - a) = **(x + 2a)(x - a)^2**

10. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(x-4)^2.$$

Answer

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$= \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} \; [R_1' = R_1 + R_2 + R_3]$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} \; [R_1' = R_1/(5x+4)]$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -x+4 & x-4 \\ 2x & 2x & x+4 \end{vmatrix} \; [R_2' = R_2 - R_3]$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -(x-4) & x-4 \\ 2x & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)(x-4)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2x & 2x & x+4 \end{vmatrix} \; [R_2' = R_2/(x-4)]$$

$= (5x + 4)(x - 4)[- (x + 4) - 2x + 2x - 0 + 0 - (- 2x)]$ [expansion by first row]

$= (5x + 4)(x - 4)(- x - 4 + 2x) =$ **$(5x + 4)(x - 4)^2$**

11. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda-x)^2.$$

Answer

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$= \begin{vmatrix} 5x+\lambda & 5x+\lambda & 5x+\lambda \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} \; [R_1' = R_1 + R_2 + R_3]$$

$$= (5x+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} \; [R_1' = R_1/(5x+\lambda)]$$

$$= (5x+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -x+\lambda & x-\lambda \\ 2x & 2x & x+\lambda \end{vmatrix} \; [R_2' = R_2 - R_3]$$

$$= (5x+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -(x-\lambda) & x-\lambda \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$= (5x+\lambda)(x-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2x & 2x & x+\lambda \end{vmatrix} \; [R_2' = R_2/(x-\lambda)]$$

$= (5x + \lambda)(x - \lambda)[- (x + \lambda) - 2x + 2x - 0 + 0 - (- 2x)]$ [expansion by first row]

$= (5x + \lambda)(x - \lambda)(- x - \lambda + 2x) =$ $\mathbf{(5x + \lambda)(x - \lambda)^2}$

12. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3.$$

Answer

$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix} \; [R_1' = R_1 - R_2 \;\&\; R_2' = R_2 - R_3]$$

$$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} \; [R_1' = R_1/(a - 1) \;\&\; R_2' = R_2/(a - 1)]$$

$= (a - 1)^2[a + 1 - 0 - 2]$ [expansion by first row]

$= \mathbf{(a - 1)^3}$

13. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y).$$

Answer

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

$$= \begin{vmatrix} 3(x+y) & 3(x+y) & 3(x+y) \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} \; [R_1' = R_1 + R_2 + R_3]$$

$$= 3(x+y) \begin{vmatrix} 1 & 1 & 1 \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} \; [R_1' = R_1/3(x + y)]$$

$$= 3(x+y) \begin{vmatrix} 1 & 1 & 1 \\ y & -2y & y \\ x+y & x+2y & x \end{vmatrix} \; [R_2' = R_2 - R_3]$$

$$= 3y(x+y) \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ x+y & x+2y & x \end{vmatrix} \; [R_2' = R_2/y]$$

$$= 3y(x+y)\begin{vmatrix} 0 & 3 & 0 \\ 1 & -2 & 1 \\ x+y & x+2y & x \end{vmatrix}$$ [$R_1' = R_1 - R_2$]

= 3y(x + y)[0 + 3(x + y) - x + 0] [expansion by first row]

= 3y(x + y)(3y) = **$9y^2(x + y)$**

14. Question

Using properties of determinants prove that:

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx).$$

Answer

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$$

$$= \begin{vmatrix} x+y+z & -x+y & -x+z \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$$ [$C_1' = C_1 + C_2 + C_3$]

$$= (x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix}$$ [$C_1' = C_1/(x + y + z)$]

$$= (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ -x+y & 3y & y-z \\ -x+z & z-y & 3z \end{vmatrix}$$ [transforming row and column]

$$= (x+y+z)\begin{vmatrix} 0 & 0 & 1 \\ -x-2y & 2y+z & y-z \\ -x+y & -y-2z & x \end{vmatrix}$$ [$C_1' = C_1 - C_2$ & $C_2' = C_2 - C_3$]

= (x + y + z)[0 + 0 + (- x - 2y)(- y - 2z) - (- x + y)(2y + z)] [expansion by first row]

= $(x + y + z)(xy + 2y^2 + 2xz + 4yz + 2xy - 2y^2 + xz - yz)$

= (x + y + z)(3xy + 3yz + 3xz)

= 3(x + y + z)(xy + yz + zx)

15. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x).$$

Answer

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$ [$C_1' = C_1/x$, $C_2' = C_2/y$ & $C_3' = C_3/z$]

$$= xyz\begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \end{vmatrix}$$ [$C_1' = C_1 - C_2$ & $C_2' = C_2 - C_3$]

$$= xyz\begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ (x+y)(x-y) & (y+z)(y-z) & z^2 \end{vmatrix}$$

$$= xyz(x-y)(y-z)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & z \\ x+y & y+z & z^2 \end{vmatrix}$$ [$C_1' = C_1/(x - y)$& $C_2' = C_2/(y - z)$]

= xyz(x - y)(y - z)(0 + 0 + y + z - x - y) [expansion by first row]

= xyz(x - y)(y - z)(z - x)

16. Question

Using properties of determinants prove that:

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3.$$

Answer

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

$$= \begin{vmatrix} 2(a+b+c) & 0 & a+b+c \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$ [$R_1' = R_1 + R_2 + R_3$]

$$= (a+b+c)\begin{vmatrix} 2 & 0 & 1 \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$ [$R_1' = R_1/(a + b + c)$]

= (a + b + c)[2(b - c)c - b(c - a) + (c + a)(c - a) - (a + b)(b - c)][expansion by first row]

= (a + b + c)(2bc - 2c^2 - bc + ab + c^2 - a^2 - ab - b^2 + ac + bc

= (a + b + c)(ab + bc + ac - a^2 - b^2 - c^2)

= 3abc - a^3 - b^3 - c^3

17. Question

Using properties of determinants prove that:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

Answer

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 2(b+c) & 2(a+c) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$ [$R_1' = R_1 + R_2 + R_3$]

$$= 2\begin{vmatrix} b+c & a+c & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} [R_1' = R_1/2]$$

$$= 2\begin{vmatrix} c & 0 & a \\ b-c & a & -a \\ c & c & a+b \end{vmatrix} [R_1' = R_1 - R_2 \ \& \ R_2' = R_2 - R_3]$$

= 2[c{a(a + b) - (- ac)} + 0 + a{c(b - c) - ac}][expansion by first row]

$= 2(a^2c + abc + ac^2 + abc - ac^2 - a^2c)$

= **4abc**

18. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} = -a^3.$$

Answer

$$\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$$

$$= \left(\frac{1}{3}\right)\begin{vmatrix} 3a & 3a+6b & 3a+6b+9c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_1' = 3R_1]$$

$$= \left(\frac{1}{3}\right)\begin{vmatrix} 0 & -a & -2a-b \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_1' = R_1 - R_2]$$

$$= \left(\frac{1}{6}\right)\begin{vmatrix} 0 & -a & -2a-b \\ 6a & 8a+12b & 10a+14b+18c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_2' = 2R_2]$$

$$= \left(\frac{1}{6}\right)\begin{vmatrix} 0 & -a & -2a-b \\ 0 & -a & -a-b \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_2' = R_2 - R_3]$$

= (1/6)[0 + 0 + 6a{a(a + b) - a(2a + b)][expansion by first column]

= **$-a^3$**

19. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

Answer

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$$= \begin{vmatrix} a+b & a+b & -(a+b) \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} [R_1' = R_1 + R_2]$$

$$= (a + b)\begin{vmatrix} 1 & 1 & -1 \\ -c & a + b + c & -a \\ -b & -a & a + b + c \end{vmatrix} [R_1' = R_1/(a + b)]$$

$$= (a + b)\begin{vmatrix} 1 & 1 & -1 \\ -c - b & b + c & b + c \\ -b & -a & a + b + c \end{vmatrix} [R_2' = R_2 + R_3]$$

$$= (a + b)\begin{vmatrix} 1 & 1 & -1 \\ -(b + c) & b + c & b + c \\ -b & -a & a + b + c \end{vmatrix}$$

$$= (a + b)(b + c)\begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -b & -a & a + b + c \end{vmatrix} [R_2' = R_1/(b + c)]$$

$$= (a + b)(b + c)\begin{vmatrix} 0 & 2 & 0 \\ -1 & 1 & 1 \\ -b & -a & a + b + c \end{vmatrix} [R_1' = R_1 + R_2]$$

= (a + b)(b + c){0 + 2(- b + a + b + c) + 0}[expansion by first row]

= 2(a + b)(b + c)(c + a)

20. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a & b & ax + by \\ b & c & bx + cy \\ ax + by & bx + cy & 0 \end{vmatrix} = \left(b^2 - ac\right)\left(ax^2 + 3bxy + cy^2\right).$$

Answer

$$\begin{vmatrix} a & b & ax + by \\ b & c & bx + cy \\ ax + by & bx + cy & 0 \end{vmatrix}$$

$$= \left(\frac{1}{xy}\right)\begin{vmatrix} ax & bx & ax^2 + bxy \\ by & cy & bxy + cy^2 \\ ax + by & bx + cy & 0 \end{vmatrix} [R_1' = xR_1 \ \& \ R_2' = yR_2]$$

$$= \left(\frac{1}{xy}\right)\begin{vmatrix} 0 & 0 & ax^2 + 2bxy + cy^2 \\ by & cy & bxy + cy^2 \\ ax + by & bx + cy & 0 \end{vmatrix} [R_1' = R_1 + R_2 - R_3]$$

= $(1/xy)[0 + 0 + (ax^2 + 2bxy + cy^2)\{by(bx + cy) - cy(ax + by)\}]$[expansion by first row] .

= $(1/xy)(ax^2 + 2bxy + cy^2)(b^2xy + bcy^2 - acxy - bcy^2)$

= $(b^2 - ac)(ax^2 + 2bxy + cy^2)$

21. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 4(a-b)(b-c)(c-a)$$

Answer

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ a^2+2a+1 & b^2+2b+1 & c^2+2c+1 \\ a^2-2a+1 & b^2-2b+1 & c^2-2c+1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ a^2-2a+1 & b^2-2b+1 & c^2-2c+1 \end{vmatrix} [R_2' = R_2 - R_3]$$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ a^2-2a+1 & b^2-2b+1 & c^2-2c+1 \end{vmatrix} [R_2' = R_2/4]$$

$$= 4\begin{vmatrix} a^2 & a & a^2-2a+1 \\ b^2 & b & b^2-2b+1 \\ c^2 & c & c^2-2c+1 \end{vmatrix} \text{[transforming row and column]}$$

$$= 4\begin{vmatrix} a^2-b^2 & a-b & (a^2-b^2)-2(a-b) \\ b^2-c^2 & b-c & (b^2-c^2)-2(b-c) \\ c^2 & c & c^2-2c+1 \end{vmatrix} [R_1' = R_1 - R_2 \ \& \ R_2' = R_2 - R_3]$$

$$= 4\begin{vmatrix} (a-b)(a+b) & a-b & (a-b)(a+b-2) \\ (b-c)(b+c) & b-c & (b-c)(b+c-2) \\ c^2 & c & c^2-2c+1 \end{vmatrix}$$

$$= 4(a-b)(b-c)\begin{vmatrix} a+b & 1 & a+b-2 \\ b+c & 1 & b+c-2 \\ c^2 & c & c^2-2c+1 \end{vmatrix} [R_1' = R_1/(a-b) \ \& \ R_2' = R_2/(b-c)]$$

$$= 4(a-b)(b-c)\begin{vmatrix} a-c & 0 & a-c \\ b+c & 1 & b+c-2 \\ c^2 & c & c^2-2c+1 \end{vmatrix} [R_1' = R_1 - R_2]$$

$$= 4(a-b)(b-c)(a-c)\begin{vmatrix} 1 & 0 & 1 \\ b+c & 1 & b+c-2 \\ c^2 & c & c^2-2c+1 \end{vmatrix} [R_1' = R_1/(a-c)]$$

$= 4(a - b)(b - c)(a - c)(c^2 - 2c + 1 - bc - c^2 + 2c + 0 + bc + c^2 - c^2)$ [expansion by first row]

= 4(a - b)(b - c)(c - a)

22. Question

Using properties of determinants prove that:

$$\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix} = -8$$

Answer

$$\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix}$$

$$= \begin{vmatrix} x^2-4x+4 & x^2-2x+1 & x^2 \\ x^2-2x+1 & x^2 & x^2+2x+1 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix}$$

$$= \begin{vmatrix} -2x+3 & -2x+1 & -2x-1 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix} [R_1' = R_1 - R_2 \ \& \ R_2' = R_2 - R_3]$$

$$= \begin{vmatrix} 2 & 2 & 2 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix}$$ [R₁' = R₁ - R₂]

$$= 2\begin{vmatrix} 1 & 1 & 1 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix}$$ [$R_1' = R_1/2$]

$$= 2\begin{vmatrix} 1 & -2x+1 & x^2 \\ 1 & -2x-1 & x^2+2x+1 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix}$$ [transforming row and column]

$$= 2\begin{vmatrix} 0 & 2 & -2x-1 \\ 0 & 2 & -2x-3 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix}$$ [$R_1' = R_1 - R_2$ & $R_2' = R_2 - R_3$]

$$= 2\begin{vmatrix} 0 & 0 & 2 \\ 0 & 2 & -2x-3 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix}$$ [$R_1' = R_1 - R_2$]

= 2{0 + 0 + 2(0 - 2)} [expansion by first row]

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23. Question

Using properties of determinants prove that:

$$\begin{vmatrix} (m+n)^2 & l^2 & mn \\ (n+l)^2 & m^2 & ln \\ (l+m)^2 & n^2 & lm \end{vmatrix} = (l^2+m^2+n^2)(l-m)$$

$$(m-n)(n-l).$$

Answer

$$\begin{vmatrix} (m+n)^2 & l^2 & mn \\ (n+l)^2 & m^2 & ln \\ (l+m)^2 & n^2 & lm \end{vmatrix}$$

$$= \left(\frac{1}{2}\right)\begin{vmatrix} m^2+2mn+n^2 & l^2 & 2mn \\ n^2+2nl+l^2 & m^2 & 2ln \\ l^2+2lm+m^2 & n^2 & 2lm \end{vmatrix}$$ [$C_3' = 2C_3$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} m^2+n^2 & l^2 & 2mn \\ n^2+l^2 & m^2 & 2ln \\ l^2+m^2 & n^2 & 2lm \end{vmatrix}$$ [$C_1' = C_1 - C_3$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} l^2+m^2+n^2 & l^2 & 2mn \\ l^2+m^2+n^2 & m^2 & 2ln \\ l^2+m^2+n^2 & n^2 & 2lm \end{vmatrix}$$ [$C_1' = C_1 + C_2$]

$$= \left(\frac{1}{2}\right)(l^2+m^2+n^2)\begin{vmatrix} 1 & l^2 & 2mn \\ 1 & m^2 & 2ln \\ 1 & n^2 & 2lm \end{vmatrix}$$ [$C_1' = C_1/(l^2 + m^2 + n^2)$]

$$= \left(\frac{1}{2}\right)(l^2+m^2+n^2)\begin{vmatrix} 1 & 1 & 1 \\ l^2 & m^2 & n^2 \\ 2mn & 2ln & 2lm \end{vmatrix}$$ [transforming row and column]

$$= \left(\frac{1}{2}\right)(l^2+m^2+n^2)\begin{vmatrix} 0 & 0 & 1 \\ l^2-m^2 & m^2-n^2 & n^2 \\ -2n(l-m) & -2l(m-n) & 2lm \end{vmatrix}$$ [$C_1' = C_1 - C_2$ & $C_2' = C_2 - C_3$]

$= (l^2 + m^2 + n^2)(l-m)(m-n)\begin{vmatrix} 0 & 0 & 1 \\ l+m & m+n & n^2 \\ -n & -l & lm \end{vmatrix}$ [$C_1' = C_1/(l - m)$ & $R_2' = C_2/(l - m)$]

$= (l^2 + m^2 + n^2)(l - m)(m - n)\{0 + 0 - l(l + m) + n(m + n)\}$ [expansion by first row]

$= (l^2 + m^2 + n^2)(l - m)(m - n)\{0 + 0 - l(l + m) + n(m + n)\}$

$= (l^2 + m^2 + n^2)(l - m)(m - n)(- l^2 - ml + mn + n^2)$

$= (l^2 + m^2 + n^2)(l - m)(m - n)\{(n^2 - l^2) + m(n - l)\}$

$= (l^2 + m^2 + n^2)(l - m)(m - n)(n - l)(l + m + n)$

24. Question

Using properties of determinants prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2+b^2+c^2)(a-b)\ (b-c)(c-a)(a+b+c).$$

Answer

$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$= \left(\frac{1}{2}\right)\begin{vmatrix} b^2+2bc+c^2 & a^2 & 2bc \\ c^2+2ac+a^2 & b^2 & 2ca \\ a^2+2ab+b^2 & c^2 & 2ab \end{vmatrix}$ [$C_3' = 2C_3$]

$= \left(\frac{1}{2}\right)\begin{vmatrix} b^2+c^2 & a^2 & 2bc \\ c^2+a^2 & b^2 & 2ca \\ a^2+b^2 & c^2 & 2ab \end{vmatrix}$ [$C_1' = C_1 - C_3$]

$= \left(\frac{1}{2}\right)\begin{vmatrix} a^2+b^2+c^2 & a^2 & 2bc \\ a^2+b^2+c^2 & b^2 & 2ca \\ a^2+b^2+c^2 & c^2 & 2ab \end{vmatrix}$ [$C_1' = C_1 + C_2$]

$= \left(\frac{1}{2}\right)(a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & 2bc \\ 1 & b^2 & 2ca \\ 1 & c^2 & 2ab \end{vmatrix}$ [$C_1' = C_1/(a^2 + b^2 + c^2)$]

$= \left(\frac{1}{2}\right)(a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ 2bc & 2ca & 2ab \end{vmatrix}$ [transforming row and column]

$= \left(\frac{1}{2}\right)(a^2+b^2+c^2)\begin{vmatrix} 0 & 0 & 1 \\ a^2-b^2 & b^2-c^2 & c^2 \\ -2c(a-b) & -2a(b-c) & 2ab \end{vmatrix}$ [$C_1' = C_1 - C_2$ & $C_2' = C_2 - C_3$]

$= (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ a+b & b+c & c^2 \\ -c & -a & ab \end{vmatrix}$ [$C_1' = C_1/(a - b)$ & $C_2' = C_2/(b - c)$]

$= (a^2 + b^2 + c^2)(a - b)(b - c)\{0 + 0 - a(a + b) + c(b + c)\}$ [expansion by first row]

$= (a^2 + b^2 + c^2)(a - b)(b - c)\{0 + 0 - a(a + b) + c(b + c)\}$

$= (a^2 + b^2 + c^2)(a - b)(b - c)(- a^2 - ba + bc + c^2)$

$= (a^2 + b^2 + c^2)(a - b)(b - c)\{(c^2 - a^2) + b(c - a)\}$

$= (a^2 + b^2 + c^2)(a - b)(b - c)(c - a)(a + b + c)$

25. Question

Using properties of determinants prove that:

$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2.$$

Answer

$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 2(b^2+c^2) & 2(c^2+a^2) & 2(a^2+b^2) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} [R_1' = R_1 + R_2 + R_3]$$

$$= 2\begin{vmatrix} (b^2+c^2) & (c^2+a^2) & (a^2+b^2) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} [R_1' = R_1/2]$$

$$= 2\begin{vmatrix} c^2 & 0 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} [R_1' = R_1 - R_2]$$

$= 2[c^2\{(c^2 + a^2)(a^2 + b^2) - b^2c^2\} + 0 + a^2\{b^2c^2 - c^2(c^2 + a^2)\}]$ [expansion by first row]

$= 2[c^2(c^2a^2 + a^4 + b^2c^2 + a^2b^2 - b^2c^2) + a^2(b^2c^2 - c^4 - a^2c^2)]$

$= 2[a^2c^4 + a^4c^2 + a^2b^2c^2 + a^2b^2c^2 - a^2c^4 - a^4c^2]$

$= \mathbf{4a^2b^2c^2}$

26. Question

Using properties of determinants prove that:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3.$$

Answer

Operating $R_1 \to R_1 + bR_3$, $R_2 \to R_2 - aR_3$

$$\begin{vmatrix} 1+a^2-b^2+2b^2 & 2ab-2ab & -2b+b-a^2b-b^3 \\ 2ab-2ab & 1-a^2+b^2+2a^2 & 2a-a+a^3+ab^2 \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -b-a^2b-b^3 \\ 0 & 1+a^2+b^2 & a+a^3+ab^2 \\ 2b & -2a & 1-a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 0 & 1+a^2+b^2 & a(1+a^2+b^2) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Taking $(1+ a^2+b^2)$ from R_1 and R_2

$$= (1+a^2+b^2)^2\begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Operating $R_3 \to R_3 - 2bR_1 + 2aR_2$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 0 & 0 & 1+a^2+b^2 \end{vmatrix}$$

Taking $(1+a^2+b^2)$ from R_3

$$(1+a^2+b^2)^3 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding with respect to C_1

$= (1+a^2+b^2)^3\ 1\times[1-0]$

$= (1+a^2+b^2)^3$

Hence proved

27. Question

Using properties of determinants prove that:

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2).$$

Answer

Operating $C_1 \to aC_1$

$$\frac{1}{a}\begin{vmatrix} a^2 & b-c & c+b \\ a^2+ac & b & c-a \\ a^2-ab & a+b & c \end{vmatrix}$$

Operating $C_1 \to C_1 + bC_2 + cC_3$

$$= \frac{1}{a}\begin{vmatrix} a^2+\ b^2-bc+c^2+bc & b-c & c+b \\ a^2+ac+b^2+c^2-ac & b & c-a \\ a^2-ab+ab+b^2+c^2 & a+b & c \end{vmatrix}$$

$$= \frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & a+b & c \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from C_1

$$= \frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & a+b & c \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$= \frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 0 & -c-a & b \\ 0 & -a & -a \\ 1 & a+b & c \end{vmatrix}$$

Operating $C_2 \to C_2 - C_3$

$$= \frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 0 & -(a+b+c) & b \\ 0 & 0 & -a \\ 1 & (a+b+c) & c \end{vmatrix}$$

Taking $(a+b+c)$ common from C_2

$$= \frac{1}{a}(a^2+b^2+c^2)(a+b+c)\begin{vmatrix} 0 & -1 & b \\ 0 & 0 & -a \\ 1 & 1 & c \end{vmatrix}$$

Expanding with respect to C_1

$$= \frac{1}{a}(a^2+b^2+c^2)(a+b+c) \times 1 \times (0 — (-a))$$

$$= \frac{1}{a}(a^2+b^2+c^2)(a+b+c)\ (a)$$

$= (a^2+b^2+c^2)(a+b+c)$

28. Question

Using properties of determinants prove that:

$$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0.$$

Answer

Expanding with R1

$=b^2c^2(a^2c+abc-abc-a^2b)-bc(a^3c^2+a^2bc^2-a^2b^2c-a^3b^2)+(b+c)(a^3bc^2-a^3b^2c)$

$=a^2b^3c^2-a^2b^3c^2-a^3bc^2-a^2b^3c^2+a^2b^3c^2+a^3b^3c+a^3b^2c^2-a^3b^3c+a^3bc^3-a^3b^2c^2$

$=0$

29. Question

Using properties of determinants prove that:

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$

Answer

$$= \begin{vmatrix} b^2+c^2+2bc & ab & ac \\ ab & a^2+c^2+2ac & bc \\ ac & bc & a^2+b^2+2ab \end{vmatrix}$$

Operating $R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3$

$$= \frac{1}{abc}\begin{vmatrix} a(b^2+c^2+2bc) & a^2b & a^2c \\ ab^2 & b(a^2+c^2+2ac) & b^2c \\ ac^2 & bc^2 & c(a^2+b^2+2ab) \end{vmatrix}$$

Taking a, b, c common from C_1, C_2, C_3 respectively

$$= \frac{abc}{abc}\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ 0 & (a+c)^2 - b^2 & b^2 \\ c^2 - (a+b)^2 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c+a)(b+c-a) & 0 & a^2 \\ 0 & (a+c+b)(a+c-b) & b^2 \\ (c-a-b)(c+a+b) & (c-a-b)(c+a+b) & (a+b)^2 \end{vmatrix}$$

Taking (a+b+c) common from R_1, R_2

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$$

Operating $R_3 \rightarrow R_3 - R_1 - R_2$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ -2b & -2a & a^2 + b^2 + 2ab - a^2 - b^2 \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

Operating $C_1 \rightarrow aC_1$, $C_2 \rightarrow bC_2$

$$\frac{(a+b+c)^2}{ab} \begin{vmatrix} a(b+c-a) & 0 & a^2 \\ 0 & b(a+c-b) & b^2 \\ -2ab & -2ab & 2ab \end{vmatrix}$$

Operating $C_1 \rightarrow C_1 + C_3$, $C_2 \rightarrow C_2 + C_3$

$$= \frac{(a+b+c)^2}{ab} \begin{vmatrix} a(b+c) & a^2 & a^2 \\ b^2 & b(a+c) & b^2 \\ 0 & 0 & 2ab \end{vmatrix}$$

Taking a, b, 2ab from R_1, R_2, R_3

$$= \frac{(a+b+c)^2 a.b.2ab}{ab} \begin{vmatrix} b+c & a & a \\ b & a+c & b \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding with R_3

$$= 2ab(a+b+c)^2 \times 1 \times (ab + ac + bc + c^2 - ab)$$

$$= 2ab(a+b+c)^2\big(c(a+b+c)\big)$$

$$= 2abc(a+b+c)^3$$

30. Question

Using properties of determinants prove that:

$$\begin{vmatrix} b^2 - ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix} = 0.$$

Answer

$$= \begin{vmatrix} b(b-a) & b-c & c(b-a) \\ a(b-a) & a-b & b(b-a) \\ c(b-a) & c-a & a(b-a) \end{vmatrix}$$

Taking (b-a) common from C_1, C_3

$$= (b-a)^2 \begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$$

Operating $R_2 \to R_2 - R_1 + R_3$

$$= \begin{vmatrix} b & b-c-b+c & c \\ a & a-b-a+b & b \\ c & c-a-c+a & a \end{vmatrix}$$

$$= (b-a)^2 \begin{vmatrix} b & 0 & c \\ a & 0 & b \\ c & 0 & a \end{vmatrix}$$

[Properties of determinants say that if 1 row or column has only 0 as its elements, the value of the determinant is 0]

= 0

Hence Proved

31. Question

Using properties of determinants prove that:

$$\begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & ab^3 & -c\left(a^2+b^2+c^2\right) \end{vmatrix} = (abc)\left(a^2+b^2+c^2\right)^3.$$

Answer

Taking a, b, c from C_1, C_2, C_3

$$= abc \begin{vmatrix} -b^2-c^2+a^2 & 2b^2 & 2c^2 \\ 2a^2 & b^2-c^2-a^2 & 2c^2 \\ 2a^2 & 2b^2 & -a^2-b^2+c^2) \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$= abc \begin{vmatrix} -b^2-c^2-a^2 & 0 & a^2+b^2+c^2 \\ 0 & -(a^2+b^2+c^2) & a^2+b^2+c^2 \\ 2a^2 & 2b^2 & -a^2-b^2+c^2 \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from R_1, R_2

$$= abc(a^2+b^2+c^2)^2 \begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -a^2-b^2+c^2 \end{vmatrix}$$

Operating $R_3 \to R_3 + R_1 + R_2$

$$= abc(a^2+b^2+c^2)^2 \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 2a^2 & 2b^2 & a^2+b^2+c^2 \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from C_3 �

$$= abc(a^2+b^2+c^2)^3 \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 2a^2 & 2b^2 & 1 \end{vmatrix}$$

Expanding with C_3

$= abc(a^2+b^2+c^2)^3 \times 1 \times (1-0)$

$= abc\ (a^2+b^2+c^2)^3$

Hence proved

32. Question

Using properties of determinants prove that:

$$\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0,$$ where α, β, γ are in AP.

Answer

Given that α, β, γ are in an AP, which means 2β=α+γ

Operating $R_3 \to R_3 - 2R_2 + R_1$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1-2x+4+x-3 & x-2-2x+6+x-4 & x-\gamma-2x+2\beta+x-\alpha \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & -\gamma+2\beta-\alpha \end{vmatrix}$$ [we know that 2β=α+γ]

Operating $R_1 \to R_1 - R_3$, $R_2 \to R$ �$_2 - R_3$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & -\gamma+\alpha+\gamma-\alpha \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & 0 \end{vmatrix}$$

[By the properties of determinants, we know that if all the elements of a row or column is 0, then the value of the determinant is also 0]

=0

Hence proved

33. Question

Using properties of determinants prove that:

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

Answer

Operating $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} (a+1)(a+2)-(a+2)(a+3) & a+2-a-3 & 0 \\ (a+2)(a+3)-(a+3)(a+4) & a+3-a-4 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a+2)(a+1-a-3) & -1 & 0 \\ (a+3)(a+2-a-4) & -1 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} -2(a+2) & -1 & 0 \\ -2(a+3) & -1 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

Expanding with C_3

= (2(a+2) – 2(a+3))

= (2a+4-2a-6)

= -2

34. Question

If x ≠ y ≠ z and $\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix} = 0$, prove that xyz (xy + yz + zx) = (x + y + z).

Answer

By properties of determinants, we can split the given determinant into 2 parts

$$\to 0 = \begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

Taking x, y, z common from R_1, R_2, R_3 respectively

$$\to 0 = xyz\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$\to 0 = xyz\begin{vmatrix} 0 & x^2-z^2 & x^3-z^3 \\ 0 & y^2-z^2 & y^3-z^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x-z & x^3-z^3 & 0 \\ y-z & y^3-z^3 & 0 \\ z & z^3 & 1 \end{vmatrix}$$

$$\to \begin{vmatrix} x-z & (x-z)(x^2+xz+z^2) & 0 \\ y-z & (y-z)(y^2+yx+z^2) & 0 \\ z & z^3 & 1 \end{vmatrix} = xyz\begin{vmatrix} 0 & (x-z)(x+z) & (x-z)(x^2+xz+z^2) \\ 0 & (y-z)(y+z) & (y-z)(y^2+yz+z^2) \\ 1 & z^2 & z^3 \end{vmatrix}$$

Taking (x-z) and (y-z) common from R_1, R_2

$$\to (x-z)(y-z)\begin{vmatrix} 1 & (x^2+xz+z^2) & 0 \\ 1 & (y^2+yz+z^2) & 0 \\ z & z^3 & 1 \end{vmatrix} = (x-z)(y-z)\begin{vmatrix} 0 & x+z & (x^2+xz+z^2) \\ 0 & y+z & (y^2+yz+z^2) \\ 1 & z^2 & z^3 \end{vmatrix}$$

Expanding with R_3

$\to y^2+yz+z^2-x^2-xz-z^2 = xyz(xy^2+xyz+xz^2+zy^2+yz^2+z^3-x^2y-xyz-yz^2-x^2z-xz^2-z^3)$

$\to$ (y-x)(y+x) +z(y-x) $= xyz(xy^2+zy^2 -x^2y -x^2z)$

$\to (y-x)(x+y+z)=xyz(xy(y-x)+z(y^2-x^2))$

→(y-x)(x+y+z)= xyz(xy(y-x)+z(x+y)(y-x))

→(y-x)(x+y+z) = xyz(xy(y-x)+(xz+yz)(y-x))

→(y-x)(x+y+z)= xyz(y-x)(xy+xz+yz)

→x+y+z = xyz(xy+xz+yz)

Hence Proved

35. Question

Prove that $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$ = - (a – b) (b – c) (c – a) (a^2 + b^2 + c^2).

Answer

Operating $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & a^2+bc-b^2-ac & a^3-b^3 \\ 0 & b^2+ca-c^2-ab & b^3-c^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & (a-b)(a+b)-c(a-b) & (a-b)(a^2+ab+b^2) \\ 0 & (b-c)(b+c)-a(b-c) & (b-c)(b^2+bc+c^2) \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Taking (a-b), (b-c) common from R_1, R_2 respectively

$$= (a-b)(b-c)\begin{vmatrix} 0 & a+b-c & a^2+ab+b^2 \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Operating $R_1 \to R_1 - R_2$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 2a-2c & a^2+ab-bc-c^2 \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 2(a-c) & (a+c)(a-c)+b(a-c) \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Taking (a-c) common from R_1

$$= (a-c)(a-b)(b-c)\begin{vmatrix} 0 & 2 & a+b+c \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Expanding with C_1

$= (a-c)(a-b)(b-c)\times(2b^2+2bc+2c^2-ab-b^2-bc-ac-bc-c^2+a^2+ab+ac)$

$=-(c-a)(b-c)(a-b)(a^2+b^2+c^2)$

Hence Proved

36. Question

Without expanding the determinant, prove that:

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Answer

Operating $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\to \begin{vmatrix} 0 & a-b & bc-ac \\ 0 & b-c & ac-ab \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\rightarrow\begin{vmatrix}0 & a-b & -c(a-b)\\0 & b-c & -a(b-c)\\1 & c & ab\end{vmatrix}=\begin{vmatrix}0 & a-b & (a-b)(a+b)\\0 & b-c & (b-c)(b+c)\\1 & c & c^2\end{vmatrix}$$

Taking (a-b) and (b-c) from R_1, R_2

$$\rightarrow(a-b)(b-c)\begin{vmatrix}0 & 1 & -c\\0 & 1 & -a\\1 & c & ab\end{vmatrix}=(a-b)(b-c)\begin{vmatrix}0 & 1 & (a+b)\\0 & 1 & (b+c)\\1 & c & c^2\end{vmatrix}$$

Method 1:

For the two determinants to be equal, their difference must be 0.

$$=\begin{vmatrix}0 & 1 & -c\\0 & 1 & -a\\1 & c & ab\end{vmatrix}-\begin{vmatrix}0 & 1 & a+b\\0 & 1 & b+c\\1 & c & c^2\end{vmatrix}$$

$$=\begin{vmatrix}0-0 & 1-1 & -(a+b+c)\\0-0 & 1-1 & -(a+b+c)\\1-1 & c-c & ab-c^2\end{vmatrix}$$

$$=\begin{vmatrix}0 & 0 & -(a+b+c)\\0 & 0 & -(a+b+c)\\0 & 0 & ab-c^2\end{vmatrix}$$

Since 2 columns have only 0 as their elements, by properties of determinants

=0

Method 2:

Expanding both with C_1

LHS

=(a-b)(b-c)(-a+c)

RHS

=(a-b)(b-c)(b+c-a-b)

=(a-b)(b-c)(-a+c)

∴ LHS = RHS

37. Question

Without expanding the determinant, prove that:

$$\begin{vmatrix}1 & a & a^2\\1 & b & b^2\\1 & c & c^2\end{vmatrix}=\begin{vmatrix}1 & bc & b+c\\1 & ca & c+a\\1 & ab & a+b\end{vmatrix}$$

Answer

Operating $R1\rightarrow R1-R_3$, $R_2\rightarrow R_2-R3$

$$\begin{vmatrix}0 & a-c & a^2-c^2\\0 & b-c & b^2-c^2\\1 & c & c^2\end{vmatrix}=\begin{vmatrix}0 & bc-ab & b+c-a-b\\0 & ac-ab & c+a-a-b\\1 & ab & a+b\end{vmatrix}$$

$$\rightarrow\begin{vmatrix}0 & a-c & (a-c)(a+c)\\0 & b-c & (b-c)(b+c)\\1 & c & c^2\end{vmatrix}=\begin{vmatrix}0 & -b(a-c) & -(a-c)\\0 & -a(b-c) & -(b-c)\\1 & ab & a+b\end{vmatrix}$$

Taking (a-c) and (b-c) common from R_1, R_2

$$\rightarrow (a-c)(b-c)\begin{vmatrix} 0 & 1 & a+c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} = (a-c)(b-c)\begin{vmatrix} 0 & -b & -1 \\ 0 & -a & -1 \\ 1 & ab & a+b \end{vmatrix}$$

Method 1:

If the determinants are equal, their difference must also be equal.

(a-c) and (b-c) get cancelled.

$$= \begin{vmatrix} 0 & 1 & a+c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 0 & -b & -1 \\ 0 & -a & -1 \\ 1 & ab & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 0-0 & 1+b & a+c+1 \\ 0-0 & 1+a & b+c+1 \\ 1-1 & c-ab & c^2+a+b \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1+b & a+c+1 \\ 0 & 1+a & b+c+1 \\ 0 & c-ab & c^2+a+b \end{vmatrix}$$

Since all elements of C_1 are 0, by properties of determinants,

=0

∴ The 2 determinants are equal.

Method 2:

Expanding with C_1

→(a-c)(b-c)(b+c-a-c) = (a-c)(b-c)(b-a)

→(a-c)(b-c)(b-a)=(a-c)(b-c)(b-a)

∴ RHS and LHS are equal

38. Question

Show that x = 2 is a root of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix} = 0.$

Answer

Operating $R_1 \rightarrow R_1 - R_2$

$$0 = \begin{vmatrix} x-2 & -6+3x & -1-x+3 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$$

$$0 = \begin{vmatrix} x-2 & 3(x-2) & -(x-2) \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$$

Taking (x-2) common from R_1

$$0 = (x-2)\begin{vmatrix} 1 & 1 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & 2+x \end{vmatrix}$$

Here, we can see that x-2 is a factor of the determinant.

We can say that when x-2 is put in the equation, we get 0.

x-2=0

$\rightarrow x=2$

39. Question

Solve the following equations:

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & x^3 \end{vmatrix} = 0$$

Answer

Operating $R1 \rightarrow R1 - R_2$, $R_2 \rightarrow R2 - R_3$

$$\begin{vmatrix} 0 & x-b & x^3-b^3 \\ 0 & b-c & b^3-c^3 \\ 1 & c & c^3 \end{vmatrix} = 0$$

$$0 = \begin{vmatrix} 0 & x-c & (x-b)^3+3xb(x-b) \\ 0 & b-c & (b-c)^3+3bc(b-c) \\ 1 & c & c^3 \end{vmatrix}$$

$$0 = (x-c)(b-c)\begin{vmatrix} 0 & 1 & (x-b)^2+3xb \\ 0 & 1 & (b-c)^2+3bc \\ 1 & c & c^3 \end{vmatrix}$$

Expanding with C_1

$0=(x-c)(b-c)(b^2-2bc+c^2+3bc-x^2+2xb-b^2-3xb)$

$0= (x-c)(b-c)(bc+c^2-x^2-xb)$

$0=(x-c)(b-c)(-b(-c+x)-(c-x)(-c-x))$

$0=(x-c)^2(b-c)(-b-c-x)$

Either $x-c=0$ or $b-c=0$ or $(-b-c-x)=0$

$\therefore x=c$ or $b=c$ or $x=-(b+c)$

If $b=c$, $x=b$

$\therefore$ $x=c$ or $x=b$ or $x=-(b+c)$

40. Question

Solve the following equations:

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ b & b & x+c \end{vmatrix} = 0$$

Answer

Operating $C1 \rightarrow C1 + C_2 + C_3$

$$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$

Taking $(x+a+b+c)$ common from C_1

$$(x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$(x+a+b+c)\begin{vmatrix} 0 & 0 & -x \\ 0 & x & -x \\ 1 & b & x+c \end{vmatrix} = 0$$

Expanding with C_1

$0=(x+a+b+c)(0+x^2)$

$0=x^2(x+a+b+c)$

Either $x^2=0$ or $(x+a+b+c)=0$

$\therefore$ x=0 or x=-(a+b+c)

41. Question

Solve the following equations:

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Answer

Operating $C1 \to C1 + C_2 + C_3$

$$0 = \begin{vmatrix} 3x-8+3+3 & 3 & 3 \\ 3+3x-8+3 & 3x-8 & 3 \\ 3+3+3x-8 & 3 & 3x-8 \end{vmatrix}$$

$$0 = \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix}$$

Taking (3x-2) common from C_1

$$0 = (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$0 = (3x-2)\begin{vmatrix} 0 & 0 & -(3x-11) \\ 0 & 3x-11 & -3x+11 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

Expanding with C_1

$0= (3x-2)(0+(3x-11)^2)$

$0=(3x-2)(3x-11)^2$

Either 3x-2=0 or 3x-11=0

$\therefore \mathbf{x = \frac{2}{3} \text{ or } x = \frac{11}{3}}$

42. Question

Solve the following equations:

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Answer

Operating $C_1 \to C_1 + C_2 + C_3$

$$0 = \begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix}$$

Taking (x+9) common from C_1

$$0 = (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$0 = (x+9)\begin{vmatrix} 0 & 0 & 1-x \\ 0 & x-1 & 1-x \\ 1 & 3 & x+4 \end{vmatrix}$$

$0= (x+9)(0-x+x^2+1-x)$

$0=(x+9)(x^2-2x+1)$

$0=(x+9)(x-1)^2$

∴ Either $x+9=0$ or $x-1=0$

$x=-9$, $x=1$

43. Question

Solve the following equations:

$$\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Answer

Operating $R1 \to R1 + R_2 + R_3$

$$0 = \begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

Taking (x+9) common from R_1

$$0 = (x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

Operating $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$

$$0 = (x+9)\begin{vmatrix} 0 & 0 & 1 \\ 0 & x-2 & 2 \\ 7-x & 6-x & x \end{vmatrix}$$

Expanding with R_1

$0=(x+9)(0-(x-2)(7-x))$

$0=(x+9)(7-x)(2-x)$

Either $x+9=0$ or $7-x=0$ or $2-x=0$

∴ $x=-9$ or $x=7$ or $x=2$

44. Question

Solve the following equations:

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

Answer

Expanding with R1

$0= x(-3x^2-6x-2x^2+6x)+6(2x+4+3x-9)-1(4x-9x)$

$0=x(-5x^2)+6(5x-5)-1(-5x)$

$0= -5x^3+30x-30+5x$

$0=-5x^3+35x-30$

$x^3-7x+6=0$

$x^3-x-6x+6=0$

$x(x^2-1)-6(x-1)=0$

x(x-1)(x+1)-6(x-1)=0

$(x-1)(x^2+x-6)=0$

$(x-1)(x^2+3x-2x-6)=0$

(x-1)(x(x+3)-2(x+3)(=0

(x-1)(x+3)(x-2)=0

Either x-1=0 or x+3=0 or x-2=0

∴ x=1 or x=-3 or x=2

45. Question

Prove that

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

Answer

Operating $C_1 \to aC_1$

$$= \frac{1}{a}\begin{vmatrix} a^2 & b-c & c+b \\ a^2+ac & b & c-a \\ a^2-ab & b+a & c \end{vmatrix}$$

Operating $C_1 \to C_1+bC_2+cC_3$

$$= \frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & b+a & c \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$

$$= \frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & b+a & c \end{vmatrix}$$

Operating $C_2 \rightarrow C_2 - bC_1$, $C_3 \rightarrow C_3 - cC_3$

$$= \frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & -c & b \\ 1 & 0 & -a \\ 1 & a & 0 \end{vmatrix}$$

Expanding with R_3

$$= \frac{a^2+b^2+c^2}{a}(ac - 0 + a^2 + ab)$$

$$= \frac{a^2+b^2+c^2}{a} a(a+b+c)$$

$= (a^2+b^2+c^2)(a+b+c)$

Hence Proved

Exercise 6C

1 A. Question

Find the area of the triangle whose vertices are:

A(3, 8), B(-4, 2) and C(5, -1)

Answer

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

Expanding with C_3

$$= \frac{1}{2}[\,(4-10) - (-3-40) + (6+32)]$$

$$= \frac{1}{2}[\,-6 + 43 + 38]$$

$$= \frac{75}{2}$$

= 37.5 sq. units

1 B. Question

Find the area of the triangle whose vertices are:

A(-2, 4), B(2, -6) and C(5, 4)

Answer

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding with C_3

$= \frac{1}{2}[(8 + 30) - (-8 - 20) + (12 - 8)]$

$= \frac{1}{2}[38 + 28 + 4]$

$= \frac{68}{2}$

= 34 sq. units

1 C. Question

Find the area of the triangle whose vertices are:

A(-8, -2), B(-4, -6) and C(-1, 5)

Answer

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -8 & -2 & 1 \\ -4 & -6 & 1 \\ -1 & 5 & 1 \end{vmatrix}$

Expanding with R_3

$= \frac{1}{2}[(-20 - 6) - (-40 - 2) + (48 - 8)]$

$= \frac{1}{2}[-26 + 42 + 40]$

$= \frac{56}{2}$

=28 sq. units

1 D. Question

Find the area of the triangle whose vertices are:

P(0, 0), Q(6, 0) and R(4, 3)

Answer

Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$

Expanding with R_1

$= \frac{1}{2}[18]$

= 9 sq. units

1 E. Question

Find the area of the triangle whose vertices are:

P(1, 1), Q(2, 7) and R(10, 8)

Answer

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 7 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

Operating $R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$$= \frac{1}{2}\begin{vmatrix} -9 & -7 & 0 \\ -8 & -1 & 0 \\ 10 & 8 & 1 \end{vmatrix}$$

Expanding with C_3

$$= \frac{1}{2}[\,9 - 56\,]$$

$$= \frac{1}{2}[-47]$$

$$= \frac{-47}{2}$$

= -23.5 sq. units = 23.5 sq units

2 A. Question

Use determinants to show that the following points are collinear.

A(2, 3), B(-1, -2) and C(5, 8)

Answer

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

Expanding with C_3

$$= \frac{1}{2}[\,(-8 + 10) - (16 - 15) + (-4 + 3)] = \frac{1}{2}\,[\,2 - 1 - 1]$$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

2 B. Question

Use determinants to show that the following points are collinear.

A(3, 8), B(-4, 2) and C(10, 14)

Answer

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 10 & 14 & 1 \end{vmatrix}$$

Expanding with C_3

$= \frac{1}{2} [(-56 - 20) - (42 - 80) + (6 + 32)]$

$= \frac{1}{2} [-76 + 38 + 38]$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

2 C. Question

Use determinants to show that the following points are collinear.

P(-2, 5), Q(-6, -7) and R(-5, -4)

Answer

$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -2 & 5 & 1 \\ -6 & -7 & 1 \\ -5 & -4 & 1 \end{vmatrix}$

Expanding with C_3

$= \frac{1}{2} [(24 - 35) - (8 + 25) + (14 + 30)] = \frac{1}{2} [-11 - 33 + 44]$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

3. Question

Find the value of k for which thepoints A(3, -2), B(k, 2) and C(8, 8) are collinear.

Answer

$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Since they are collinear, the area will be 0

$\to 0 = \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$

Expanding with C_3

$\to 0 = \frac{1}{2} [(8k - 16) - (24 + 16) + (6 + 2k)]$

$\to 0 = \frac{1}{2} [10k - 50]$

→ 10k – 50=0

→ 10k=50

∴ k=5

4. Question

Find the value of k for which thepoints P(5, 5), Q(k, 1) and R(11, 7) are collinear.

Answer

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$\rightarrow 0 = \frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 11 & 7 & 1 \end{vmatrix}$$

Expanding with C_3

→ 0 = (7k-11)-(35-55)+(5-5k)

→ 0= 2k-14

→ 2k=14

∴ k=7

5. Question

Find the value of k for which thepoints A(1, -1), B(2, k) and C(4, 5) are collinear.

Answer

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$\rightarrow 0 = \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & k & 1 \\ 4 & 5 & 1 \end{vmatrix}$$

Expanding with C_3

→ 0 = (10-4k)-(5+4)+(k+2)

→ 0=-3k+3

→ 3k=3

∴ k= 1

6. Question

Find the value of k for which the area of aABC having vertices A(2, -6), B(5, 4) and C(k, 4) is 35 sq units.

Answer

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$35 = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

Expanding with C_3

→ 70 = (20-4k)-(8+6k)+(8+30)

→ 70= -10k+50

→ 20=-2k

→ k=-2

7. Question

If A(-2, 0), B(0, 4) and C(0, k) be three points such that area of a ABC is 4 sq units, find the value of k.

Answer

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$4 = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

Expanding with C_1

→ 8=-2(4-k)

→ -4=4-k

→ k=8

8. Question

If the points A(a, 0), B(0, b) and C(1, 1) are collinear, prove that $\frac{1}{a}+\frac{1}{b}=1.$

Answer

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since the points are collinear, the area they enclose is 0

$$0 = \frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Expanding with C_1

→ 0 = a(b-1)+(-b)

→ 0= ab-a-b

→ a+b=ab

→ $\frac{a+b}{ab} = 1$

→ $\frac{1}{a} + \frac{1}{b} = 1$

Hence proved

Objective Questions

1. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \cos 70^{\circ} & \sin 20^{\circ} \\ \sin 70^{\circ} & \cos 20^{\circ} \end{vmatrix} = ?$$

A. 1

B. 0

C. cos 50^{o}

D. sin 50^{o}

Answer

To find: Value of $\begin{vmatrix} \cos 70° & \sin 20° \\ \sin 70° & \cos 20° \end{vmatrix}$

Formula used: (i) $\cos \theta = \sin (90 - \theta)$

We have, $\begin{vmatrix} \cos 70° & \sin 20° \\ \sin 70° & \cos 20° \end{vmatrix}$

On expanding the above,

⇒ {cos 70°} {cos 20°} – {sin 70°} {sin 20°}

On applying formula $\cos \theta = \sin (90 - \theta)$

⇒ {sin (90 – 70)} {sin (90 – 20)} - {sin 70°} {sin 20°}

⇒ {sin 20°} {sin 70°} - {sin 70°} {sin 20°}

= 0

2. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix} = ?$$

A. 1

B. $\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$

D. none of these

Answer

To find: Value of $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 15° & \cos 15° \end{vmatrix}$

Formula used: (i) $\cos (A + B) = \cos A \cos B - \sin A \sin B$

We have, $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 15° & \cos 15° \end{vmatrix}$

On expanding the above,

⇒ {cos 15°} {cos 15°} – {sin 15°} {sin 15°}

On applying formula $\cos (A + B) = \cos A \cos B - \sin A \sin B$

= cos (15 + 15)

= cos (30°)

$= \frac{\sqrt{3}}{2}$

3. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \sin 23^\circ & -\sin 7^\circ \\ \cos 23^\circ & \cos 7^\circ \end{vmatrix} = ?$$

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{2}$

C. sin 16^o

D. cos 16^o

Answer

To find: Value of $\begin{vmatrix} \sin 23° & -\sin 7° \\ \cos 23° & \cos 7° \end{vmatrix}$

Formula used: (i) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

We have, $\begin{vmatrix} \sin 23° & -\sin 7° \\ \cos 23° & \cos 7° \end{vmatrix}$

On expanding the above,

⇒ (sin 23°) (cos 7°) – (cos 23°) (-sin 7°)

⇒ (sin 23°) (cos 7°) + (cos 23°) (sin 7°)

On applying formula $\sin(A + B) = \sin A \cos B + \cos A \sin B$

= sin (23 + 7)

= sin (30°)

$= \frac{1}{2}$

4. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a + ib & c + id \\ -c + id & a - id \end{vmatrix} = ?$$

A. $(a^2 + b^2 - c^2 - d^2)$

B. $(a^2 - b^2 + c^2 - d^2)$

C. $(a^2 + b^2 + c^2 + d^2)$

D. none of these

Answer

To find: Value of $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$

Formula used: $i^2 = -1$

We have, $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$

On expanding the above,

⇒ (a + ib) (a – ib) – (-c + id) (c + id)

$\Rightarrow (a^2 - iab + iba - i^2b^2) - (-c^2 - icd + icd + i^2d^2)$

$\Rightarrow \{a^2 - iab + iba - (-1)b^2\} - \{-c^2 - icd + icd + (-1)d^2\}$

$\Rightarrow \{a^2 - iab + iba + 1b^2\} - \{-c^2 - icd + icd - 1d^2\}$

$\Rightarrow a^2 + b^2 + c^2 + d^2$

5. Question

Mark the tick against the correct answer in the following:

If ω is a complex root of unity then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = ?$

A. 1

B. -1

C. 0

D. none of these

Answer

To find: Value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

Formula used: $\omega^3 = 1$

We have, $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

On expanding the above along 1st column

$\Rightarrow 1 \begin{vmatrix} \omega^2 & 1 \\ 1 & \omega \end{vmatrix} - \omega \begin{vmatrix} \omega & \omega^2 \\ 1 & \omega \end{vmatrix} + \omega^2 \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix}$

$\Rightarrow [1\{(\omega^2)(\omega)-(1)(1)\}] - [\omega\{(\omega)(\omega)-(\omega^2)(1)\}] + [\omega^2\{(\omega)(1)-(\omega^2)(\omega^2)\}]$

$\Rightarrow [1\{\omega^3-1\}]-[\omega\{\omega^2-\omega^2\}]+[\omega^2\{\omega-\omega^4\}]$ ... (i)

As $\omega^3 = 1$,

$\Rightarrow \omega^3.\omega = 1.\,\omega$

$\Rightarrow \omega^4 = \omega$

Using the above obtained value of ω^4 in eqn. (i)

$\Rightarrow [1\{\omega^3-1\}] - [\omega\{\omega^2-\omega^2\}] + [\omega^2\{\omega-\omega\}]$

$\Rightarrow 1\{\omega^3-1\}$

$\Rightarrow \omega^3-1$

$\Rightarrow 1 - 1 = 0$

6. Question

Mark the tick against the correct answer in the following:

If ω is a complex cube root of unity then the value of $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$ is

A. 2

B. 4

C. 0

D. -3

Answer

To find: Value of $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$

Formula used: (i) $\omega^3 = 1$

(ii) $1+\omega+\omega^2 = 0$

We have, $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$

On expanding the above along 1^{st} column

$\Rightarrow 1 \begin{vmatrix} \omega^2 & 1 \\ 1 & \omega \end{vmatrix} - \omega \begin{vmatrix} \omega & \omega^2 \\ 1 & \omega \end{vmatrix} + \omega^2 \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix}$

$\Rightarrow [1\{(\omega^2)(\omega)-(1)(1)\}]-[\omega\{(\omega)(\omega)-(\omega^2)(1)\}]+[\omega^2\{(\omega)1-(\omega^2)(\omega^2)\}]$

$\Rightarrow [1\{\omega^3-1\}]-[\omega\{\omega^2-\omega^2\}]+[\omega^2\{\omega-\omega^4\}]$ ... (i)

As $\omega^3 = 1$,

$\Rightarrow \omega^3.\omega=1.\ \omega$

$\Rightarrow \omega^4= \omega$

Using the above obtained value of ω^4 in eqn. (i)

$\Rightarrow [1\{\omega^3-1\}]-[\omega\{\omega^2-\omega^2\}]+[\omega^2\{\omega-\omega\}]$

$\Rightarrow 1\{\omega^3-1\}$

$\Rightarrow \omega^3-1$

$\Rightarrow 1 - 1 = 0$

7. Question

Mark the tick against the correct answer in the following:

$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = ?$

A. 8

B. -8

C. 16

D. 142

Answer

To find: Value of $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

We have, $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$

Applying $R_1 \rightarrow R_3 - R_1$

$\Rightarrow \begin{vmatrix} 8 & 12 & 16 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$

Applying $R_2 \rightarrow R_1 - R_2$

$\Rightarrow \begin{vmatrix} 8 & 12 & 16 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$

Taking 4 common from R_1

$\Rightarrow 4 \begin{vmatrix} 2 & 3 & 4 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$

Applying $R_1 \rightarrow R_1 - R_2$

$\Rightarrow 4 \begin{vmatrix} -2 & 0 & 4 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$

Taking -2 common from R_1

$\Rightarrow (4)\,(-2) \begin{vmatrix} 1 & 0 & -2 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$

Applying $R_1 \rightarrow 9R_1$

$\Rightarrow \frac{-8}{9} \begin{vmatrix} 9 & 0 & -18 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$

Applying $R_3 \rightarrow R_3 - R_1$

$\Rightarrow \frac{-8}{9} \begin{vmatrix} 9 & 0 & -18 \\ 4 & 3 & 0 \\ 0 & 16 & 43 \end{vmatrix}$

Taking 9 common from R_1

$\Rightarrow -8 \begin{vmatrix} 1 & 0 & -2 \\ 4 & 3 & 0 \\ 0 & 16 & 43 \end{vmatrix}$

Expanding along R_1

$\Rightarrow -8\ [1[(3)(43)-(16)(0)] – 0\ [(4)(43)-(0)(0)] - 2\ [(4)(16)-(3)(0)]]$

⇒ -8 [[(129)-(0)] - 2 [(64)-(0)]]

⇒ -8 [129 – 128]

⇒ -8

8. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = ?$$

A. 2

B. 6

C. 24

D. 120

Answer

To find: Value of $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$

We have, $\left|\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}\right|$

$\Rightarrow \begin{vmatrix} 1 & 2 & 6 \\ 2 & 6 & 24 \\ 6 & 24 & 120 \end{vmatrix}$

Taking 2 common from R_2

$\Rightarrow 2 \begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 12 \\ 6 & 24 & 120 \end{vmatrix}$

Taking 6 common from R_3

$\Rightarrow 2 \times 6 \begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 12 \\ 1 & 4 & 20 \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_1$

$\Rightarrow 12 \begin{vmatrix} 1 & 2 & 6 \\ 0 & 1 & 6 \\ 1 & 4 & 20 \end{vmatrix}$

Applying $R_3 \rightarrow R_3 - R_1$

$\Rightarrow 12 \begin{vmatrix} 1 & 2 & 6 \\ 0 & 1 & 6 \\ 0 & 2 & 14 \end{vmatrix}$

Expanding column 1

⇒ 12 [1{(1)(14)-(6)(2)}]

⇒ 12 [1{(14)-(12)}]

⇒ 12[2]

$\Rightarrow 24$

9. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = ?$$

A. (a + b + c)

B. 3(a + b + c)

C. 3abc

D. 0

Answer

To find: Value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

We have, $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2$

$$\Rightarrow \begin{vmatrix} a-b+b-c & b-c+c-a & c-a+a-b \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a-c & b-a & c-b \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$

$$\Rightarrow \begin{vmatrix} a-c+c-a & b-a+a-b & c-b+b-c \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

If every element of a row is 0 then the value of the determinant will be 0

10. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = ?$$

A. 0

B. 1

C. -1

D. none of these

Answer

To find: Value of $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

We have, $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - 2R_1$

$\Rightarrow \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & p-2 \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

Applying $R_3 \rightarrow R_3 - 3R_1$

$\Rightarrow \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & p-2 \\ 0 & 3 & 3p-2 \end{vmatrix}$

Expanding along C_1

$\Rightarrow [1\{(1)(3p-2)-(3)(p-2)\}]$

$\Rightarrow 1$

11. Question

Mark the tick against the correct answer in the following:

$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = ?$

A. (a – b) (b – c) (c – a)

B. -(a – b) (b – c) (c – a)

C. (a – b) (b – c) (c – a) (a + b + c)

D. abc (a – b)(b – c) (c – a)

Answer

To find: Value of $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

We have, $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

Applying $C_2 \rightarrow C_2 - C_1$

$\Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ a & b-a & c \\ a^3 & b^3-a^3 & c^3 \end{vmatrix}$

Applying $C_3 \rightarrow C_3 - C_1$

$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix}$

We know, $x^3 - y^3 = (x - y)(x^2+xy+y^2)$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & (b-a)(b^2+ab+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$$

Taking (b-a) common from C_2

$$\Rightarrow (b-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & c-a \\ a^3 & (b^2+ab+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$$

Taking (c-a) common from C_2

$$\Rightarrow (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & (b^2+ab+a^2) & (c^2+ca+a^2) \end{vmatrix}$$

Expanding along C_1

$\Rightarrow (b - a)(c - a)[1\{(1)(c^2 + ca + a^2) - (b^2 + ab + a^2)(1)\}]$

$\Rightarrow (b - a)(c - a)[c^2 + ca + a^2 - b^2 - ab - a^2]$

$\Rightarrow (b - a)(c - a)[c^2 - b^2 + ca - ab]$

$\Rightarrow (b - a)(c - a)[(c - b)(c + b) + a(c - b)]$

$\Rightarrow (b - a)(c - a)[(a + b + c)(c - b)]$

$\Rightarrow (a - b)(b - c)(c - a)(a + b + c)$

12. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix} = ?$$

A. 0

B. 1

C. $\sin(\alpha + \delta) + \sin(\beta + \delta) + \sin(\gamma + \delta)$

D. none of these

Answer

To find: Value of $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$

Formula Used: $\sin(A+B) = \sin A\cos B+\cos A\sin B$

We have, $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$

Applying $C_1 \rightarrow \cos(\delta)C_1$

$$\Rightarrow \begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta\cos\delta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma\cos\delta & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$$

Applying $C_2 \rightarrow \sin(\delta)C_2$

$$\Rightarrow \begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha\sin\delta & \sin(\alpha+\delta) \\ \sin\beta\cos\delta & \cos\beta\sin\delta & \sin(\beta+\delta) \\ \sin\gamma\cos\delta & \cos\gamma\sin\delta & \sin(\gamma+\delta) \end{vmatrix}$$

We know, sin(A+B) = sinAcosB+cosAsinB

$$\Rightarrow \begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha\sin\delta & \sin\alpha\cos\delta + \cos\alpha\sin\delta \\ \sin\beta\cos\delta & \cos\beta\sin\delta & \sin\beta\cos\delta + \cos\beta\sin\delta \\ \sin\gamma\cos\delta & \cos\gamma\sin\delta & \sin\gamma\cos\delta + \cos\gamma\sin\delta \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - C_1$

$$\Rightarrow \begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha\sin\delta & \sin\alpha\cos\delta + \cos\alpha\sin\delta - \sin\alpha\cos\delta \\ \sin\beta\cos\delta & \cos\beta\sin\delta & \sin\beta\cos\delta + \cos\beta\sin\delta - \sin\beta\cos\delta \\ \sin\gamma\cos\delta & \cos\gamma\sin\delta & \sin\gamma\cos\delta + \cos\gamma\sin\delta - \sin\gamma\cos\delta \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha\sin\delta & \cos\alpha\sin\delta \\ \sin\beta\cos\delta & \cos\beta\sin\delta & \cos\beta\sin\delta \\ \sin\gamma\cos\delta & \cos\gamma\sin\delta & \cos\gamma\sin\delta \end{vmatrix}$$

= 0

When two columns are identical then the value of determinant is 0

13. Question

Mark the tick against the correct answer in the following:

If a, b, c be distinct positive real numbers then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

A. positive

B. negative

C. a perfect square

D. 0

Answer

To find: Nature of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

We have, $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} a+b+c & b+c+a & c+a+b \\ b & c & a \\ c & a & b \end{vmatrix}$$

Taking (a+b+c) common from R_1

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

Expanding along R_1

⇒ (a+b+c)[1{(b)(c)-(a)(a)} – 1{(b)(b)-(c)(a)} + 1{(a)(b)-(c)(c)}]

⇒ (a+b+c)[1{bc-a^2} – 1{b^2-ca} + 1{ba - c^2}]

⇒ (a+b+c)[bc - a^2 –b^2 + ca + ab - c^2]

⇒ -(a+b+c)[c^2 + a^2 + b^2 - ca - bc – ba]

⇒ $-\frac{1}{2}$(a+b+c) 2[c^2 + a^2 + b^2 - ca - bc – ba]

⇒ $-\frac{1}{2}$(a+b+c) [2c^2 + 2a^2 + 2b^2 - 2ca - 2bc – 2ba]

⇒ $-\frac{1}{2}$(a+b+c) [c^2 + a^2 - 2ca + c^2 + b^2 - 2bc + a^2 + b^2 - 2ba]

⇒ $-\frac{1}{2}$(a+b+c) [$(c-a)^2$ +$(c-b)^2$ +$(a-b)^2$]

Clearly, we can see that the answer is negative

14. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = ?$$

A. 0

B. x^3

C. y^3

D. none of these

Answer

To find: Value of $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$

We have, $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$

Applying $R_2 \to 2R_2$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x+y & x & x \\ 10x+8y & 8x & 4x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

Applying $R_2 \to R_2 - R_3$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x+y & x & x \\ 0 & 0 & x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

Applying $R_1 \to 8R_1$

$$\Rightarrow \frac{1}{2\times 8}\begin{vmatrix} 8x+8y & 8x & 8x \\ 0 & 0 & x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$

$\Rightarrow \frac{1}{16}\begin{vmatrix} 8x+8y & 8x & 8x \\ 0 & 0 & x \\ 2x & 0 & -5x \end{vmatrix}$

Expanding along R_2

$\Rightarrow \frac{1}{16}$ [x{(2x)(8x) - (8x+8y)(0)}]

$\Rightarrow \frac{1}{16}$ [x{16x^2}]

$\Rightarrow x^3$

15. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = ?$$

A. (a – 1)

B. $(a - 1)^2$

C. $(a - 1)^3$

D. none of these

Answer

To find: Value of $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

We have, $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

Applying $R_1 \rightarrow R_1 - R_2$

$\Rightarrow \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_3$

$\Rightarrow \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$

Expanding along C_3

$\Rightarrow [1\{(a^2-1)(a-1) - (a-1)(2a - 2)\}]$

$\Rightarrow [1\{(a-1)(a+1)(a-1) - (a-1)2(a - 1)\}]$

$\Rightarrow [\{(a+1)(a-1)^2 - 2(a-1)^2\}]$

$\Rightarrow [\{(a-1)^2 (a+1-2)\}]$

$\Rightarrow [\{(a-1)^2 (a-1)\}]$

$\Rightarrow (a-1)^3$

16. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} = ?$$

A. a^3

B. $-a^3$

C. 0

D. none of these

Answer

To find: Value of $\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$

We have, $\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$

Applying $R_3 \rightarrow R_3 - 2R_2$

$\Rightarrow \begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 0 & a & a+b \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - 3R_1$

$\Rightarrow \begin{vmatrix} a & a+2b & a+2b+3c \\ 0 & a & 2a+b \\ 0 & a & a+b \end{vmatrix}$

Expanding along C_1

$\Rightarrow [a\{(a)(a+b) - (a)(2a+b)\}]$

$\Rightarrow [a\{(a^2 + ab) - (2a^2+ab)\}]$

$\Rightarrow [a\{a^2 + ab - 2a^2 - ab\}]$

$\Rightarrow [a\{-a^2\}]$

$\Rightarrow -a^3$

17. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix} = ?$$

A. $(a + b + c)(a - c)$

B. $(a + b + c)(b - c)$

C. $(a + b + c)(a - c)^2$

D. $(a + b + c)(b - c)^2$

Answer

To find: Value of $\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

We have, $\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$

$\Rightarrow \begin{vmatrix} b+c+c+a+a+b & a+c+b & b+a+c \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 2(a+b+c) & a+b+c & a+b+c \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

$\Rightarrow (a+b+c)\begin{vmatrix} 2 & 1 & 1 \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

Expanding along R_1

$\Rightarrow (a+b+c)[2\{(c)(c) – (b)(a)\} -1\{(c+a)(c)-(a+b)(a)\} + 1\{(c+a)(b)-(a+b)(c)\}]$

$\Rightarrow (a+b+c)[2\{c^2 – ab\} -1\{c^2+ac-a^2-ab\} + 1\{bc+ba-ac-bc\}]$

$\Rightarrow (a+b+c)[2c^2 – 2ab - c^2 – ac + a^2 + ab + ba - ac]$

$\Rightarrow (a+b+c)[c^2 + a^2 - 2ac]$

$\Rightarrow (a+b+c)(c – a)^2$

18. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = ?$$

A. (x + y)

B. (x – y)

C. xy

D. none of these

Answer

To find: Value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

We have, $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Applying $R_1 \rightarrow R_2 - R_1$

$\Rightarrow \begin{vmatrix} 0 & -x & 0 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Expanding along R_1

⇒ [x{(1)(1+y)-(1)(1)}]

⇒ [x{1+y-1}]

⇒ xy

19. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = ?$$

A. (a – b) (b – c) (c – a)

B. -(a – b) (b – c) (c – a)

C. (a + b) (b + c) (c + a)

D. None of these

Answer

To find: Value of $\begin{vmatrix} bc & b+c & 1 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$

We have, $\begin{vmatrix} bc & b+c & 1 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$

Applying $R_1 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{vmatrix} bc-ca & b-a & 0 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} c(b-a) & b-a & 0 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

Taking (b – a) common

$$\Rightarrow (b-a) \begin{vmatrix} c & 1 & 0 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_3$

$$\Rightarrow (b-a) \begin{vmatrix} c & 1 & 0 \\ ca-ab & c-b & 0 \\ ab & a+b & 1 \end{vmatrix}$$

$$\Rightarrow (b-a) \begin{vmatrix} c & 1 & 0 \\ a(c-b) & c-b & 0 \\ ab & a+b & 1 \end{vmatrix}$$

Taking (c - b) common

$$\Rightarrow (b-a)(c-b) \begin{vmatrix} c & 1 & 0 \\ a & 1 & 0 \\ ab & a+b & 1 \end{vmatrix}$$

Expanding along C_3

⇒ (b – a) (c – b) [1{(c) (1) – (a) (1)}]

$\Rightarrow$ (b – a) (c – b) (c – a)

$\Rightarrow$ (a – b) (b – c) (c – a)

20. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = ?$$

A. 4abc

B. 2(a + b + c)

C. (ab + bc + ca)

D. none of these

Answer

To find: Value of $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

We have, $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} b+c+b+c & a+c+a+c & a+b+a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking 2 common

$$\Rightarrow 2\begin{vmatrix} b+c & a+c & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow 2\begin{vmatrix} c & 0 & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Expanding along R_1

$\Rightarrow$ 2 [c{(c + a) (a + b) – (b) (c)} + a{(b)(c) – (c) (c + a)}]

$\Rightarrow$ 2 [c{(ac + cb +a^2 + ab – bc} + a{(bc – c^2 - ac)}]

$\Rightarrow$ 2 [c{(ac + a^2 + ab)} + a{(bc – c^2 - ac)}]

$\Rightarrow$ 2 [ac^2 + ca^2 + abc + abc – ac^2 – a^2c]

$\Rightarrow$ 2 [2abc]

$\Rightarrow$ 4abc

21. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix} = ?$$

A. a + b+ c

B. 2(a + b + c)

C. 4abc

D. $a^2b^2c^2$

Answer

To find: Value of $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

We have, $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_1$

$\Rightarrow \begin{vmatrix} a & 1 & b+c \\ b-a & 0 & a-b \\ c & 1 & a+b \end{vmatrix}$

Taking (a - b) common

$\Rightarrow (a-b)\begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ c & 1 & a+b \end{vmatrix}$

Applying $R_3 \rightarrow R_3 - R_1$

$\Rightarrow (a-b)\begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ c-a & 0 & a-c \end{vmatrix}$

Taking (c-a) common

$\Rightarrow (b-a)(c-a)\begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ 1 & 0 & -1 \end{vmatrix}$

Expanding along R_1

= (b – a)(c – a)[0 – 1(1 – (a – b)) + (b + c)(0)]

= (b – a)(c – a)(-1 + a – b)

= (b – a)(c – a)(a – b – 1)

= $(b - a)(ac - bc - c - a^2 + ab + a)$

= $(abc - b^2c - bc - a^2b + ab^2 + ab - a^2c + abc + ac + a^3 + a^2b + a^2)$

= 4abc

22. Question

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix} = ?$$

A. -2

B. 2

C. x^2 – 2

D. x^2 + 2

Answer

To find: Value of $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

We have, $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

Applying $R_1 \rightarrow R_2 - R_1$

$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

Applying $R_2 \rightarrow R_3 - R_2$

$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ x+7 & x+10 & x+14 \end{vmatrix}$

Expanding along R_1

⇒ [2{(5)(x+14) – (6)(x+10)} – 3{(4)(x+14) – (6)(x+7)} + 4 {(4)(x+10) – (5)(x+7)}]

⇒ [2{5x + 70 – 6x – 60} – 3{4x + 56 -6x - 42} + 4 {4x + 40 – 5x - 35}]

⇒ [2{10 - x} – 3{14 -2x} + 4 {5 - x}]

⇒ [20 – 2x – 42 + 6x + 20 - 4x]

⇒ -2

23. Question

Mark the tick against the correct answer in the following:

If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$ then x = ?

A. 0

B. 6

C. -6

D. 9

Answer

To find: Value of x

We have, $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$

Applying $R_1 \rightarrow 2R_1$

$\Rightarrow \begin{vmatrix} 10 & 6 & -2 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$

Applying $R_1 \rightarrow R_1 - R_3$

$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$

Expanding along R_1

⇒ [1{(x)(-2) – (6)(2)}] = 0

⇒ [1{-2x – 12}] = 0

⇒ -2x-12 = 0

⇒ -2x = 12

⇒ x = -6

24. Question

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ is

A. {2, -3. 7}

B. {2, 7. -9}

C. [-2, 3, -7}

D. none of these

Answer

To find: Value of x

We have, $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

Applying $R_1 \rightarrow 2R_1$

$\Rightarrow \begin{vmatrix} 2x & 6 & 14 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

Applying $R_1 \rightarrow R_1 - R_3$

$\Rightarrow \begin{vmatrix} 2x-7 & 0 & 14-x \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

Expanding along R_1

⇒ [(2x-7){(x)(x) – (6)(2)} + (14-x){(2)(6) – (x)(7)] = 0

⇒ [(2x-7){x^2– 12} + (14-x){12 – 7x}] = 0

⇒ [2x^3 – 24x – 7x^2 + 84 + 168 – 98x -12x + 7x^2] = 0

⇒ [2x^3 – 134x + 252] = 0

⇒ [x^3 – 67x + 126] = 0

By Hit and trial x = -2, 3, -7

25. Question

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 2x-64 \end{vmatrix} = 0$ is

A. {4}

B. {2, 4}

C. {2, 8}

D. {4, 8}

Answer

To find: Value of x

We have, $\begin{vmatrix} x\text{-}2 & 2x\text{-}3 & 3x\text{-}4 \\ x\text{-}4 & 2x\text{-}9 & 3x\text{-}16 \\ x\text{-}8 & 2x\text{-}27 & 3x\text{-}64 \end{vmatrix} = 0$

Applying $C_2 \rightarrow C_2 - 2C_1$

⇒ $\begin{vmatrix} x\text{-}2 & 1 & 3x\text{-}4 \\ x\text{-}4 & \text{-}1 & 3x\text{-}16 \\ x\text{-}8 & \text{-}11 & 3x\text{-}64 \end{vmatrix} = 0$

Applying $C_3 \rightarrow C_3 - 3C_1$

⇒ $\begin{vmatrix} x\text{-}2 & 1 & 2 \\ x\text{-}4 & \text{-}1 & \text{-}4 \\ x\text{-}8 & \text{-}11 & \text{-}40 \end{vmatrix} = 0$

Expanding along R_1

⇒ [x-2{(-1)(-40) – (-4)(-11)} -1 {(x-4)(-40) – (-4)(x-8)} + 2 {(x-4)(-11) – (-1)(x-8)] = 0

⇒ [(x-2){40-44} -1 {(-40x + 160 + 4x – 32} + 2 {-11x + 44 + x - 8}] = 0

⇒ [(x-2){-4} -1 {(- 36x + 128} + 2 {-10x+36}] = 0

⇒ [-4x + 8 + 36x - 128 - 20x + 72] = 0

⇒ 12x – 48 = 0

⇒ x = 4

26. Question

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ is

A. {a, 0}

B. {3a, 0}

C. {a, 3a}

D. None of these

Answer

To find: Value of x

We have, $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Applying $R_1 \rightarrow R_1 - R_2$

$\Rightarrow \begin{vmatrix} 2x & -2x & 0 \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Applying $R_2 \rightarrow R_2 - R_3$

$\Rightarrow \begin{vmatrix} 2x & -2x & 0 \\ 0 & 2x & -2x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Taking 2 common from R_1

$\Rightarrow 2\begin{vmatrix} x & -x & 0 \\ 0 & 2x & -2x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Taking 2 common from R_2

$\Rightarrow 2 \times 2\begin{vmatrix} x & -x & 0 \\ 0 & x & -x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Applying $R_3 \rightarrow R_1 + R_3$

$\Rightarrow 4\begin{vmatrix} x & -x & 0 \\ 0 & x & -x \\ a & a-2x & a+x \end{vmatrix} = 0$

Expanding along R_1

$\Rightarrow 4[x\{(x)(a+x) – (-x)(a-2x)\}] – (-x)\{(0)(a+x) – (-x)(a)\}] = 0$

$\Rightarrow 4[x\{ax + x^2 + ax - 2x^2\}] – (-x)\{ax\}] = 0$

$\Rightarrow 4[x\{2ax - x^2\}] + ax^2] = 0$

$\Rightarrow 4[2ax^2 – x^3 + ax^2] = 0$

$\Rightarrow – x^2 + 3ax = 0$

$\Rightarrow -x(x – 3a) = 0$

$\Rightarrow x = 0$, or $x = 3a$

27. Question

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$ is

A. $\left\{\frac{2}{3}, \frac{8}{3}\right\}$

B. $\left\{\frac{2}{3}, \frac{11}{3}\right\}$

C. $\left\{\frac{3}{2}, \frac{8}{3}\right\}$

D. None of these

Answer

To find: Value of x

We have, $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Applying $R_1 \rightarrow R_1 - R_2$

$\Rightarrow \begin{vmatrix} 3x-11 & 11-3x & 0 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Applying $R_2 \rightarrow R_2 - R_3$

$\Rightarrow \begin{vmatrix} 3x-11 & 11-3x & 0 \\ 0 & 3x-11 & 11-3x \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Expanding along R_1

$\Rightarrow$ (3x-11){(3x-11)(3x-8) – (3)(11-3x)} – (11-3x){(0)((3x-8) – (11-3x)(3)} = 0

$\Rightarrow$ (3x-11){(3x-11)(3x-8+3)} – (11-3x){–(11-3x)(3)} = 0

$\Rightarrow$ $(3x-11)^2(3x-5))\} + (3x-11)\{(3x-11)(3)\} = 0$

$\Rightarrow$ $(3x-11)^2(3x-5))\} + (3x-11)^2(3)\} = 0$

$\Rightarrow$ $(3x-11)^2(3x-5+3) = 0$

$\Rightarrow$ $(3x-11)^2(3x-2) = 0$

$\Rightarrow x = \frac{11}{3}$, Or, $x = \frac{2}{3}$

28. Question

Mark the tick against the correct answer in the following:

The vertices of a a ABC are A(-2, 4), B(2, -6) and C(5, 4). The area of a ABC is

A. 17.5 sq units

B. 35 sq units

C. 32 sq units

D. 28 sq units

Answer

To find: Area of ABC

Given: A(-2,4), B(2,-6) and C(5,4)

Formula used: $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

We have, A(-2,4), B(2,6) and C(5,4)

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding along R_1

$\Rightarrow \frac{1}{2}$ [-2{(-6)(1)-(4)(1)-4 {(2)(1)-(5)(1)}+1 {(2)(4)-(5)(-6)}]

$\Rightarrow \frac{1}{2}$ [-2{-6-4} -4 {2-5}+1 {8+30}]

$\Rightarrow \frac{1}{2}$ [-2{-10} - 4{-3}+1 {38}]

$\Rightarrow \frac{1}{2}$ [20 + 12 + 38]

$\Rightarrow \frac{1}{2}$ [70]

⇒ 35 sq. units

29. Question

Mark the tick against the correct answer in the following:

If the points A(3, -2), B(k, 2) and C(8, 8) are collinear then the value of k is

A. 2

B. -3

C. 5

D. -4

Answer

To find: Area of ABC

Given: A(3,-2), B(k,2) and C(8,8)

The formula used: $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

We have, A(3,-2), B(k,2) and C(8,8)

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$$

Expanding along R_1

$\Rightarrow \frac{1}{2}$ [3{(2)(1)-(8)(1)} - (-2){(k)(1)-(8)(1)}+ 1{(k)(8)-(2)(8)}] = 0

$\Rightarrow \frac{1}{2}$ [3{2-8} + 2{k-8}+ 1{8k-16}]=0

⇒ -18 +2k - 16 + 8k -16 = 0

$\Rightarrow 10k - 50 = 0$

$\Rightarrow k = 5$