CHAPTER -06

APPLICATION OF DERIVATIVES

Rate of Change of Quantities:

FIVE MARK QUESTIONS

1. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.(U)

2. The radius of an air bubble is increasing at the rate of $\frac{1}{2} cm/s$. At what rate is the volume of the bubble increasing when the radius is $1 cm$?(U)

3. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x .(U)

4. The length x of a rectangle is decreasing at the rate of $3\ cm/minute$ and the width y is increasing at the rate of $2\ cm/minute$. When $x=10\ cm\ and\ \ y=6\ cm$, find the rate of change of (i) the perimeter and (ii) the area of the rectangle.(U)

5.The length x of a rectangle is decreasing at the rate of $5\ cm/minute$ and the width y is increasing at the rate of $4\ cm/minute$. When $x = 8\ cm$ and $y=6\ cm$, find the rates of change of **(i)** the perimeter, **(ii)** the area of the rectangle.(U)

6. The volume of a cube is increasing at the rate of $8\ cm^3/s$. How fast is the surface area increasing when the length of an edge is $12\ cm$?(A)

7. The volume of a cube is increasing at a rate of 9 cubic centimeters persecond. How fast is the surface area increasing when the length of an edge is 10centimeters?(U)

8. An edge of a variable cube is increasing at the rate of $3\ cm/s$. How fast is the volume of the cube increasing when the edge is $10\ cm$ long?(U)

9. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meter per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m.(A)

10. A sand is pouring from a pipe at the rate of 12cm^3/s. The falling sand form a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.

How fast is the height of the sand cone increasing when the height is $4cm$?(A)

11.A ladder $5m$ long is leaning against a wall. The bottom of the ladder is Pulled along the ground, Away from the wall at the rate of $2m/sec$.How fast is its height on the wall decreasing when the foot of the ladder is $4m$ away from the wall? (A)

12.A ladder 24 feet long leansagainst a vertical wall. Thelower end ismoving away at the rate of 3feet/sec. findthe rate at which the top of the ladder ismoving downwards, if its foot is 8 feet from the wall.(A)

13.A man of height 2 meters walks at a uniform speed of 5 km/hour, away from a lamp post which is 6 meters high. Find the rate at which the length of the his shadow increases.(A)

14.A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the $y-$coordinate is changing 8 times as fast as the $x-$coordinate.(U)

15. The total cost $C(x)$ in Rupees, associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost wemean the instantaneous rate of change of total cost at any level of output.(U)

16. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue, when $x = 5$, where by marginal revenue we mean the rate of change of total revenue with respect to thenumber of items sold at an instant.(U)

17. The total cost $C(x)$ in Rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$ .Find the marginal cost when 17 units are produced.(U)

18. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.(U)

19. Find the total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$.(U)

20. The radius of a circle is increasing uniformly at the rate of $3\ cm/s$. Find the rate at which the area of the circle is increasing when the radius is $10\ cm$.(U)

21. Find the rate of change of the area of a circle with respect to its radius r at $r = 6\ cm$.(U)

22. A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is $10\ cm$, how fast is the enclosed area increasing?(U)

23. A car starts from a point P at time $t = 0$ seconds and stops at point Q. The distance x , in meters, covered by it, in t seconds is given by $x = t^2\left(2 - \frac{t}{3}\right)$. Find the time taken by it to reach Q and also find distance between P and Q.(U)

24. Find the rate of change of the area of a circle per second with respect to its radius r when $r = 5\ cm$.(U)

25. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?(U)

TANGENT AND NORMAL:

Two mark questions:

1. Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$. (U)

2. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$. (U)

3. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$, $x \neq 2$ at $x = 10$. (U)

4. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose $x-$ coordinate is 2. (K)

5. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose $x-$ coordinate is 3. (K)

6. Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.(U)

7. Find the slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$. (K)

THREE MARK QUESTIONS

1. Find the point at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope $\frac{2}{3}$. (K)

2. Find the equation of all lines having slope 2 and being tangent to the curve $y + \frac{2}{x-3} = 0$. (K)

3. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are parallel to $x - axis$. (U)

4. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are parallel to $y - axis$. (U)

5. Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the $x - axis$. (U)

6. Find the equations of the tangent and normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at $(1, 1)$. (U)

7. Find the equation of tangent to the curve given by $x = a\sin^3 t, \quad y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$. (K)

8. Find the slope of the normal to the curve $x = a\cos^3\theta, y = b\sin^3\theta$ at $\theta = \frac{\pi}{4}$. (K)

9. Find the slope of the normal to the curve $x = 1 - a\sin\theta$, $y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$. (K)

10. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the $X - axis$. (K)

11. Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the points $(2, 0)$ and $(4, 4)$. (S)

12. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$. (K)

13. Find the equation of all lines having slope -1 that are tangents to the curve $y = \frac{1}{x-1}$, $x \neq 1$. (K)

14. Find the equation of all lines having slope 2 which are tangents to the curve $y=\frac{1}{x-3}$, $x\neq 3$. (K)

15. Find the equations of all lines having slope 0 which are tangent to the curve $y=\frac{1}{x^2-2x+3}$. (K)

16. Find points on the curve $\frac{x^2}{9}+\frac{y^2}{16}=1$ at which the tangents are parallel to $X-axis$.(K)

17. Find points on the curve $\frac{x^2}{9}+\frac{y^2}{16}=1$ at which the tangents are parallel to $Y-axis$.(K)

19. Find the equations of the tangent and normal to the given curve $y=x^4-6x^3+13x^2-10x+5$ at $(0,5)$. (U)

20. Find the equations of the tangent and normal to the given curve $y=x^4-6x^3+13x^2-10x+5$ at $(1,3)$. (U)

21. Find the equations of the tangent and normal to the given curve $y=x^3$ at $(1,1)$. (U)

22. Find the equations of the tangent and normal to the given curve $y=x^3$ at $(0,0)$. (U)

23. Find the equations of the tangent and normal to the given curve $x=\cos t$, $y=\sin t$ at $t=\frac{\pi}{4}$. (K)

24. Find the equation of the tangent line to the curve $y=x^2-2x+7$ which is parallel to the line $2x-y+9=0$.(K)

25. Find the equation of the tangent line to the curve $y=x^2-2x+7$ which is perpendicular to the line $5y-15x=13$.(K)

26. Show that the tangents to the curve $y=7x^3+11$ at the points where $x=2$ and $x=-2$ are parallel. (K)

27. Find the points on the curve $y=x^3$ at which the slope of the tangent is equal to the $y-$ coordinate of the point. (K)

28. For the curve $y=4x^3-2x^5$, find all the points at which the tangent passes through the origin. (K)

29. Find the points on the curve $x^2+y^2-2x-3=0$ at which the tangents are parallel to the $x-axis$. (K)

29. Find the equation of the normals to the curve $y=x^3+2x+6$ which are parallel to the line $x+14y+4=0$. (K)

30. Find the equations of the tangent and normal to the parabola $y^2=4ax$ at the point $(at^2,2at)$. (U)

31. Prove that the curves $x=y^2$ and $xy=k$ cut at right angle if $8k^2=1$. (A)

32. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point (x_0,y_0). (U)

33. Find the equation of the tangent to the curve $y=\sqrt{3x-2}$ which is parallel to the line $4x-2y+5=0$. (K)

34. Find the point at which the line $y=x+1$ is a tangent to the curve $y^2=4x$. (U)

35. Find the equation of the normal to curve $y^2=4x$ which passes through the point $(1,2)$. (U)

36. Show that the normal at any point θ to the curve $x=a\cos\theta+a\theta\sin\theta,\ y=a\sin\theta-a\theta\cos\theta$ is at a constant distance from the origin. (U)

37. Find the slope of the tangent to the curve $x=t^2+3t-8,\ y=2t^2-2t-5$ at the point $(2,-1)$. (K)

38. Find the value of 'm' so that the line $y=mx+1$ is a tangent to the curve $y^2=4x$. (U)

39. Find the equation of the normal to the curve $2y+x^2=3$ at the point $(1,1)$.(U)

40. Find the equation of the normal to the curve $x^2=4y$ at $(1,2)$. (U)

41. Find the points on the curve $9y^2=x^3$, where the normal to the curve makes equal intercepts with their axes. (U)

42. Find the equation of tangents to the curve $y=\cos(x+y),\ -2\pi\le x\le 2\pi$ that are parallel to the line $x+2y=0$. (K)

Increasing and Decreasing Functions

TWO MARK QUESTIONS

1.Show that the function given by $f(x)=7x-3$ is strictly increasing on R . (U)

2.Show that the function f given by $f(x)=x^3-3x^2+4x, x\in R$ is strictly increasing on R .(U)

3. Find the interval in which the function f given by $f(x)=2x^2-3x$ is strictly increasing. (U)

4. Prove that the function given by $f(x)=\cos x$ is strictly decreasing in $(0,\pi)$.(U)

5. Prove that the function given by $f(x)=\cos x$ is strictly increasing in $(\pi,\ 2\pi)$.(U)

6. Prove that the function given by $f(x)=\cos x$ is neither increasing nor decreasing in $(0,2\pi)$.(U)

7.Show that the function given by $f(x)=3x+17$ is strictly increasing on R .(U)

8.Show that the function given by $f(x)=e^{2x}$ is strictly increasing on R .(U)

9.Show that the function given by $f(x)=\sin x$ is strictly increasing in $\left(0,\frac{\pi}{2}\right)$.(U)

10.Show that the function given by $f(x)=\sin x$ is strictly decreasing in $\left(\frac{\pi}{2},\pi\right)$ (U)

11.Show that the function given by $f(x)=\sin x$ is neither increasing nor decreasing in $(0,\pi)$.(U)

12.Find the intervals in which the function f given by $f(x)=2x^2-3x$ is strictly increasing.(U)

13.Find the intervals in which the function f given by $f(x)=2x^2-3x$ is strictly decreasing.(U)

14.Prove that the logarithmic function is strictly increasing on $(0,\infty)$.(U)

15. Show that the function given by $f(x)=\cos x$ is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$.(U)

16. Show that the function given by $f(x)=\cos 2x$ is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$.(U)

17. Show that the function given by $f(x)=\cos 3x$ is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$.(U)

18. Show that the function given by $f(x)=\tan x$ is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$.(U)

19.Prove that the function f given by $f(x)=x^3-3x^2-3x-100$ is increasing in R .(U)

20.Prove that the function f given by $f(x)=x^2e^{-x}$ is increasing in $(0,2)$.(U)

THREE MARK QUESTIONS

1. Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is strictly increasing. (K)

2. Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is strictly decreasing. (K)

3. Find the intervals in which the function f given by $f(x)=4x^3-6x^2-72x+30$ is strictly increasing. (K)

4. Find the intervals in which the function f given by $f(x)=4x^3-6x^2-72x+30$ is strictly decreasing. (K)

5.Find intervals in which the function given by $f(x)=\sin 3x,\ x\in\left[0,\frac{\pi}{2}\right]$ is increasing. (U)

6.Find intervals in which the function given by $f(x)=\sin 3x,\ x\in\left[0,\frac{\pi}{2}\right]$ is decreasing. (U)

7.Find the intervals in which the function f given by $f(x)=2x^3-3x^2-36x+7$ is strictly increasing.(U)

8.Find the intervals in which the function f given by $f(x)=2x^3-3x^2-36x+7$ is strictly decreasing. (U)

9. Find the intervals in which the function f given by x^2+2x-5 is strictly increasing. (U)
10. Find the intervals in which the function f given by x^2+2x-5 is strictly decreasing. (U)
11. Find the intervals in which the function f given by $10-6x-2x^2$ is strictly increasing. (U)
12. Find the intervals in which the function f given by $10-6x-2x^2$ is strictly decreasing. (U)
13. Find the intervals in which the function f given by $6-9x-x^2$ is strictly increasing. (U)
14. Find the intervals in which the function f given by $6-9x-x^2$ is strictly decreasing. (U)
15. Prove that the function f given by $f(x)=x^2-x+1$ is neither strictly increasing nor strictly decreasing on $(-1,1)$. (U)
16. Show that the function given by $f(x)=x^{100}+\sin x-1$ is strictly decreasing in $(0,1)$. (K)
17. Show that the function given by $f(x)=x^{100}+\sin x-1$ is strictly decreasing in $\left(\frac{\pi}{2},\pi\right)$.(K)
18. Show that the function given by $f(x)=x^{100}+\sin x-1$ is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$.(K)
19. Find the least value of 'a' such that the function f given by $f(x)=x^2+ax+1$ is strictly increasing on $(1,2)$. (K)
20. Prove that the function f given by $f(x)=\log(\sin x)$ is strictly increasing on $\left(0,\frac{\pi}{2}\right)$.(U)
21. Prove that the function f given by $f(x)=\log(\sin x)$ is strictly decreasing on $\left(\frac{\pi}{2},\pi\right)$. (U)
22. Prove that the function f given by $f(x)=\log(\cos x)$ is strictly decreasing on $\left(0,\frac{\pi}{2}\right)$.(U)
23. Prove that the function f given by $f(x)=\log(\cos x)$ is strictly increasing on $\left(\frac{\pi}{2},\pi\right)$. (U)
24. Find the intervals in which the function f given by $f(x)=\sin x+\cos x$, $0\le x\le 2\pi$ is strictly increasing or strictly decreasing.
25. Find the intervals in which the following functions are strictly increasing or decreasing:
(i) $-2x^3-9x^2-12x+1$ (ii) $(x+1)^3(x-3)^3$.
26. Show that $y=\log(1+x)-\frac{2x}{2+x}$, $x>-1$, is an increasing function of x throughout its domain.
27. Find the values of x for which $y=[x(x-2)]^2$ is an increasing function.
28. Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$.
29. Find intervals in which the function given by $f(x)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$ is
(a) strictly increasing **(b)** strictly decreasing.
30. Show that the function f given by $f(x)=\tan^{-1}(\sin x+\cos x)$, $x>0$ is always an strictly increasing function in $\left(0,\frac{\pi}{4}\right)$.
31. Find the intervals in which the function f given by $f(x)=\frac{4\sin x-2x-x\cos x}{2+\cos x}$ is
(i) increasing (ii) decreasing.
32. Find the intervals in which the function f given by $f(x)=x^3+\frac{1}{x^3}$, $x\ne 0$ is
(i) increasing (ii) decreasing.

ADDITIONAL QUESTIONS

1. Find the intervals in which the function f given by $f(x)=\sin x+\cos x$, $0\le x\le 2\pi$ is strictly increasing or strictly decreasing.

2. Find the intervals in which the following functions are strictly increasing or decreasing: (i) $-2x^3-9x^2-12x+1$ (ii) $(x+1)^3(x-3)^3$.

3. Show that $y=\log(1+x)-\frac{2x}{2+x}$, $x>-1$, is an increasing function of x throughout its domain.

4. Find the values of x for which $y=[x(x-2)]^2$ is an increasing function.

5. Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$.

6. Find intervals in which the function given by

$f(x)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$ is **(a)** strictly increasing **(b)** strictly decreasing.

7. Show that the function f given by $f(x)=\tan^{-1}(\sin x+\cos x)$, $x>0$ is always an strictly increasing function in $\left(0,\frac{\pi}{4}\right)$.

8. Find the intervals in which the function f given by

$f(x)=\frac{4\sin x-2x-x\cos x}{2+\cos x}$ is (i) increasing (ii) decreasing.

9. Find the intervals in which the function f given by $f(x)=x^3+\frac{1}{x^3}$, $x\ne 0$ is (i) increasing (ii) decreasing.

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Maxima and Minima

TWO MARK QUESTIONS:

1.Find the maximum and the minimum values of the function f given by $f(x) = x^2,\ x \in R$. (U)

2.Find the maximum and minimum values of the function f given by $f(x) = |x|,\ x \in R$. (U)

3. Find the maximum and minimum values of the function f given by $f(x) = x,\ x \in (0,1)$. (K)

4.Prove that the function $f(x) = e^x$ do not have maxima or minima. (U)

5.Prove that the function $g(x) = \log x$ do not have maxima or minima. (U)

6.Prove that the function $h(x) = x^3 + x^2 + x + 1$ do not have maxima or minima. (U)

7.It is given that at $x = 1$, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval $[0, 2]$. Find the value of 'a'. (U)

8. Find all points of local maxima and local minima of the function f given by $f(x) = x^3 - 3x + 3$. (U)

9. Find all the points of local maxima and local minima of the function $f(x) = 2x^3 - 6x^2 + 6x + 5$.(U)

10. Find local minimum value of the function f given by $f(x) = 3 + |x|,\ x \in R$.(U)

11. Find all the points of local maxima and local minima of the function $f(x) = 2x^3 - 6x^2 + 6x + 5$. (U)

12. Show that the function given by $f(x) = \dfrac{\log x}{x}$ has maximum at $x = e$.(U)

13. At what points in the interval $[0, 2\pi]$, does the function $\sin 2x$ attain its maximum value? (U)

14. Prove that the function f(x)= e^x do not have maxima or minima.

15. Prove that the function f(x)=logx do not have maxima or minima.

THREE MARK QUESTIONS

1 . Find local maximum and local minimum values of the function f given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$ (U) 2. Find the maximum and minimum values of the function given by $f(x) = (2x-1)^2 + 3$. (U)

2. Find the maximum and minimum values of the function given by $f(x) = 9x^2 + 12x + 2$.(U)

3. Find the maximum and minimum values of the function given by $f(x) = -(x-1)^2 + 10$ (U)

4. Find the maximum and minimum values of the function given by $g(x) = x^3 + 1$.(U)

5. Find the maximum and minimum values of the function given by $f(x) = |x+2| - 1$.(A)

6. Find the maximum and minimum values of the function given by $g(x) = -|x+1| + 3$ (A)

7. Find the maximum and minimum values of the function given by $h(x) = \sin(2x) + 5$.(U)

8. Find the maximum and minimum values of the function given by $h(x) = |\sin 4x + 3| + 5$.(U)

9. Find both the maximum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$.(U)

10. Find the maximum and minimum values of the function given by $h(x) = x + 1,\ x \in (-1, 1)$. (U)

11. Find the maximum value of $[x(x-1)+1]^{\frac{1}{3}},\ 0 \le x \le 1$. (A)

12. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum. (A)

13. Find two numbers whose sum is 24 and whose product is as large as possible. (A)

14. Find two positive numbers $x\ and\ y$ such that $x + y = 60$ and xy^3 is maximum. (A)

15. Find two positive numbers $x\ and\ y$ such that their sum is 35 and the product x^2y^5 is a maximum. (A)

16. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum. (A)

17. Find the local maxima and local minima of the function $g(x) = x^3 - 3x$. Also find the local maximum and the local minimum values. (U)

18. Find the local maxima and local minima of the function $f(x) = x^2$. Also find the local maximum and the local minimum values. (U)

19. Find the local maxima and local minima of the function $h(x) = \sin x + \cos x,\ 0 < x < \frac{\pi}{2}$. Also find the local maximum and the local minimum values. (U)

20. Find the local maxima and local minima of the function $f(x) = x^3 - 6x^2 + 9x + 15$. Also find the local maximum and the local minimum values. . (U)

21. Find the local maxima and local minima of the function $g(x) = \frac{x}{2} + \frac{2}{x},\ x > 0$. Also find the local maximum and the local minimum values. (U)

22. Find the local maxima and local minima of the function $f(x) = \sin x - \cos x,\ 0 < x < 2\pi$. Also find the local maximum and the local minimum values. (U)

23. Find the local maxima and local minima of the function $f(x) = x\sqrt{1-x},\ x > 0$. Also find the local maximum and the local minimum values. (U)

24. Find the local maxima and local minima of the function $g(x) = \frac{1}{x^2 + 2}$. Also find the local maximum and the local minimum values. (U)

25. Find the absolute maximum value and the absolute minimum value of the function $f(x) = (x-1)^2 + 3,\ x \in [-3, 1]$. (U)

26. Find the absolute maximum value and the absolute minimum value of the function $f(x) = x^3,\ x \in [-2, 2]$. (U)

27. Find the absolute maximum and minimum values of a function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval $[1, 5]$. (U)

28. Find absolute maximum and minimum values of a function f given by

$f(x)=12x^{\frac{4}{3}}-6x^{\frac{1}{3}},\ x\in[-1,1]$. (U)

29. Find the absolute maximum value and the absolute minimum value of the function

$f(x)=\sin x+\cos x,\ x\in[0,\pi]$. (U)

30. Find the absolute maximum value and the absolute minimum value of the function

$f(x)=4x-\frac{1}{2}x^2,\ x\in\left[-2,\frac{9}{2}\right]$. (U)

31. Find the absolute maximum and minimum values of the function f given by

$f(x)=\cos^2 x+\sin x,\ x\in[0,\pi]$. (U)

32. Manufacturer can sell x items at a price of rupees $\left(5-\frac{x}{100}\right)$ each.

The cost price of x items is Rs $\left(\frac{x}{5}+500\right)$.

Find the number of items he should sell to earn maximum profit. (A)

33. Find the maximum profit that a company can make, if the profit function is given by

$p(x)=41-72x-18x^2$. (A)

34. What is the maximum value of the function $\sin x+\cos x$?(U)

35.Find the maximum value of $2x^3-24x+107$ in the interval $[1,3]$. (U)

36. Find the maximum value of $2x^3-24x+107$ in $[-3,-1]$.(U)

37.Find the maximum and minimum values of $x+\sin 2x$ on $[0,2\pi]$.(U)

38. Find the points at which the function f given by $f(x)=(x-2)^4(x+1)^3$ has

(i) local maxima(ii) local minima. (U)

39.For all real values of x, find the minimum value of $\frac{1-x+x^2}{1+x+x^2}$. (U)

ADDITIONAL QUESTIONS

1. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum. (A)

2. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone. (A)

3. An Apache helicopter of enemy is flying along the curve given by $y=x^2+7$.

A soldier, placed at $(3,7)$, wants to shoot down the helicopter when it is nearest to him.

Find the nearest distance. (A)

4.A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible. (A)

5.A rectangular sheet of tin $45\ cm$ by $24\ cm$ is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ? (A)

6.Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.(A)

7.Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base. (A)

8.Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimeters, find the dimensions of the can which has the minimum surface area? (A)

9.A wire of length $28\ m$ is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum? (A)

10.Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. (A)

11.Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base. (A)

12.Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.(A)

13.Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.(A)

14.An open topped box is to be constructed by removing equal squares from each corner of a 3 meter by 8 meter rectangular sheet of aluminum and folding up the sides. Find the volume of the largest such box. (A)

15.The two equal sides of an isosceles triangle with fixed base 'b' are decreasing at the rate of $3\ cm\ per\ second$. How fast is the area decreasing when the two equal sides are equal to the base ? (A)

16.Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one end of the major axis. (A)

17.A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is $2\ m$ and volume is $8\ m^3$. If building of tank costs $Rs\ 70\ per\ sq\ metres$ for the base and $Rs\ 45\ per\ square\ metre$ for sides. What is the cost of least expensive tank? (A)

18.The sum of the perimeter of a circle and square is k, where k is some constant .Prove that the sum of their areas is least when the side of square is double the radius of the circle. (A)

19.A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is $10\ m$. Find the dimensions of the window to admit maximum light through the whole opening. (A)

20. A point on the hypotenuse of a triangle is at distance 'a' and 'b' from the sides of the triangle .

Show that the maximum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.(A)

21.Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.(A)

22.Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume. (A)

23.Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.(A)

24. Find the point on the curve $x^2 = 2y$ which is nearest to the point $(0,5)$.(U)

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Approximations

TWO MARK QUESTIONS

1.Use differential to approximate $\sqrt{36.6}$. (U)

2.Using differentials, find the approximate value of $(25)^{\frac{1}{3}}$. (U)

3.Using differentials, find the approximate value of $\sqrt{25.3}$ up to 3 places of decimal.(K)

4.Use differential to approximate $\sqrt{25.3}$.(U)

5.Use differential to approximate $\sqrt{0.6}$.(U)

6. Using differentials, find the approximate value of $\sqrt{49.5}$. (U)

7. Using differentials, find the approximate value of $(0.009)^{\frac{1}{3}}$.(U)

8. Using differentials, find the approximate value of $(0.999)^{\frac{1}{10}}$.(U)

9.Use differential to approximate $(15)^{\frac{1}{4}}$.(U)

10.Use differential to approximate $(26)^{\frac{1}{3}}$.(U)

11.Use differential to approximate $(255)^{\frac{1}{4}}$.(U)

12.Use differential to approximate $(82)^{\frac{1}{4}}$ up to 3 places of decimal.(K)

13.Use differential to approximate $(401)^{\frac{1}{2}}$ up to 3 places of decimal.(K)

14.Use differential to approximate $(0.0037)^{\frac{1}{2}}$.(U)

15.Use differential to approximate $(26.57)^{\frac{1}{3}}$.(U)

16.Use differential to approximate $(81.5)^{\frac{1}{4}}$.(U)

17.Use differential to approximate $(3.968)^{\frac{3}{2}}$. (U)

18.Use differential to approximate $(32.15)^{\frac{1}{5}}$. (U)

19.Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%. (A)

20.If the radius of a sphere is measured as $9\ cm$ with an error of $0.03\ cm$, then find the approximate error in calculating its volume.(A)

21.Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 1%.(A)

22.Find the approximate change in the surface area of a cube of side x meters caused by decreasing the side by 1%.(A)

23.If the radius of a sphere is measured as $7\,m$ with an error of $0.02\ m$, then find the approximate error in calculating its volume.(A)

24.If the radius of a sphere is measured as $9\ m$ with an error of $0.03\ m$, then find the approximate error in calculating its surface area.(A)

25.Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 3%. (A)

26.Find the approximate value of $f(3.02)$, where $f(x)=3x^2+5x+3$.(A)

27. Find the approximate value of $f(3.02)$, where $f(x)=3x^2+5x+3$.(A)

28.Find the approximate value of $f(2.01)$, where $f(x)=4x^2+5x+2$.(A)

29.Find the approximate value of $f(5.001)$, where $f(x)=x^3-7x^2+15$.(A)

30.Using differentials, find the approximate value of $\left(\frac{17}{81}\right)^{\frac{1}{4}}$.(U)

31.Using differentials, find the approximate value of $(33)^{-\frac{1}{5}}$.(U)

32.A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of $0.05\ cm/s$. Find the rate at which its area is increasing when radius is $3.2\ cm$.(A)

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