Roll No …………………….. 12th **028/B**

Mathematics

(Common for Humanities, Sc & Agri Groups)

(Evening Session)

Time : Three Hours] [Maximum Marks : 90

I. i If $\tan^{-1} x = y$,then 1

(a) $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$ (b) $0 < y < \pi$

(c) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (d) $0 \leq y \leq \pi$

ii If A is any square matrix then A+A' is a : 1

(a) Skew-symmetric matric (b) symmetric matric

(c) null matric (d) identity matric

iii If $f(x) = \begin{cases} kx + 2 & , \quad x \leq 2 \\ 3x - 4 & , \quad x > 2 \end{cases}$ is continuous at x=5 then value of k is : 1

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$

(c) $\frac{8}{5}$ (d) $\frac{9}{5}$

iv $\int_{-1}^{1} x^5 \ cos^6 x \, dx \ is \ equal \ to$: 1

(a) $\frac{1}{9}$ (b) $\frac{-1}{9}$

(c) 0 (d) $\frac{1}{8}$

v $\int_0^1 \frac{dx}{1+x^2}$ is equal to : 1

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{12}$ (d) $\frac{7\pi}{12}$

vi Degree of differential equation $\left(\frac{d^2y}{dx^3}\right)^3 + \left(\frac{dy}{dx}\right)^2 = 0$ is; 1

(a) 0 (b) 1

(c) 2 (d) 3

vii Position vector of mid-point of vector joining the points P (2,3,6) and Q (4,5,-2) is 1

(a) $3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ (b) $6\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$

(c) $8\hat{\imath} + 3\hat{\jmath} - 8\hat{k}$ (d) $3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$

viii If line makes angles $90^0, 45^0, 135^0$ with X,Y and Z axes respectively then its direction ratios are : 1

(a) <1,-1,1> (b) <0,-1,1>

(c) <-1,0,1> (d) <0,1,-1>

2 Show that function $Z \to Z$ given by f(x)= x^2 is neither one – one nor onto. 3

OR

Show that relation $R + \{(P_1, P_2): P_2\ is\ parallel\ to\ P_2\}$, defined on the set of all lines, is an equivalence relation.

3 Prove that $2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{2} = \tan^{-1}\frac{22}{19}$ 3

4 Express $\begin{bmatrix} 5 & 1 \\ 7 & 0 \end{bmatrix}$as sum of symmetric and skew-symmetric matrices 3

OR

Using determinates find the equation on line passing from the points (2,4) and (6,10).

5 If $y = (x\tan x)^x\ then\ find \frac{dy}{dx}$. 3

6 Verify Rolle's theorem for $f(x) = x^2 - 9x + 14, x\ \epsilon\ [2,7]$. 3

7 Using differentials find the approximate value of $(0.065)^{1/3}$. 3

8 Evaluate $\int \frac{7x+1}{\sqrt{x^2+4x+11}} dx$ 3

OR

Evaluate $\int \frac{(x^2-3)e^x}{(x-1)^2} dx$

9 Evaluate $\int_{\pi/6}^{\pi/3} \frac{1}{1+\tan^{3/2} x} dx$ 3

10 Using integration find the area of triangle with vertices (4,3) , (4,1) and (6,0). 3

11 Find the particular solution of the differential equation $\cot x \frac{dy}{dx} = y; y = 1\ when\ x = 0$ 3

12 $Solve\ \frac{dy}{dx} + 2y = \sin 2x$ 3

13 If : $|\vec{a}| = 7, |\vec{b}| = 1\ and\ |\vec{c}| = 5$ and each of them is perpendicular to the sum of other two them find value is $|\vec{a} + \vec{b} + \vec{c}|$. 3

OR

Using vectors find the area of the triangle whose vertices are (0,1,-2),(2,1,5)and (1,5,2).

14 Probability of solving specific problem independently by A,B and C are $\frac{1}{2}, \frac{1}{3}\ and\ \frac{1}{5}$ respectively, if all of them try to solve the problem independently then find the probability that problem will be solved. 3

15 A pair of dice thrown 4 times. If getting a doublet is considred success then find the probability of exactly 3 successes, 3

16 Solve the following system of linear equation by matrix method: 5

$x + 2y + z = 6, 2x + y + 2z = 6, x - y - z = 2$

OR

Using elementary transformation find the inverse of$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 1 & 2 \end{bmatrix}$.

17 A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 30 m. find the dimensions of the window to admit maximum light through the whole opening. 5

18 Find the distance of the (-1,-5,-10) from the point of intersection of the line 5

$$\vec{r} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k} + \lambda(3\hat{\imath} + \hat{\jmath} + 2\hat{k}) \text{ and the plane } \vec{r}.(2\hat{\imath} - 2\hat{\jmath} + \hat{k}) = 7.$$

OR

Find the shortest distance between the lines:

$$\vec{r} = \hat{\imath} + 2\hat{\jmath} + \hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k}) \text{ and } \vec{r} = 2\hat{\imath} - \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} + \hat{\jmath} + 2\hat{k})$$

19 Graphically minimize Z= 7x + 4y subject to the constraints 5

$3x + y \leq 90, x + 5y \geq 100, 9x + 8y \leq 400, x \geq 0, y \geq 0.$