

## Differential Equation and Applications

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### EXERCISE 8.1 [PAGE 162]

#### Exercise 8.1 | Q 1.1 | Page 162

Determine the order and degree of the following differential equations.

$$\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 + 8 = 0$$

**Solution:**

$$\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 + 8 = 0$$

By definition of order and degree,

Order : 2 ; Degree : 1

#### Exercise 8.1 | Q 1.2 | Page 162

Determine the order and degree of the following differential equations.

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = a^x$$

**Solution:**

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = a^x$$

By definition of order and degree,

Order : 2 ; Degree : 2

#### Exercise 8.1 | Q 1.3 | Page 162

Determine the order and degree of the following differential equations.

$$\frac{d^4y}{dx^4} + \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^3 = 0$$

**Solution:**

$$\frac{d^4y}{dx^4} + \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^3 = 0$$

By definition of order and degree,

Order : 4 ; Degree : 1

**Exercise 8.1 | Q 1.4 | Page 162**

Determine the order and degree of the following differential equations.

$$(y''')^2 + 2(y'')^2 + 6y' + 7y = 0$$

**Solution:**

$$(y''')^2 + 2(y'')^2 + 6y' + 7y = 0$$

By definition of order and degree,

Order : 3 ; Degree : 2

**Exercise 8.1 | Q 1.5 | Page 162**

Determine the order and degree of the following differential equations.

$$\sqrt{1 + \frac{1}{\left( \frac{dy}{dx} \right)^2}} = \left( \frac{dy}{dx} \right)^{\frac{3}{2}}$$

**Solution:**

$$\sqrt{1 + \frac{1}{\left(\frac{dy}{dx}\right)^2}} = \left(\frac{dy}{dx}\right)^{\frac{3}{2}}$$

Squaring on both sides, we get

$$1 + \frac{1}{\left(\frac{dy}{dx}\right)^2} = \left(\frac{dy}{dx}\right)^3$$
$$\therefore \left(\frac{dy}{dx}\right)^2 + 1 = \left(\frac{dy}{dx}\right)^5$$

By definition of order and degree,

Order : 1 ; Degree : 5

### Exercise 8.1 | Q 1.6 | Page 162

Determine the order and degree of the following differential equations.

$$\frac{dy}{dx} = 7 \frac{d^2y}{dx^2}$$

**Solution:**

$$\frac{dy}{dx} = 7 \frac{d^2y}{dx^2}$$

By definition of order and degree,

Order : 2 ; Degree : 1

### Exercise 8.1 | Q 1.7 | Page 162

Determine the order and degree of the following differential equations.

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}} = 9$$

**Solution:**

$$\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}} = 9$$

Taking sixth power on both sides, we get

$$\frac{d^3y}{dx^3} = 9^6$$

By definition of order and degree,

Order : 3 ; Degree : 1

**Exercise 8.1 | Q 2.1 | Page 162**

In each of the following examples, verify that the given function is a solution of the corresponding differential equation.

<b>Solution</b>	<b>D.E.</b>
$xy = \log y + k$	$y'(1-xy) = y^2$

**Solution:**  $xy = \log y + k$

Differentiating w.r.t.  $x$ , we get

$$x \frac{dy}{dx} + y(1) = \frac{1}{y} \cdot \frac{dy}{dx}$$

$$\therefore xy \frac{dy}{dx} + y^2 = \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} - xy = \frac{dy}{dx} = y^2$$

$$\therefore (1 - xy) \frac{dy}{dx} = y^2$$

$$\therefore y'(1 - xy) = y^2$$

$\therefore$  Given function is a solution of the given differential equation.

### Exercise 8.1 | Q 2.2 | Page 162

In the following example, verify that the given function is a solution of the corresponding differential equation.

Solution	D.E.
$y = x^n$	$x^2 \frac{d^2y}{dx^2} - n \times \frac{xdy}{dx} + ny = 0$

**Solution:**

$$y = x^n$$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = nx^{n-1}$$

Again, differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = n(n-1)x^{n-2}$$

$$\therefore x^2 \frac{d^2y}{dx^2} - nx \frac{dy}{dx} + ny$$

$$= n(n-1)x^2x^{n-2} - nx \cdot nx^{n-1} + nx^n$$

$$= n(n-1)x^n - n^2x^n + nx^n$$

$$= [n(n-1) - n^2 + n]x^n$$

$$= 0$$

$$\therefore x^2 \frac{d^2 y}{dx^2} - nx \frac{dy}{dx} + ny = 0$$

$\therefore$  Given function is a solution of the given differential equation.

### Exercise 8.1 | Q 2.3 | Page 162

In each of the following examples, verify that the given function is a solution of the corresponding differential equation.

Solution	D.E.
$y = e^x$	$\frac{dy}{dx} = y$

**Solution:**

$$y = e^x$$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = e^x$$

$$\therefore \frac{dy}{dx} = y$$

$\therefore$  Given function is a solution of the given differential equation.

### Exercise 8.1 | Q 2.4 | Page 162

Determine the order and degree of the following differential equations.

Solution	D.E.
$y = 1 - \log x$	$x^2 \frac{d^2 y}{dx^2} = 1$

**Solution:**

$$y = 1 - \log x$$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = -\frac{1}{x}$$

Again, differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = \frac{1}{x^2}$$

$$\therefore x^2 \frac{d^2y}{dx^2} = 1$$

$\therefore$  Given function is a solution of the given differential equation.

**Exercise 8.1 | Q 2.5 | Page 162**

Determine the order and degree of the following differential equations.

Solution	D.E
$y = ae^x + be^{-x}$	$\frac{d^2y}{dx^2} = 1$

**Solution:**  $y = ae^x + be^{-x}$  .....(1)

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = ae^x - be^{-x}$$

$$\frac{dy}{dx} = ae^x - be^{-x}$$

Again, differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = ae^x - be^{-x}$$

$$\therefore \frac{d^2y}{dx^2} = y \dots [\text{From (i)}]$$

$\therefore$  Given function is a solution of the given differential equation.

### Exercise 8.1 | Q 2.6 | Page 162

Determine the order and degree of the following differential equations.

Solution	D.E.
$ax^2 + by^2 = 5$	$xy \frac{d^2y}{dx^2} + x \left( \frac{dy}{dx} \right)^2 = y \frac{dy}{dx}$

**Solution:**

$$ax^2 + by^2 = 5$$

Differentiating w.r.t. x, we get

$$2ax + 2by \frac{dy}{dx} = 0 \dots (i)$$

Again, differentiating w.r.t. x, we get

$$2a + 2b \left( \frac{dy}{dx} \right)^2 + 2by \left( \frac{d^2y}{dx^2} \right) = 0 \dots (ii)$$

From (i), we get

$$a = -\frac{by}{x} \left( \frac{dy}{dx} \right)$$

Substituting the value of a in (ii), we get

$$-2\frac{by}{x} \left( \frac{dy}{dx} \right) + 2b \left( \frac{dy}{dx} \right)^2 + 2by \left( \frac{d^2y}{dx^2} \right) = 0$$

$$\therefore -\frac{y}{x} \left( \frac{dy}{dx} \right) + \left( \frac{dy}{dx} \right)^2 + y \left( \frac{d^2y}{dx^2} \right) = 0$$

$$\therefore -y \left( \frac{dy}{dx} \right) + x \left( \frac{dy}{dx} \right)^2 + xy \left( \frac{d^2y}{dx^2} \right) = 0$$

$$\therefore xy \left( \frac{d^2y}{dx^2} \right) + x \left( \frac{dy}{dx} \right)^2 = y \left( \frac{dy}{dx} \right)$$

$\therefore$  Given function is a solution of the given differential equation.

### EXERCISE 8.2 [PAGE 163]

#### Exercise 8.2 | Q 1.1 | Page 163

Obtain the differential equation by eliminating arbitrary constants from the following equations.

$$y = Ae^{3x} + Be^{-3x}$$

**Solution:**

$$y = Ae^{3x} + B.e^{-3x} \dots\dots(i)$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 3Ae^{3x} - 3Be^{-3x}$$

Again, differentiating w.r. t. x, we get

$$\begin{aligned}
\frac{d^2y}{dx^2} &= 3A \frac{d}{dx} e^{3x} - 3B \frac{d}{dx} (e^{-3x}) \\
&= 3A(3e^{3x}) - 3b(-3e^{-3x}) \\
&= 9Ae^{3x} + 9Be^{-3x} \\
&= 9(Ae^{3x} + Be^{-3x}) = 9y \dots\dots[\text{From(i)}] \\
\therefore \frac{d^2y}{dx^2} &= 9y
\end{aligned}$$

**Exercise 8.2 | Q 1.2 | Page 163**

Obtain the differential equations by eliminating arbitrary constants from the following equation.

$$y = c_2 + \frac{c_1}{x}$$

**Solution:**

$$y = c_2 + \frac{c_1}{x}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{-c_1}{x^2}$$

$$\therefore x^2 \frac{dy}{dx} = -c_1$$

Again, differentiating w.r.t. x, we get

$$x^2 \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot 2x = 0$$

$$\therefore x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = 0$$

### Exercise 8.2 | Q 1.3 | Page 163

Obtain the differential equation by eliminating arbitrary constants from the following equations.

$$y = (c_1 + c_2 x) e^x$$

**Solution:**  $y = (c_1 + c_2 x) e^x$

$$\therefore y e^{-x} = c_1 + c_2 x$$

Differentiating w.r.t.  $x$ , we get

$$y(-e^{-x}) + e^{-x} \frac{dy}{dx} = 0 + c_2$$

$$\therefore e^{-x} \left( \frac{dy}{dx} - y \right) = c_2$$

Again, differentiating w.r.t.  $x$ , we get

$$e^{-x} \left( \frac{d^2 y}{dx^2} - \frac{dy}{dx} \right) - e^{-x} \left( \frac{dy}{dx} - y \right) = 0$$

$$\therefore e^{-x} \left( \frac{d^2 y}{dx^2} - \frac{dy}{dx} - \frac{dy}{dx} + y \right) = 0$$

$$\therefore \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = 0$$

### Exercise 8.2 | Q 1.4 | Page 163

Obtain the differential equations by eliminating arbitrary constants from the following equations.

$$y = c_1 e^{3x} + c_2 e^{2x}$$

**Solution:**  $y = c_1 e^{3x} + c_2 e^{2x}$

Dividing throughout by  $e^{2x}$ , we get

$$y e^{-2x} = c_1 e^x + c_2$$

Differentiating w.r.t.  $x$ , we get

$$-2ye^{-2x} + e^{-2x} \frac{dy}{dx} = c_1 e^x$$

$$\therefore e^{-2x} \left( \frac{dy}{dx} - 2y \right) = c_1 e^x$$

Dividing throughout by  $e^x$ , we get

$$e^{-3x} \left( \frac{dy}{dx} - 2y \right) = c_1$$

Again, differentiating w.r.t.  $x$ , we get

$$e^{-3x} \left( \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} \right) - 3e^{-3x} \left( \frac{dy}{dx} - 2y \right) = 0$$

$$\therefore e^{-(3x)} \left( \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - 3 \frac{dy}{dx} + 6y \right) = 0$$

$$\therefore \frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

### Exercise 8.2 | Q 1.5 | Page 163

Obtain the differential equation by eliminating arbitrary constants from the following equations.

$$y^2 = (x + c)^3$$

**Solution:**  $y^2 = (x + c)^3 \dots(i)$

Differentiating w.r.t.  $x$ , we get

$$2y \frac{dy}{dx} = 3(x + c)^2 \dots(ii)$$

Dividing (i) by (ii), we get

$$2y \frac{dy}{dx} = 3(x + c)^2 \dots(ii)$$

Dividing (i) by (ii), we get

$$\frac{y^2}{2y \left( \frac{dy}{dx} \right)} = \frac{(x + c)^3}{3(x + c)^2}$$

$$\therefore \frac{y}{2 \left( \frac{dy}{dx} \right)} = \frac{x + c}{3}$$

$$\therefore x + c = \frac{3y}{2 \left( \frac{dy}{dx} \right)}$$

$$\therefore c = -x + \frac{3y}{2 \left( \frac{dy}{dx} \right)}$$

Substituting the value of c in (i), we get

$$y^2 = \left[ X + \left( -x + \frac{3y}{2 \left( \frac{dy}{dx} \right)} \right) \right]^3$$

$$= \left( \frac{3y}{2 \left( \frac{dy}{dx} \right)} \right)^3$$

$$\therefore y^2 = \frac{27y^3}{8 \left( \frac{dy}{dx} \right)^3}$$

$$\therefore \left( \frac{dy}{dx} \right)^3 = \frac{27y}{8}$$

$$\therefore \frac{dy}{dx} = \frac{3}{2} 3\sqrt{y}$$

### Exercise 8.2 | Q 2 | Page 163

Find the differential equation by eliminating arbitrary constants from the relation

$$x^2 + y^2 = 2ax$$

**Solution:** Given relation is

$$x^2 + y^2 = 2ax \dots(i)$$

Differentiating w.r.t.  $x$ , we get

$$2x + 2y \frac{dy}{dx} = 2a \dots(ii)$$

Substituting (ii) in (i), we get

$$x^2 + y^2 = \left(2x + 2y \frac{dy}{dx}\right)x$$

$$\therefore x^2 + y^2 = 2x^2 + 2xy \frac{dy}{dx}$$

$$\therefore 2xy \frac{dy}{dx} = y^2 - x^2, \text{ which is the required differential equation.}$$

### Exercise 8.2 | Q 3 | Page 163

Form the differential equation by eliminating arbitrary constants from the relation

$$bx + ay = ab.$$

**Solution:** Given relation is

$$bx + ay = ab$$

Differentiating w.r.t.  $x$ , we get

$$b + a \frac{dy}{dx} = 0$$

Again, differentiating w.r.t.  $x$ , we get

$$a \frac{d^2y}{dx^2} = 0$$

$\therefore \frac{d^2y}{dx^2} = 0$ , which is the required differential equation.

### Exercise 8.2 | Q 4 | Page 163

Find the differential equation whose general solution is

$$x^3 + y^3 = 35ax.$$

**Solution:**

$$x^3 + y^3 = 35ax \dots(i)$$

Differentiating w.r.t.  $x$ , we get

$$3x^2 + 3y^2 \frac{dy}{dx} = 35a \dots(ii)$$

Substituting (ii) in (i), we get

$$x^3 + y^3 = \left( 3x^2 + 3y^2 \frac{dy}{dx} \right) x$$

$$\therefore x^3 + y^3 = 3x^3 + 3x \cdot y^2 \frac{dy}{dx}$$

$$\therefore 2x^3 - y^3 + 3xy^2 \frac{dy}{dx} = 0, \text{ which is the required differential equation.}$$

### Exercise 8.2 | Q 5 | Page 163

Form the differential equation from the relation

$$x^2 + 4y^2 = 4b^2$$

**Solution:**

Given relation is

$$x^2 + 4y^2 = 4b^2$$

Differentiating w.r.t.  $x$ , we get

$$2x + 4 \cdot 2y \frac{dy}{dx} = 0$$

$\therefore x + 4y \frac{dy}{dx} = 0$ , which is the required differential equation.

### EXERCISE 8.3 [PAGE 165]

#### Exercise 8.3 | Q 1.1 | Page 165

Solve the following differential equation.

$$\frac{dy}{dx} = x^2y + y$$

**Solution:**

$$\frac{dy}{dx} = x^2y + y = (x^2 + 1)y$$

$$\therefore \frac{1}{y} dy = (x^2 + 1) dx$$

Integrating on both sides, we get

$$\int \frac{1}{y} dy = \int (x^2 + 1) dx$$

$$\therefore \log |y| = \frac{x^3}{3} + x + c$$

#### Exercise 8.3 | Q 1.2 | Page 165

Solve the following differential equation.

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

**Solution:**

$$\frac{d\theta}{dt} = -k(\theta - \theta_0), \text{ k is constant.}$$

$$\therefore \frac{d\theta}{\theta - \theta_0} = -k dt$$

Integrating on both sides, we get

$$\int \frac{d\theta}{\theta - \theta_0} = -k \int dt$$

$$\therefore \log |\theta - \theta_0| = -kt + c$$

$$\therefore \theta - \theta_0 = e^{kt+c}$$

**Exercise 8.3 | Q 1.3 | Page 165**

Solve the following differential equation

$$(x^2 - yx^2) dy + (y^2 + xy^2) dx = 0$$

**Solution:**

$$(x^2 - yx^2) dy + (y^2 + xy^2) dx = 0$$

$$\therefore x^2(1 - y) dy = -y^2(1 + x) dx$$

$$\therefore \left( \frac{1 - y}{y^2} \right) dy = - \left( \frac{1 + x}{x^2} \right) dx$$

Integrating on both sides, we get

$$\int \left( \frac{1}{y^2} - \frac{1}{y} \right) dy = - \int \left( \frac{1}{x^2} + \frac{1}{x} \right) dx$$

$$\therefore -\frac{1}{y} - \log|y| = - \left( -\frac{1}{x} + \log|x| \right) + c$$

$$\therefore -\frac{1}{y} - \log|y| = \frac{1}{x} - \log|x| + c$$

$$\therefore \log|x| - \log|y| = \frac{1}{x} + \frac{1}{y} + c$$

**Exercise 8.3 | Q 1.4 | Page 165**

Solve the following differential equation.

$$y^3 - \frac{dy}{dx} = x \frac{dy}{dx}$$

**Solution:**

$$y^3 - \frac{dy}{dx} = x \frac{dy}{dx}$$

$$\therefore y^3 = (1+x) \frac{dy}{dx}$$

$$\therefore \frac{dx}{(1+x)} = \frac{dy}{y^3}$$

Integrating on both sides, we get

$$\int \frac{dx}{1+x} = \int \frac{dy}{y^3}$$

$$\therefore \log|1+x| = -\frac{1}{2y^2} + c$$

$$\therefore 2y^2 \log|1+x| = -1 + 2y^2c$$

**Exercise 8.3 | Q 2.1 | Page 165**

For each of the following differential equations find the particular solution.

$$(x - y^2 x) dx - (y + x^2 y) dy = 0, \text{ when } x = 2, y = 0$$

**Solution:**

$$(x - y^2 x)dx - (y + x^2 y) dy = 0, \text{ when } x = 2, y = 0$$

$$\therefore x(1 - y^2) dx = y(1 + x^2) dy$$

$$\therefore \frac{x dx}{1 + x^2} = \frac{y dy}{1 - y^2}$$

Integrating on both sides, we get

$$\int \frac{2x}{1 + x^2} dx = \int \frac{2y}{1 - y^2} dy$$

$$\therefore \int \frac{2x}{1 + x^2} dx = - \int \frac{-2y}{1 - y^2} dy$$

$$\therefore \log|1 + x^2| = -\log|1 - y^2| + \log|c|$$

$$\therefore \log|1 + x^2| = \log\left|\frac{c}{1 - y^2}\right|$$

$$\therefore (1 + x^2)(1 - y^2) = c \dots(i)$$

When  $x = 2, y = 0$ , we have

$$(1 + 4)(1 - 0) = c$$

$$\therefore c = 5$$

Substituting  $c = 5$  in (i), we get

$$(1 + x^2)(1 - y^2) = 5,$$

which is the required particular solution.

**Exercise 8.3 | Q 2.2 | Page 165**

For each of the following differential equations find the particular solution.

$$(x + 1) \frac{dy}{dx} - 1 = 2e^{-y},$$

when  $y = 0, x = 1$

**Solution:**

$$(x + 1) \frac{dy}{dx} - 1 = 2e^{-y}$$

$$\therefore (x + 1) \frac{dy}{dx} = \frac{2}{e^y} + 1$$

$$\therefore (x + 1) \frac{dy}{dx} = \frac{(2e^y)}{e^y}$$

$$\therefore \frac{e^y}{2 + e^y} dy = \frac{dx}{1 + x}$$

Integrating on both sides, we get

$$\int \frac{e^y}{2 + e^y} dy = \int \frac{dx}{1 + x}$$

$$\therefore \log |2 + e^y| = \log |1 + x| + \log |c|$$

$$\therefore \log |2 + e^y| = \log |c(1 + x)|$$

$$\therefore 2 + e^y = c(1 + x) \dots\dots(i)$$

When  $y = 0, x = 1$ , we have

$$2 + e^0 = c(1 + 1)$$

$$\therefore 2 + 1 = 2c$$

$$\therefore c = \frac{3}{2}$$

Substituting  $c = \frac{3}{2}$  in (i), we get

$$2 + e^y = \frac{3}{2}(1 + x)$$

$$\therefore 4 + 2e^y = 3 + 3x$$

$\therefore 3x - 2e^y - 1 = 0$ , which is the required particular solution

### Exercise 8.3 | Q 2.3 | Page 165

For each of the following differential equations find the particular solution.

$$y(1 + \log x) \frac{dx}{dy} - x \log x = 0,$$

when  $x=e$ ,  $y = e^2$ .

**Solution:**

$$y(1 + \log x) \frac{dx}{dy} - x \log x = 0,$$

when  $x=e$ ,  $y = e^2$ .

$$\therefore y(1 + \log x) \frac{dx}{dy} = x \log x$$

$$\therefore y(1 + \log x) dx = x \log x dy$$

$$\therefore \frac{1}{y} dy = \frac{1 + \log x}{x \log x} dx$$

Integrating on both sides, we get

$$\int \frac{1}{y} dy = \int \frac{1 + \log x}{x \log x} dx$$

$$\therefore \log |y| = \log |x \log x| + \log |c|$$

$$\therefore \log |y| = \log |cx \log x|$$

When  $x = e$ ,  $y = e^2$ , we have

$$\therefore y = cx \log x \dots(i)$$

$$e^2 = ce \log e$$

$$\therefore e^2 = ce$$

$$\therefore c = e$$

Substituting  $c = e$  in (i), we get

$y = ex \log x$ , which is the required particular solution.

### Exercise 8.3 | Q 2.4 | Page 165

For the following differential equation find the particular solution.

$$dy/dx = (4x + y + 1),$$

when  $y = 1$ ,  $x = 0$

**Solution:**

$$\frac{dy}{dx} = (4x + y + 1) \dots(i)$$

$$\text{Put } 4x + y + 1 = t \dots(ii)$$

Differentiating w.r.t.  $x$ , we get

$$4 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{dt}{dx} - 4 \dots \text{(iii)}$$

Substituting (ii) and (iii) in (i), we get

$$\frac{dt}{dx} - 4 = t$$

$$\therefore \frac{dt}{dx} = t + 4$$

$$\therefore \frac{dt}{t + 4} = dx$$

Integrating on both sides, we get

$$\int \frac{dt}{t + 4} = \int dx$$

$$\therefore \log |t + 4| = x + c$$

$$\therefore \log |(4x + y + 1) + 4| = x + c$$

$$\therefore \log |4x + y + 5| = x + c \dots \text{(iv)}$$

When  $y = 1$ ,  $x = 0$ , we have

$$\log |4(0) + 1 + 5| = 0 + c$$

$$\therefore c = \log |6|$$

Substituting  $c = \log |6|$  in (iv), we get

$$\log |4x + y + 5| = x + \log |6|$$

$$\therefore \log |4x + y + 5| - \log |6| = x$$

$$\therefore \log \left| \frac{4x + y + 5}{6} \right| = x,$$

which is the required particular solution.

## EXERCISE 8.4 [PAGE 167]

### Exercise 8.4 | Q 1.1 | Page 167

Solve the following differential equation.

$$x dx + 2y dy = 0$$

**Solution:**

$$x dx + 2y dy = 0$$

Integrating on both sides, we get

$$\int x dx + 2 \int y dy = 0$$

$$\therefore \frac{x^2}{2} + \frac{2y^2}{2} = c_1$$

$$\therefore x^2 + 2y^2 = c \quad \dots\dots[2c_1 = c]$$

### Exercise 8.4 | Q 1.2 | Page 167

Solve the following differential equation.

$$y^2 dx + (xy + x^2) dy = 0$$

**Solution:**

$$y^2 dx + (xy + x^2) dy = 0$$

$$\therefore (xy + x^2) dy = -y^2 dx$$

$$\therefore \frac{dy}{dx} = \frac{y^2}{xy + x^2} \quad \dots(i)$$

Put  $y = tx$  ... (ii)

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \dots (iii)$$

Substituting (ii) and (iii) in (i), we get

$$t + x \frac{dt}{dx} = \frac{-t^2 x^2}{x \cdot tx + x^2}$$

$$\therefore t + x \frac{dt}{dx} = \frac{-t^2 x^2}{tx^2 + x^2}$$

$$\therefore t + x \frac{dt}{dx} = \frac{-t^2 x^2}{x^2(t+1)}$$

$$\therefore x \frac{dt}{dx} = \frac{-t^2}{t+1} - t$$

$$\therefore x \frac{dt}{dx} = \frac{-t^2 - t^2 - t}{t+1}$$

$$\therefore x \frac{dt}{dx} = \frac{(-2t^2 + t)}{t+1}$$

$$\therefore \frac{t+1}{2t^2+t} dt = -\frac{1}{x} dx$$

Integrating on both sides, we get

$$\int \frac{t+1}{2t^2+t} dt = - \int \frac{1}{x} dx$$

$$\therefore \int \frac{2t+1-t}{t(2t+1)} dt = - \int \frac{1}{x} dx$$

$$\therefore \int \frac{1}{t} dt - \int \frac{1}{2t+1} dt = - \int \frac{1}{x} dx$$

$$\therefore \log |t| - \frac{1}{2} \log |2t+1| = - \log |x| + \log |c|$$

$$\therefore 2 \log |t| - \log |2t+1| = -2 \log |x| + 2 \log |c|$$

$$\therefore 2 \log \left| \frac{y}{x} \right| - \log \left| \frac{2y}{x} + 1 \right| = -2 \log |x| + 2 \log |c|$$

$$\therefore 2 \log |y| - 2 \log |x| - \log |2y+x| + \log |x|$$

$$= -2 \log |x| + 2 \log |c|$$

$$\therefore \log |y^2| + \log |x| = \log |c^2| + \log |2y+x|$$

$$\therefore \log |y^2 x| = \log |c^2 (x+2y)|$$

$$\therefore xy^2 = c^2 (x+2y)$$

#### Exercise 8.4 | Q 1.3 | Page 167

Solve the following differential equation.

$$x^2 y dx - (x^3 + y^3) dy = 0$$

**Solution:**

$$x^2 y dx - (x^3 + y^3) dy = 0$$

$$\therefore x^2 y dx = (x^3 + y^3) dy$$

$$\therefore \frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3} \dots(i)$$

$$\text{Put } y = tx \dots(ii)$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \dots(\text{iii})$$

Substituting (ii) and (iii) in (i), we get

$$\begin{aligned}t + x \frac{dt}{dx} &= \frac{x^2 \cdot tx}{x^3 + t^3 x^3} \\ \therefore t + x \frac{dt}{dx} &= \frac{x^3 \cdot t}{x^3(1 + t^3)} \\ \therefore x \frac{dt}{dx} &= \frac{t}{1 + t^3} - t \\ \therefore x \frac{dt}{dx} &= \frac{t - t - t^4}{1 + t^3} \\ \therefore x \frac{dt}{dx} &= -\frac{t^4}{1 + t^3} \\ \therefore \frac{1 + t^3}{t^4} dt &= -\frac{dx}{x}\end{aligned}$$

Integrating on both sides, we get

$$\begin{aligned}\int \frac{1 + t^3}{t^4} &= -\int \frac{1}{x} dx \\ \therefore \int \left( \frac{1}{t^4} + \frac{1}{t} \right) dt &= -\int \frac{1}{x} dx \\ \therefore \int t^{-4} dt + \int \frac{1}{t} dt &= -\int \frac{1}{x} dx \\ \therefore \frac{t^{-3}}{-3} + \log|t| &= -\log|x| + \log|c_1| \\ \therefore -\frac{1}{-3} t^3 + \log|t| &= -\log|x| + \log|c_1|\end{aligned}$$

$$\therefore -\frac{1}{3} \cdot \frac{1}{\left(\frac{y}{x}\right)^3} + \log\left|\frac{y}{x}\right| = -\log|x| + \log|c_1|$$

$$\therefore -\frac{x^3}{3y^3} + \log|y| - \log|x| = -\log|x| + \log|c_1|$$

$$\therefore \log|y| + \log|c| = \frac{x^3}{3y^3} \dots\dots\dots [-\log|c_1| = \log|c|]$$

$$\therefore \log|yc| = \frac{x^3}{3y^3}$$

**Exercise 8.4 | Q 1.4 | Page 167**

**Solve the following differential equation.**

$$\frac{dy}{dx} + \frac{x - 2y}{2x - y} = 0$$

**Solution:**

$$\frac{dy}{dx} + \frac{x - 2y}{2x - y} = 0 \dots(i)$$

Put  $y = tx \dots(ii)$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \dots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$t + x \frac{dt}{dx} + \frac{x - 2tx}{2x - tx} = 0$$

$$\therefore x \frac{dt}{dx} + t + \frac{1 - 2t}{2} - t = 0$$

$$\therefore x \frac{dt}{dx} + \frac{2t - t^2 + 1 - 2t}{2} - t = 0$$

$$\therefore x \frac{dt}{dx} + \frac{1 - t^2}{2 - t} = 0$$

$$\therefore x \frac{dt}{dx} = -\frac{1 - t^2}{2 - t}$$

$$\therefore = \frac{2 - t}{1 - t^2} dt = \frac{dx}{x}$$

$$\therefore \frac{2 - t}{t^2 - 1} dt = \frac{dx}{x}$$

Integrating on both sides, we get

$$\int \frac{2 - t}{t^2 - 1} dt = \int \frac{dx}{x}$$

$$\therefore \int \frac{2 - t}{(t + 1)(t - 1)} dt = \int \frac{dx}{x}$$

$$\text{Let } 2 - \frac{t}{(t + 1)(t - 1)} = \frac{A}{t + 1} + \frac{B}{t - 1}$$

$$\therefore 2 - t = A(t - 1) + B(t + 1)$$

Putting  $t = 1$ , we get

$$\therefore 2 - 1 = A(1 - 1) + B(1 + 1)$$

$$\therefore B = \frac{1}{2}$$

Putting  $t = -1$ , we get

$$2 - (-1) = A(-1 - 1) + B(-1 + 1)$$

$$\therefore A = \frac{-3}{2}$$

$$\begin{aligned}
\therefore \int \frac{-\frac{3}{2}}{t+1} dt + \int \frac{\frac{1}{2}}{t-1} dt &= \int \frac{dx}{x} \\
\therefore \frac{-3}{2} \int \frac{1}{t+1} dt + \frac{1}{2} \int \frac{1}{t-1} dt &= \int \frac{dx}{x} \\
\therefore \frac{-3}{2} \log|t+1| + \frac{1}{2} \log|t-1| &= \log|x| + \log|c_1| \\
\therefore -3 \log \left| \frac{y+x}{x} \right| + \log \left| \frac{y-x}{x} \right| &= 2 \log|x| + 2 \log|c_1| \\
\therefore -3 \log|y+x| + 3 \log|x| + \log|y-x| - \log|x| & \\
= 2 \log|x| + 2 \log|c_1| & \\
\therefore \log|y-x| = 3 \log|y+x| + 2 \log|c_1| & \\
\therefore \log|y-x| = \log|(y+x)^3| + \log|c_1^2| & \\
\therefore \log|y-x| = \log|c_1^2 (x+y)^3| & \\
\therefore (y-x) = c(x+y)^3 \dots |c_1^2 c| &
\end{aligned}$$

## NOTES

Answer given in the textbook is  $\log \left| \frac{x+y}{x-y} \right| - \frac{1}{2} \log|x^2 - y^2| + 2 \log x = \log c$ .

However, as per our calculation it is  $(y-x) = c(x+y)^3$ .

## Exercise 8.4 | Q 1.5 | Page 167

Solve the following differential equation.

$$(x^2 - y^2) dx + 2xy dy = 0$$

**Solution:**

$$(x^2 - y^2) dx + 2xy dy = 0$$

$$\therefore 2xy dy = (y^2 - x^2) dx$$

$$\therefore \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \dots\dots(i)$$

$$\text{Put } y = tx \dots(ii)$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \dots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$t + x \frac{dt}{dx} = \frac{t^2 x^2 - x^2}{2tx^2}$$

$$\therefore x \frac{dt}{dx} = \frac{t^2 - 1}{2t} - t = \frac{-(1 + t^2)}{2t}$$

$$\therefore 2 \frac{t}{1 + t^2} dt = -\frac{dx}{x}$$

Integrating on both sides, we get

$$\int 2 \frac{t}{1 + t^2} dt = - \int \frac{dx}{x}$$

$$\therefore \log |1 + t^2| = -\log |x| + \log |c|$$

$$\therefore \log \left| 1 + \frac{y^2}{x^2} \right| = \log \left| \frac{c}{x} \right|$$

$$\therefore \frac{x^2 + y^2}{x^2} = \frac{c}{x}$$

$$\therefore x^2 + y^2 = cx$$

**Solve the following differential equation.**

$$xy \frac{dy}{dx} = x^2 + 2y^2$$

**Solution:**

$$xy \frac{dy}{dx} = x^2 + 2y^2$$

$$\therefore \frac{dy}{dx} = \frac{x^2 + 2y^2}{xy} \dots (i)$$

Put  $y = tx$  ... (ii)

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \dots (iii)$$

Substituting (ii) and (iii) in (i), we get

$$t + x \frac{dt}{dx} = \frac{x^2 + 2t^2x^2}{x(tx)}$$

$$\therefore t + x \frac{dt}{dx} = \frac{x^2(1 + 2t^2)}{x^2t}$$

$$\therefore x \frac{dt}{dx} = \frac{1 + 2t^2}{t} - t = \frac{1 + t^2}{t}$$

$$\therefore \frac{t}{1 + t^2} dt = \frac{1}{x} dx$$

Integrating on both sides, we get

$$\frac{1}{2} \int \frac{2t}{1 + t^2} dt = \int \frac{dx}{x}$$

$$\begin{aligned} \therefore \log |1 + t^2| &= 2 \log |x| + 2 \log |c_1| \\ &= \log |x^2| + \log |c| \quad \dots [\log c_1^2 = \log c] \\ \therefore \log |1 + t^2| &= \log |cx^2| \\ \therefore 1 + t^2 &= cx^2 \\ \therefore 1 + \frac{y^2}{x^2} &= cx^2 \\ \therefore x^2 + y^2 &= cx^4 \end{aligned}$$

#### Exercise 8.4 | Q 1.7 | Page 167

Solve the following differential equation.

$$x^2 \frac{dy}{dx} = x^2 + xy - y^2$$

**Solution:**

$$\begin{aligned} x^2 \frac{dy}{dx} &= x^2 + xy - y^2 \\ \therefore \frac{dy}{dx} &= \frac{x^2 + xy - y^2}{x^2} \quad \dots(i) \end{aligned}$$

Put  $y = tx$  ... (ii)

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \quad \dots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$t + x \frac{dt}{dx} = \frac{x^2 + x(tx) - (tx)^2}{x^2}$$

$$\therefore t + x \frac{dt}{dx} = \frac{x^2 + tx^2 - t^2x^2}{x^2}$$

$$\therefore t + x \frac{dt}{dx} = 1 + t - t^2$$

$$\therefore x \frac{dt}{dx} = 1 + t - t^2$$

$$\therefore x \frac{dt}{dx} = 1 - t^2$$

$$\therefore \frac{dt}{1^2 - t^2} = \frac{dx}{x}$$

Integrating on both sides, we get

$$\int \frac{dt}{(1)^2 - (t)^2} = \int \frac{dx}{x}$$

$$\therefore \frac{1}{2(1)} \log \left| \frac{1+t}{1-t} \right| = \log |x| + \log |c_1| \quad \dots \left[ Q \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c \right]$$

$$\therefore \log \left| \frac{1+t}{1-t} \right| = \log |x| + \log |c_1|$$

$$\therefore \log \left| \frac{1+t}{1-t} \right| = \log |c_1^2 x^2|$$

$$\therefore \frac{1 + \left(\frac{y}{x}\right)}{1 - \left(\frac{y}{x}\right)} = c_1^2 x^2$$

$$\therefore \frac{x+y}{x-y} = cx^2 \dots [c_1^2 = c]$$

### EXERCISE 8.5 [PAGE 168]

#### Exercise 8.5 | Q 1.1 | Page 168

Solve the following differential equation.

$$\frac{dy}{dx} + y = e^{-x}$$

**Solution:**

$$\frac{dy}{dx} + y = e^{-x}$$

The given equation is of the form

$$\frac{dy}{dx} + py = Q$$

where,  $P = 1$  and  $Q = e^{-x}$

$$\therefore \text{I.F.} = e^{\int p dx} = e^{\int 1 \cdot dx} = e^x$$

$\therefore$  Solution of the given equation is

$$y(\text{I.F.}) = \int Q(\text{I.F.}) dx + c$$

$$\therefore ye^x = \int e^{-x} e^x dx + c$$

$$\therefore ye^x = \int 1 dx + c$$

$$\therefore ye^x = x + c$$

**Exercise 8.5 | Q 1.2 | Page 168**

Solve the following differential equation.

$$\frac{dy}{dx} + y = 3$$

**Solution:**

$$\frac{dy}{dx} + y = 3$$

The given equation is of the form

$$\frac{dy}{dx} + py = Q$$

where,  $P = 1$  and  $Q = 3$

$$\therefore \text{I.F.} = e^{\int p dx} = e^{\int 1 dx} = e^x$$

$\therefore$  Solution of the given equation is

$$y(\text{I.F.}) = \int Q(\text{I.F.}) dx + c$$

$$\therefore ye^x = \int 3e^x dx + c$$

$$\therefore ye^x = 3e^x + c$$

**Exercise 8.5 | Q 1.3 | Page 168**

Solve the following differential equation.

$$x \frac{dy}{dx} + 2y = x^2 \log x$$

**Solution:**

$$x \frac{dy}{dx} + 2y = x^2 \log x$$

Dividing throughout by  $x$ , we get

$$\frac{dy}{dx} + \frac{2}{x}y = x \log x$$

The given equation is of the form

$$\frac{dy}{dx} + py = Q$$

where,  $P = \frac{2}{x}$  and  $Q = x \log x$

$$\therefore \text{I.F.} = e^{\int p dx} = e^2 \int \frac{1}{x} dx = e^2 \log|x| = e \log|x^2| = x^2$$

$\therefore$  Solution of the given equation is

$$y(\text{I.F.}) = \int Q(\text{I.F.}) dx + c$$

$$\therefore yx^2 = \int (x \log x)x^2 dx + c$$

$$= \int x^3 \log x dx + c$$

$$= \log x \int x^3 dx - \int \left( \frac{d}{dx} \log x \int x^3 dx \right) dx + c$$

$$= \frac{x^4}{4} \log x - \int \frac{1}{x} \left( \frac{x^4}{4} \right) dx + c$$

$$= \frac{x^4}{4} \log x - \frac{1}{4} \int x^3 dx + c$$

$$\therefore yx^2 = \frac{x^4}{4} \log x - \frac{X^4}{16} + c$$

**Exercise 8.5 | Q 1.4 | Page 168**

Solve the following differential equation.

$$(x + y) \frac{dy}{dx} = 1$$

**Solution:**

$$(x + y) \frac{dy}{dx} = 1$$

$$\therefore \frac{dy}{dx} = \frac{1}{x + y}$$

$$\therefore \frac{dy}{dx} = (x + y)$$

$$\therefore \frac{dx}{dy} - x = y$$

The given equation is of the form  $\frac{dx}{dy} + Px = Q$

where,  $P = -1$  and  $Q = y$

$$\therefore I.F. = e^{\int P dy} = e^{\int -1 dy} = e^{-y}$$

$\therefore$  Solution of the given equation is

$$x(I.F.) = \int Q(I.F.) dy + c$$

$$\therefore xe^{-y} = \int ye^{-y} dy + c$$

$$\therefore xe^{-y} = y \int e^{-y} dy - \int \left[ \frac{d}{dy}(y) \int e^{-y} dy \right] dy + c$$

$$\therefore xe^{-y} = -y(e^{-y}) - \int 1 \times (-e^{-y}) dy + c$$

$$\therefore xe^{-y} = -ye^{-y} - e^{-y} + c$$

$$\therefore x = -y - 1 + ce^y$$

$$\therefore x + y + 1 = c e^y$$

### Exercise 8.5 | Q 1.5 | Page 168

Solve the following differential equation.

$$y dx + (x - y^2) dy = 0$$

**Solution:**

$$y dx + (x - y^2) dy = 0$$

$$\therefore y dx = (y^2 - x) dy$$

$$\therefore \frac{dx}{dy} = \frac{y^2 - x}{y} = y - \frac{x}{y}$$

$$\therefore \frac{dx}{dy} + \frac{x}{y} = y$$

The given equation is of the form

$$\frac{dx}{dy} + Px = Q$$

where,  $P = \frac{1}{y}$  and  $Q = y$

$$\therefore \text{I.F.} = e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log|y|} = y$$

$\therefore$  Solution of the given equation is

$$x(\text{I.F.}) = \int Q(\text{I.F.}) dy + c_1$$

$$\therefore xy = \int y(y) dy = \frac{y^3}{3} + c_1$$

$$\therefore 3xy = y^3 + c \quad \dots [3c_1 = c]$$

Exercise 8.5 | Q 1.6 | Page 168

Solve the following differential equation.

$$\frac{dy}{dx} + 2xy = x$$

**Solution:**

$$\frac{dy}{dx} + 2xy = x$$

The given equation is of the form

$$\frac{dy}{dx} + py = Q$$

where,  $P = 2x$  and  $Q = x$

$$\therefore I.F. = e^{\int P dx} = e^{\int 2x dx} = e^{x^2}$$

$\therefore$  Solution of the given equation is

$$y(I.F.) = \int Q(I.F.) dx + c$$

$$\therefore ye^{x^2} = \int xe^{x^2} dx + c$$

In R. H. S., put  $x^2 = t$

Differentiating w.r.t.  $x$ , we get

$$2x dx = dt$$

$$\therefore ye^{x^2} = \int e^t \frac{dt}{2} + c$$

$$= \frac{1}{2} \int e^t dt + c$$

$$= \frac{e^t}{2} + c$$

$$\therefore ye^{x^2} = \frac{1}{2}e^{x^2} + c$$

**Exercise 8.5 | Q 1.7 | Page 168**

Solve the following differential equation.

$$(x + a) \frac{dy}{dx} = -y + a$$

**Solution:**

$$(x + a) \frac{dy}{dx} = -y + a$$

$$\therefore \frac{dy}{dx} + \frac{y}{(x + a)} = \frac{a}{(x + a)}$$

The given equation is of the form

$$\frac{dy}{dx} + py = Q$$

$$\text{where, } P = \frac{1}{(x + a)} \text{ and } Q = \frac{a}{(x + a)}$$

$$\therefore \text{I.F.} = e^{\int p dx} = e^{\int \frac{1}{x+a} dx}$$

$$= e^{\log|x+a|} = (x + a)$$

$\therefore$  Solution of the given equation is

$$y(\text{I.F.}) = \int Q(\text{I.F.}) dx + c$$

$$\therefore y(x + a) = \int \frac{a}{(x + a)} (x + a) dx + c$$

$$\therefore y(x + a) = a \int 1 dx + c$$

$$\therefore y(x + a) = ax + c$$

### Exercise 8.5 | Q 1.8 | Page 168

Solve the following differential equation.

$$dr + (2r)d\theta = 8d\theta$$

**Solution:**

$$dr + (2r)d\theta = 8d\theta$$

$$\frac{dr}{d\theta} + 2r = 8$$

The given equation is of the form

$$\frac{dr}{d\theta} + Pr = Q$$

where,  $P = 2$  and  $Q = 8$

$$\text{I.F.} = e^{\int P d\theta} = e^{\int 2 d\theta} = e^{2\theta}$$

Solution of the given equation is

$$r(\text{I.F.}) = \int Q(\text{I.F.})d\theta + c$$

$$re^{2\theta} = \int 8e^{2\theta} d\theta + c$$

$$re^{2\theta} = 8 \int e^{2\theta} d\theta + c$$

$$re^{2\theta} = 8 \frac{e^{2\theta}}{2} + c$$

$$re^{2\theta} = 4e^{2\theta} + c$$

### EXERCISE 8.6 [PAGE 170]

#### Exercise 8.6 | Q 1 | Page 170

In a certain culture of bacteria, the rate of increase is proportional to the number present. If it is found that the number doubles in 4 hours, find the number of times the bacteria are increased in 12 hours.

**Solution:**

Let  $x$  be the number of bacteria in the culture at time  $t$ .

Then the rate of increase is  $\frac{dx}{dt}$  which is proportional to  $x$ .

$$\therefore \frac{dx}{dt} \propto x$$

$$\therefore \frac{dx}{dt} = kx, \text{ where } k \text{ is a constant}$$

$$\therefore \frac{dx}{x} = k dt$$

On integrating, we get

$$\int \frac{dx}{x} = k \int 1 dt + c$$

$$\therefore \log x = kt + c$$

Initially, i.e. when  $t = 0$ , let  $x = x_0$

$$\therefore \log x_0 = k \times 0 + c$$

$$\therefore c = \log x_0$$

$$\therefore \log x = kt + \log x_0$$

$$\therefore \log x - \log x_0 = kt$$

$$\therefore \log\left(\frac{x}{x_0}\right) = kt \quad \dots(1)$$

Since the number doubles in 4 hours, i.e. when  $t = 4$ ,

$$x = 2x_0$$

$$\therefore \log\left(\frac{2x_0}{x_0}\right) = 4k$$

$$\therefore k = \frac{1}{4} \log 2$$

$$\therefore (1) \text{ becomes, } \log\left(\frac{x}{x_0}\right) = \frac{t}{4} \log 2$$

When  $t = 12$ , we get

$$\log\left(\frac{x}{x_0}\right) = \frac{12}{4} \log 2 = 3 \log 2$$

$$\therefore \log\left(\frac{x}{x_0}\right) = \log 8$$

$$\therefore \frac{x}{x_0} = 8$$

$$\therefore x = 8x_0$$

$\therefore$  number of bacteria will be 8 times the original number in 12 hours.

The population of a town increases at a rate proportional to the population at that time. If the population increases from 40 thousands to 60 thousands in 40 years, what will be the population in another 20 years?

$$\left( \text{Given: } \sqrt{\frac{3}{2}} = 1.2247 \right)$$

**Solution:**

Let 'x' be the population at time 't'

$$\therefore \frac{dx}{dt} \propto x$$

$$\therefore \frac{dx}{dt} = kx, \text{ where } k \text{ is the constant of proportionality.}$$

$$\therefore \frac{dx}{dt} = k dt$$

Integrating on both sides, we get

$$\int \frac{dx}{x} = k \int 1 dt$$

$$\therefore \log x = kt + c \dots(i)$$

When  $t = 0$ ,  $x = 40000$

$$\therefore \log (40000) = k(0) + c$$

$$\therefore c = \log (40000)$$

$$\therefore \log x = kt + \log (40000) \dots(ii) \text{ [From (i)]}$$

When,  $t = 40$ ,  $x = 60000$

$$\therefore \log (60000) = 40k + \log (40000)$$

$$\therefore 40k = \log (60000) - \log (40000)$$

$$\therefore 40k = \log \left( \frac{60000}{40000} \right)$$

$$\therefore k = \frac{1}{40} \log \left( \frac{3}{2} \right) \dots(\text{iii})$$

When  $t = 60$ , we get

$$\log x = k(60) + \log(40000) \dots[\text{From (ii)}]$$

$$\therefore \log x = \left[ \frac{1}{40} \log \left( \frac{3}{2} \right) \right] (60) + \log(40000) \dots[\text{From (iii)}]$$

$$\therefore \log x = \frac{3}{2} \log \left( \frac{3}{2} \right) + \log(40000)$$

$$= \frac{3}{2} \log \left( \frac{3}{2} \right)^{\frac{3}{2}} + \log(40000)$$

$$= \frac{3}{2} \log \left( \sqrt{\frac{3}{2}} \right)^3 + \log(40000)$$

$$= \log \left( \frac{3}{2} \sqrt{\frac{3}{2}} \times \log 40000 \right)$$

$$\therefore \log x = \log \left( \frac{3 \times 1.2247}{2} \times 40000 \right)$$

$$\therefore x = 73482$$

$\therefore$  Population in another 20 years, i.e., in 60 years will be 73482.

### Exercise 8.6 | Q 3 | Page 170

The rate of growth of bacteria is proportional to the number present. If initially, there were 1000 bacteria and the number doubles in 1 hour, find the number of bacteria after  $5/2$  hours.

(Given:  $\sqrt{2} = 1.414$ )

**Solution:**

Let 'x' be the number of bacteria present at time 't'.

$$\therefore \frac{dx}{dt} \propto x$$

$$\therefore \frac{dx}{dt} = kx, \text{ where } k \text{ is the constant of proportionality.}$$

$$\therefore \frac{dx}{x} = k dt$$

Integrating on both sides, we get

$$\int \frac{dx}{x} = k \int dt$$

$$\therefore \log x = kt + c \dots(i)$$

When  $t = 0$ ,  $x = 1000$

$$\therefore \log (1000) = k(0) + c$$

$$\therefore c = \log (1000)$$

$$\therefore \log x = kt + \log (1000) \dots(ii)[\text{From (i)}]$$

When  $t = 1$ ,  $x = 2000$

$$\therefore \log (2000) = k(1) + \log (1000)$$

$$\therefore \log (2000) - \log (1000) = k$$

$$\therefore k = \log \left( \frac{2000}{1000} \right) = \log 2 \dots(iii)$$

When  $t = \frac{5}{2}$ , we get

$$\log x = \frac{5}{2}k + \log(1000) \dots[\text{From (ii)}]$$

$$\therefore \log x = \left(\frac{5}{2}\right) \log 2 + \log(1000) \dots[\text{From (iii)}]$$

$$= \log \left(2^{\frac{5}{2}}\right) + \log(1000)$$

$$= \log(4\sqrt{2}) + \log(1000)$$

$$= \log(4000\sqrt{2})$$

$$= \log(4000 \times 1.414)$$

$$\therefore \log x = \log(5656)$$

$$\therefore x = 5656$$

Thus, there will be 5656 bacteria after  $\frac{5}{2}$  hours.

### Exercise 8.6 | Q 4 | Page 170

Find the population of a city at any time  $t$ , given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years, the population increased from 30,000 to 40,000.

**Solution:** Let  $P$  be the population of the city at time  $t$ .

Then  $\frac{dP}{dt}$ , the rate of increase of population, is proportional to  $P$ .

$$\therefore \frac{dP}{dt} \propto P$$

$$\therefore \frac{dP}{dt} = kP, \text{ where } k \text{ is a constant.}$$

$$\therefore \frac{dP}{P} = k dt$$

On integrating, we get

$$\int \frac{1}{P} dP = k \int dt + c$$

$$\therefore \log P = kt + c$$

Initially, i.e. when  $t = 0$ ,  $P = 30000$

$$\therefore \log 30000 = k \times 0 + c \quad \therefore c = \log 30000$$

$$\therefore \log P = kt + \log 30000$$

$$\therefore \log P - \log 30000 = kt$$

$$\therefore \log \left( \frac{P}{30000} \right) = kt \quad \dots(1)$$

Now, when  $t = 40$ ,  $P = 40000$

$$\therefore \log \left( \frac{40000}{30000} \right) = k \times 40$$

$$\therefore k = \frac{1}{40} \log \left( \frac{4}{3} \right)$$

$$\therefore (1) \text{ becomes, } \log \left( \frac{P}{30000} \right) = \frac{t}{40} \log \left( \frac{4}{3} \right) = \log \left( \frac{4}{3} \right)^{\frac{t}{40}}$$

$$\therefore \frac{P}{30000} = \left( \frac{4}{3} \right)^{\frac{t}{40}}$$

$$\therefore P = 30000 \left( \frac{4}{3} \right)^{\frac{t}{40}}$$

∴ the population of the city at time  $t = 30000 \left(\frac{4}{3}\right)^{\frac{t}{40}}$

### Exercise 8.6 | Q 5 | Page 170

The rate of depreciation  $dV/dt$  of a machine is inversely proportional to the square of  $t + 1$ , where  $V$  is the value of the machine  $t$  years after it was purchased. The initial value of the machine was ₹ 8,00,000 and its value decreased ₹1,00,000 in the first year. Find its value after 6 years.

**Solution:** According to the given condition,

$$\frac{dV}{dt} \propto \frac{1}{(t+1)^2}$$

$$\therefore \frac{dV}{dt} = \frac{-k}{(t+1)^2} \dots [\text{Negative sign indicates disintegration}]$$

$$\therefore dV = \frac{-kdt}{(t+1)^2}$$

Integrating on both sides, we get

$$\int dV = -k \int \frac{dt}{(t+1)^2}$$

$$\therefore V = \frac{k}{t+1} + c$$

when  $t = 0, V = 8,00,000$

$$\therefore 8,00,000 = \frac{k}{(0+1)} + c$$

$$\therefore 8,00,000 = k + c \dots (i)$$

when  $t = 1, V = 7,00,000$

$$\therefore 7,00,000 = \frac{k}{(1+1)} + c$$

$$\therefore 7,00,000 = \frac{k}{2} + c \dots(ii)$$

From (i) – (ii), we get

$$1,00,000 = \frac{k}{2}$$

$$\therefore k = 2,00,000 \dots(iii)$$

Substituting (iii) in (i), we get

$$c = 6,00,000 \dots(iv)$$

when  $t = 6$ , we get

$$\begin{aligned} V &= \frac{k}{(6+1)} + c \\ &= \frac{2,00,000}{7} + 6,00,000 \end{aligned}$$

$$= 6,28,571.4286$$

$$\approx 6,28,571$$

$\therefore$  Value of the machine after 6 years is ₹ 6,28,571.

### MISCELLANEOUS EXERCISE 8 [PAGES 171 - 173]

#### Miscellaneous Exercise 8 | Q 1.01 | Page 171

**Choose the correct alternative.**

The order and degree of  $\left(\frac{dy}{dx}\right)^3 - \frac{d^3y}{dx^3} + ye^x = 0$  are respectively.

1. 3, 1
2. 1, 3

3. 3, 3

4. 1, 1

**Solution:**

The order and degree of  $\left(\frac{dy}{dx}\right)^3 - \frac{d^3y}{dx^3} + ye^x = 0$  are respectively - **3, 1**

**Miscellaneous Exercise 8 | Q 1.02 | Page 171**

**Choose the correct alternative.**

The order and degree of  $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{2}{3}} = 8\frac{d^3y}{dx^3}$  are respectively.

1. 3, 1

2. 1, 3

**3. 3, 3**

4. 1, 1

**Solution:**

The order and degree of  $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{2}{3}} = 8\frac{d^3y}{dx^3}$  are respectively - **3, 3**

**Explanation**

$$\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{2}{3}} = 8\frac{d^3y}{dx^3}$$

Taking cube on both sides, we get

$$\left[1 + \left(\frac{dy}{dx}\right)^3\right]^2 = 8^3\left(\frac{d^3y}{dx^3}\right)^3$$

∴ By definition of order and degree,

Order : 3; Degree : 3

**Choose the correct alternative.**

The differential equation of  $y = k_1 + \frac{k_2}{x}$  is

Options

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$$

$$x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$$

**Solution:**

The differential equation of  $y = k_1 + \frac{k_2}{x}$  is  $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

**Explanation**

$$y = k_1 + \frac{k_2}{x}$$

$$\therefore xy = xk_1 + k_2$$

Differentiating w.r.t.  $x$ , we get

$$y + x\frac{dy}{dx} = k_1$$

Again, differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} + \frac{dy}{dx} + x \frac{d^2y}{dx^2} = 0$$

$$\therefore x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 0$$

Miscellaneous Exercise 8 | Q 1.04 | Page 171

**Choose the correct alternative.**

The differential equation of  $y = k_1e^x + k_2e^{-x}$  is

Options

$$\frac{d^2y}{dx^2} - y = 0$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} + y \frac{dy}{dx} = 0^*$$

$$\frac{d^2y}{dx^2} + y = 0$$

**Solution:**

The differential equation of  $y = k_1e^x + k_2e^{-x}$  is  $\frac{d^2y}{dx^2} - y = 0$

**Explanation**

$$y = k_1e^x + k_2e^{-x}$$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = k_1 e^x - k_2 e^{-x}$$

Again, differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = k_1 e^x + k_2 e^{-x}$$

$$\therefore \frac{d^2y}{dx^2} = y$$

$$\therefore \frac{d^2y}{dx^2} - y = 0$$

### Miscellaneous Exercise 8 | Q 1.05 | Page 171

**Choose the correct alternative.**

The solution of  $dy/dx = 1$  is

1.  $x + y = c$
2.  $xy = c$
3.  $x^2 + y^2 = c$
4.  $y - x = c$

**Solution:**

The solution of  $\frac{dy}{dx} = 1$  is  $y - x = c$

### **Explanation**

$$\frac{dy}{dx} = 1$$

$$\therefore dy = dx$$

Integrating on both sides, we get

$$\int 1dy = \int 1dx$$

$$\therefore y = x + c$$

$$\therefore y - x = c$$

Miscellaneous Exercise 8 | Q 1.06 | Page 171

**Choose the correct alternative.**

The solution of  $\frac{dy}{dx} + \frac{x^2}{y^2} = 0$  is

1.  $x^3 + y^3 = 7$
2.  $x^2 + y^2 = c$
3.  $x^3 + y^3 = c$
4.  $x + y = c$

**Solution:**

The solution of  $\frac{dy}{dx} + \frac{x^2}{y^2} = 0$  is  $x^3 + y^3 = c$

Miscellaneous Exercise 8 | Q 1.07 | Page 172

**Choose the correct alternative.**

The solution of  $x \frac{dy}{dx} = y \log y$  is

1.  $y = ae^x$
2.  $y = be^{2x}$
3.  $y = be^{-2x}$
4.  $y = e^{ax}$

**Solution:**

The solution of  $x \frac{dy}{dx} = y \log y$  is  $y = e^{ax}$

$$x \frac{dy}{dx} = y \log y$$

$$\therefore \frac{dy}{y \log y} = \frac{dx}{x}$$

Integrating on both sides, we get

$$\int \frac{dy}{y \log y} = \int \frac{1}{x} dx$$

$$\therefore \log \log(y) = \log x + \log a$$

$$\therefore \log \log(y) = \log xa$$

$$\therefore \log(y) = ax$$

$$\therefore y = e^{ax}$$

### Miscellaneous Exercise 8 | Q 1.08 | Page 172

**Choose the correct alternative.**

Bacteria increases at the rate proportional to the number present. If the original number  $M$  doubles in 3 hours, then the number of bacteria will be  $4M$  in

1. 4 hours
2. 6 hours
3. 8 hours
4. 10 hours

**Solution:** Bacteria increases at the rate proportional to the number present. If the original number  $M$  doubles in 3 hours, then the number of bacteria will be  $4M$  in **6 hours**

### Miscellaneous Exercise 8 | Q 1.09 | Page 172

**Choose the correct alternative.**

The integrating factor of  $\frac{dy}{dx} + y = e^{-x}$

1.  $x$

2.  $-x$
3.  $e$
4.  $e^{-x}$

**Solution:**

The integrating factor of  $\frac{dy}{dx} + y = e^{-x} - e$

### Explanation

$$\frac{dy}{dx} + y = e^{-x}$$

The given equation is of the form  $\frac{dy}{dx} + py = Q$

where,  $P = 1$  and  $Q = e^{-x}$

$$\therefore \text{I.F.} = e^{\int p dx} = e^{\int 1 dx} = e^x$$

Miscellaneous Exercise 8 | Q 1.1 | Page 172

**Choose the correct alternative.**

The integrating factor of  $\frac{dy}{dx} - y = e^x$  is  $e^x$ , then its solution is

1.  $ye^{-x} = x + c$
2.  $ye^x = x + c$
3.  $ye^x = 2x + c$
4.  $ye^{-x} = 2x + c$

**Solution:**

The integrating factor of  $\frac{dy}{dx} - y = e^x$  is  $e^x$ , then its solution is  $ye^{-x} = x + c$

## Explanation

$$\frac{dy}{dx} - y = e^x$$

Here, I.F. =  $e^{-x}$ , Q =  $e^x$

∴ Solution of the given equation is

$$y(I.F.) = \int Q(I.F.)dx + c$$

$$\therefore ye^{-x} = \int e^x e^{-x} dx + c$$

$$\therefore ye^{-x} = \int 1 dx + c$$

$$\therefore ye^{-x} = x + c$$

### Miscellaneous Exercise 8 | Q 2.1 | Page 172

**Fill in the blank:**

The order of highest derivative occurring in the differential equation is called \_\_\_\_\_ of the differential equation.

**Solution:** The order of highest derivative occurring in the differential equation is called order of the differential equation.

### Miscellaneous Exercise 8 | Q 2.2 | Page 172

**Fill in the blank:**

The power of the highest ordered derivative when all the derivatives are made free from negative and / or fractional indices if any is called \_\_\_\_\_ of the differential equation.

**Solution:** The power of the highest ordered derivative when all the derivatives are made free from negative and/or fractional indices if any is called degree of the differential equation.

### Miscellaneous Exercise 8 | Q 2.3 | Page 172

**Fill in the blank:**

A solution of differential equation which can be obtained from the general solution by giving particular values to the arbitrary constants is called \_\_\_\_\_ solution.

**Solution:** A solution of differential equation which can be obtained from the general solution by giving particular values to the arbitrary constants is called **particular** solution.

**Miscellaneous Exercise 8 | Q 2.4 | Page 172**

**Fill in the blank:**

Order and degree of a differential equation are always \_\_\_\_\_ integers.

**Solution:** Order and degree of a differential equation are always **positive** integers.

**Miscellaneous Exercise 8 | Q 2.5 | Page 172**

**Fill in the blank:**

The integrating factor of the differential equation  $dy/dx - y = x$  is \_\_\_\_\_

**Solution:**

The integrating factor of the differential equation  $\frac{dy}{dx} - y = x$  is  **$e^{-x}$**

**Miscellaneous Exercise 8 | Q 2.6 | Page 172**

**Fill in the blank:**

The differential equation by eliminating arbitrary constants from  $bx + ay = ab$  is \_\_\_\_\_.

**Solution:**

The differential equation by eliminating arbitrary constants from  $bx + ay = ab$  is  $\frac{d^2y}{dx^2} = 0$

### **Explanation**

$$bx + ay = ab$$

Differentiating w.r.t.  $x$ , we get

$$b + a \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = \frac{-b}{a}$$

Again, differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = 0$$

### **Miscellaneous Exercise 8 | Q 3.1 | Page 172**

**State whether the following is True or False:**

The integrating factor of the differential equation  $\frac{dy}{dx} - y = x$  is  $e^{-x}$

1. True
2. False

**Solution:**

The integrating factor of the differential equation  $\frac{dy}{dx} - y = x$  is  $e^{-x}$  - **True**

### **Miscellaneous Exercise 8 | Q 3.2 | Page 172**

**State whether the following is True or False:**

Order and degree of a differential equation are always positive integers.

1. True

2. False

**Solution:** Order and degree of a differential equation are always positive integers.- **True**

### Miscellaneous Exercise 8 | Q 3.3 | Page 172

**State whether the following is True or False:**

The degree of a differential equation is the power of the highest ordered derivative when all the derivatives are made free from negative and/or fractional indices if any.

1. True

2. False

**Solution:** The degree of a differential equation is the power of the highest ordered derivative when all the derivatives are made free from negative and/or fractional indices if any. - **True**

### Miscellaneous Exercise 8 | Q 3.4 | Page 172

**State whether the following is True or False:**

The order of highest derivative occurring in the differential equation is called degree of the differential

1. True

2. False

**Solution:** The order of highest derivative occurring in the differential equation is called degree of the differential equation. - **False**

### Miscellaneous Exercise 8 | Q 3.5 | Page 172

**State whether the following is True or False:**

The power of the highest ordered derivative when all the derivatives are made free from negative and / or fractional indices if any is called order of the differential equation.

1. True

2. False

**Solution:** The power of the highest ordered derivative when all the derivatives are made free from negative and / or fractional indices if any is called order of the differential equation. - **False**

Miscellaneous Exercise 8 | Q 3.6 | Page 172

**State whether the following is True or False:**

The degree of the differential equation  $e^{\frac{dy}{dx}} = \frac{dy}{dx} + c$  is not defined.

1. True
2. False

**Solution:**

The degree of the differential equation  $e^{\frac{dy}{dx}} = \frac{dy}{dx} + c$  is not defined. - **True**

Miscellaneous Exercise 8 | Q 4.01 | Page 172

**Find the order and degree of the following differential equation:**

$$\left[ \frac{d^3 y}{dx^3} + x \right]^{\frac{3}{2}} = \frac{d^2 y}{dx^2}$$

**Solution:**

$$\left[ d^3 \frac{y}{dx^3} + x \right]^{\frac{3}{2}} = \frac{d^2 y}{dx^2}$$

Squaring on both sides, we get

$$\left[ \frac{d^3 y}{dx^3} + x \right]^3 = \left( \frac{d^2 y}{dx^2} \right)^2$$

By definition of order and degree,

Order : 3 ; Degree : 3

Miscellaneous Exercise 8 | Q 4.01 | Page 172

**Find the order and degree of the following differential equation:**

$$x + \frac{dy}{dx} = 1 + \left(\frac{dy}{dx}\right)^2$$

**Solution:**

$$x + \frac{dy}{dx} = 1 + \left(\frac{dy}{dx}\right)^2$$

By definition of order and degree,

Order : 1 ; Degree : 2

**Miscellaneous Exercise 8 | Q 4.02 | Page 172**

Verify  $y = \log x + c$  is a solution of the differential equation

$$x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

**Solution:**

$$y = \log x + c$$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\therefore x \frac{dy}{dx} = 1$$

Again, differentiating w.r.t.  $x$ , we get

$$x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

$\therefore$  Given function is a solution of the given differential equation.

**Miscellaneous Exercise 8 | Q 4.03 | Page 172**

**Solve the differential equation:**

$$\frac{dy}{dx} = 1 + x + y + xy$$

**Solution:**

$$\frac{dy}{dx} = 1 + x + y + xy$$

$$= (1 + x) + y(1 + x)$$

$$= (1 + x)(1 + y)$$

$$\therefore \frac{dy}{1 + y} = (1 + x)dx$$

Integrating on both sides, we get

$$\int \frac{dy}{1 + y} = \int (1 + x)dx$$

$$\therefore \log|1 + y| = x + \frac{x^2}{2} + c$$

**Miscellaneous Exercise 8 | Q 4.03 | Page 172**

**Solve the differential equation:**

$$e^{\frac{dy}{dx}} = x$$

**Solution:**

$$e^{\frac{dy}{dx}} = x$$

$$\therefore \frac{dy}{dx} = \log x$$

$$\therefore dy = \log x \, dx$$

Integrating on both sides, we get

$$\int dy = \int (\log x) \, dx$$

$$\therefore y = \log x \int 1 \, dx - \int \left[ \frac{d}{dx} (\log x) \int 1 \, dx \right] dx$$

$$= x \log x - \int \frac{1}{x} \cdot x \, dx$$

$$= x \log x - \int dx$$

$$\therefore y = x \log x - x + c$$

### Miscellaneous Exercise 8 | Q 4.03 | Page 173

**Solve the differential equation:**

$$dr = a r \, d\theta - \theta \, dr$$

**Solution:**

$$dr = a r \, d\theta - \theta \, dr$$

$$\therefore (1 + \theta) \, dr = a r \, d\theta$$

$$\therefore \frac{dr}{r} = a \frac{d\theta}{(1 + \theta)}$$

Integrating on both sides, we get

$$\int \frac{dr}{r} = a \int \frac{d\theta}{1 + \theta}$$

$$\log | r | = a \log | 1 + \theta | + \log | c |$$

$$\therefore \log | r | = \log | c(1 + \theta)^a |$$

$$\therefore r = c (1 + \theta)^a$$

### Miscellaneous Exercise 8 | Q 4.03 | Page 173

**Solve the differential equation:**

Find the differential equation of family of curves  $y = e^x (ax + bx^2)$ , where A and B are arbitrary constants.

**Solution:**  $y = e^x (ax + bx^2)$

$$\therefore y = xe^x (a + bx)$$

$$\therefore \frac{y}{xe^x} = a + bx$$

Differentiating w.r.t. x, we get

$$\frac{xe^x \frac{dy}{dx} - (y(e^x + xe^x))}{x^2(e^x)^2} = b$$

$$\therefore \frac{x \frac{dy}{dx} - y - xy}{x^2 e^x} = b$$

Again, differentiating w.r.t. x, we get

$$\frac{\cdot x^2 e^x \left( \frac{dy}{dx} + x \frac{d^2y}{dx^2} - \frac{dy}{dx} - y - x \frac{dy}{dx} \right) - \left( x \frac{dy}{dx} - y - xy \right) (x^2 e^x + 2xe^x)}{(x^2 e^x)^2} = 0$$

$$\therefore xe^x \left[ x \left( x \frac{d^2y}{dx^2} - y - x \frac{dy}{dx} \right) - \left( x \frac{dy}{dx} - y - xy \right) (x + 2) \right] = 0$$

$$\therefore x^2 \frac{d^2y}{dx^2} - xy - x^2 \frac{dy}{dx} - \left( x^2 \frac{dy}{dx} - xy - x^2 y + 2x \frac{dy}{dx} - 2y - 2xy \right) = 0$$

$$\therefore x^2 \frac{d^2y}{dx^2} - xy - x^2 \frac{dy}{dx} - x^2 \frac{dy}{dx} + xy + x^2y - 2x \frac{dy}{dx} + 2y + 2xy = 0$$

$$\therefore x^2 \frac{d^2y}{dx^2} - 2x^2 \frac{dy}{dx} - 2x \frac{dy}{dx} + x^2y + 2y + 2xy = 0$$

**Miscellaneous Exercise 8 | Q 4.04 | Page 173**

**Solve**

$$\frac{dy}{dx} = \frac{x + y + 1}{x + y - 1} \text{ when } x = \frac{2}{3} \text{ and } y = \frac{1}{3}$$

**Solution:**

$$\frac{dy}{dx} = \frac{x + y + 1}{x + y - 1} \dots(i)$$

$$\text{Put } x + y = t \dots(ii)$$

$$\therefore y = t - x$$

Differentiating w.r.t. x, we get

$$\therefore \frac{dy}{dx} = \frac{dt}{dx} - 1 \dots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$\frac{dt}{dx} - 1 = \frac{t + 1}{t - 1}$$

$$\therefore \frac{dt}{dx} = \frac{t + 1}{t - 1} + 1 = \frac{t + 1 + t - 1}{t - 1}$$

$$\therefore \frac{dt}{dx} = \frac{2t}{t - 1}$$

$$\therefore \left( \frac{t-1}{t} \right) dt = 2dx$$

$$\therefore \left( 1 - \frac{1}{t} \right) dt = 2dx$$

Integrating on both sides, we get

$$\int \left( 1 - \frac{1}{t} \right) dt = 2 \int dx$$

$$\therefore t - \log |t| = 2x + c$$

$$\therefore x + y - \log |x + y| = 2x + c$$

$$\therefore -\log |x + y| = x - y + c$$

Putting  $x = \frac{2}{3}$  and  $y = \frac{1}{3}$ , we get

$$-\log(1) = \frac{1}{3} + c$$

$$\therefore c = \frac{1}{3}$$

$$\therefore -\log|x + y| = x - y + \frac{1}{3}$$

$$\therefore \log|x + y| = y - x + \frac{1}{3}$$

### Miscellaneous Exercise 8 | Q 4.05 | Page 173

**Solve**

$$y dx - x dy = -\log x dx$$

**Solution:**  $y dx - x dy = -\log x dx$

Dividing throughout by  $dx$ , we get

$$y - x \frac{dy}{dx} = -\log x$$

$$\therefore -x \frac{dy}{dx} + y = -\log x$$

$$\therefore \frac{dy}{dx} - \frac{1}{x}y = \frac{\log x}{x}$$

The given equation is of the form

$$\frac{dy}{dx} + py = Q$$

$$\text{where, } P = -\frac{1}{x} \text{ and } Q = \frac{\log x}{x}$$

$$\therefore \text{I.F.} = e^{\int p dx} = e^{\int -\frac{1}{x} dx} = e^{-\log x}$$

$$= e^{\log x^{-1}} = x^{-1} = \frac{1}{x}$$

$\therefore$  Solution of the given equation is

$$y(\text{I.F.}) = \int Q(\text{I.F.}) dx + c$$

$$\therefore \frac{y}{x} = \int \frac{\log x}{x} \times \frac{1}{x} dx + c$$

In R. H. S., put  $\log x = t$  ... (i)

$$\therefore x = e^t$$

Differentiating (i) w.r.t.  $x$ , we get

$$\frac{1}{x} dx = dt$$

$$\therefore \frac{y}{x} = \int \frac{t}{e^t} dt + c$$

$$\therefore \frac{y}{x} = \int te^t dt + c$$

$$= t \int e^{-t} dt - \int \left( \frac{d}{dt}(t) \times \int e^{-t} dt \right) dt + c$$

$$= -te^{-t} - \int (-e^{-t}) dt + c$$

$$= -te^{-t} + \int e^{-t} dt + c$$

$$= -te^{-t} - e^{-t} + c$$

$$= \frac{-t - 1}{e^t} + c$$

$$= \frac{-\log x - 1}{x} + c$$

$$\therefore y = cx - (1 + \log x)$$

$$\therefore \log x + y + 1 = cx$$

Miscellaneous Exercise 8 | Q 4.06 | Page 173

**Solve**

$$y \log y \frac{dy}{dx} + x - \log y = 0$$

**Solution:**

$$y \log y \frac{dy}{dx} + x - \log y = 0$$

$$\therefore \frac{dx}{dy} + \frac{1}{y \log y} x = \frac{1}{y}$$

The given equation is of the form  $\frac{dx}{dy} + px = Q$

where,  $P = \frac{1}{y \log y}$  and  $Q = \frac{1}{y}$

$$\therefore I.F. = e^{\int P dy} = e^{\int \frac{1}{y \log y} dy} = e^{\log |\log y|} = \log y$$

$\therefore$  Solution of the given equation is

$$x(I.F.) = \int Q(I.F.) dy + c_1$$

$$\therefore x \cdot \log y = \int \frac{1}{y} \log y dy + c_1$$

In R. H. S., put  $\log y = t$

Differentiating w.r.t.  $x$ , we get

$$\frac{1}{y} dy = dt$$

$$\therefore x \log y = \int t dt + c_1 = \frac{t^2}{2} + c_1$$

In R. H. S., put  $\log y = t$

Differentiating w.r.t.  $x$ , we get

$$\frac{1}{y} dy = dt$$

$$\therefore x \log y = \int t dt + c_1 = \frac{t^2}{2} + c_1$$

$$\therefore x \log y = \frac{(\log y)^2}{2} + c_1$$

$$\therefore 2x \log y = (\log y)^2 + c \quad \dots [2c_1 = c]$$

Miscellaneous Exercise 8 | Q 4.07 | Page 173

**Solve:**

$$(x + y) dy = a^2 dx$$

**Solution:**

$$(x + y) dy = a^2 dx$$

$$\therefore \frac{dy}{dx} = \frac{a^2}{x + y} \dots(i)$$

$$\text{Put } x + y = t \dots(ii)$$

$$\therefore y = t - x$$

Differentiating w.r.t.  $x$ , we get

$$\therefore \frac{dy}{dx} = \frac{dt}{dx} - 1 \dots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$\frac{dt}{dx} - 1 = \frac{a^2}{t}$$

$$\therefore \frac{dt}{dx} = \frac{a^2}{t} + 1$$

$$\therefore \frac{dt}{dx} = \frac{a^2 + t}{t}$$

$$\therefore \frac{t}{a^2 + t} dt = dx$$

Integrating on both sides, we get

$$\int \frac{(a^2 + t) - a^2}{a^2 + t} dt = \int dx$$

$$\therefore \int 1 dt - a^2 \int \frac{1}{a^2 + t} dt = \int dx$$

$$\therefore t - a^2 \log |a^2 + t| = x + c_1$$

$$\therefore x + y - a^2 \log |a^2 + x + y| = x + c_1$$

$$\therefore y - a^2 \log |a^2 + x + y| = c_1$$

$$\therefore y - c_1 = a^2 \log |a^2 + x + y|$$

$$\therefore \frac{y}{a^2} - \frac{c_1}{a^2} = \log |a^2 + x + y|$$

$$\therefore a^2 + x + y = e^{a \frac{y}{a^2}} \cdot e^{a \frac{-c_1}{a^2}}$$

$$\therefore a^2 + x + y = ce^{a \frac{y}{a^2}} \dots \left[ c = e^{a \frac{-c_1}{a^2}} \right]$$

### Miscellaneous Exercise 8 | Q 4.08 | Page 173

#### Solve

$$\frac{dy}{dx} + \frac{2}{x}y = x^2$$

**Solution:**

$$\frac{dy}{dx} + \frac{2}{x}y = x^2$$

The given equation is of the form

$$\frac{dy}{dx} + py = Q$$

$$\text{where, } P = \frac{2}{x} \text{ and } Q = x^2$$

$$\therefore \text{I.F.} = e^{\int p dx} = e^{2 \int \frac{1}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$$

$\therefore$  Solution of the given equation is

$$y(\text{I.F.}) = \int Q(\text{I.F.}) dx + c_1$$

$$y(x^2) = \int x^2 \times x^2 dx + c_1$$

$$\therefore x^2 y = x^4 \int dx + c_1$$

$$\therefore x^2 y = \frac{x^5}{5} + c_1$$

$$\therefore 5x^2 y = x^5 + c \quad \dots [c = 5c_1]$$

### Miscellaneous Exercise 8 | Q 4.09 | Page 173

The rate of growth of population is proportional to the number present. If the population doubled in the last 25 years and the present population is 1 lac, when will the city have population 4,00,000?

**Solution:**

Let 'x' be the population at time 't' years.

$$\therefore \frac{dx}{dt} \propto x$$

$$\therefore \frac{dx}{dt} = kx, \text{ where } k \text{ is the constant of proportionality.}$$

Integrating on both sides, we get

$$\int \frac{dx}{x} = k \int dt$$

$$\therefore \log x = kt + c \dots(i)$$

When  $t = 0$ ,  $x = 50000$

$$\therefore \log(50000) = k(0) + c$$

$$\therefore c = \log(50000)$$

$$\therefore \log x = kt + \log(50000) \dots(ii)[\text{From (i)}]$$

When  $t = 25$ ,  $x = 100000$ , we have

$$\log(100000) = 25k + \log(50000)$$

$$\therefore \log 2 = 25k$$

$$\therefore k = \frac{1}{25} \log 2 \dots(iii)$$

When  $x = 400000$ , we get

$$\log(400000) = \left[ \frac{1}{25} \log(2) \right] t + \log(50000) \dots[\text{From (ii) and (iii)}]$$

$$\therefore \log \left[ \frac{400000}{50000} \right] = \frac{t}{25} \log 2$$

$$\therefore \log 8 = \frac{t}{25} \log 2$$

$$\therefore 3 \log 2 = \frac{t}{25} \log 2$$

$$\therefore \frac{t}{25} = 3$$

$$\therefore t = 75 \text{ years.}$$

Thus, the population will be 4,00,000 after  $75 - 25 = 50$  years from present date.

### Miscellaneous Exercise 8 | Q 4.1 | Page 173

The resale value of a machine decreases over a 10 year period at a rate that depends on the age of the machine. When the machine is  $x$  years old, the rate at which its value is changing is ₹ 2200  $(x - 10)$  per year. Express the value of the machine as a function of its age and initial value. If the machine was originally worth ₹1,20,000, how much will it be worth when it is 10 years old?

**Solution:** Let 'y' be the value of the machine when machine is 'x' years old.

$\therefore$  According to the given condition,

$$\frac{dy}{dx} = 2200(x - 10)$$

$$\therefore dy = 2200(x - 10) dx$$

Integrating on both sides, we get

$$\int 1 dy = 2200 \int (x-10) dx$$

$$\therefore y = 2200 \left( \frac{x^2}{2} - 10x \right) + c$$

$$\therefore y = 1100 x^2 - 22,000 x + c$$

$$\text{when } x = 0, y = 1,20,000$$

$$\therefore 1,20,000 = 1100(0)^2 - 22,00(0) + c$$

$$\therefore c = 1,20,000$$

$\therefore$  The value of the machine can be expressed as a function of its age as

$$y = 1,100x^2 - 22,000x + 1,20,000$$

Initial value: when  $x = 0$ ,  $y = 1,20,000$

$\therefore$  when  $x = 10$ ,

$$y = 1100(10)^2 - 22,000(10) + 1,20,000$$

$$= 10,000$$

$\therefore$  The machine will worth ₹ 10,000 when it is 10 years old.

### Miscellaneous Exercise 8 | Q 4.11 | Page 173

$$y^2 dx + (xy + x^2)dy = 0$$

**Solution:**

$$y^2 dx + (xy + x^2)dy = 0$$

$$\therefore (xy + x^2) dy = -y^2 dx$$

$$\therefore \frac{dy}{dx} = -\frac{y^2}{xy + x^2} \dots(i)$$

Put  $y = tx \dots(ii)$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \dots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$\therefore t + x \frac{dt}{dx} = \frac{-t^2 x^2}{x \cdot tx + x^2}$$

$$\therefore t + x \frac{dt}{dx} = \frac{-t^2 x^2}{x^2(t+1)}$$

$$\therefore x \frac{dt}{dx} = \frac{-t^2}{t+1} - t$$

$$\therefore x \frac{dt}{dx} = \frac{-t^2 - t^2 - t}{t+1}$$

$$\therefore x \frac{dt}{dx} = \frac{-(2t^2 + t)}{t+1}$$

$$\therefore \frac{t+1}{2t^2 + t} dt = -\frac{1}{x} dx$$

Integrating on both sides, we get

$$\int \frac{t+1}{2t^2 + t} dt = - \int \frac{1}{x} dx$$

$$\therefore \int \frac{2t+1-t}{t(2t+1)} dt = - \int \frac{1}{x} dx$$

$$\therefore \int \frac{1}{t} dt - \int \frac{1}{2t+1} dt = - \int \frac{1}{x} dx$$

$$\therefore \log|t| - \frac{1}{2} \log|2t+1| = -\log|x| + \log|c|$$

$$\therefore 2\log|t| - \log|2t+1| = -2\log|x| + 2\log|c|$$

$$\therefore 2\log\left|\frac{y}{x}\right| - \log\left|\frac{2y}{x} + 1\right| = -2\log|x| + 2\log|c|$$

$$\therefore 2\log|y| - 2\log|x| - \log|2y+x| + \log|x| = -2\log|x| + 2\log|c|$$

$$\therefore \log|y^2| + \log|x| = \log|c^2| + \log|2y+x|$$

$$\therefore \log|y^2 x| = \log|c^2(x+2y)|$$

$$\therefore \log |y^2x| = \log |c^2(x + 2y)|$$

$$\therefore xy^2 = c^2(x + 2y)$$

### Miscellaneous Exercise 8 | Q 4.12 | Page 173

$$x^2y \, dx - (x^3 + y^3) \, dy = 0$$

**Solution:**  $x^2y \, dx - (x^3 + y^3) \, dy = 0$

$$\therefore x^2y \, dx - (x^3 + y^3) \, dy = 0$$

$$\therefore \frac{dy}{dx} = \frac{x^2y}{x^3 + y^3} \quad \dots(i)$$

Put  $y = tx \quad \dots(ii)$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \quad \dots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$t + x \frac{dt}{dx} = \frac{x^2 \cdot tx}{x^3 + t^3x^3}$$

$$\therefore t + x \frac{dt}{dx} = \frac{x^3 \cdot t}{x^3(1 + t^3)}$$

$$\therefore x \frac{dt}{dx} = \frac{t}{1 + t^3} - t$$

$$\therefore x \frac{dt}{dx} = \frac{-t^4}{1 + t^3}$$

$$\therefore \frac{1 + t^3}{t^4} dt = -\frac{dx}{x}$$

Integrating on both sides, we get

$$\int \frac{1+t^3}{t^4} dt = - \int \frac{1}{x} dx$$

$$\therefore \int \left( \frac{1}{t^4} + \frac{1}{t} \right) dt = - \int \frac{1}{x} dx$$

$$\therefore \int t^{-4} dt + \int \frac{1}{t} dt = - \int \frac{1}{x} dx$$

$$\therefore \frac{t^3}{-3} + \log|t| = -\log|x| + c$$

$$\therefore -\frac{1}{3t^3} + \log|t| = -\log|x| + c$$

$$\therefore -\frac{1}{3} \cdot \frac{1}{\left(\frac{y}{x}\right)^3} + \log\left|\frac{y}{x}\right| = -\log|x| + c$$

$$\therefore \frac{x^3}{3y^3} + \log|y| - \log|x| = -\log|x| + c$$

$$\therefore \log|y| - \frac{x^3}{3y^3} = c$$

**Miscellaneous Exercise 8 | Q 4.13 | Page 173**

$$xy \frac{dy}{dx} = x^2 + 2y^2$$

**Solution:**

$$xy \frac{dy}{dx} = x^2 + 2y^2$$

$$\therefore \frac{dy}{dx} = x^2 + \frac{2y^2}{xy} \quad \dots(i)$$

Put  $y = tx \quad \dots(ii)$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \dots(\text{iii})$$

Substituting (ii) and (iii) in (i), we get

$$t + x \frac{dt}{dx} = \frac{x^2 + 2t^2x^2}{x(tx)}$$

$$\therefore t + x \frac{dt}{dx} = \frac{x^2(1 + 2t^2)}{x^2t}$$

$$\therefore x \frac{dt}{dx} \frac{1 + 2t^2}{t} - t = \frac{1 + t^2}{t}$$

$$\therefore \frac{t}{1 + t^2} dt = \frac{1}{x} dx$$

Integrating on both sides, we get

$$\frac{1}{2} \int \frac{2t}{1 + t^2} dt = \int \frac{dx}{x}$$

$$\therefore \frac{1}{2} \log|1 + t^2| = \log|x| + \log|c|$$

$$\therefore \log|1 + t^2| = 2 \log|x| + 2\log|c|$$

$$= \log|x^2| + \log|c^2|$$

$$\therefore \log|1 + t^2| = \log|c^2 x^2|$$

$$\therefore 1 + t^2 = c^2 x^2$$

$$\therefore 1 + \frac{y^2}{x^2} = c^2 x^2$$

$$\therefore x^2 + y^2 = c^2 x^4$$

$$(x + 2y^3) \frac{dy}{dx} = y$$

**Solution:**

$$(x + 2y^3) \frac{dy}{dx} = y$$

$$\therefore \frac{x}{y} + 2y^2 = \frac{dx}{dy}$$

$$\therefore \frac{dx}{dy} - \frac{1}{y}x = 2y^2$$

The given equation is of the form

$$\frac{dx}{dy} + px = Q$$

where,  $P = -\frac{1}{y}$  and  $Q = 2y^2$

$$\therefore \text{I.F.} = e^{\int p dy} = e^{-\int \frac{1}{y} dy}$$

$$= e^{-\log y} = e^{\log y - 1}$$

$$= y^{-1} = 1/y$$

$\therefore$  Solution of the given equation is

$$x(\text{I.F.}) = \int Q(\text{I.F.}) dy + c$$

$$\therefore \frac{x}{y} = 2 \int \frac{y^2}{y} dy + c$$

$$\therefore \frac{x}{y} 2 \int y dy + c$$

$$\therefore \frac{x}{y} 2 \int \frac{y^2}{y} + c$$

$$\therefore x = y(c + y^2)$$

### Miscellaneous Exercise 8 | Q 4.15 | Page 173

$$y dx - x dy + \log x dx = 0$$

**Solution:**  $y dx - x dy + \log x dx = 0$

$$y dx - x dy = - \log x dx$$

Dividing throughout by dx, we get

$$y - x \frac{dy}{dx} = - \log x$$

$$\therefore -x \frac{dy}{dx} + y = - \log x$$

$$\therefore \frac{dy}{dx} - \frac{1}{xy} = \frac{\log x}{x}$$

The given equation is of the form

$$\frac{dy}{dx} + py = Q$$

where,  $P = -\frac{1}{x}$  and  $Q = \frac{\log x}{x}$

$$\therefore \text{I.F.} = e^{\int p dx} = e^{\int -\frac{1}{x} dx} = e^{-\log x}$$

$$= e^{\log x^{-1}} = x^{-1} = \frac{1}{x}$$

∴ Solution of the given equation is

$$y(I.F.) = \int Q(I.F.)dx + c$$

$$\therefore \frac{y}{x} = \int \frac{\log x}{x} \times \frac{1}{x} dx + c$$

In R. H. S., put  $\log x = t \dots(i)$

$$\therefore x = e^t$$

Differentiating (i) w.r.t.  $x$ , we get

$$\frac{1}{x} dx = dt$$

$$\therefore \frac{y}{x} = \int \frac{t}{e^t} dt + c$$

$$\therefore \frac{y}{x} = \int te^t dt + c$$

$$= t \int e^{-t} dt - \int \left( \frac{d}{dt}(t) \times \int e^{-t} dt \right) dt + c$$

$$= -te^{-t} - \int (-e^{-t}) dt + c$$

$$= -te^{-t} + \int e^{-t} dt + c$$

$$= -te^{-t} - e^{-t} + c$$

$$= \frac{-t - 1}{e^t} + c$$

$$= \frac{-\log x - 1}{x} + c$$

$$\therefore y = cx - (1 + \log x)$$

**Miscellaneous Exercise 8 | Q 4.16 | Page 173**

$$\frac{dy}{dx} = \log x$$

**Solution:**

$$\frac{dy}{dx} = \log x$$

$$\therefore dy = \log x \, dx$$

Integrating on both sides, we get

$$\int 1 \, dy = \int (\log x \times 1) \, dx$$

$$\therefore y = \log x \left( \int 1 \, dx \right) - \int \left[ \frac{d}{dx} (\log x) \int 1 \, dx \right]$$

$$\therefore y = \log x(x) - \int \left( \frac{1}{x} \times x \right) dx$$

$$= x \log x - \int 1 \, dx$$

$$\therefore y = x \log x - x + c$$

**Miscellaneous Exercise 8 | Q 4.17 | Page 173**

**Solve**

$$y \log y \frac{dx}{dy} = \log y - x$$

**Solution:**

$$y \log y \frac{dx}{dy} = \log y - x$$

$$y \log y \frac{dx}{dy} + x = \log y$$

$$\therefore \frac{dx}{dy} + \frac{1}{y \log y} x = \frac{1}{y}$$

The given equation is of the form  $\frac{dx}{dy} + px = Q$

$$\text{where, } P = \frac{1}{y \log y} \text{ and } Q = \frac{1}{y}$$

$$\therefore I.F. = e^{\int p dy} = e^{\int \frac{1}{y \log y} dy} = e^{\log |\log y|} = \log y$$

$\therefore$  Solution of the given equation is

$$x(I.F.) = \int Q(I.F.) dy + c_1$$

$$\therefore x \cdot \log y = 1y \int \log y dy + c_1$$

In R. H. S., put  $\log y = t$

Differentiating w.r.t.  $x$ , we get

$$\frac{1}{y} dy = dt$$

$$\therefore x \log y = t dt \int + c_1 = \frac{t^2}{2} + c_1$$

$$\therefore x \log y = \frac{(\log y)^2}{2} + c_1$$

$$\therefore 2x \log y = (\log y)^2 + c \dots [2c_1 = c]$$

$$x \log y = \frac{1}{2} (\log y)^2 + c$$