3. Electrostatics

3.1. Three charges are placed at the vertices of an isosceles right triangle, with charges $+Q$ and $-Q$ at the acute angles and a charge $+2Q$ at the right angle. Determine which of the numbered vectors coincides in direction with the field produced by these charges at a point that is the middle of the hypotenuse.

3.2. Two point-like charges a and b whose strengths are equal in absolute value are positioned at a certain distance from each other. Assuming the field strength is positive in the direction coinciding with the positive direction of the r axis, determine the signs of the charges for each distribution of the field strength between the charges shown in Figures (a), (b), (c), and (d).

3.3. Two point-like charges are positioned at points a and b. The field strength to the right of the charge Q_b on the line that passes through the two charges varies according to a law that is represented schematically in the figure accompanying the problem (without employing a definite scale). The field strength is assumed to be positive if its direction coincides with the positive direction on the x axis. The distance between the charges is l.

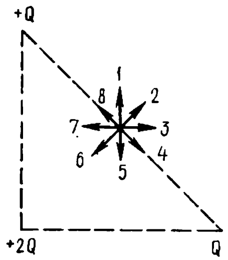


Fig. 3.1

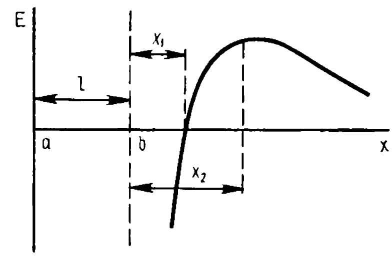


Fig. 3.3

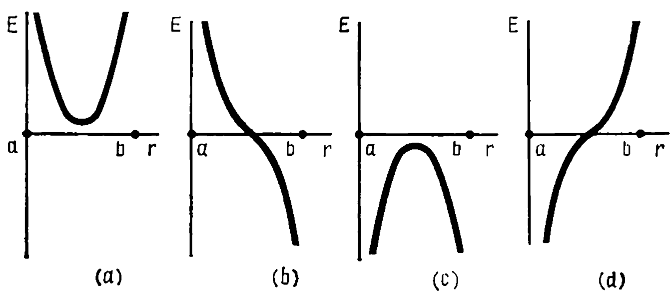


Fig. 3.2

Find the signs of the charges and, bearing in mind that the field strength at a point x_1 is zero, the ratio of the absolute values of charges Q_a and Q_b and the coordinate x_2 of the point where the field strength is maximal.

3.4. Two mutually perpendicular straight conductors carry evenly distributed charges with linear densities τ_1 and τ_2. Among the lines of force representing the field generated by these conductors there is a straight line passing through the point of intersection of the conductors.

At what angle α with respect to the conductor with the charge density τ_2 does this line pass?*

* The statement of the problem is not quite proper. The electrostatic interaction between the charges makes it impossible to maintain an even distribution of charge on the conductors. The same situation is present in other problems (e.g. see Problems 3.5 and 3.6). The difficulty can be overcome by assuming that each conductor consists of a large number of sufficiently small sections isolated from each other.

3.5. An infinitely long straight conductor carrying a charge with a linear density $+\tau$ and a point charge

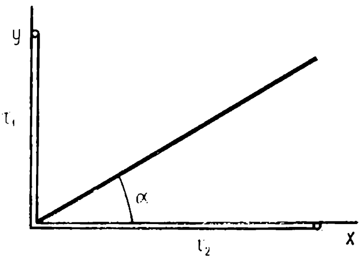


Fig. 3.4

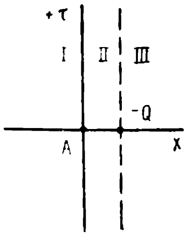


Fig. 3.5

$-Q$ are at a certain distance from each other. In which of the three regions (I, II, or III) are there points that (a) lie on the line passing through the point charge perpendicular to the conductor and (b) at which the field strength is zero?

3.6. Two mutually perpendicular infinitely long straight conductors carrying uniformly distributed charges of linear densities τ_1 and τ_2 are positioned at a distance a from each other. How does the interaction between the conductors depend on a?

3.7. Near an infinitely large flat plate with a surface charge density σ on each side, the field strength is**

$$E = \frac{\sigma}{\varepsilon_0 \varepsilon},$$

while the field produced by a point charge at a distance r from the charge is

$$E = \frac{Q}{4\pi\varepsilon_0 \varepsilon r^2}.$$

Prove that for a uniformly charged disk with a surface charge density σ (on each side), the electric field strength on the axis of the disk is the same as for an infinitely large flat plate if the distances are small in comparison with the disk's radius R, and is the same as for a point charge if the distances are large.

** Usually the value of the field strength given in textbooks is half the one given here, since there it is assumed that the charge is on a geometric plane.

3.8. At a certain distance r from an infinitely long straight conductor with a uniformly distributed linear charge τ there is a dipole with an electric moment p_{el} directed along the line of force representing the field generated by the conductor at the point where the dipole is

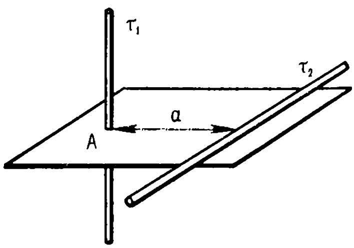


Fig. 3.6

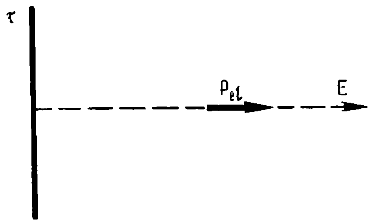


Fig. 3.8

located. Assuming the arm of the dipole is very small compared to the distance r, find the force with which the field acts on the dipole.

3.9. The figure shows the schematic of an absolute electrometer. The potential difference that is to be measured is applied between the plates *1* and *2*, with the upper plate connected to one arm of a balance beam.* The pan connected to the other arm is loaded with weights until balance is achieved, that is, when the upper plate begins to move upward. In this way the force acting between the charged plates is measured, and this enables one to determine the magnitude of the potential difference between the plates. It the equilibrium in the electrometer stable or unstable?

* The figure does not show the protecting rings around plates *1* and *2* with the same potentials. These are used to ensure that the field is as uniform as possible.

3.10. A small thin metal strip lies on the lower plate of a parallel-plate capacitor positioned horizontally. The voltage across the capacitor plates is increased gradually to a value at which the electric force acting on the strip becomes greater than the strip's weight and makes the strip move toward the upper plate. Does the force acting on the strip remain constant during the lifting process?

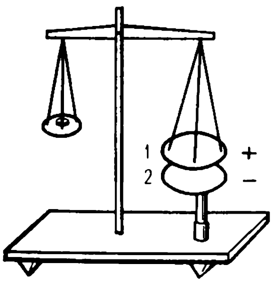


Fig. 3.9

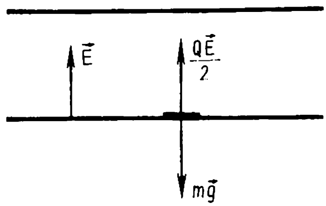


Fig. 3.10

3.11. Into the region of space between the plates of a parallel-plate capacitor there flies (a) an electron and (b) a negatively charged ion with a velocity directed parallel to the plates. Both the electron and the ion have received

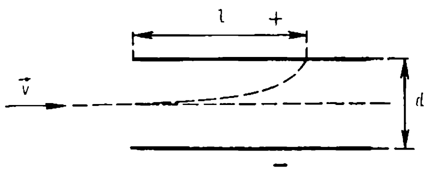


Fig. 3.11

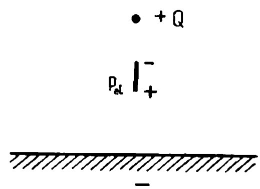


Fig. 3.12

their initial kinetic energy by passing the same potential difference U_0, and the potental difference across the capacitor is U. The distance between the plates is d. Which of the two particles will travel a greater distance before hitting the positively charged plate if both fly into the capacitor at a point that is exactly in the middle of the distance between the plates?

3.12. An electric dipole is positioned between a point-like charge and a uniformly charged conducting plate. In which direction will the dipole move?

3.13. A point-like charge Q and a dipole with an electric moment p_{el} are separated by a distance that is considerably larger than the arm of the dipole, with the result that the dipole may be considered as being point. The dipole's axis lies along the lines of force of the point charge. Compare the force acting on the dipole in the field of the point charge with that acting on the point charge in the field of the dipole.

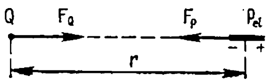


Fig. 3.13

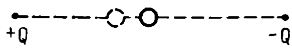


Fig. 3.14

3.14. A small uncharged sphere is positioned exactly in the midpoint between two charges whose absolute values are the same but whose signs are opposite. Suppose the sphere is shifted somewhat. Will it remain in the new position or will it move in some direction?

3.15. A small uncharged metal sphere is suspended by a long nonconducting string in the region between the

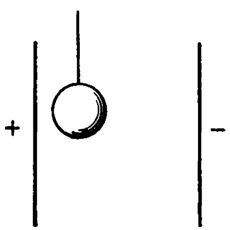


Fig. 3.15

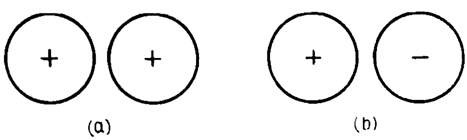


Fig. 3.16

vertically positioned plates of a parallel-plate capacitor, closer to one plate than to the other. How will the sphere behave?

3.16. Two conducting spheres carry equal charges. The distance between the spheres cannot be considered large in comparison with the diameters of the spheres. In which case will the force of interaction between the spheres be greater (in absolute value): when they carry like charges (Figure (a)) or when they carry unlike charges (Figure (b))?

3.17. A point charge is surrounded by two spherical layers (Figure (a)), with the electric field strength as a function of distance having the form depicted in Figure (b) (on the log-log scale). In what layer (the inner or the outer) is the dielectric constant greater and by what factor?

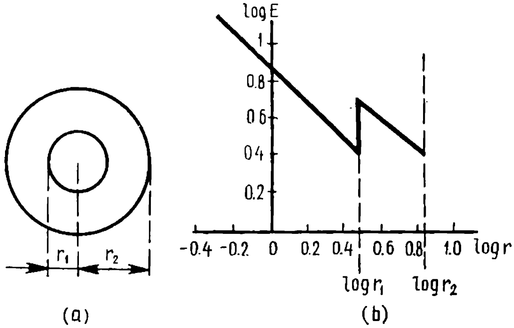


Fig. 3.17

3.18. The region of space between the plates of a parallel-plate capacitor is filled with a liquid dielectric with a dielectric constant ε_1. A solid dielectric with a dielectric constant ε_2 is immersed in the liquid. The lines of

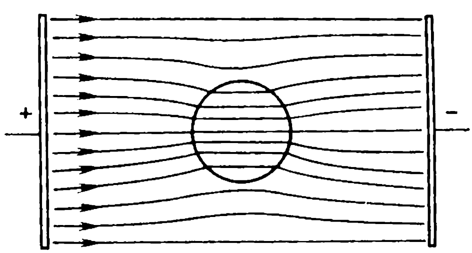


Fig. 3.18

force in the liquid have the shape shown in the figure. Which of the two dielectric constants is greater?

3.19. Various potential distributions between two point charges are shown in Figures (a)-(d) (the charges are equal in absolute value). Determine the signs of the charges for each case.

3.20. Two point charges, Q_1 and Q_2, are positioned at a certain distance from each other. The curves in the figure represent the distribution of the potential along the straight line connecting the two charges. At which points (*1*, *2*, and/or *3*) is the electric field strength zero?

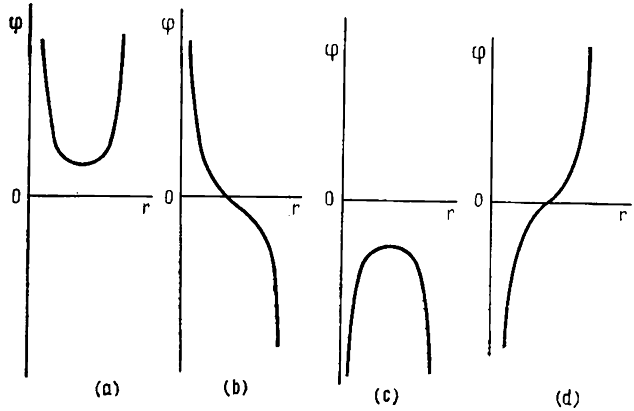


Fig. 3.19

What are the signes of the charges Q_1 and Q_2 and which of the two is greater in magnitude?

3.21. Two equal like charges are positioned at a certain distance from each other. How do the electric field

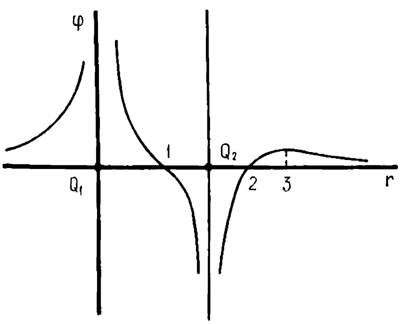


Fig. 3.20

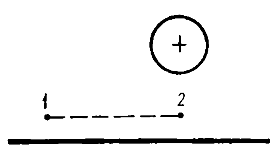


Fig. 3.22

strength and the potential vary along the axis that passes through the midpoint of the distance between the charges at right angles to the line connecting the charges?

3.22. A potential difference is applied between a conducting sphere and a conducting plate ("plus" on the sphere and "minus" on the plate). The dimensions of the plate are much larger than the distance between sphere and plate. A point positive charge is moved from point *1* to point *2* parallel to the plate. Is any work done in the process?

3.23. Two parallel-plate capacitors with different distances between the plates are connected in parallel to a voltage source. A point positive charge is moved from a point *1* that is exactly in the middle between the plates of a capacitor *C1* to a point *2* (or a capacitor *C2*) that lies

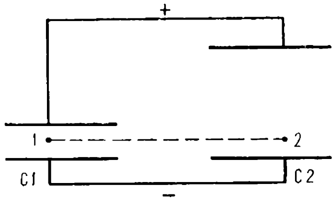


Fig. 3.23

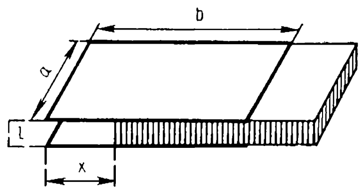


Fig. 3.24

at a distance from the negative plate of *C2* equal to half the distance between the plates of *C1*. Is any work done in the process?

3.24. The space between the rectangular plates (with sides a and b) of a parallel-plate capacitor (the distance between the plates is l) is filled with a solid dielectric whose dielectric constant is ε. The capacitor is charged to a certain potential difference and disconnected from the voltage source. After that the dielectric is slowly moved out of the capacitor, which means that the section x not filled with the dielectric gradually increases in size. How will the potential difference between the plates and the surface charge densities on both parts of the capacitor (with and without the dielectric) change in the process?

3.25. At which of the two points, *1* or *2*, of a charged capacitor with nonparallel plates is the surface charge density greater?

3.26. The diameter of the outer conductor of a cylindrical capacitor is D_2. What should the diameter of the core, D_1, of this capacitor be so that for a given potential difference between the outer conductor and the core the electric field strength at the core is minimal?

3.27. Four capacitors, *C1*, *C2*, *C3*, and *C4*, are connected as shown in the figure. A potential difference is applied between points A and B. What should the relationship between the capacitances of the capacitors be so that the potential difference between points a and b is zero?

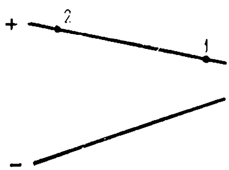


Fig. 3.25

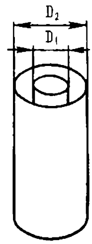


Fig. 3.26

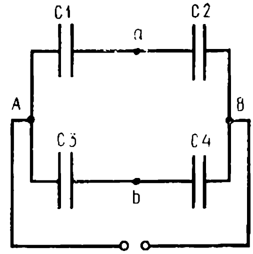


Fig. 3.27

3.28. An electric charge with a constant volume density ρ is distributed within a solid sphere of radius R. Determine and represent graphically the radial distributions of the electric field strength and the potential inside and outside the sphere.

3.29. In the region of space between the plates of a parallel-plate capacitor there is a uniformly distributed positive charge with a volume density ρ. The plates are connected electrically and their potential is set at zero. Calculate and draw a sketch of the distributions of the potential and electric field strength between the plates.

3.30. Two series-connected capacitors of the same size, one filled with air and the other with a dielectric, are

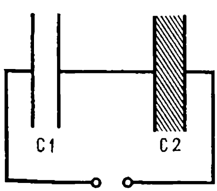


Fig. 3.30

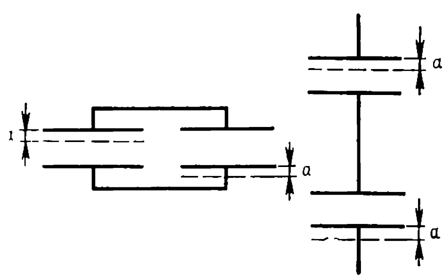


Fig. 3.32

connected to a voltage source. To which of the capacitors a higher voltage is applied?

3.31. Two identical air capacitors are connected in series. How will the charge on and potential difference across each capacitor change when the distance between the plates of one capacitor is increased in the following cases: when the capacitors are connected to a DC source, and when the capacitors are first charged and then disconnected from the DC source?

3.32. Two identical parallel-plate air capacitors are connected in one case in parallel and in the other in series. In each case the plates of one capacitor are brought closer together by a distance a and the plates of the other are moved apart by the same distance a. How will the total capacitance of each system change as a result of such manipulations?

3.33. A parallel-plate capacitor is filled with a dielectric up to one-half of the distance between the plates.

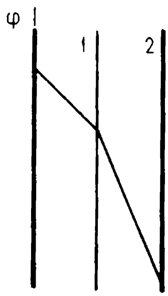


Fig. 3.33

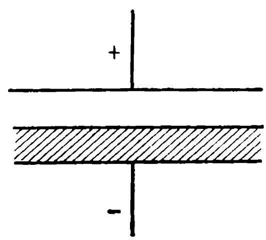


Fig. 3.34

The manner in which the potential between the plates varies is illustrated in the figure. Which half (*1* or *2*) of the space between the plates is filled with the dielectric and what will be the distribution of the potential after the dielectric is taken out of the capacitor provided that (a) the charges on the plates are conserved or (b) the potential difference across the capacitor is conserved?

3.34. A capacitor is partially filled with a dielectric. In which of its parts is the electric field strength greater? What about the electric displacement and the energy density?

3.35. Two parallel-plate capacitors, one filled with air and the other with a dielectric, have the same geometric dimensions, are connected in parallel, and are charged to a certain potential difference. In which of the two capacitors is the electric field strength greater, in which is the electric displacement greater, in which is the energy density greater, and in which is the surface charge density on the plates greater?

3.36. Three point-like charges are positioned at the vertices of an equilateral triangles. Two are equal in magnitude and are like, while the third is opposite in sign.

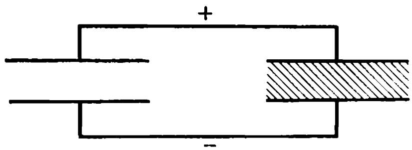


Fig. 3.35

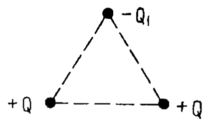


Fig. 3.36

What should the magnitude of the third charge be so that the total interaction energy of the charges is zero?

3.37. The dielectric filling the space between the plates of a capacitor that has been charged and then disconnected from the voltage source is removed. How should the distance between the plates be changed so that the energy stored in the capacitor remains the same? Explain the origin of the change in energy.

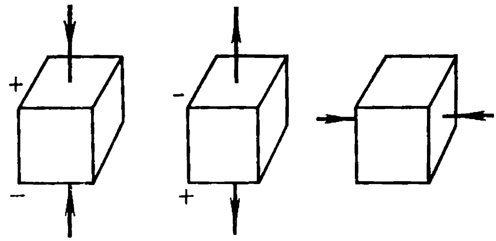
3.38. A capacitor between whose plates there is a dielectric with a dielectric constant ε is connected to a DC source. How will the energy stored in the capacitor change if the dielectric is removed? Explain the cause of this change.

3.39. A parallel-plate capacitor that has been first charged and then disconnected from the voltage source is submerged in the vertical position into a liquid dielectric. How does the level of the dielectric between the plates change in the process?

3.40. A parallel-plate capacitor with vertical plates is connected to a voltage source and then submerged into a liquid dielectric. How does the level of the dielectric between the plates change in the process? Explain the change of the energy stored by the capacitor.

3.41. A cube has been cut out from a piezoelectric crystal. When the cube was compressed, it exhibited electric charges on the faces: a positive charge on the upper face and a negative charge on the lower (Figure (a)). When the cube was stretched, the charges were found to change their signs (Figure (b)). What will be the signs of the charges on these faces if pressure is applied as shown in Figure (c)?

3.42. The relationship that exists between the electric displacement and the electric field strength in a ferroelectric is given by the curve of primary polarization and a hysteresis loop. Are there any points on the hysteresis



(a) (b) (c)

Fig. 3.41

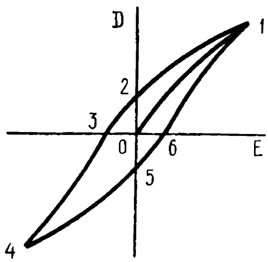


Fig. 3.42

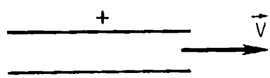


Fig. 3.43

loop to which we might formally assign a dielectric constant equal to zero or to infinity?

3.43. A charged parallel-plate capacitor is moving with respect to a certain system of coordinates with a velocity **v** directed parallel to the plates. What is the ratio of the electric field between the plates in this coordinate system to the same quantity in the system of coordinates in which the capacitor is at rest?

3. Electrostatics

3.1. The components of the electric field strength that are generated by the charges at the acute angles are equal and are directed toward the negative charge. If we denote the length of the hypotenuse by $2a$, each of these components is $Q/4\pi\varepsilon_0\varepsilon a^2$ and the sum is $Q/2\pi\varepsilon_0\varepsilon a^2$. The component of the electric field strength generated by the charge $+2Q$ is the same. It is directed at right angles to the hypotenuse away from the right angle. The resultant field strength is directed parallel to the leg connecting the charges $+2Q$ and $-Q$ along vector *3*.

3.2. Since in the case at hand all the electric field vectors lie on a single straight line, the vector sum may be replaced with the scalar sum. For unlike charges the direction of the resultant vector does not change while for like charges it does. In the case illustrated by Figure (a), the electric field strength is positive everywhere. Allowing for the signs specified in the problem, we conclude that the left charge is positive and the right charge is negative. Similarly, for the case illustrated by Figure (c), the left charge is negative and the right charge is positive. In Figures (b) and (d) the electric field strength changes its sign at the midpoint of the distance between the charges. Obviously, this can only occur if the charges are like. Bearing in mind the aforesaid and allowing for the relationship between the direction of the electric field vector and the sign of the charge generating the field, we conclude that for the case depicted in Figure (b) both charges are positive, while for the case depicted in Figure (d) both charges are negative.

3.3. Since both electric field vectors lie on a single straight line, they can be added algebraically, just as we did in the previous problem. The electric field strength to the right of charge Q_b in the immediate vicinity of the charge is negative; hence, the charge is negative (the electric field vector is directed toward the charge). The electric field strength may be positive to the right of Q_b only if Q_a is positive and greater (in absolute value) than Q_b. The electric field strength is zero at point x_1 if

$$\frac{Q_a}{(l+x_1)^2}-\frac{Q_b}{x_1^2}=0,$$

whence

$$\frac{Q_a}{Q_b}=\left(\frac{l+x_1}{x_1}\right)^2.$$

At all points that are to the right of Q_b the electric field strength is specified by the equation

$$E_x=\frac{Q_a}{(l+x)^2}-\frac{Q_b}{x^2}.$$

Taking the derivative with respect to x and nullifying it, we find that the maximum is at the point

$$x_2=\frac{l}{(Q_a/Q_b)^{1/3}-1}.$$

3.4. The direction of the electric field vector at a point with coordinates x and y (see the figure accompanying the answer) is determined by the two components, E_x and E_y:

$$E_x=\frac{\tau_2}{2\pi\varepsilon_0 x},\quad E_y=\frac{\tau_1}{2\pi\varepsilon_0 y}.$$

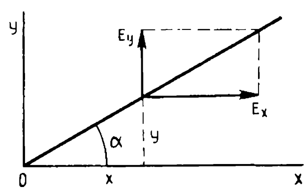


Fig. 3.4

For the extension of the resultant vector to pass through the origin, which is where the conductors intersect, the slope of the vector must be equal to y/x, that is,

$$\frac{E_y}{E_x}=\frac{\tau_1 x}{\tau_2 y}=\frac{y}{x}.$$

Thus,

$$\tan\alpha = y/x = \sqrt{\tau_1/\tau_2}.$$

3.5. No such point can exist in region *II*, since the electric field vectors of the two charges point in the same direction—from the linear charge to the point charge. In regions *III* and *I* the electric field vectors of these charges

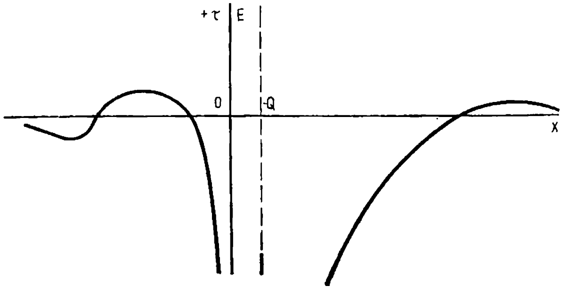


Fig. 3.5

point in different directions. Let us examine each region separately. At a certain point to the right of the point charge, the electric field strength produced by this charge is

$$E_1 = -Q/4\pi\varepsilon_0 x^2,$$

where x is the distance from the charge to the point. The linear charge produces the following field at the same point:

$$E_2 = \tau/2\pi\varepsilon_0 (x + a).$$

The sum of these fields is zero if

$$\frac{Q}{2x^2} = \frac{\tau}{a+x},$$

whence

$$x = \frac{Q}{4\tau} \pm \sqrt{\frac{Q^2}{16\tau^2} + \frac{aQ}{2\tau}}.$$

Only the plus sign in front of the radical sign has any meaning, since the minus sign corresponds to a point to the left of the point charge, where the electric field strengths of both charges are added rather than subtracted from each other (the quantities are equal in absolute value). Now let us turn to region I, that is, to the left of the linear charge. To see whether there are points in this region where the electric field strength is zero, we determine the electric field strengths produced by the two charges in this region. For the sake of convenience we direct the x axis to the left and take point A on the linear conductor as the origin (see the figure accompanying the problem). Then the field produced by the point charge is

$$E_1 = -\frac{Q}{4\pi\varepsilon_0 (a+x)^2},$$

while that produced by the linear charge is

$$E_2 = \frac{\tau}{2\pi\varepsilon_0 x}.$$

The two vectors point in opposite directions, obviously. The condition that their sum is zero yields the following equation for x:

$$x^2 + \left(2a - \frac{Q}{2\tau}\right) x + a^2 = 0,$$

whence

$$x = \frac{1}{2}\left(\frac{Q}{2\tau} - 2a\right) \pm \sqrt{\frac{1}{4}\left(\frac{Q}{2\tau} - 2a\right)^2 - a^2}.$$

The net field strength in region I is zero if the radicand is positive, obviously, that is, if

$$Q \geqslant 8a\tau.$$

If this condition is met, region I contains two points where the electric field is zero. The distribution of the electric field strength along the x axis is shown schematically (without a definite scale) in the figure accompanying the answer.

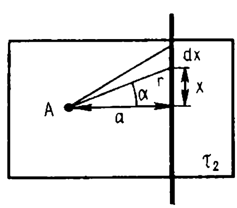


Fig. 3.6

3.6. Let us first solve this problem by dimensional considerations. Here are the quantities on which the interaction force between the conductors might depend: the charge densities, the distance between the conductors, and the "absolute" permittivity

$$\varepsilon_a = \varepsilon_0 \varepsilon,$$

which obviously has the same dimensions as the permittivity of empty space ε_0, since the dielectric constant ε is dimensionless. The SI dimensions of these quantities are

$$[F] = LMT^{-2}, \quad [\tau] = L^{-1}TI, \quad [\varepsilon_a] = L^{-3}M^{-1}T^4I^2,$$
$$[a] = L.$$

Assuming that these quantities enter the expression for force F with exponents p, q, and r, we can write

$$F = C\tau^p\varepsilon_a^q a^r$$

(C is a dimensionless constant), and the equation for the dimensions is

$$LMT^{-2} = [L^{-1}TI]^p \times [L^{-3}M^{-1}T^4I^2]^q \times L^r.$$

This yields the following equations for the exponents:

$$1 = -p - 3q + r, \quad 1 = -q, \quad -2 = p + 4q,$$
$$0 = p + 2q.$$

Hence,

$$p = 2, \quad q = -1, \quad r = 0,$$

or

$$F = C\frac{\tau_1\tau_2}{\varepsilon_0\varepsilon}. \tag{3.6.1}$$

We have found, therefore, that the interaction does not depend on the distance between the conductors.

It goes without saying that C cannot be determined by dimensional analysis alone. The same problem can be solved by direct integration via the Coulomb law. In the figure accompanying the answer, A stands for the point where the plane of the drawing "cuts" the conductor with linear density τ_1. The electric field generated by this conductor at the point with the element dx of the second conductor distant r from the first is

$$E = \frac{\tau_1}{2\pi\varepsilon_0\varepsilon r}.$$

The following force acts on element dx of the second conductor:

$$dF = E\tau_2\, dx.$$

We are interested, however, in the component of the force that is perpendicular to the second conductor, or $dF \cos\alpha$, since the longitudinal component is canceled out by an equal component acting on the symmetrical element. Let us express all linear quantities in terms of distance a and angle α:

$$r=\frac{a}{\cos\alpha}, \qquad dx=\frac{a}{\cos^2\alpha}\,d\alpha.$$

Substituting these quantities into the expression for the perpendicular component of the force acting on element dx, we get (after canceling out like terms)

$$dF=\frac{\tau_1\tau_2}{2\pi\varepsilon_0\varepsilon}\,d\alpha.$$

Integration from $-\pi/2$ to $+\pi/2$ yields

$$F=\frac{\tau_1\tau_2}{2\varepsilon_0\varepsilon},$$

that is, we arrive at an expression of the (3.6.1) type. Hence $C = 1/2$.

3.7. The element of the disk bounded by radii ρ and $\rho + d\rho$ and angle $d\varphi$ carries a charge (taking into account both sides of the disk) equal to $2\sigma\rho d\rho d\varphi$. At a distance z from this element and, hence, at a distance r from the disk's center (Figure (a)), the electric field generated by this charge is

$$E=\frac{2\sigma\rho\,d\rho\,d\varphi}{4\pi\varepsilon_0\varepsilon z^2}.$$

Only the component of this field that points in the direction of r is of any interest to us since the perpendicular component is canceled out by an equal component (pointing in the opposite direction) from the symmetrically situated charge. For this reason, the charge on the disk limited by the radii ρ and $\rho + d\rho$ creates an electric field

$$dE=\frac{\rho\,d\rho\,\sigma\cos\alpha}{z^2}. \qquad (3.7.1)$$

We express all geometric quantities in terms of distance r and angle α:

$$z=\frac{r}{\cos\alpha}, \qquad \rho=r\tan\alpha, \qquad d\rho=\frac{r\,d\alpha}{\cos^2\alpha}.$$

After substituting into (3.7.1) and canceling out like terms, we get

$$dE = \frac{\sigma \sin\alpha \, d\alpha}{\varepsilon_0 \varepsilon}.$$

Integration from $\alpha = 0$ to the value α_m corresponding to the edge of the disk yields

$$E = \frac{\sigma}{\varepsilon_0 \varepsilon}(1 - \cos\alpha_m) = \frac{\sigma}{\varepsilon_0 \varepsilon}\left(1 - \frac{r}{\sqrt{R^2 + r^2}}\right). \quad (3.7.2)$$

For $r \ll R$, angle α is close to 90°. In this case, $E \approx \sigma/\varepsilon_0\varepsilon$, just as in the case with an infinitely large plate.

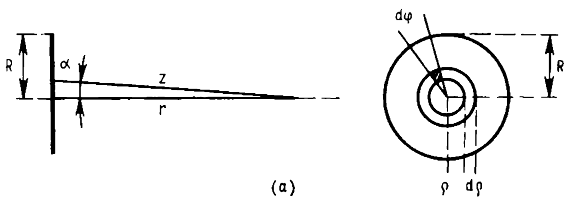


Fig. 3.7a

Let us calculate E for $r \gg R$. To this end we express $\cos\alpha_m$ in terms of r and R:

$$\cos\alpha_m = \frac{r}{\sqrt{R^2 + r^2}}.$$

Using the rules of approximate calculations, we arrive at

$$1 - \frac{r}{\sqrt{R^2 + r^2}} = 1 - \frac{1}{\sqrt{1 + R^2/r^2}} \approx \frac{R}{2r^2}.$$

Substituting this into (3.7.2), we get

$$E = \frac{\sigma R^2}{2\varepsilon_0 \varepsilon r^2}.$$

Since $\sigma = Q/2\pi R^2$, we have

$$E = \frac{Q}{4\pi\varepsilon_0 \varepsilon r^2},$$

just as for a point charge (see the problem).

Figure (b) shows the variation of the electric field of the disk with distance (curve *1*); for comparison, the straight line *3* corresponds to the field created by an infinitely large plate with a surface charge density equal to that of the disk, while curve *2* corresponds to the field of a point charge whose magnitude coincides with the charge of the disk. Dimensionless coordinates are

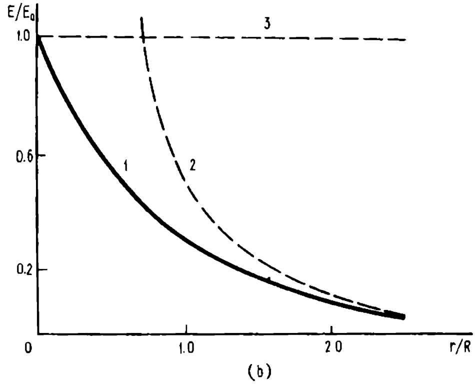


Fig. 3.7b

employed in Figure (b): r/R along the horizontal axis and E/E_0 along the vertical axis (E_0 is the electric field strength generated by the infinitely large plate).

3.8. The force with which an electric field acts on a dipole is

$$F = p_{\text{el}} \frac{dE}{dr}. \tag{3.8.1}$$

Since an infinitely long straight conductor with an evenly distributed charge (density) generates an electric field

$$E = \frac{\tau}{2\pi\varepsilon_0\varepsilon r},$$

we have (according to (3.8.1))

$$F = -\frac{\tau p_{\text{el}}}{2\pi\varepsilon_0\varepsilon r^2}. \tag{3.8.2}$$

Nothing was said in the problem about the sign of the charge on the conductor. Obviously, if the charge is positive and the dipole moment coincides in direction with the positive direction of the electric field vector, the dipole will move toward the conductor, which agrees with the "minus" sign in (3.8.2).

3.9. If the field in the region between the plates can be assumed to be uniform, the plates of the parallel-plate capacitor interact with a force

$$F = \varepsilon_0 \varepsilon E^2 S/2,$$

where S is the area of the plates of the capacitor. Since $E = U/l$, with U the potential difference between the plates, we have

$$F = \varepsilon_0 \varepsilon U^2 S/2l^2.$$

Thus, for a given potential difference between the plates, the attractive force is the greater the smaller the distance between the plates. If the upper plate is balanced by weights, a small decrease in the distance between the plates leads to an increase in the attractive force, while a small increase in the distance leads to a decrease in the force. In both cases the balance will be violated. This means that the plate equilibrium is unstable. There is a special set screw in the electrometer that does not allow the upper plate to move below the level at which the measurement is taken.

3.10. The force acting on the strip when the strip lies on the lower plate is determined by the formula for the attractive force between the plates of a parallel-plate capacitor,

$$F = \frac{\varepsilon_0 \varepsilon E^2}{2} S = \frac{QE}{2},$$

where S is the area of the strip, as if it was part of the lower plate of the capacitor. When this force becomes greater than the weight of the strip, the strip begins to move upward, but retains its charge $Q = \sigma S$.

When the distance between the strip and the lower plate becomes great, the strip will not only be attracted by the upper plate but will also be repulsed by the lower plate where the charge density will gradually become even. As a result, the force on the strip increases in magnitude. If we ignore the distortions introduced by the charge of the strip into the field (this can be done if the strip is small), we can assume that the strip is in a field of strength E and that the following force acts on it: $F = QE$. The charged strip induces a charge on the upper plate as it approaches the plate. This leads to a distortion in the field and a slight increase in F. Although in the above discussion we have considered a flat strip, the same line of reasoning is valid qualitatively for any small conductor lying, at the initial stage, on the lower plate of the capacitor.

3.11. Let us first solve this problem by dimensional analysis. The following quantities are present in the problem: the initial potential difference U that the electron or ion has to pass, the potential difference U_0 between the plates, the distance d between the plates, the sought distance l that the electron or ion has to travel before it hits the plate, the charge Q of the particle, and the particle's mass m. The equation for the dimensions can be written as follows:

$$[l] = [d]^a[U_0]^b[U]^c[Q]^x[m]^y,$$

or

$$L = L^a[L^2MT^{-3}I^{-1}]^{b+c}[IT]^xM^y.$$

For the exponents we have the following four equations:

$$a + 2b + 2c = 1,\ b + c + y = 0,$$
$$x - 3b - 3c = 0,\ x - b - c = 0,$$

whence

$$a = 1,\quad b = -c,\quad x = 0,\quad y = 0.$$

We see that the distance traveled by the particle (an electron or an ion) does not depend on the charge-to-mass ratio.

We arrive at the same result if we solve the equation of motion of the particle. Under the potential difference U_0, the particle acquires a velocity

$$v = \sqrt{2QU_0/M},$$

with which it moves parallel to the plates, while the acceleration with which the particle moves transversely to the plates is

$$w = QU/md.$$

The particle takes a time interval

$$t = \sqrt{\frac{d}{w}} = d\sqrt{\frac{m}{QU}}$$

to cover the distance

$$l = vt = d\sqrt{2U_0/U}.$$

This conclusion has a broader meaning than the one obtained earlier. It follows that for a given initial energy, a charged particle moves in an electric field along a trajectory that does not depend on the particle's charge-to-mass ratio.

3.12. A dipole that is placed in a nonuniform electric field and is oriented along the field's direction is under a force

$$F = p_{el} \frac{dE}{dr},$$

where p_{el} is the dipole electric moment. If the direction of the dipole's axis is taken as the positive direction, the direction of the force will be determined by the sign of the derivative. In the case at hand the derivative is negative and, hence, the dipole is moving toward the point charge.

3.13. A point dipole oriented along the lines of force of the field created by a point charge is under a force

$$F_p = p_{el} \frac{dE_Q}{dr}.$$

Since the electric field created by a point charge is

$$E_Q = \frac{Q}{4\pi\varepsilon_0\varepsilon r^2},$$

we can write

$$\frac{dE_Q}{dr} = -\frac{Q}{2\pi\varepsilon_0\varepsilon r^3},$$

with the result that the force acting on the dipole is

$$F_p = -\frac{Qp_{el}}{2\pi\varepsilon_0\varepsilon r^3}.$$

At points that lie on the axis of the point dipole, the electric field of the dipole is

$$E_p = \frac{p_{el}}{2\pi\varepsilon_0\varepsilon r^3}.$$

When a point charge Q is in this field, the force acting on it is

$$F_Q = \frac{Qp_{el}}{2\pi\varepsilon_0\varepsilon r^3}.$$

In accordance with Newton's third law, this force must coincide in magnitude with, but be opposite to, force F_p.

The positive direction in the figure accompanying the problem is the one from the point charge to the dipole. Therefore, the "minus" sign in the force acting on the dipole implies that this force is directed toward the point charge. The field created by the dipole at the point where the point charge is positioned has a "plus" sign, that is, is directed toward the dipole. The force acting on the point charge points in the same direction.

3.14. The electric field in which the sphere is placed induces charges of opposite sign on the sphere, in view of which the sphere becomes a dipole. After the sphere is shifted, it finds itself in a nonuniform field, which forces it to move toward the charge to which it was shifted. Thus, the equilibrium of the sphere at the midpoint between the charges is unstable.

3.15. Due to electrostatic induction, one side of the sphere becomes positively charged, while the other becomes negatively charged, and the sphere becomes a dipole. At first glance it might seem that since the dipole is oriented along the lines of force of the field and the field of the capacitor is uniform, no forces act on the sphere. But this is not so. The presence of the sphere will distort the field. The charge density, and hence the field strength, at the points of the plates that lie on the straight line that is perpendicular to the plates and passes through the center of the sphere will increase. The dipole will find itself in a nonuniform field and will be attracted to the plate that is closer to it. If the string enables the sphere to touch the plate, the sphere will lose its charge, which is opposite to the one on the plate. But the sphere will then acquire a charge that is of the same sign as that on the plate it has just touched. This leads to a repulsive force between sphere and plate, with the result that the sphere will move toward the other plate. After touching this plate (if the string enables it to do this), the sphere will reverse the sign of its charge and will move in the direction of the first plate, and so on.

3.16. If the distance between the spheres is not very large, the charges on the spheres are not evenly distributed over the surfaces. The effect of the spheres on each other results in that in the case of like charges the sections of the spheres that are farthest from each other will have an enhanced charge density, while in the case of unlike charges the sections of the spheres that are closest to each other will have an enhanced charge density. For this reason, the distance between the "centers of charge" for like charges is greater than that for unlike charges. Hence, the attractive force between the unlike charges will be greater (in magnitude) than the repulsive force between the like charges.

3.17. The field strength in each layer is

$$E = \frac{Q}{4\pi\varepsilon_0\varepsilon r^2}.$$

On the log-log scale,

$$\log E_1 = \log \frac{Q}{4\pi\varepsilon_0} - \log \varepsilon_1 - 2 \log r_1 \qquad (3.17.1)$$

and

$$\log E_2 = \log \frac{Q}{4\pi\varepsilon_0} - \log \varepsilon_2 - 2 \log r_1 \qquad (3.17.2)$$

in each layer at the boundary between the layers. Subtracting (3.17.1) from (3.17.2) and bearing in mind that the difference of the logarithms of two quantities equals the logarithm of the ratio of these quantities, we have

$$\log (E_2/E_1) = \log (\varepsilon_1/\varepsilon_2).$$

Hence, in the inner layer the dielectric constant is higher than in the outer. The difference of the logarithms of the field strengths in Figure (b) accompanying the problem is about 0.3, which corresponds to the ratio of the dielectric constants of about 2.

3.18. The lines of force of electric induction become denser as one moves closer to the solid dielectric, which means that the density of bound charges on the surface of the solid dielectric becomes enhanced. This density is the higher the greater the dielectric constant. Whence $\varepsilon_2 > \varepsilon_1$.

3.19. The potential at each point is the algebraic sum of potentials of the field of each charge. For a point charge, the potential at distance r from the charge is

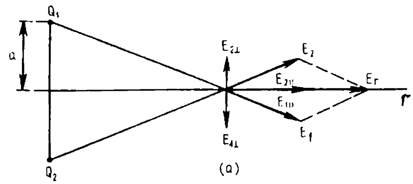
$$\varphi = \frac{Q}{4\pi\varepsilon_0\varepsilon r}$$

(it is assumed that the potential at infinity is zero). When the charges are like, the absolute value of the potential at a point r distant from one of the charges is

$$\varphi = \frac{Q}{4\pi\varepsilon_0\varepsilon}\left(\frac{1}{r} + \frac{1}{l-r}\right).$$

The sign of the potential coincides with that of the charge. Hence, in Figure (a) both charges are positive, while in Figure (c) both are negative. When the charges are unlike, the potential at midpoint between the charges is zero. The potential is positive closer to the positive charge to the left in the case shown in Figure (b) and to the right in the case shown in Figure (d).

3.20. The field strength vanishes only at one point, *3*, where the derivative $d\varphi/dr$ is zero. Since near charge Q_2



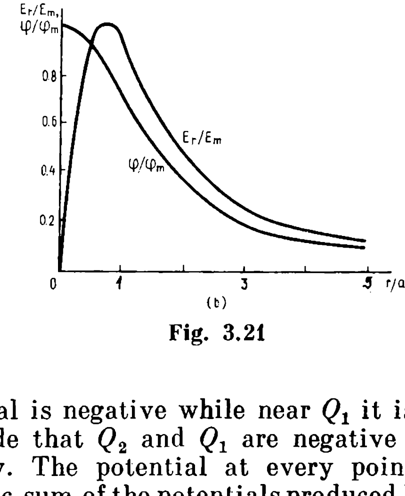


Fig. 3.21

the potential is negative while near Q_1 it is positive, we can conclude that Q_2 and Q_1 are negative and positive, respectively. The potential at every point in space is the algebraic sum of the potentials produced by all charges. To the right of Q_2 (except in the immediate vicinity of Q_2) the potential is positive. This implies that in the entire region to the right of Q_2 the potential produced by Q_1 is greater in absolute value than the potential produced by Q_2. Hence, the absolute value of Q_1 is greater than that of Q_2, too.

3.21. Since potentials must be added algebraically, we conclude that at a point removed from the middle of the distance between the charges by an interval of r the potential is

$$\varphi = \frac{Q}{2\pi\varepsilon_0\varepsilon\,(a^2+r^2)^{1/2}}$$

(the potential at infinity is assumed to be equal to zero). Hence, the potential falls off as r increases in exactly the same manner on both sides of the straight line connecting the charges. At great distances ($r \gg a$), φ varies in exactly the same way as the potential produced by a point charge equal to $2Q$ does.

There are two ways in which one can determine the electric field in this problem: either directly calculating the values of the vectors and adding the vectors geometrically, just as shown in Figure (a), or employing the formula that links the electric field strength and the potential, $E = -\mathrm{d}\varphi/\mathrm{d}r$. Both methods yield

$$E_r = \frac{Qr}{2\Omega\varepsilon_0\varepsilon\,(a^2+r^2)^{3/2}}.$$

The electric field strength vanishes at exactly the middle of the distance between the charges and at an infinite distance from them. It is at its maximum, which can be found by nullifying the derivative $\mathrm{d}E_r/\mathrm{d}r$:

$$\frac{\mathrm{d}E_r}{\mathrm{d}r} = \frac{2Q}{\pi\varepsilon_0\varepsilon}\left[\frac{(a^2+r^2)^{3/2} - 3r^2\,(a^2+r^2)^{1/2}}{(a^2+r^2)^{5/2}}\right] = 0.$$

The electric field strength is maximal at $r = a/\sqrt{2}$, with

$$E_{\mathrm{m}} = \frac{0.77Q}{4\pi\varepsilon_0\varepsilon a^2}.$$

Figure (b) shows the behavior of E and φ in dimensionless coordinates: $\varphi/\varphi_{\mathrm{m}}$, E/E_{m}, and r/a.

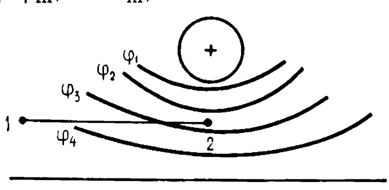


Fig. 3.22

3.22. All the equipotential surfaces of the field between the sphere and the plate are convex downward (that is, toward the plate). Hence, on any straight line parallel to the plate, the points farther from the sphere have a potential lower than those closer to the sphere. Hence, the point charge is moved from a point with a lower potential to a point with a higher potential. This requires doing work against the forces of the electric field.

3.23. Point *1* has a positive potential with respect to the negatively charged plate of *C1*. This potential is half the difference in potential between the plates of *C1* (and of *C2*). Since point *2* lies in capacitor *C2* closer to the negatively charged plate, its potential is lower than that at point *1*. When the point charge is moved from point *1* with a higher potential to point *2* with a lower potential, the electric field performs work equal to the product of the strength of the point charge by the potential difference between points *1* and *2*:

$$A = Q(\varphi_1 - \varphi_2) > 0.$$

3.24. Initially the capacitance of the capacitor (filled with the dielectric) is $C = \varepsilon_0 \varepsilon ab/l$. After the dielectric is moved out of the capacitor by a distance x, the capacitance becomes

$$C = \varepsilon_0 a\,[x + \varepsilon(b - x)]/l.$$

Since the total charge on the plates of the capacitor remains unchanged, the potential difference between the plates becomes

$$U = \frac{Ql}{\varepsilon_0 a\,[x + \varepsilon(b - x)]},$$

where Q is the charge on the plates. Since initially the potential difference was $U = Ql/\varepsilon_0 \varepsilon ab$, we have

$$\frac{U}{U_0} = \frac{\varepsilon b}{x + \varepsilon(b - x)} = \frac{\varepsilon b}{\varepsilon b - (\varepsilon - 1)x}.$$

The field strength between the plates will increase by the same factor. The charge density in the part without the dielectric is

$$\sigma_1 \varepsilon = {}_0E = \frac{\varepsilon_0 Q}{a\,[\varepsilon b - (\varepsilon - 1)x]},$$

while on the part with the dielectric it is

$$\sigma_2 = \varepsilon_0 \varepsilon E = \frac{\varepsilon_0 \varepsilon Q}{a\,[\varepsilon b - (\varepsilon - 1)x]}.$$

Initially the charge density on each plate was

$$\sigma_0 = Q/ab,$$

or, respectively

$$\frac{\sigma_1}{\sigma_0} = \frac{b}{\varepsilon b - (\varepsilon - 1)\, x/b} \quad \text{and} \quad \frac{\sigma_2}{\sigma_0} = \frac{\varepsilon}{\varepsilon - (\varepsilon - 1)\, x/b}.$$

In the part filled with the dielectric, the charge density gradually grows in the same proportion as the electric field strength and the potential difference between the plates, while the total charge of this part gradually decreases due to the increase in x. In the part not filled with the dielectric, the charge density first drops ε-fold (at $x \ll b$) and then gradually grows, approaching the value it had when the dielectric filled the entire space between the plates.

3.25. Being a conductor, each plate has the same potential at each point, while the electric field strength, which is minus one multiplied by the gradient of the potential, is highest where the plates are closest to each other. At the same time, the electric field strength near the surface of a conductor is linked with the local surface charge density through the formula $E = \sigma/\varepsilon_0\varepsilon$. For this reason, the surface charge density at point *1* is higher than that at point *2*.

3.26. The electric field strength at the core is

$$E_1 = \frac{2U}{D_1 \ln (D_2/D_1)}.$$

To find the extremum of E_1 we take the derivative,

$$\frac{dE_1}{dD_1} = -2U \frac{(\ln D_2 - \ln D_1) - 1}{[D_1 (\ln D_2 - \ln D_1)]^2},$$

and nullify it. The result is

$$\ln D_2 - \ln D_1 = 1,$$

or

$$D_1 = D_2/\mathrm{e}.$$

This corresponds to a minimum, since E_1 tends to ∞ as $D_1 \to 0$ and $D_1 \to D_2$.

3.27. Since the charges on the capacitors *C1* and *C2* are equal, the potential difference across these capacitors and the capacitance of each capacitor are linked through the following formula:

$$C_1U_1 = C_2U_2. \tag{3.27.1}$$

For capacitors *C3* and *C4* there is a similar formula:

$$C_3U_3 = C_4U_4. \tag{3.27.2}$$

For a potential difference between points a and b to be zero, we must make sure that $U_1 = U_3$ and $U_2 = U_4$. Dividing (3.27.1) by (3.27.2) termwise and canceling equal potential differences, we get

$$C_1/C_3 = C_2/C_4.$$

Note that if a constant potential difference is applied between points A and B and the capacitors leak some charge (i.e. their resistance is not very high), the distribution of potential between the capacitors is the same as in the Wheatstone bridge, that is, is proportional to the resistances.*

* These considerations must be taken into account in some other problems, too (e.g. see Problems 3.30 and 3.31).

3.28. The charge of the solid sphere is

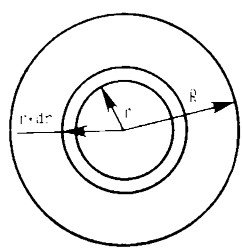
$$Q = \frac{4}{3}\pi\rho R^3,$$

where ρ is the volume charge density. Outside the sphere, that is, for $r > R$, the electric field strength coincides with the electric field strength of the same charge Q concentrated, however, at the center of the sphere:

$$E = \frac{Q}{4\pi\varepsilon_0\varepsilon r^2} = \frac{1}{3}\frac{\rho R^3}{\varepsilon_0\varepsilon r^2}. \tag{3.28.1}$$

On the surface of the sphere,

$$E_R = \frac{\rho R}{3\varepsilon_0\varepsilon}. \tag{3.28.2}$$



(a)

Fig. 3.28a

To find the electric field inside the sphere, we isolate a sphere of radius $r < R$ inside the sphere (Figure (a) accompanying the answer). The charge contained in this smaller sphere is $4\pi\rho r^3/3$. According to Gauss's theorem, the electric field at the boundary of the isolated sphere is

$$E = \frac{4\pi\rho r^3}{3\times 4\pi\varepsilon_0\varepsilon r^2} = \frac{1}{3}\frac{\rho r}{\varepsilon_0\varepsilon}. \tag{3.28.3}$$

Thus, the electric field along r behaves in two ways: inside the sphere it increases linearly with r according to (3.28.3) from zero to the value given by formula (3.28.2), while outside the sphere it decreases by a quadratic (hyperbolic) law, just as in the case of a point charge.

The behavior of the potential inside and outside the sphere must also be considered separately. Inside the sphere,

$$\int_{\varphi_0}^{\varphi} d\varphi = -\frac{\rho}{3\varepsilon_0\varepsilon}\int_0^r r\,dr = -\frac{\rho}{6\varepsilon_0\varepsilon} r^2, \quad \varphi = \varphi_0 - \frac{1}{6}\frac{\rho r^2}{\varepsilon_0\varepsilon}.$$

At the boundary of the sphere,

$$\varphi_R = \varphi_0 - \frac{1}{6}\frac{\rho R^2}{\varepsilon_0\varepsilon}.$$

Finally, outside the sphere the potential is distributed thus:

$$\int_{\varphi_R}^{\varphi} d\varphi = -\frac{1}{3}\frac{\rho R^3}{\varepsilon_0\varepsilon}\int_R^r \frac{dr}{r^2}, \quad \varphi - \varphi_R = \frac{1}{3}\frac{\rho R^3}{\varepsilon_0\varepsilon}\left(\frac{1}{r} - \frac{1}{R}\right).$$

Putting $\varphi = 0$ at $r = \infty$, we get

$$\varphi_R = \frac{1}{3}\frac{\rho R^2}{\varepsilon_0\varepsilon}. \tag{3.28.4}$$

If this is taken into account, we can write for the potential outside the sphere the following formula:

$$\varphi = \frac{1}{3}\frac{\rho R^3}{\varepsilon_0\varepsilon r}.$$

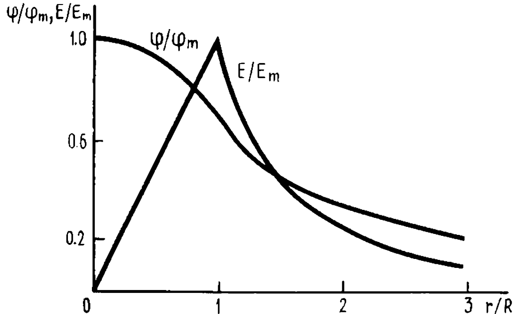
Formula (3.28.4) can also be used to find the potential at the center of the sphere:

$$\varphi_0 = \frac{1}{2}\frac{\rho R^2}{\varepsilon_0\varepsilon}.$$

For the potential distribution inside the sphere we then get

$$\varphi = \frac{\rho}{2\varepsilon_c\varepsilon}\left(R^2 - \frac{r^2}{3}\right).$$

Figure (b) accompanying the answer shows the behavior of the electric field and the potential inside and outside



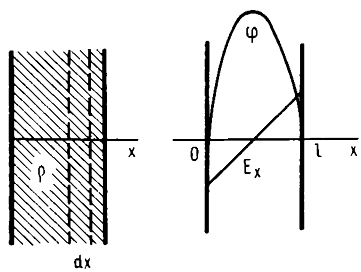
(b)

Fig. 3.28b

the sphere. Dimensionless coordinates φ/φ_m, E/E_m, and r/R are employed.

3.29. Let us isolate a thin layer of thickness dx parallel to the plates and lying between them (Figure (a) accompanying the answer). A unit area of this layer carries a volume charge ρdx. According to Gauss's theorem, the electric field generated by this layer is equal in absolute value (on each side of the layer) to

$$dE^* = \rho dx/2\varepsilon_0.$$



(a) (b)

Fig. 3.29

If all the charges to the left of the isolated layer generate a field of strength E, the resultant electric field strength is $E - dE^*$ at the left boundary of the layer and $E + dE^*$ at the right. Thus, over a distance of dx the electric field strength increases by

$$dE = 2dE^* = \rho\,(dx/\varepsilon_0).$$

Integration yields

$$E = \rho x/\varepsilon_0 + E_0,$$

with E_0 the electric field at the left plate. According to the basic equation of electrostatics (the one that links the electric field strength with the potential),

$$\frac{\rho x}{\varepsilon_0} + E_0 = -\frac{d\varphi}{dx}.$$

Integration from 0 to x yields

$$\varphi_1 - \varphi_2 = \frac{\rho x^2}{2\varepsilon_0} + E_0 x, \qquad (3.29.1)$$

where φ_1 is the potential of the left plate, which is zero by hypothesis. The potential is zero also at $x = l$. Hence,

$$E_0 = -\rho l/2\varepsilon_0.$$

Substituting this into (3.29.1), we arrive at the relationship between φ and x:

$$\varphi = \frac{\rho}{2\varepsilon_0} x (l - x).$$

This function represents a parabola with a maximum at $x = l/2$. The sketches of the φ vs. x and E vs. x curves are shown in Figure (b) accompanying the answer.

3.30. When the capacitors are connected in series, the charges on them are the same. Since these charges are

$$Q = C_1 U_1 = C_2 U_2,$$

the capacitor voltages are inversely proportional to the capacitances. Hence, the voltage applied to the capacitor filled with the dielectric is smaller than that applied to the air capacitor by a factor equal to the ratio of the dielectric constant to unity (the dielectric constant of air, roughly).

3.31. If C_0 is the initial capacitance of each capacitor, the total initial capacitance of the two capacitors is $C = C_0/2$. After the distance between the plates of one capacitor is increased, the capacitance of this capacitor, C', becomes smaller than C_0. The voltage U_0 applied to the capacitors is distributed among the capacitors in inverse proportion to the capacitances, since the charge on the plates is

$$Q = U_1 C_0 = U_2 C'.$$

Since U_0 remains unchanged, the voltage across the capacitor whose plates are not moved will decrease, while that across the second capacitor will increase.

If the capacitors are first charged and then disconnected from the DC source, the charge on them will remain unchanged. The voltage across each capacitor will be

$$U_1 = Q/C_0, \quad U_2 = Q/C_1.$$

For this reason, the potential difference across the capacitor whose plates are not moved remains unchanged, while that across the second capacitor increases.

3.32. When the capacitors are connected in parallel, the initial capacitance is $C = 2\varepsilon_0\varepsilon S/l$. After the distance between the plates is changed, the capacitance becomes

$$C = \frac{\varepsilon_0\varepsilon S}{l+a} + \frac{\varepsilon_0\varepsilon S}{l-a} = \frac{2\varepsilon_0\varepsilon S}{l-a^2/l}.$$

The new capacitance is greater than the initial one.

When the capacitors are connected in series,

$$\frac{1}{C} = \frac{2l}{\varepsilon_0\varepsilon S}.$$

After the distance between the plates is changed,

$$\frac{1}{C} = \frac{l+a}{\varepsilon_0\varepsilon S} + \frac{l-a}{\varepsilon_0\varepsilon S} = \frac{2l}{\varepsilon_0\varepsilon S},$$

that is, the capacitance remains unchanged.

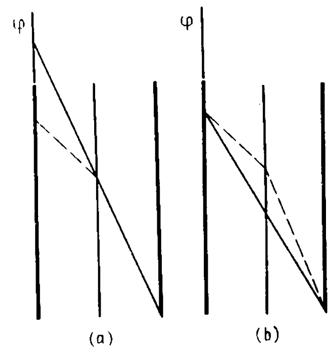


Fig. 3.33

3.33. The electric displacement vector has the same length in both halves, and since $E = D/\varepsilon_0\varepsilon$, the electric field strength is lower in the half filled with the dielectric (where the potential gradient is smaller in absolute value), that is, part *1* (see the figure accompanying the ploblem). If removal of the dielectric does not alter the charge on the plates, the potential behaves in the same way as it did in part *2* prior to removal of dielectric and the total potential difference will increase (Figure (a) accompanying the answer). If removal of the dielectric does not alter the potential difference, the points representing the potentials on the plates (φ and 0) will remain unchanged, while the slope of the straight line will acquire a value intermediate between the one it had in the dielectric and in the air prior to removal of dielectric (Figure (b) accompanying the answer).

3.34. Since the lines of force of the electric displacement vector are continuous and the field in each part is uniform, with the lines of force being perpendicular to the vacuum-dielectric interface, the electric displacements is the same in both parts. The electric field strength, which is defined by the formula $E = D/\varepsilon_0\varepsilon$, is higher in the vacuum. The electric-field energy density is determined via the formula $w = ED/2$, which shows that this quantity is higher in the vacuum.

3.35. Since the potential difference between the plates of the two capacitors is the same and so is the distance between the plates, the electric field, which for a parallel-plate capacitor is $E = U/l$, is the same for both capacitors. According to its definition, the electric displacement $D = \varepsilon_0\varepsilon E$, is greater in the capacitor with the dielectric. In a parallel-plate capacitor, the surface charge density is numerically equal to the electric displacement and therefore must be higher in the capacitor with the dielectric. This also follows from the fact that the capacitor filled with the dielectric has a higher capacitance, which means that, with a fixed potential difference, the charge on its plates is greater than that on the plates of the air capacitor. The electric-field energy density, determined via the formula $w = ED/2$, is also higher in the capacitor with the dielectric.

3.36. The total energy is the sum of the interaction energies of each charge with the other charges in the system, or

$$W = \frac{Q^2}{4\pi\varepsilon_0\varepsilon r} - 2\,\frac{QQ_1}{4\pi\varepsilon_0\varepsilon r}\,.$$

By hypothesis, $W = 0$, whence $Q_1 = Q/2$.

3.37. The energy stored by a capacitor is determined by the electric-field energy density in the capacitor and the capacitor's volume: $W = wSl$. Since the energy density is

$$w = D^2/\varepsilon_0\varepsilon, \qquad (3.37.1)$$

after the dielectric is removed, the energy of the capacitor will increase ε-fold. Since the charge on the capacitor remains unchanged, the value of the electric displacement vector remains unchanged, too. If prior to removal of the dielectric the distance between the plates was l_1 and after removal it was changed and became equal to l_2, the fact that the energy remained unchanged in the process can be expressed as follows:

$$\frac{D^2Sl_1}{\varepsilon_0\varepsilon_1}=\frac{D^2Sl_2}{\varepsilon_0\varepsilon_2}.$$

Hence the distance between the plates must be decreased ε-fold. Formula (3.37.1) shows that after the dielectric is removed (but prior to changing the distance between the plates) the capacitor increases its energy. This increase in energy is due to the work performed in removing the dielectric. The work is done against the forces of attraction of the free charges on the plates of the capacitor and the bound charges on the surface of the dielectric.

3.38. Since the capacitor voltage remains constant, the energy stored in the capacitor, $W = U^2C/2$, decreases because when the dielectric is removed, the capacitance decreases ε-fold. If the entire system consisting of the DC source and the capacitor is considered, it can be seen that the charge flows from the capacitor to the source when the dielectric is being removed. A fraction of the energy stored in the capacitor is spent on heating the leads that connect the capacitor with the source of potential, while still another fraction goes into the source. Note that removing the dielectric from the capacitor requires performing mechanical work, which must be included in the general energy balance. It is expedient, for the sake of comparison, to consider the reverse process, the introduction of a dielectric into the capacitor. Since in this case the capacitance of the capacitor grows, the energy grows, too. This growth is provided by the energy stored in the source (a DC source), which supplies the capacitor with the necessary charge as the capacitance is increased.

3.39. The problem can be related to Problem 3.38. The answer can be obtained from the general formula for the energy stored in a charged capacitor: $W = Q^2/2C$. When the capacitor is submerged into liquid dielectric, its capacitance increases, with the result that the energy stored by the capacitor decreases, since the charge on it remains unchanged. Thus, if the liquid dielectric is "sucked" into the capacitor, the capacitor-dielectric system goes over to a state with a lower energy. This process continues until the decrease in energy is compensated for by the increase in the potential energy of the layer of dielectric between the plates in the gravitation field of the earth. It must also be noted that work is done against viscosity forces when the capacitor is drawn out or submerged into the dielectric. After the capacitor is submerged into the dielectric, its capacitance will increase, while the potential difference between the plates will drop. The electric field strength, which is the same in the parts with and without the dielectric, decreases too, while the electric displacement proves to be greater by a factor of ε in the part with the dielectric as compared to the value in the part without the dielectric.

3.40. The problem can be related to Problem 3.38. There we found that into the general energy balance one must include the energy flow through the current source, which uses a fraction of its energy to increase the energy stored in the capacitor when the capacitor is submerged into the dielectric. The liquid dielectric must be "sucked" into the capacitor, just as in the previous problem. The effect of the capacitor's field on the dielectric can also be taken into account by considering the polarization of the dielectric. As a result of this process, each volume element of the dielectric becomes a dipole and is pulled into the field at the edge of the capacitor. The strength of this field is higher than that in the dielectric at a certain distance from the plates.

3.41. When the cube is compressed in the transverse direction, it is stretched in the longitudinal direction, as a result of which the upper face becomes negatively charged and the lower face becomes positively charged.

3.42. Formally, such points are determined by the expression

$$\varepsilon = D/\varepsilon_0 E.$$

Obviously, at the point where $E = 0$ and $D \neq 0$ (point 0), the dielectric constant is formally equal to infinity, while at points where $D = 0$ and $E \neq 0$ it is zero (points *3* and *6*). Of course, such values of ε are of a purely formal nature.

3.43. If l is the length of the plates of the capacitor in the system where the capacitor is at rest, in a system where the capacitor is moving with a velocity v this length is $l\sqrt{1 - v^2/c^2}$. Since the transverse dimensions of the plates do not change, the area ratio is also $1/\sqrt{1 - v^2/c^2}$. Since the charge on the capacitor remains unchanged, the surface charge density increases, with the result that $E/E_0 = 1/\sqrt{1 - v^2/c^2}$.