CBSE Class 10th Mathematics
Basic Sample Paper - 03

Maximum Marks:
Time Allowed: 3 hours

General Instructions:

a. All questions are compulsory
b. The question paper consists of 40 questions divided into four sections A, B, C & D.
c. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises 6 questions of 4 marks each.
d. There is no overall choice. However internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

Section A

1. For every natural number ‘n’, 6^n always ends with the digit
    a. 4
    b. 8
    c. 6
    d. 0
2. If HCF(72, 120) = 24, then LCM(72, 120) is
    a. 2880
    b. 240

c. 1728

d. 360

3. Which of the following is not a rational number?

$\sqrt{8}$, $\sqrt{16}$, $\sqrt{9}$, $\sqrt{25}$

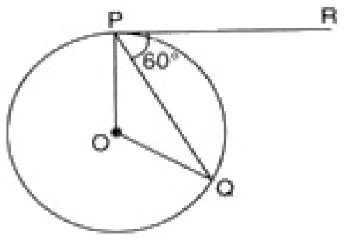
a. $\sqrt{25}$

b. $\sqrt{16}$

c. $\sqrt{8}$

d. $\sqrt{9}$

4. If O is the centre of a circle, PQ is a chord and tangent PR at P makes an angle of 60° with PQ, then $\angle$POQ is equal to



a. 110°

b. 120°

c. 100°

d. 90°

5. The mode of 4, 5, 6, 8, 5, 4, 6, 5, 6, x, 8 is 6. The value of ‘x’ is

a. 8

b. 6

c. 5

d. 4

6. An unbiased die is thrown once. The probability of getting a multiple of 3 is

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{2}{5}$

d. $\frac{2}{3}$

7. A real number ‘k’ is said to be a zero of a polynomial p(x), if p(k) =

a. 0

b. 2

c. 3

d. 1

8. A polynomial whose sum and product of zeroes are – 4 and 3 is

a. $x^2 + 4x + 3$

b. $x^2 - 4x - 3$

c. None of these

d. $x^2 - 4x + 3$

9. The distance of a point from the x – axis is called

a. None of these

b. origin

c. abscissa

d. ordinate

10. If the co – ordinates of a point are (– 5, 11), then its abscissa is

a. – 5

b. 11

c. 5

d. – 11

11. Fill in the blanks:

Three points are said to be collinear, if area of triangle formed by these points is ________.

12. Fill in the blanks:

The equation $ax^n + by^n + c = 0$ represents a straight line if 'n' = ________.

OR

Fill in the blanks:

A system of two linear equations in two variables has no solution, if their graphs _____________ at any point.

13. Fill in the blanks:

The value of $\sin\theta\ \cos\theta$, for $\theta = 30^o$ is ________.

14. Fill in the blanks:

If A and B are acute angles and sin A = cos B, then the value of (A + B) is ________.

15. Fill in the blanks:

Two polygons of the same number of sides are similar, if their corresponding angles are ________ and their corresponding sides are ________.

16. Write the value of $\sin\theta\cos(90° - \theta) + \cos\theta\sin(90° - \theta)$.

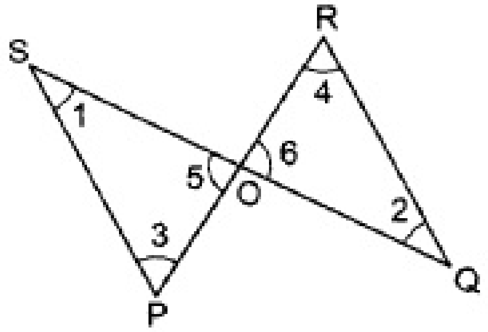
OR

Solve: $2\cos 3\theta = 1$

17. Find the radius of a circle whose circumference is equal to the sum of the circumference of two circles of diameter 36 cm and 20 cm.

18. A black die and a white die are thrown at the same time. Write all the possible outcomes. What is the probability that the numbers obtained have a product less than 16?

19. In Fig. if ΔPOS ~ ΔROQ, prove that $PS || QR$.



20. The first and last terms of an AP are a and l respectively. Show that the sum of the nth term from the beginning and the nth term from the end is (a + l).

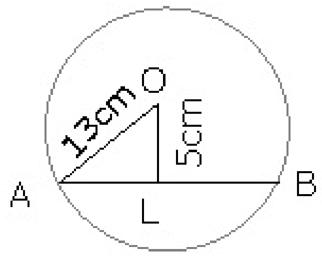
Section B

21. In a musical chair game, the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the first half minute after starting?

22. A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One of the pieces is lost at random. Find the probability that it is a

    i. triangle
    ii. square

23. In figure, if OL = 5 cm, OA = 13 cm, then length of AB is



OR

Prove that the perpendicular at the point of contact of the tangent to a circle passes through the centre.

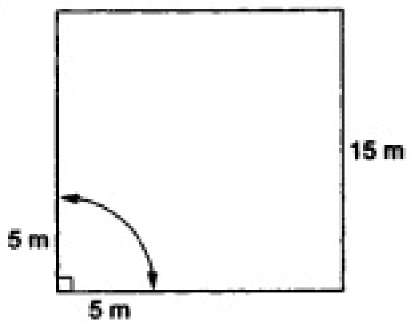
24. If $\tan\theta = \frac{1}{\sqrt{3}}$, then evaluate $\left[\frac{cosec^2\theta - \sec^2\theta}{cosec^2\theta + \sec^2\theta}\right]$

OR

If $\sin\theta = \frac{3}{5}$, find the value of $(\tan\theta + \sec\theta)^2$

25. A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5-m-long rope (as shown in the figure).

i. Find the area of that part of the field in which the horse can graze,
ii. the increase in the grazing area if the rope were 10 m long instead of 5 m. [Use π= 3.14.]



26. In the class test of mathematics, a teacher asked his students to write different kinds of polynomials. 6 students wrote the following polynomials. Identify the type of polynomials written by these students:

i. $f(p) = 3 - p^2 + \sqrt{7}p$
ii. $p(v) = \sqrt{3}v^4 - \frac{2}{3}v + 7$
iii. $q(x) = \frac{\sqrt{2}}{5}x^3 + 1$

iv. $p(z) = \sqrt{5}z + 2\sqrt{2}$

v. $r(t) = \frac{-t+3t^2-4t^3}{t}$

Section C

27. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

28. Draw a pair of tangents to a circle of radius 3 cm which are inclined to each other at angle of 60°.

OR

Construct a triangle ABC in which AB = 5 cm, BC = 6 cm and AC = 7 cm. Construct another triangle similar to $\triangle$ABC such that its sides are $\frac{3}{5}$ of the corresponding sides of $\triangle$ABC.

29. A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cms. Find the height to which the water rises.

30. Prove the identity:

$(\tan A + \operatorname{cosec} B)^2 - (\cot B - \sec A)^2 = 2\tan A \cot B (\operatorname{cosec} A + \sec B)$

OR

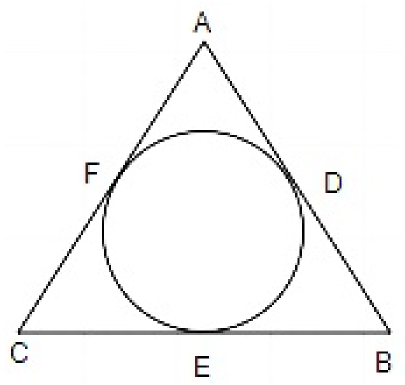
If $\sin 3\theta = \cos(\theta - 6°)$, where 3θ and $\theta - 6°$ are both acute angles, find the value of θ.

31. Use Euclid's division algorithm to find the HCF of 184, 230 and 276.

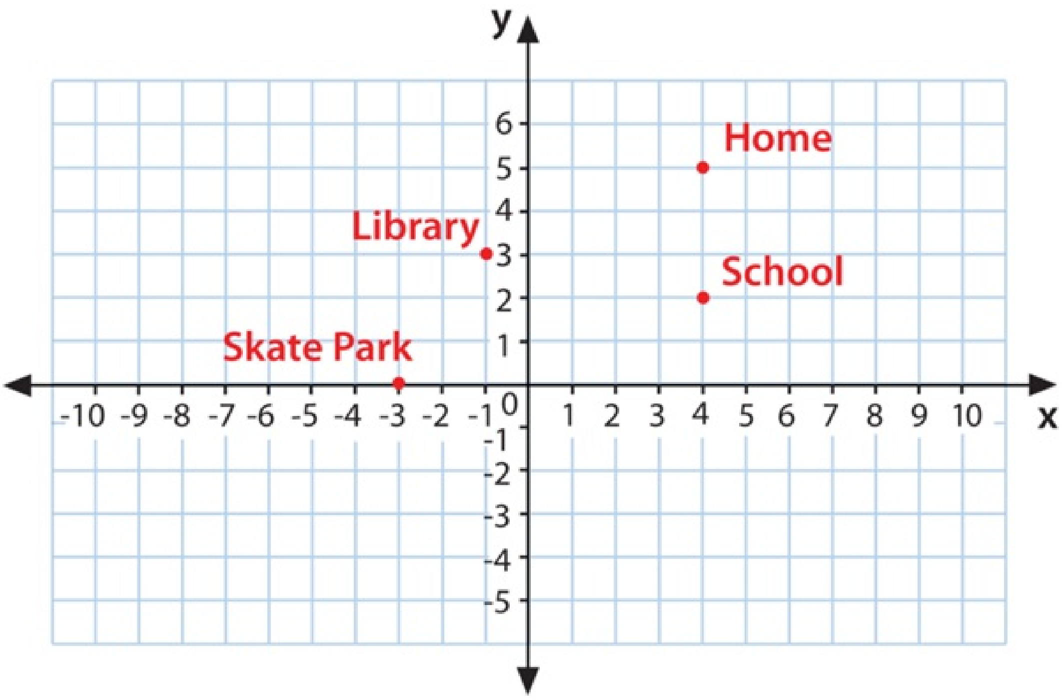
OR

Prove that $3 + \sqrt{5}$ is an irrational number.

32. A circle is inscribed in a $\triangle ABC$ having sides AB = 8 cm, BC = 10 cm and CA = 12 cm as shown in the figure. Find AD, BE and CF.



33. Two brothers Ramesh and Pulkit were at home and have to reach School. Ramesh went to Library first to return a book and then reaches School directly whereas Pulkit went to Skate Park first to meet his friend and then reaches School directly.



i. How far is School from their Home?
ii. What is the extra distance travelled by Ramesh in reaching his School?
iii. What is the extra distance travelled by Pulkit in reaching his School? (All distances are measured in metres as straight lines)

34. A number consisting of two digits is seven times the sum of its digits. When 27 is subtracted from the number, the digits are reversed. Find the number.

Section D

35. A train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr

from its usual speed. Find the usual speed of the train.

36. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n^{th} term and S_n the sum of first n terms, find n and d, if a = 8, a_n = 62 and S_n = 210.

OR

Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n^{th} term and S_n the sum of first n terms, find n and a_n, if a = 2, d = 8 and S_n = 90.

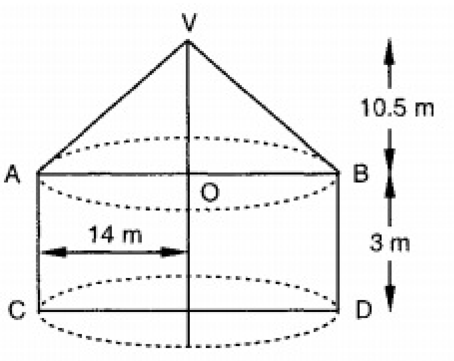
37. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it.

38. P is a point in the interior of rectangle ABCD. If P is joined with the vertices of the rectangle, then show that $PB^2 + PD^2 = PA^2 + PC^2$.

OR

ABC is an isosceles triangle with AB = AC and D is a point on AC such that $BC^2 = AC \times CD.$ Prove that BD = BC.

39. A tent is of the shape of a right circular cylinder upto a height of 3 metres and then becomes a right circular cone with a maximum height of 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs 2 per square metre, if the radius of the base is 14 metres.



OR

A cone of radius 10cm divided into two parts by drawing a plane through the mid-point of its axis, parallel to its base. Compare the volume of the two parts.

40. In an orchard, the numbers of apples on trees are given below :

Number of apples	more than or equal to 50	more than or equal to 60	more than or equal to 70	more than or equal to 80	more than or equal to 90	more than or equal to 100	more than or equal to 110
Number of trees	60	55	39	29	10	6	2

Draw a 'more than type' ogive and hence obtain median from the curve.

Solution

Section A

1. (c) 6

   Explanation:

   Let us assume for some integer k we have $6^k = 10x + 6$

   $\therefore 6^{k+1} = 6.6^k$ = $6\left(10x+6\right)$

   $\Rightarrow 6^{k+1} = 60x + 36$

   $\Rightarrow 6^{k+1} = 60x + 30 + 6$

   $\Rightarrow 6^{k+1} = 10\left(6x+3\right) + 6$

   $\therefore$ If 6^k ends with 6,

   then 6^{k+1} ends with 6 for all natural numbers.

2. (d) 360

   Explanation:

   Using the result, HCF × LCM = Product of two natural numbers

   $\Rightarrow$ LCM (72, 120) = $\frac{72\times120}{24}$ = 360

3. (c) $\sqrt{8}$

   Explanation:

   $\sqrt{1}6$= $\sqrt{4} \times \sqrt{4}$ = 4 is a rational number

   $\sqrt{9}$ = $\sqrt{3} \times \sqrt{3}$ = 3 is a rational number

   $\sqrt{2}5$ = $\sqrt{5}$ x$\times\sqrt{5}$ = 5 is also a rational number

   but $\sqrt{8}$ is not a rational number

because $\sqrt{8} = 2\sqrt{2}$

and $\sqrt{2}$ is an irrational number

therefore $\sqrt{8}$ is also an irrational number.

4. (b) 120°

   Explanation:

   Here ∠RPO = 90°

   ∠RPQ = 60° (given)

   ∴∠OPQ = 90° - 60° = 30° ∠PQO = 30° Also [Opposite angles of equal radii] Now, In triangle OPQ,

   $\angle\text{OPQ} + \angle\text{PQO} + \angle\text{QOP} = 180^\circ$

   ⇒30° + 30° + ∠QOP = 180°

   ⇒ ∠QOP = 120°

5. (b) 6

   Explanation:

   Here Observation is 6 has more frequency than that of other numbers.

   ∴ 6 could repeat itself at least once more.

   ⇒ x should be 6.

6. (b) $\frac{1}{3}$

   Explanation:

   Number of multiple of 3 on a dice = {3, 6}, = 2

   Number of possible outcomes = 2

   Number of Total outcomes = 6

   ∴ Required Probability = $\frac{2}{6} = \frac{1}{3}$

7. (a) 0

   Explanation:

   A real number ‘k’ is said to be a zero of a polynomial p(x), if p(k) is equals to 0.

   Explanation: if P(x) is a Polynomial in x and k is any real number, then value of P(k) at x = k is denoted by P(k) is found by replacing x by k in P(x).

e.g., In the polynomial $x^2–3x+2$,

Replacing x by 1 gives,

P(1) = 1–3+2 = 0

Similarly, replacing x by 2 gives,

P(2) = 4–6+2 = 0

For a polynomial P(x), real number k is said to be zero of polynomial P(x), if P(k) = 0.

8. (a) $x^2 + 4x + 3$
Explanation:

x^2- (Sum the Zeroes)x + (Product of Zeroes)

x^2 - (-4)x + 3

= x^2 + 4x + 3

9. (d) ordinate
Explanation:

The distance of a point from the x – axis is the y (vertical) coordinate of the point and is called ordinate.

10. (a) – 5
Explanation:

Since $x-$coordinate of a point is called abscissa.
Therefore, abscissa is -5.

11. zero

12. n = 1

OR

do not intersect

13. $\frac{\sqrt{3}}{4}$

14. 90^{o}

15. equal, proportional

16. We know that Sin(90° - x) = Cosx & Cos(90°-x) = Sinx. Using these in the given expression,
we get,
sinθ cos(90° - θ) + cosθ sin(90° - θ)
= sinθ $\times$ sinθ + cosθ $\times\theta$ cos
= sin$^2\theta$ + cos$^2\theta$
= 1

OR

Now we have,
$2\cos 3\theta = 1$
$\Rightarrow \cos 3\theta = \frac{1}{2} \Rightarrow \cos 3\theta = \cos 60^\circ$ [Since, cos60°=(1/2)]
$\Rightarrow 3\theta = 60^\circ$
$\therefore \theta = 20^\circ$

17. $d_1 = 36$cm and $d_2 = 20$ cm
So, $r_1 = 18$cm and $r_2 = 10$ cm
$\because$ Circumference of the circle = $2\pi r$
According to the question,
$2\pi r = 2\pi \times r_1 + 2\pi \times r_2$
or, $2\pi r = 2\pi \times 18 + 2\pi \times 10$
or, $2\pi r = 2\pi(18 + 10)$
or,$r = 28$cm
Hence radius of given circle = 28 cm.

18. Consider the set of ordered pairs
{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)

(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)

(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)

(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)

(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)}

Clearly, there are 36 elementary events.

∴ n(Total number of throws) = 36

Number of pairs such that the numbers obtained have a product less than 16 can be selected as listed below:

{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)

(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)

(3,1)(3,2)(3,3)(3,4)(3,5)

(4,1)(4,2)(4,3)

(5,1)(5,2)(5,3)

(6,1)(6,2)}

Therefore, n(Favourable events) =25

P(the number obtained appearing have a product less than 16)=

$\frac{number\ obtained\ have\ a\ product\ less\ than\ 16}{Total\ number\ throws} = \frac{25}{36}$

19. We have,

ΔPOS ~ ΔROQ

$\Rightarrow \angle$3 = $\angle$4 and $\angle$1 = $\angle$2

Thus, PS and QR are two lines and the transversal PR cuts them in such a way that $\angle$3 = $\angle$4 i.e., alternate angles are equal. Hence, $PS || QR$.

20. Suppose a be the first term and d be the common difference

∴ nth term from the beginning $= a + (n - 1)d$ .....(1)

nth term from end$= l - (n - 1)d$.....(2)

Adding equation (1) and (2),

Sum of the nth term from the beginning and nth term from the end

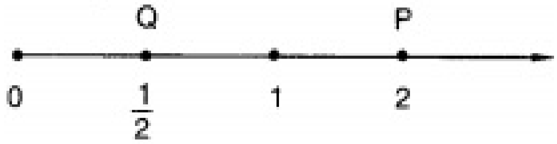
$= [a + (n - 1)d] + [l - (n - 1)d] = a + l$

Section B

21. The possible outcomes are all the numbers between $0\ and\ 2$.

Suppose A be the event 'music is stopped within the first half minute'.

$\therefore$ Outcomes favourable to the event A are all points on the number line from O to Q i.e. from $0\ to\ \frac{1}{2}$



Total number of outcomes are the points on the number line from O to P i.e. from 0 to 2.

$\therefore P(A)=\frac{\text{Length } OQ}{\text{Length } OP}=\frac{1/2}{2}=\frac{1}{4}$

22. According to question, we have 8 triangles in which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red i.e

Triangles with blue colour = 3

Triangles with red colour = 8 - 3 = 5

Total no. of squares = 10

Squares with blue colour = 6

Squares with red colour = 10 - 6 = 4

Now,

i. Number of favorable outcomes for the event that lost figure is triangle i.e., F(E) = 8
Total figures (square and triangle) = 8 + 10 = 18
i.e., T(E) = 18
Therefore, Probability (getting a triangle) P(E) = $\frac{F(E)}{T(E)}=\frac{8}{18}=\frac{4}{9}$
ii. Number of favourable outcomes for the events that squares is lost i.e., F(E) = 10
T(E) = 8 + 10 = 18 i.e T(E)=18
Therefore P(getting a square) = P(E) $=\frac{10}{18}=\frac{5}{9}$

23. AB = 2 AL = 2$\sqrt{OA^2-OL^2}$
= 2 $\sqrt{13^2-5^2}$
= 2 $\sqrt{169-25}$ = 2$\sqrt{144}$
= 2 $\times$ 12 = 24 cm

OR

Let O be the centre of the given circle.
AB is the tangent drawn touching the circle at A.
Draw AC ⊥ AB at point A, such that point C lies on the given circle.
∠OAB = 90° (Radius of the circle is perpendicular to the tangent)
Given ∠CAB = 90°
∴ ∠OAB = ∠CAB
This is possible only when centre O lies on the line AC.
Hence, perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

24. $\tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta$ = 30°

$\frac{cosec^2\theta - \sec^2\theta}{cosec^2\theta + \sec^2\theta} = \frac{cosec^2 30^\circ - \sec^2 30^\circ}{cosec^2 30^\circ + \sec^2 30^\circ}$

$= \frac{(2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2}{(2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2}$

$= \frac{4 - \frac{4}{3}}{4 + \frac{4}{3}} = \frac{1}{2}$

OR

We have, $\sin\theta = \frac{3}{5}$

$\therefore \quad \cos\theta = \sqrt{1 - \sin^2\theta} \Rightarrow \cos\theta = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$ and, $\sec\theta = \frac{1}{\cos\theta} \Rightarrow \sec\theta = \frac{5}{4}$

Hence, $(\tan\theta + \sec\theta)^2 = \left(\frac{3}{4} + \frac{5}{4}\right)^2 = \left(\frac{8}{4}\right)^2 = 4$

25. i. Clearly, the required area is the area of a quadrant of a circle of radius 5 m.

$\therefore$ required area $= \frac{1}{4} \times \pi r^2$ = 19.625 m^2

ii. Let the length of the rope be 10 m. Then,

area grazed $= \frac{1}{4} \times \pi R^2$

$= \left(\frac{1}{4} \times 3.14 \times 10^2\right) m^2$

= 78.50 m^2

Increase in grazing area = (78.50 -19.625) m^2

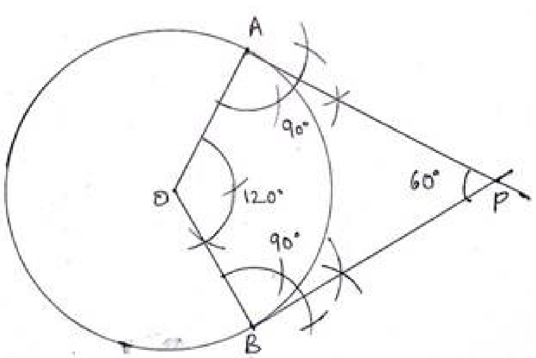
= 58.875 m^2.

26. i. Quadratic polynomial
    ii. Biquadratic polynomial
    iii. Cubic polynomial
    iv. Linear polynomial
    v. Quadratic polynomial

Section C

27. If α and β are the zeroes of polynomial $2x^2$ - 3x + 1,

a=2, b=-3, c=1
then $\alpha+\beta=\frac{-b}{a}=\frac{3}{2}$
$\alpha\beta=\frac{c}{a}=\frac{1}{2}$
General equation of the quadratic polynomial = x^2 - (Sum of the roots) x + Product of the roots
$\therefore$The quadratic polynomial whose zeroes are 3α and 3β is :
$=x^2-(3\alpha+3\beta)x+3\alpha\times3\beta$
$=x^2-3(\alpha+\beta)x+9\alpha\beta$
$=x^2-3\left(\frac{3}{2}\right)x+9\left(\frac{1}{2}\right)$
$=x^2-\frac{9}{2}x+\frac{9}{2}$
$=\frac{1}{2}\left(2x^2-9x+9\right)$
$\therefore$ the required quadratic polynomial is $\frac{1}{2}\left(2x^2-9x+9\right)$.

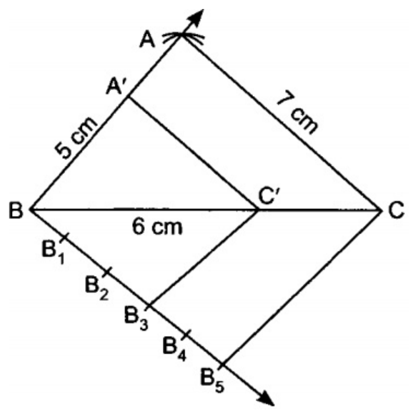
28. 

Steps of construction:

i. Draw a circle of radius 3 cm with center O.
ii. Take a point A on the circumference of the circle and join OA.
iii. Draw a perpendicular to OA at point A.
iv. Draw a radius OB, making an angle of 120° (180° -60°) i.e. $\angle AOB = 120^\circ$
v. Draw a perpendicular to OB at point B..
vi. Let the two perpendiculars intersect each other at P. Then, PA and PC are required tangents.

Justification:
Since OA is the radius, so PA has to be a tangent to the circle. Similarly, PC is also tangent to the circle.
$\angle APC = 360^\circ - (\angle OAP + \angle OCP + \angle AOC)$
= 360° - (90° + 90° + 120°)
= 360° - 300°
= 60°
Hence, tangents PA and PC are inclined to each other at an angle of 60°.

OR



Steps of Construction:

i. Draw a line segment BC = 6 cm.
ii. Draw an arc from B of radius 5 cm.
iii. Again draw an arc from C of radius 7 cm, cutting the first arc at A.
iv. Join AB and AC. We get the required ΔABC.
v. An acute angle CBX is drawn below BC.
vi. On BX, points B_1, B_2, B_3, B_4, B_5 are taken such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

vii. Join B_5 and C.

viii. Draw B_3C' parallel to B_5C meeting BC at C'.

ix. And draw C'A' is parallel to CA, meeting BA at A'.

x. Then $\triangle$A'BC' is the required triangle similar to $\triangle$ABC, where sides are $\frac{3}{5}$ of the corresponding sides of $\triangle$ABC.

29. A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cms.We have to find the height to which the water rises.

We have,

r_1 = radius of the conical vessel = 5 cm, h_1 = height of the conical vessel = 24 cm and, r_2 = radius of the cylindrical vessel = 10 cm

Suppose water rises upto the height of h_2 cm in the cylindrical vessel. Clearly, Volume of water in conical vessel = Volume of water in cylindrical vessel

$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2$

$\Rightarrow \quad r_1^2 h_1 = 3r_2^2 h_2$

$\Rightarrow 5 \times 5 \times 24 = 3 \times 10 \times 10 \times h_2$

$\Rightarrow \quad h_2 = \frac{5\times5\times24}{3\times10\times10} = 2\text{cm}$

Hence, the height of water in the cylindrical vessel is 2 cm.

30. We have,

LHS = (tanA + cosec B)^2 - (cotB - sec A)^2

$\Rightarrow$ LHS = (tan^{2}A + cosec^{2}B + 2tanA cosecB) - (cot^{2}B + sec^{2}A - 2cotB secA)

$\Rightarrow$ LHS = (tan^{2}A - sec^{2}A)+(cosec^{2}B - cot^{2}B)+2tanA cosecB +2cotB secA

But, Sec^{2}A - tan^{2}A =1 & cosec^{2}A - cot^2 A = 1

$\therefore$ LHS = -1 + 1 + 2 tanA cosecB + 2cotB secA

$\Rightarrow$ LHS = 2 (tanA cosecB + cotB secA)

$\Rightarrow$ LHS = 2 tanA cotB$\left(\frac{cosec\ B}{\cot B} + \frac{\sec A}{\tan A}\right)$ [Dividing and multiplying by tanA cotB]

$\Rightarrow$ LHS = 2tan A cotB$\left\{\frac{\frac{1}{\sin B}}{\frac{\cos B}{\sin B}} + \frac{\frac{1}{\cos A}}{\frac{\sin A}{\cos A}}\right\}$ [Since, CosecA.SinA =1 , SecA.cosA =1, (sinA/cosA)= tanA & (cosA/SinA) =cotA]

$\Rightarrow$ LHS = 2 tanA cotB$\left(\frac{1}{\cos B}+\frac{1}{\sin A}\right)$ = 2tanA cotB (secB + cosecA) = RHS. Hence, proved.

OR

Given,

sin 3θ = cos (θ - 6°)

cos (90° - 3θ) = cos (θ - 6°)

90° - 3θ = θ - 6 °

4θ = 90° + 6° = 96°

$\therefore \quad \theta=\frac{96^{\circ}}{4}=24^{\circ}$

31. Given numbers are 184, 230, and 276.

Applying Euclid's division lemma to 184 and 230, we get

230 = 184 $\times$ 1 + 46

184 = 46 $\times$ 4 + 0

The remainder at this stage is zero.

So, the divisor at this or the remainder at the previous stage i.e., 46 is the HCF of 184 and 230.

Also,

276 = 46 $\times$ 6 + 0

$\therefore$ HCF of 276 and 46 is 46

HCF (184, 230, 276) = 46

Hence, the required HCF of 184, 230 and 276 is 46.

OR

Let 3 + $\sqrt{5}$ is a rational number.

$3+\sqrt{5}=\frac{p}{q}, q \neq 0$

$3+\sqrt{5}=\frac{p}{q}$

$\Rightarrow \quad \sqrt{5}=\frac{p}{q}-3$

$\Rightarrow \quad \sqrt{5}=\frac{p-3q}{q}$

Now in RHS {\frac{p-3q}p}} is rational

This shows that $\sqrt{5}$ is rational
But this contradict the fact that $\sqrt{5}$ is irrational, This is because we assumed that 3 + $\sqrt{5}$ is a rational number.
$\therefore 3+\sqrt{5}$ is an irrational number.

32. We know that, tangents drawn from an exterior point to a circle are equal in length.
$\therefore AD = AF = x\ cm$
$BD = BE = y\ cm$
$CE = CF = z\ cm$
Given, AB = 8 cm
$\Rightarrow AD + BD = 8\ cm \Rightarrow x + y = 8$....(i)
$BC = 10\ cm \Rightarrow BE + CE = 10 \Rightarrow y + z = 10$....(ii)
and $CA = 12\ cm \Rightarrow CF + AF = 12\ cm \Rightarrow z + x = 12$...(iii)
On adding Eqs(i), (ii) and (iii), we get
$2(x + y + z) = 30$
$\Rightarrow x + y + z = 15$....(iv)
On subtracting Eq (ii) from Eq (iv), we get
$x = 15 - 10 = 5$
On subtracting Eq (iii) from Eq (iv), we get
$y = 15 - 12 = 3$
On subtracting Eq (i) from Eq (iv), we get
$z = 15 - 8 = 7$
$\therefore AD = x\ cm = 5\ cm$
$BE = y\ cm = 3\ cm$
and $CF = z\ cm = 7\ cm$
Hence, the length of AD, BE and CE are 5 cm, 3 cm and 7 cm, respectively.

33. Let Home represented by point H(4, 5), Library by point L(-1, 3), Skate Park by point P(3, 0) and School by S(4, 2).

    i. Distance between Home and School, HS = $\sqrt{(4-4)^2 + (2-5)^2} = 3\ metres$
    ii. Now, HL = $\sqrt{(-1-4)^2 + (3-5)^2} = \sqrt{25+4} = \sqrt{29}$

LS = $\sqrt{[4-(-1)]^2+(2-3)^2} = \sqrt{25+1} = \sqrt{26}$

Thus, HL + LS = $\sqrt{29}+\sqrt{26} = 10.48\ metres$

So, extra distance covered by Ramesh is = HL + LS - HS = 10.48 - 3 = 7.48 metres

iii. Now, HP = $\sqrt{(3-4)^2+(0-5)^2} = \sqrt{1+25} = \sqrt{26}$

PS = $\sqrt{[4-3]^2+(2-0)^2} = \sqrt{1+4} = \sqrt{5}$

Thus, HP + PS = $\sqrt{26}+\sqrt{5} = 7.33\ metres$

So, extra distance covered by Ramesh is = HP + PS - HS = 7.33 - 3 = 4.33 metres

34. Let the ten's digit of required number be x and its unit digit be y respectively.

Required number = 10x + y

According to the question, it is given that a number consisting of two digits is 7 times the sum of its digits.

∴ 10x + y = 7(x + y)

10x + y = 7x + 7y

10x - 7x - 7y + y = 0

3x - 6y = 0............(i)

when 27 is subtracted from the number the digits are reversed.

After reversing the digits, the number = 10y + x

∴(10x + y) - 27 = 10y + x

10x - x + y - 10y = 27

9x - 9y = 27

x - y = 3............(ii)

Multiplying (i) by 1 and (ii) by 6, we get

3x - 6y = 0........(iii)

6x - 6y = 18.......(iv)

Subtracting (iii) from (iv), we get

3x = 18

$x = \frac{18}{3} = 6$

Put the value of x = 6 in equation (i), we get

3 $\times$ 6 - 6y = 0

18 - 6y = 0

$-6y = -18 \Rightarrow y = \frac{-18}{-6} = 3$

Number = 10x + y

= 10 $\times$ 6 + 3

= 60 + 3

= 63

Hence the number is 63.

Section D

35. Let the usual speed of train be x km/hr

$\frac{300}{x} - \frac{300}{x+5} = 2$

300 (x+5 - x) = 2x(x+5)

150(5) = x^2 + 5x

750 = x^2 + 5x

or, $x^2 + 5x - 750 = 0$

or, $x^2 + 30x - 25x - 750 = 0$

or, (x + 30) (x - 25) = 0

or, x = - 30 or x = 25

Since, speed cannot be negative.

$\therefore \quad x \neq -30, x = 25\text{km/hr}$

$\therefore$ Speed of train = 25 km/hr

36. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n^{th} term and S_n the sum of first n terms, therefore

First term(a) = 8

and, nth term (a_n) = 62

$\Rightarrow$a + (n - 1)d = 62

$\Rightarrow$8 + (n - 1)(d) = 62

$\Rightarrow$(n - 1)d = 62 - 8

$\Rightarrow$(n - 1)d = 54................(i) since, S_n = 210

$\Rightarrow \quad \frac{n}{2}[a + a_n] = 210$

$\Rightarrow \quad \frac{n}{2}[8 + 62] = 210$

$\Rightarrow \quad \frac{n}{2} \times 70 = 210$

$\Rightarrow \quad n = \frac{210 \times 2}{70}$

$\Rightarrow$n = 6

Put the value of n in eq(i),

(6 - 1)d = 54

$\Rightarrow$ 5d = 54

$\Rightarrow \quad d = \frac{54}{5}$

OR

Given that, a = 2, d = 8 and S_n = 90.

As, S_n= $\frac{n}{2}[2a + (n-1)d]$

$90 = \frac{n}{2}[4 + (n-1)8]$

90 = n[2 + (n - 1)4]

90 = n[2 + 4n - 4]

90 = n(4n - 2) = $4n^2$ - 2n

$4n^2$ - 2n - 90 = 0

$4n^2$ - 20n + 18n - 90= 0

4n(n - 5) + 18(n - 5) = 0

(n - 5)(4n + 18) = 0

Either n = 5 or $n = -\frac{18}{4} = \frac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, n = 5

a_n = a + (n - 1)d

a_5 = 2 + (5 - 1)8

= 2 + 4(8)

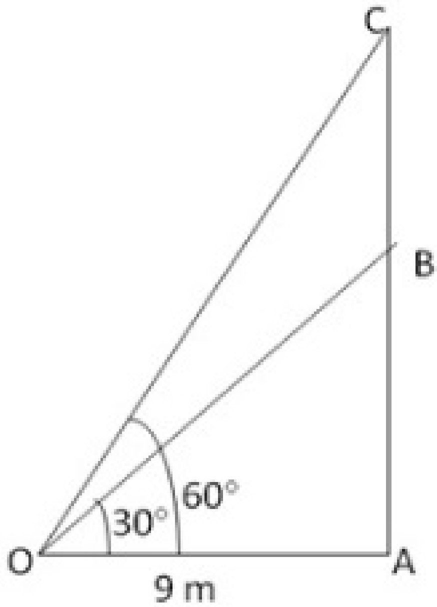
= 2 + 32 = 34

37. Let us suppose that AB be the tower and BC be flagpole

Let us suppose that O be the point of observation.Then, OA = 9m

According to question it is given that

$\angle AOB = 30^\circ$ and $\angle AOC = 60^\circ$



From right angled $\triangle \mathrm{BOA}$

$\frac{AB}{OA} = \tan 30^\circ$

$\Rightarrow \frac{AB}{9} = \frac{1}{\sqrt{3}} \Rightarrow AB = 3\sqrt{3}$

From right angled ΔOAC

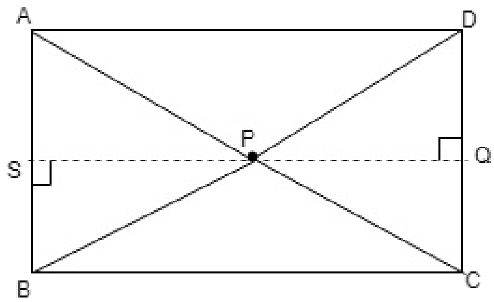
$\frac{AC}{OA} = \tan 60^\circ$

$\frac{AC}{9} = \sqrt{3} \Rightarrow AC = 9\sqrt{3}\mathrm{m}$

$\therefore$ BC = (AC - AB) $= 6\sqrt{3}\mathrm{m}$

Thus $AB = 3\sqrt{3}m = 5.196m$ and $BC = 6\sqrt{3}m = 10.392m$

Hence, height of the tower is 5.196m and the height of the flagpole is 10.392 m

38. 

Given: ABCD is a rectangle and P is inside the rectangle.

Let us draw a line SQ || BC.

$\Rightarrow$ SQ || AD [Opposite sides of the rectangle are parallel]

$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90^\circ$ [Angle of a rectangle]

Since ABCD is a rectangle,

$\Rightarrow$ ADQS and SQCB also rectangles [By construction]

$\Rightarrow \angle PSB = \angle PQC = \angle PQD = \angle PSA = 90^\circ$

For right $\triangle$PSB,

$PB^2 = PS^2 + SB^2$ ....(i) [By Pythagoras theorem]

For right $\triangle$PQD,

$PD^2 = PQ^2 + QD^2$ ....(ii) [By Pythagoras theorem]

Add eq. (i) and (ii), we get,

$PB^2 + PD^2 = PS^2 + SB^2 + PQ^2 + QD^2$....(iii)

For right $\triangle$PSA,

$PA^2 = AS^2 + PS^2$....(iv) [By Pythagoras theorem]

For right $\triangle$PQC,

$PC^2 = PQ^2 + QC^2$....(v) [By Pythagoras theorem]

Adding eq. (iv) and (v), we get,

$PA^2 + PC^2 = AS^2 + PS^2 + PQ^2 + QC^2$

$PA^2 + PC^2 = PS^2 + QC^2 + PQ^2 + AS^2$

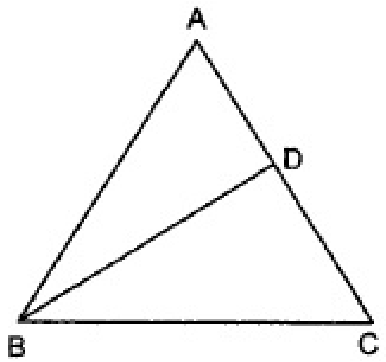
$PA^2 + PC^2 = PS^2 + SB^2 + PQ^2 + QD^2$....(vi) [$\because$ SB = QC, AS = QD, opposite sides of rectangle]

Equating eq. (iii) and (vi) we get,

$PB^2 + PD^2 = PA^2 + PC^2$

Hence Proved.

OR



We have,

$BC^2 = AC \times CD$ and AB = AC

$\Rightarrow \quad BC \times BC = AC \times CD$ and $\angle B = \angle C$[angles opposite to equal sides of a triangle are equal]

$\Rightarrow \quad \frac{BC}{AC} = \frac{CD}{BC}$ and $\angle B = \angle C$

$\Rightarrow \quad \frac{BC}{CA} = \frac{DC}{CB}$ and $\angle B = \angle C$

So, by SAS-criterion of similarity, we obtain

$\Delta BCA \sim \Delta DCB$

$\Rightarrow \quad \frac{BC}{DC} = \frac{CA}{CB} = \frac{BA}{DB}$

$\Rightarrow \quad \frac{CA}{CB} = \frac{BA}{DB}$

$\Rightarrow \quad \frac{BA}{CA} = \frac{DB}{CB}$

$\Rightarrow \quad 1 = \frac{DB}{CB}$ [$\because$ AB = AC]

$\Rightarrow \quad DB = CB \Rightarrow BD = BC$

39. Height of the cylinder = 3 m.

Total height of the tent above the ground = 13.5 m

$\therefore$ height of the cone = (13.5 - 3)m = 10.5 m

Radius of the cylinder = radius of cone = 14 m

Curved surface area of the cylinder = $2\pi rh \mathrm{m}^2 = \left(2 \times \frac{22}{7} \times 14 \times 3\right) \mathrm{m}^2 = 264 \mathrm{m}^2$

$\therefore \quad l = \sqrt{r^2 + h^2} = \sqrt{14^2 + (10.5)^2} = \sqrt{196 + 110.25} = \sqrt{306.25} = 17.5$

$\therefore$ Cured surface area of the cone = $\pi rl = \left(\frac{22}{7} \times 14 \times 17.5\right) \mathrm{m}^2 = 770 \mathrm{m}^2$

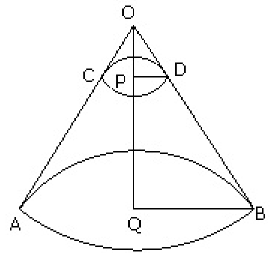
Let S be the total area which is to be painted.Then,

S = Curved surface area of the cylinder + Curved surface area of the cone

$\Rightarrow$ S = (264 + 770) m^2 = 1034 m^2

Hence, Cost of painting = S $\times$ Rate = Rs(1034×2)= Rs2068

OR



Let OAB be the cone and OQ be its axis and P be the mid-point of OQ

Let OQ = h cm

Then OP = PQ = $\frac{h}{2}$ cm

And QB = 10cm

Also $\triangle$OPD $\sim$ $\triangle$OQB

$\therefore \frac{OP}{OQ} = \frac{PD}{QB}$

$\frac{\frac{h}{2}}{h} = \frac{PD}{10\text{cm}}$

$\Rightarrow$ PD = 5 cm

i. A smaller cone of radius = 5cm and height = $\frac{h}{2}$ cm
ii. Frustum of a cone in which
    R = 10cm, r = 5 cm, height = $\frac{h}{2}$ cm
    Volume of smaller cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 5 \times 5 \times \frac{h}{2} = \frac{25\pi h}{6}\text{cm}^3$
    The volume of a frustum of the cone =
    $\frac{1}{3}\pi\frac{\text{h}}{2}\left[(10)^2 + (5)^2 + 10 \times 5\right] \text{cm}^3 = \frac{175\pi\text{h}}{6}\text{cm}^3$
    Ratio of required volume= $\frac{25\pi\text{h}}{6} : \frac{175\pi\text{h}}{6}$
    = 25 : 175 = 1 : 7

40.

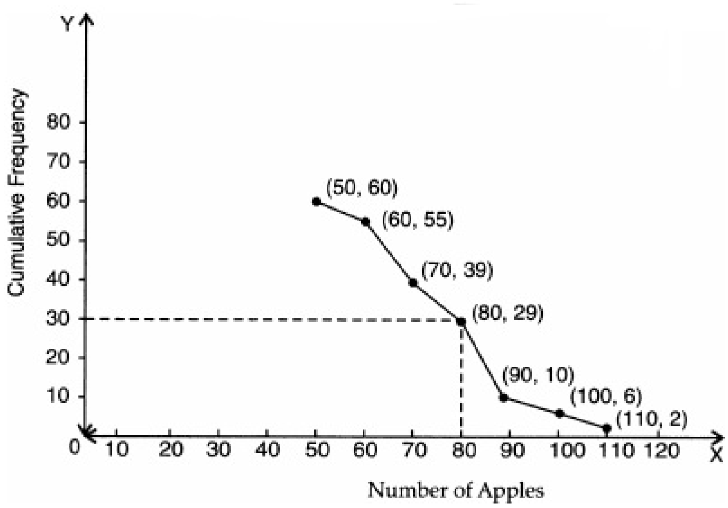
Apples	c.f.
More than 50	60
More than 60	55
More than 70	39

More than 80	29
More than 90	10
More than 100	6
More than 110	2

This curve shows cumulative frequency on an ogive of the 'more than type'.

Units : x-axis 1 cm = 10

y-axis 1 cm = 10



Here N = 60,

Units on: x-axis 1 cm = 10

x-axis 1 cm = 10

So $\frac{N}{2} = \frac{60}{2}$ = 30

Now, we locate the point on the ogive whose ordinate is 30. The x- coordinate corresponding to this ordinate is 79.

Hence, the required median on the graph is 79.