Short Answer Type Questions – II

[3 marks]

Que 1. In Fig. 7.11, DE || BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x 0- 1, find the value of x.

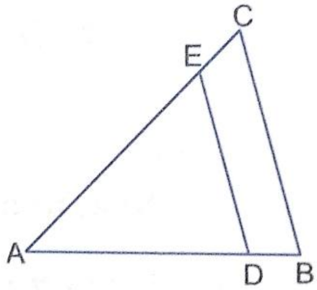

Fig. 7.11

Sol. In ΔABC, we have

DE|| BC.

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [By Basic Proportionality Theorem]

$\Rightarrow \quad \frac{x}{x-2} = \frac{x+2}{x-1} \quad \Rightarrow \quad x(x-1) = (x-2)(x+2)$

$\Rightarrow \quad x^2 - x = x^2 - 4 \quad \Rightarrow x = 4$

Que 2. E and Fare points on the sides PQ and PR respectively of a ΔPQR. Show that EF || QR. If PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.

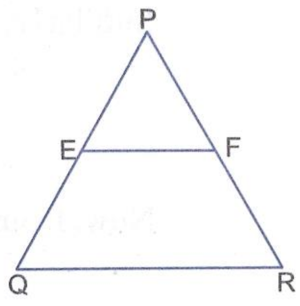

Fig. 7.12

Sol. We have, PQ = 1.28, PR = 2.56 cm

PE = 0.18 cm, PF = 0.36 cm

Now, EQ = PQ = PQ = 1.28 – 0.18 = 1.10 cm

And FR = PR – PF = 2.56 – 0.36 = 2.20 cm

Now, $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$

And, $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ $\therefore \frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF||QR [By the converse of basic proportionality Theorem]

Que 3. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol. Let AB be a vertical pole of length 6 m and BC be its shadow and DE be tower and EF be its shadow.
Join AC and DF.

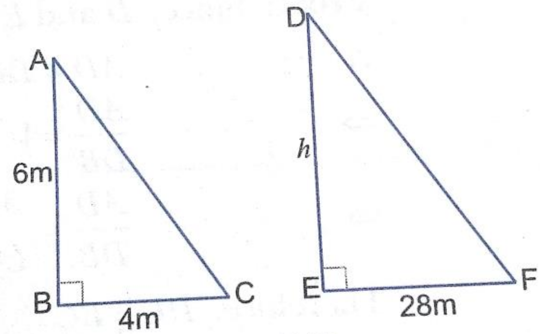

Fig. 7.13

Now, in ΔABC and ΔDEF, we have

$\angle B = \angle E = 90°$

$\angle C = \angle F$ (Angle of elevation of the sun)

$\therefore \Delta ABC \sim \Delta DEF$ (By AA criterion of similarity)

Thus, $\frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{6}{h} = \frac{4}{28}$ (Let DE = h)

$\Rightarrow \frac{6}{h} = \frac{1}{7}$ $\Rightarrow$ h = 42

Hence, height of tower, DE = 42 m

Que 4. In Fig. 7.14, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.

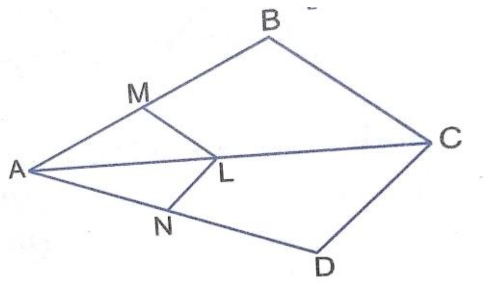

Fig. 7.14

Sol. Firstly, in ΔABC, we have

LM||CB (Given)

Therefore, by Basic proportionality Theorem, we have

$$\frac{AM}{AB} = \frac{AL}{AC} \quad .....(i)$$

Again, in ΔACD, we have

LN||CD (Given)

∴ By Basic proportionality Theorem, we have

$$\frac{AN}{AD} = \frac{AL}{AC} \quad .....(ii)$$

Now, from (i) and (ii), we have $\frac{AM}{AB} = \frac{AN}{AD}$.

Que 5. In Fig. 7.15, DE || OQ and DF || OR, Show that EF || QR.

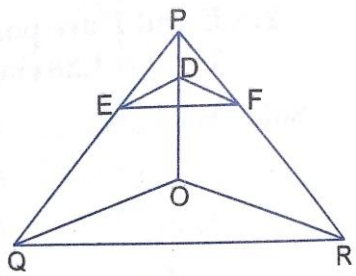

Fig. 7.15

Sol. In ΔPOQ, we have

DE || OQ (Given)

∴ By Basic proportionality Theorem, we have

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad ....(i)$$

Similarly, in ΔPOR, we have

DF || OR (Given)

$\therefore \qquad \frac{PD}{DO} = \frac{PF}{FR} \qquad$ ….(ii)

Now, from (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR} \quad \Rightarrow \quad EF \parallel QR$$

[Applying the converse of Basic proportionality Theorem in ΔPQR]

Que 6. Using converse of Basic proportionality Theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

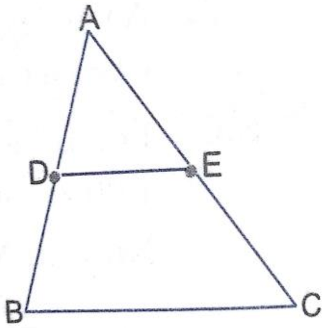

Fig. 7.16

Sol. Given: ΔABC in which D and E are the mid-points of sides AB and AC respectively.
To prove: DE || BC
Proof: Since, D and E are the mid-points of AB and AC respectively

$\therefore \qquad$ AD = DB and AE = EC

$\Rightarrow \qquad \frac{AD}{DB} = 1 \quad and \quad \frac{AE}{EC} = 1$

$\Rightarrow \qquad \frac{AD}{DB} = \frac{AE}{EC}$

Therefore, DE || BC (By the converse of Basic proportionality Theorem)

Que 7. State which pairs of triangles in the following figures are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form.

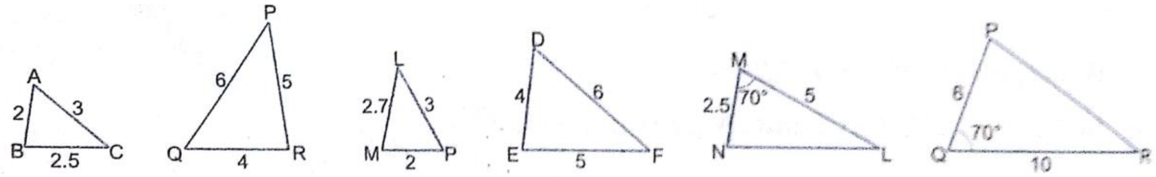

Fig. 7.17

Sol. (i) In ΔABC and ΔPQR, we have

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \quad \frac{AC}{PQ} = \frac{3}{6} = \frac{1}{2}$$

Hence, $\frac{AB}{QR} = \frac{AC}{PQ} = \frac{BC}{PR}$

$\therefore\ \Delta ABC \sim \Delta QRP$ by SSS criterion of similarity.

(ii) In ΔLMP and ΔFED, we have

$$\frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}, \quad \frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}, \quad \frac{LM}{EF} = \frac{27}{5}$$

Hence, $\frac{LP}{DF} = \frac{MP}{DE} \neq \frac{LM}{EF}$

$\therefore\ \Delta LMP$ is not similar to ΔFED.

(iii) In ΔNML and ΔPQR, we have

$$\angle M = \angle Q = 70°$$

Now, $\frac{MN}{PQ} = \frac{2.5}{6} = \frac{5}{12} \quad and \quad \frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$

Hence $\frac{MN}{PQ} \neq \frac{ML}{QR}$

$\therefore \Delta NML\ \ is\ not\ similar\ to\ \Delta PQR.$

Que 8. In Fig. 7.18, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 5 cm. Find the value of DC.

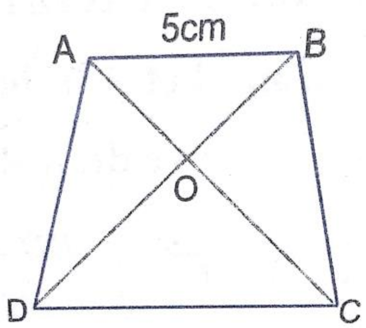

Fig. 7.18

Sol. In ΔAOB and ΔCOD, we have

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\frac{AO}{OC} = \frac{BO}{OD}$ [Given]

So, by SAS criterion of similarity, we have

$\Delta AOB \sim \Delta COD$

$\Rightarrow \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC} \qquad \Rightarrow \frac{1}{2} = \frac{5}{DC} \qquad [\because\ AB = 5\ cm]$

⇒ DC = 10 cm

Que 9. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE~ ΔCFB.

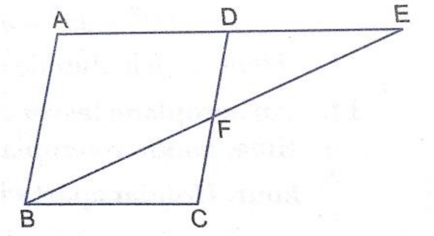

Fig. 7.19

Sol. In ΔABE and ΔCFB, we have

$\angle AEB = \angle CBF$ $(Alternate\ angles)$

$\angle A = \angle C$ (Opposite angles of a parallelogram)

∴ ΔABE~ ΔCFB (By AA criterion of similarity)

Que 10. S and T are points on sides PR and QR if ΔPQR such that $\angle P = \angle RTS$. Show that ΔRPQ ~ ΔRTS.

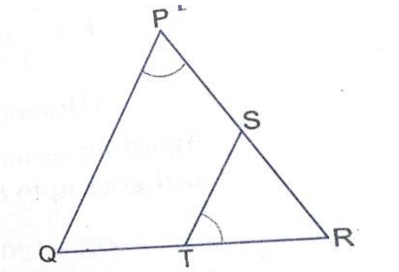

Fig. 7.20

Sol. In ΔRPQ and ΔRTS, we have

∠RPQ = ∠RTS (Given)

∠PRQ = ∠TRS = ∠R (Common)

∴ ∠RPQ ~ ΔRTS (By AA criterion of similarity)

Que 11. In Fig. 7.21, ABC and AMP are two right triangles right-angled at B and M respectively. Prove that:

(i) ΔABC ~ ΔAMP **(ii)** $\frac{CA}{PA} = \frac{BC}{MP}$

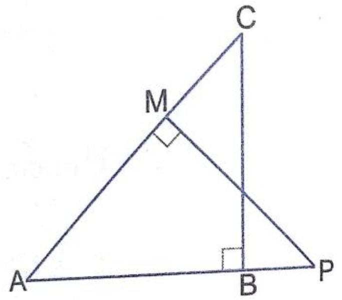

Fig. 7.21

Sol. (i) In ΔABC and ΔAMP, we have

$\angle ABC = \angle AMP = 90°$ (Given)

And, $\angle BAC = \angle MAP$ (Common angle)

∴ $\Delta ABC \sim \Delta AMP$ (By AA criterion of similarity)

(ii) As $\Delta ABC \sim \Delta AMP$ (Proved above)

∴ $\frac{CA}{PA} = \frac{BC}{MP}$ (Sides of similar triangles are proportional)

Que 12. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that CA^2 = CB.CD.

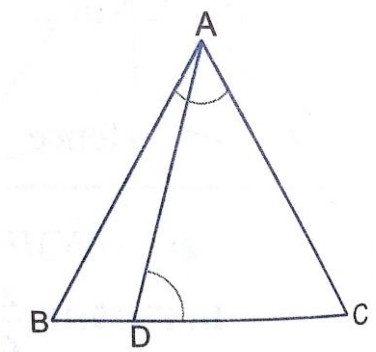

Fig. 7.22

Sol. In ΔABC and ΔDAC, we have

$\angle BAC = \angle ADC$ (Given)

and $\angle C = \angle C$ (Common)

∴ $\Delta ABC \sim \Delta DAC$ (By AA criterion of similarity)

⇒ $\frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$

⇒ $\frac{CB}{CA} = \frac{CA}{CD}$ ⇒ $CA^2 = CB \times CD$

Que 13. ABC is an equilateral triangle of side 2a. Find each of its altitudes.

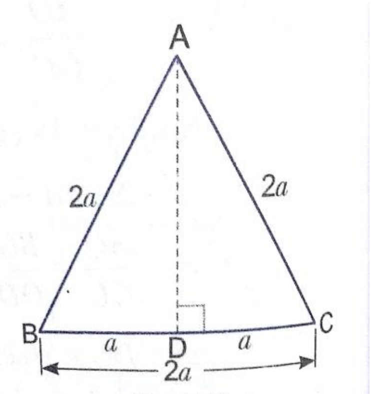

Fig. 7.23

Sol. Let ABC be an equilateral triangle of side 2a units.
We draw AD ⊥ BC. Then D is the mid-point of BC.

$\Rightarrow \quad BD = \frac{BC}{2} = \frac{2a}{2} = a$

Now, ABD is a right triangle right-angled at D.

$\therefore \quad AB^2 = AD^2 + BD^2 \quad [By\ Pythagoras\ Theorem]$

$\Rightarrow \quad (2a)^2 = AD^2 + a^2$

$\Rightarrow \quad AD^2 = 4a^2 - a^2 = 3a^2 \qquad \Rightarrow AD = \sqrt{3}a$

Hence, each altitude = $\sqrt{3}a$ unit.

Que 14. An aeroplane leaves an airport an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

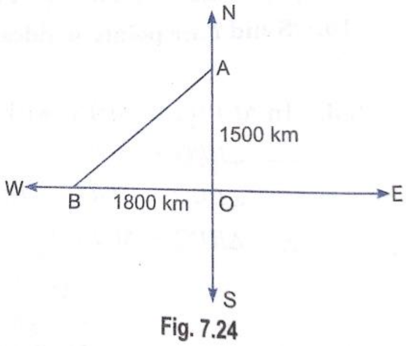

Fig. 7.24

Sol. Let the first aeroplane starts from O and goes upto A towards north where

$$OA = \left(1000 \times \frac{3}{2}\right) km = 1500\ km$$

(Distance = Speed x Time)

Again let second aeroplane starts from O at the same time and goes upto B towards west where

$$OB = 1200 \times \frac{3}{2} = 1800\ km$$

Now, we have to find AB.
In right angled ΔABO, we have

$AB^2 = OA^2 + OB^2$ [By using Pythagoras Theorem]

$\Rightarrow$ $AB^2 = (1500)^2 + (1800)^2$

$\Rightarrow$ $AB^2 = 2250000 + 3240000 \Rightarrow$ $AB^2 = 5490000$

$\therefore$ $AB = 100\sqrt{549} = 100 \times 234307 = 2343.07$ km.

Que 15. In the given Fig. 7.25, ΔABC and ΔDBC are on the same base BC. If AD intersects BC at O. prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.

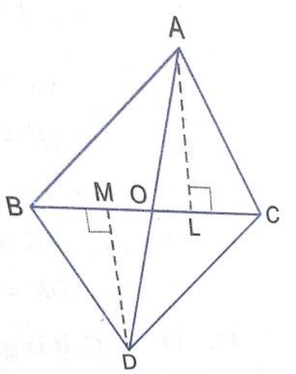

Fig. 7.25

Sol. Given: ΔABC and ΔDBC are on the same base BC and AD intersects BC at O.

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DP}$

Construction: Draw AL ⊥ BC and DM ⊥ BC
Proof: In ΔALO and ΔDMO, we have

$\angle ALO = \angle DMO = 90°$ and

$\angle AOL = \angle DOM$ (Vertically opposite angles)

$\therefore$ $\Delta ALO \sim \Delta DMO$ (By AA-Similarity)

$\Rightarrow$ $\frac{AL}{DM} = \frac{AO}{DO}$ …(i)

$\therefore$ $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}BC \times AL}{\frac{1}{2}BC \times DM} = \frac{AL}{DM} = \frac{AO}{DO}$ $(Using\ (i))$

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Que 16. In Fig. 7.26, AB || PQ || CD, AB = x units, CD = y units and PQ = z units. Prove that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$.

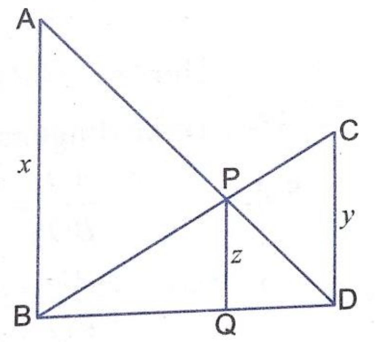

Fig. 7.26

Sol. In ΔADB and ΔPDQ,
Since AB || PQ

∠ABQ = ∠PQD (Corresponding∠'s)

∠ADB = ∠PDQ (Common)

By AA-Similarity

ΔADB ~ ΔPDQ

∴ $\frac{DQ}{DB} = \frac{PQ}{AB}$ ⇒ $\frac{DQ}{DB} = \frac{Z}{x}$ ..(i)

Similarly, ΔPBQ ~ ΔCBD

And $\frac{BQ}{DB} = \frac{z}{x}$ …(ii)

Adding (i) and (i), we get

$$\frac{z}{x} + \frac{z}{y} = \frac{DQ+BQ}{DB} = \frac{BD}{BD}$$

$$\frac{z}{x} + \frac{z}{y} = 1 \qquad \Rightarrow \qquad \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

Que 17. In Fig. 7.27, if ΔABC ~ ΔDEF and their sides are of length (in cm) as marked along them, then find the length of the sides of each triangle.

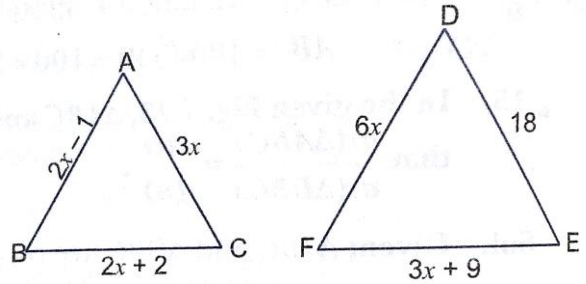

Fig. 7.27

Sol. ΔABC ~ ΔDEF (Given)

Therefore, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

So, $\frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$

Now, taking $\frac{2x-1}{18} = \frac{1}{2}$

$\Rightarrow \quad 4x - 2 = 18 \quad \Rightarrow \quad x = 5$

$\therefore \quad AB = 2 \times 5 - 1 = 9, BC = 2 \times 5 + 2 = 12$

$CA = 3 \times 5 = 15, DE = 18, EF = 3 \times 5 + 9 = 24 \; and \; FD = 6 \times 5 = 30$

Hence, AB = 9 cm, BC = 12 cm, CA = 15 cm
DE = 18 cm, EF = 24 cm, FD = 30 cm

Que 18. In ΔABC, it is given that $\frac{AB}{AC} = \frac{BD}{DC}$. If ∠B = 70° and ∠C = 50° thwen find ∠BAD.

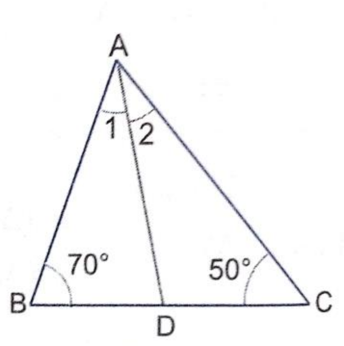

Fig. 7.28

Sol. In ΔABC

$\because \quad \angle A + \angle B + \angle C = 180°$ (Angle sum property)

$\angle A + 70° + 50° = 180°$

$\Rightarrow \quad \angle A = 180° - 120° \quad \Rightarrow \quad \angle A = 60°$

$\because \quad \frac{AB}{AC} = \frac{BD}{DC}$ (Given)

$\therefore \quad \angle 1 = \angle 2$ ….(i)

[Because a line through one vertex of a triangle divides the opposite sides in the ratio of other two sides, then the line bisects the angle at the vertex.]

But $\angle 1 + \angle 2 = 60°$ ….(ii)

From (i) and (ii) we get,

$2\angle 1 = 60° \quad \Rightarrow \quad \angle 1 = \frac{60°}{2} = 30°$

Hence, ∠BAD = 30°

Que 19. If the diagonals of a quadrilateral divides each other proportionally, prove that it is a trapezium.

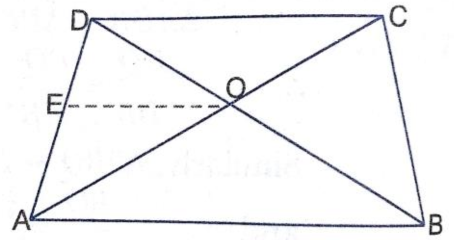

Fig. 7.29

Sol. $\frac{AO}{BO} = \frac{CO}{DO}$ (Given)

$\Rightarrow \quad \frac{AO}{CO} = \frac{BO}{DO}$ …(i)

In ΔABD, EQ || AB (Construction)

$\therefore \quad \frac{AE}{ED} = \frac{BO}{DO}$ (By BPT) …(ii)

From equations (i) and (ii)

$\frac{AE}{ED} = \frac{AO}{CO}$ ⇒ EO || DC (Converse of BPT)

But EO || AB (Construction)

∴ AB || DC

⇒ In quad ABCDD AB || DC ⇒ ABCD is a trapezium.

Que 20. In the given Fig. 7.30, $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ. Prove that PQR is an isosceles triangle.

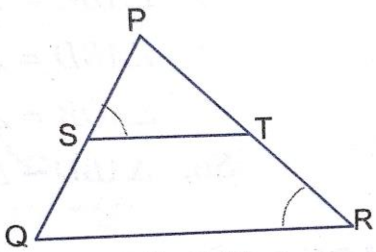

Fig. 7.30

Sol. Given: $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ

To Prove: PQR is isosceles triangle.

Proof: $\frac{PS}{SQ} = \frac{PT}{TR}$

By converse of BPT we get

ST || QR

∴ $\angle PST = \angle PQR$ (Corresponding angles) …(i)

But, ∠PST = ∠PRQ (Given) …(ii)

From equation (i) and (ii)

∠PQR = ∠PRQ ⇒ PR = PQ

So, ΔPQR is an isosceles triangle.

Que 21. The diagonals of a trapezium ABCD in which AB || DC, intersect at O. If AB = 2 cd then find the ratio of areas of triangles AOB and COD.

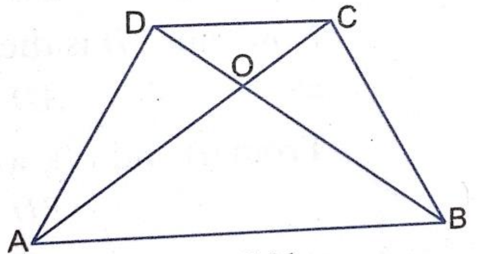

Fig. 7.31

Sol. In ΔAOB and ΔCOD

∠COD = ∠AOB (Vertically opposite angles)

∠CAB = ∠DCA (Alternate angles)

∴ ΔAOB ~ ΔCOD (B AA-similarity)

By area of theorem

$$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{DC^2} \quad \Rightarrow \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{(2CD)^2}{CD^2} = \frac{4}{1}$$

Hence, ar (ΔAOB): ar (ΔCOD) = 4: 1.

Que 22. In the given Fig. 7.32, find the value of x in terms of a, b and c.

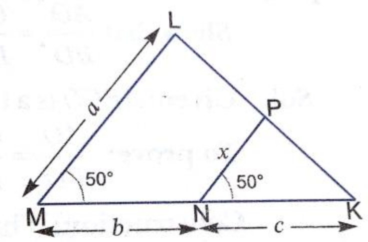

Fig. 7.32

Sol. In ΔLMK and ΔPNK

We have, ∠M = ∠N = 50° and ∠K = ∠K (Common)

ΔLMK ~ ΔPNK (AA – Similarity)

$$\frac{LM}{PN} = \frac{KM}{KN}$$

$$\frac{a}{x} = \frac{b+c}{c} \qquad \Rightarrow \quad x = \frac{ac}{b+c}$$

Que 23. In the given Fig. 7.33, CD || LA and DE || AC. Find the length of CL if BE = 4 cm and EC = 2 cm.

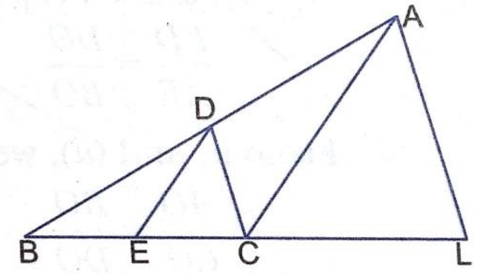

Fig. 7.33

Sol. In ΔABC, DE || AC (Given)

$\Rightarrow \qquad \frac{BD}{DA} = \frac{BE}{EC}$ (By BPT) …(i)

In ΔABL DC || AL

$\Rightarrow \qquad \frac{BD}{Da} = \frac{BC}{CL}$ (By BPT) …(ii)

From (i) and (ii) we get

$$\frac{BE}{EC} = \frac{BC}{CL} \qquad \Rightarrow \qquad \frac{4}{2} = \frac{6}{CL} \qquad \Rightarrow \quad CL = 3\ cm$$

Que 24. In the given Fig. 7.34, AB = AC. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC, prove that ΔABD is similar to ΔCEF.

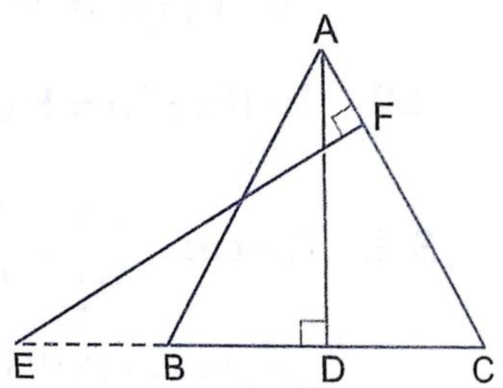

Fig. 7.34

Sol. In ΔABD and ΔCEF

AB = AC (Given)

$\Rightarrow \ \angle ABC = \angle ACB$ (Equal sides have equal opposite angles)

$\angle ABD = \angle ECF$

$\angle ADB = \angle EFC$ (Each 90°)

So, $\Delta ABD \sim \Delta CEF$ (AA – Similarity)