15. Structure of Atoms and Nuclei

Can you recall?

1. What is Dalton's atomic model?
2. What are atoms made of?
3. What is wave particle duality?
4. What are matter waves?

15.1. Introduction:

Greek philosophers Leucippus (-370 BC) and Democritus (460 – 370 BC) were the first scientists to propose, in the 5th century BC, that matter is made of indivisible parts called atoms. Dalton (1766-1844) gave his atomic theory in early nineteenth century. According to his theory (i) matter is made up of indestructible particles, (ii) atoms of a given element are identical and (iii) atoms can combine with other atoms to form new substances. That atoms were indestructible was shown to be wrong by the experiments of J. J. Thomson (1856-1940) who discovered electrons in 1887. He then proceeded to give his atomic model which had some deficiencies and was later improved upon by Ernest Rutherford (1871-1937) and Niels Bohr (1885-1962). We will discuss these different models in this Chapter. You have already studied about atoms and nuclei in XIth Std. in chemistry. This chapter will enable you to consolidate your concepts in this subject.

We will learn that, an atom contains a tiny nucleus whose size (radius) is about 100000 times smaller than the size of an atom. The nucleus contains all the positive charge of the atom and also 99.9% of its mass. In this Chapter we will also study properties of the nucleus, the forces that keep it intact, its radioactive decays and about the energy that can be obtained from it.

15.2. Thomson's Atomic Model:

Thomson performed several experiments with glass vacuum tube wherein a voltage was applied between two electrodes inside an evacuated tube. The cathode was seen to emit rays which produced a glow when they struck the glass behind the anode. By studying the properties of these rays, he concluded that the rays are made up of negatively charged particles which he called electrons. This demonstrated that atoms are not indestructible. They contain electrons which are emitted by the cathode.

Thomson proposed his model of an atom in 1903. According to this model an atom is a sphere having a uniform positive charge in which electrons are embedded. This model is referred to as Plum-pudding model. The total positive charge is equal to the total negative charge of electrons in the atom, rendering it electrically neutral. As the whole solid sphere is uniformly positively charged, the positive charge cannot come out and only the negatively charged electrons which are small, can be emitted. The model also explained the formation of ions and ionic compounds. However, further experiments on structure of atoms which are described below, showed the distribution of charges to be very different than what was proposed in Thomson's model.

15.3 Geiger-Marsden Experiment:

In order to understand the structure of atoms, Rutherford suggested an experiment for scattering of alpha particles by atoms. Alpha particles are helium nuclei and are positively charged (having charge of two protons). The experiment was performed by his colleagues Geiger (1882-1945) and Marsden (1889-1970) between 1908 and 1913. A sketch of the experimental set up is shown in Fig.15.1.

Alpha particles from a source were collimated, i.e., focused into a narrow beam, and were made to fall on a gold foil. The scattered particles produced scintillations on the

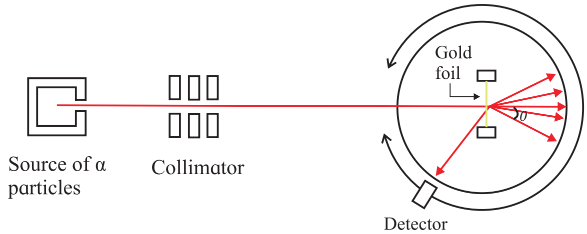

Fig.15.1: Geiger-Marsden experiment.

surrounding screen. The scintillations could be observed through a microscope which could be moved to cover different angles with respect to the incident beam. It was found that most alpha particles passed straight through the foil while a few were deflected (scattered) through various scattering angles. A typical scattering angle is shown by θ in the figure. Only about 0.14% of the incident alpha particles were scattered through angles larger than 0.1^o. Even out of these, most were deflected through very small angles. About one alpha particle in 8000 was deflected through angle larger than 90^o and a fewer still were deflected through angles as large as 180^o.

15.4. Rutherford's Atomic Model:

Results of Geiger-Marsden's experiment could not be explained by Thomson's model. In that model, the positive charge was uniformly spread over the large sphere constituting the atom. The volume density of the positive charge would thus be very small and all of the incident alpha particles would get deflected only through very small angles. Rutherford argued that the alpha particles which were deflected back must have encountered a massive particle with large positive charge so that it was repelled back. From the fact that extremely small number of alpha particles turned back while most others passed through almost undeflected, he concluded that the positively charged particle in the atom must be very small in size and must contain most of the mass of the atom. From the experimental data, the size of this particle was found to be about 10 fm (femtometre, 10^{-15}) which is about 10^{-5} times the size of the atom. The volume of this particle was thus found to be about 10^{-15} times that of an atom. He called this particle the nucleus of an atom.

He proposed that the entire positive charge and most (99.9%) of the mass of an atom is concentrated in the central nucleus and the electrons revolve around it in circular orbits, similar to the revolution of the planets around the Sun in the Solar system. The revolution of the electrons was necessary as without it, the electrons would fall into the positively charged nucleus and the atom would collapse. The space between the orbits of the electrons (which decide the size of the atom) and the nucleus is mostly empty. Thus, most alpha particles pass through this empty space undeflected and a very few which are in direct line with the tiny nucleus or are extremely close to it, get repelled and get deflected through large angles. This model also explains why no positively charged particles are emitted by atoms while negatively charged electrons are. This is because of the large mass of the nucleus which does not get affected when force is applied on the atom.

15.4.1. Difficulties with Rutherford's Model:

Though this model in its basic form is still accepted, it faced certain difficulties. We know from Maxwell's equations that an accelerated charge emits electromagnetic radiation. An electron in Rutherford's model moves uniformly along a circular orbit around the nucleus. Even though the magnitude of its velocity is constant, its direction changes continuously and so the motion is an accelerated motion. Thus, the electron should emit electromagnetic radiation continuously. Also, as it emits radiation, its energy would decrease and consequently, the radius of its orbit would decrease continuously. It would then spiral into the nucleus, causing the atom to collapse and lose its atomic properties. As the electron loses energy, its velocity changes continuously and the frequency of the radiation emitted would also change continuously as it moves towards the nucleus. None of these things are observed. Firstly, most atoms are very stable and secondly, they do not constantly emit electromagnetic radiation and definitely not of varying frequency. The atoms have to be given energy, e.g., by heating, for them to be able to emit radiation and even then, they emit electromagnetic radiations of particular frequencies as will be seen in the next section. Rutherford's model failed on all these counts.

15.5 Atomic Spectra:

We know that when a metallic object is heated, it emits radiation of different wavelengths. When this radiation is passed through a prism, we get a continuous spectrum. However, the case is different when we heat hydrogen gas inside a glass tube to high temperatures. The emitted radiation has only a few selected wavelengths and when passed through a prism we get what is called a line spectrum as shown for the visible range in Fig.15.2. It shows that hydrogen emits radiations of wavelengths 410, 434, 486 and 656 nm and does not emit any radiation with wavelengths in between these wavelengths. The lines seen in the spectrum are called emission lines.

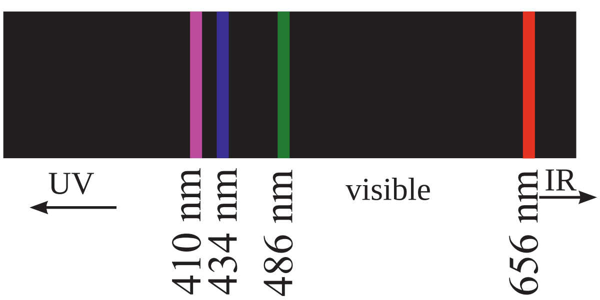

Fig.15.2: Hydrogen spectrum.

Hydrogen atom also emits radiation at some other values of wavelengths in the ultraviolet (UV), the infrared (IR) and at longer wavelengths. The spectral lines can be divided into groups known as series with names of the scientists who studied them. The series, starting from shorter wavelengths and going to larger wavelengths are called Lyman series, Balmer series, Paschen series, Brackett series, Pfund series, etc. In each series, the separation between successive lines decreases as we go towards shorter wavelength and they reach a limiting value.

Schematic diagrams for the first three series are shown in Fig.15.3. The limiting value of the wavelength for each series is shown by dotted lines in the figure.

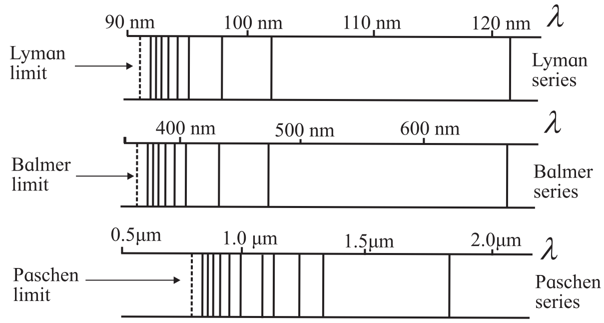

Fig.15.3: Lyman, Balmer and Paschen series in hydrogen spectrum.

The observed wavelengths of the emission lines are found to obey the relation.

$$\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right] \quad \text{--- (15.1)}$$

Here λ is the wavelength of a line, R is a constant and n and m are integers. n = 1, 2, 3,…. respectively, for Lyman, Balmer, Paschen…. series, while m takes all integral values greater than n for that series. The wavelength decreases with increase in m.

The difference in wavelengths of successive lines in each series (fixed value of n) can be calculated from Eq. (15.1) and shown to decrease with increase in m. Thus, the successive lines in a given series come closer and closer and ultimately reach the values of $\lambda = \frac{n^2}{R}$ in the limit $m \to \infty$, for different values of n. Atoms of other elements also emit line spectra. The wavelengths of the lines emitted by each element are unique, so much so that we can identify the element from the wavelengths of the spectral lines that it emits. Rutherford's model could not explain the atomic spectra.

15.6. Bohr's Atomic Model:

Niels Bohr modified Rutherford's model by applying ideas of quantum physics which were being developed at that time. He realized that Rutherford's model is essentially correct and all that it needs is stability of the orbits. Also, the electrons in these stable orbits should not emit electromagnetic waves as required by classical (Maxwell's) electromagnetic theory. He made three postulates which defined his atomic model. These are given below.

1. The electrons revolve around the nucleus in circular orbits.

This is the same assumption as in Rutherford's model and the centripetal force necessary for the circular motion is provided by the electrostatic force of attraction between the electron and the nucleus.

2. The radius of the orbit of an electron can only take certain fixed values such that the angular momentum of the electron in these orbits is an integral multiple of $h/2\pi$, h being the Planck's constant.

Such orbits are called stable orbits or stable states of the electrons and electrons in these orbits do not emit radiation as is demanded by classical physics. Thus, different orbits have different and definite values of angular momentum and therefore, different values of energies.

3. An electron can make a transition from one of its orbit to another orbit having lower energy. In doing so, it emits a photon of energy equal to the difference in its energies in the two orbits.

15.6.1. Radii of the Orbits:

Using first two postulates we can study the entire dynamics of the circular motion of the electron, including its energy. Let the mass of the electron be m_e, its velocity in the n^{th} stable orbit be v_n and the radius of its orbit be r_n. The angular momentum is then $m_e v_n r_n$ and according to the second postulate above, we can write

$$m_e v_n r_n = n\frac{h}{2\pi} \quad \text{--- (15.2)}$$

The positive integer n is called the ***principal quantum number*** of the electron. The centripetal force necessary for the circular motion of the electron is provided by the electrostatic force of attraction between the electron and the nucleus. Assuming the atomic number (number of electrons) of the atom to be Z, the total positive charge on the nucleus is Ze and we can write,

$$\frac{m_e v_n^2}{r_n} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2} \quad \text{--- (15.3)}$$

Here, ε_0 is the permeability of vacuum and e is the electron charge. Eliminating v_n from the Eq.(15.2) and Eq.(15.3), we get,

$$\frac{m_e n^2 h^2}{4\pi^2 m_e^2 r_n^3} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2}$$

$$\therefore\ r_n = \frac{n^2 h^2 \varepsilon_0}{\pi\, m_e\, Ze^2} \quad \text{--- (15.4)}$$

Similarly, eliminating r_n from Eq.(15.2) and Eq.(15.3), we get,

$$v_n = \frac{Ze^2}{2\varepsilon_0 h\, n} \quad \text{--- (15.5)}$$

Equation (15.4) shows that the radius of the orbit is proportional to n^2, i.e., the square of the principal quantum number. The radius increases with increase in n. The hydrogen atom has only one electron, i.e., Z is 1. Substituting the values of the constants h, ε_0, m and e in Eq.(15.4), we get, for $n = 1$, $r_1 = 0.053$ nm. This is called the Bohr radius and is denoted by $a_0 = \frac{h^2\varepsilon_0}{\pi\, m_e e^2}$.

This is the radius of the smallest orbit of the electron in hydrogen atom. From Eq. (15.4), we can write,

$$r_n = a_0 n^2 \quad \text{--- (15.6)}$$

Example 15.1: Calculate the radius of the 3rd orbit of the electron in hydrogen atom.

Solution: The radius of n^{th} orbit is given by $r_n = a_0 n^2$. Thus, the radius of the third orbit ($n = 3$) is

$$r_3 = a_0 3^2 = 9a_0 = 9\times 0.053 \text{ nm}$$
$$= 0.477 \text{ nm}.$$

Example 15.2: In a Rutherford scattering experiment, assume that an incident alpha particle (radius 1.80 fm) is moving directly toward a target gold nucleus (radius 6.23 fm). If the alpha particle stops right at the surface of the gold nucleus, how much energy did it have to start with?

Solution: Initially when the alpha particle is far away from the gold nucleus, its total energy is equal to its kinetic energy. As it comes closer to the nucleus, more and more of its kinetic energy gets converted to potential energy. By the time it reaches the surface of the nucleus, its kinetic energy is completely converted into potential energy and it stops moving. Thus, the initial kinetic energy K, of the alpha particle is equal to the potential energy when it is at the surface of the nucleus, i.e., when the distance between the gold nucleus and the alpha particle is equal to the sum of the radii of the gold nucleus and alpha particle.

$\therefore K = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2e\,Ze}{(r_1+r_2)}$, where, Z is the atomic number of gold and r_1 and r_2 are the radii of the gold nucleus and alpha particle respectively. For gold $Z = 79$.

$$\therefore K = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{(r_1+r_2)}$$

$$= 9\times10^9\,\frac{2\times79\times\left(1.6\times10^{-19}\right)^2}{(6.23+1.80)\times10^{-15}}$$

$$= 4.533\times10^{-12}\,\text{J} = 28.33\ \text{MeV}$$

15.6.2. Energy of the Electrons:

The total energy of an orbiting electron is the sum of its kinetic energy and its electrostatic potential energy. Thus,

$E_n = K.E. + P.E.$, E_n being the total energy of an electron in the n^{th} orbit.

$$E_n = \frac{1}{2}m_e v_n^2 + \left(-\frac{Ze^2}{4\pi\varepsilon_0 r_n}\right).$$

Using Eq. (15.3) and (15.4) this gives

$$E_n = -\frac{m_e Z^2 e^4}{8\varepsilon_0 h^2 n^2} \qquad \text{--- (15.7)}$$

The negative value of the energy of the electron indicates that the electron is bound inside the atom and it has to be given energy so as to make the total energy zero, i.e., to make the electron free from the nucleus. The energy increases (becomes less negative) with increase in n. Substituting the values of the constants m, e, h and ε_0 in the above equation, we get

$$E_n = -13.6\frac{Z^2}{n^2}\ \text{eV} \qquad \text{--- (15.8)}$$

The first orbit $(n=1)$ which has minimum energy, is called the ***ground state*** of the atom. Orbits with higher values of n and therefore, higher values of energy are called the ***excited states*** of the atom. If the electron is in the n^{th} orbit, it is said to be in the n^{th} energy state. For hydrogen atom (Z = 1) the energy of the electron in its ground state is -13.6 eV and the energies of the excited states increase as given by Eq.(15.8). The energy levels of hydrogen atom are shown in Fig.15.4. The energies of the levels are given in eV.

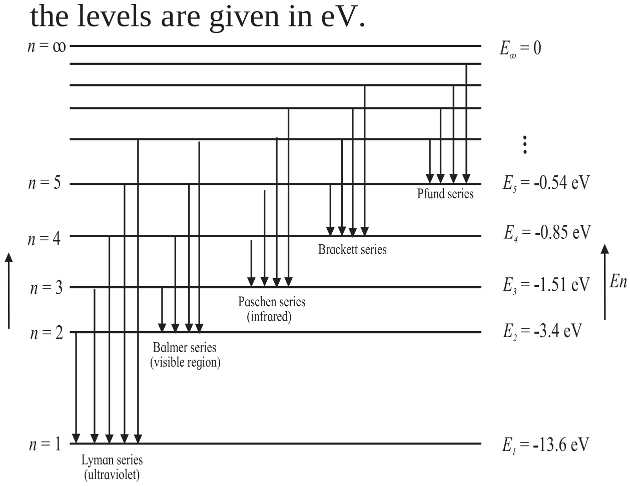

Fig.15.4: Energy levels and transitions between them for hydrogen atom (energy not to scale).

The energy levels come closer and closer as n increases and their energy reaches a limiting value of zero as n goes to infinity. The energy required to take an electron from the ground state to an excited state is called the ***excitation energy*** of the electron in that state. For hydrogen atom, the minimum excitation energy (of n = 2 state) is -3.4-(-13.6) =10.2 eV.

In order to remove or take out the electron in the ground state from a hydrogen atom, i.e., to make it free (and have zero energy), we have to supply 13.6 eV energy to it. This energy is called the ***ionization energy*** of the hydrogen atom. ***The ionization energy of an atom is the minimum amount of energy required to be given to an electron in the ground state of that atom to set the electron free.*** It is the ***binding energy*** of hydrogen atom. If we form a hydrogen atom by bringing a proton and an electron from infinity and combine them, 13.6 eV energy will be released.

According to the third postulate of Bohr, when an electron makes a transition from m^{th} to n^{th} orbit ($m > n$), the excess energy $E_m - E_n$ is emitted in the form of a photon. The energy of the photon which can be written as $h\nu$, ν being its frequency, is therefore given by,

$$h\nu = \frac{m_e Z^2 e^4}{8\varepsilon_0 h^2}\left(\frac{1}{n^2}-\frac{1}{m^2}\right)$$ which can be written in terms of the wavelength as

$$\frac{1}{\lambda}=\frac{m_e Z^2 e^4}{8c\,\varepsilon_0 h^3}\left(\frac{1}{n^2}-\frac{1}{m^2}\right) \qquad \text{---(15.9)}$$

Here c is the velocity of light in vacuum. We define a constant called the ***Rydberg's constant, R_H*** as

$$R_H = \frac{m_e e^4}{8c\,\varepsilon_0 h^3} = 1.097 \times 10^7 \text{ m}^{-1}. \qquad \text{---(15.10)}$$

In terms of R_H, the wavelength is given by

$$\frac{1}{\lambda}=R_H Z^2\left(\frac{1}{n^2}-\frac{1}{m^2}\right) \qquad \text{---(15.11)}$$

This is called the *Rydberg's formula*. Remember that for hydrogen Z is 1. Thus, Eq.(15.11) correctly describes the observed spectrum of hydrogen as given by Eq.(15.1).

Example 15.3: Determine the energies of the first two excited states of the electron in hydrogen atom. What are the excitation energies of the electrons in these orbits?

Solution: The energy of the electron in the n^{th} orbit is given by $E_n = -13.6\frac{1}{n^2}$ eV.

The first two excited states have n = 2 and 3. Their energies are

$$E_2 = -13.6\frac{1}{2^2} = -3.4 \text{ eV} \text{ and}$$

$$E_3 = -13.6\frac{1}{3^2} = -1.51 \text{ eV}\cdot$$

Excitation energy of an electron in n^{th} orbit is the difference between its energy in that orbit and the energy of the electron in its ground state, i.e. -13.6 eV. Thus, the excitation energies of the electrons in the first two excited states are 10.2 eV and 12.09 eV respectively.

Example 15.4: Calculate the wavelengths of the first three lines in Paschen series of hydrogen atom.

Solution: The wavelengths of lines in Paschen series (n=3) are given by

$$\frac{1}{\lambda}=R_H\left(\frac{1}{n^2}-\frac{1}{m^2}\right)=1.097\times10^7\left(\frac{1}{3^2}-\frac{1}{m^2}\right)$$

m^{-1} for m = 4,5,….

For the first three lines in the series, m = 4, 5 and 6. Substituting in the above formula we get,

$$\frac{1}{\lambda_1}=1.097\times10^7\left(\frac{1}{3^2}-\frac{1}{4^2}\right)$$

$$=1.097\times10^7\times7/(9\times16)$$

$$=0.0533\times10^7 \text{ m}^{-1}$$

$$\lambda_1 = 1.876 \text{ x } 10^{-6} \text{ m}$$

$$\frac{1}{\lambda_2}=1.097\times10^7\left(\frac{1}{3^2}-\frac{1}{5^2}\right)$$

$$=1.097\times10^7\times16/(9\times25)$$

$$=0.078\times10^7 \text{ m}^{-1}$$

$$\lambda_2 = 1.282\times10^{-6} \text{ m}$$

$$\frac{1}{\lambda_3}=1.097\times10^7\left(\frac{1}{3^2}-\frac{1}{6^2}\right)$$

$$=1.097\times10^7\times27/(9\times36)$$

$$=0.09142\times10^7 \text{ m}^{-1}$$

$$\lambda_3 = 1.094\times10^{-6} \text{ m}$$

15.6.3. Limitations of Bohr's Model:

Even though Bohr's model seemed to explain hydrogen spectrum, it had a few shortcomings which are listed below.

(i) It could not explain the line spectra of atoms other than hydrogen. Even for hydrogen, more accurate study of the observed spectra showed multiple components in some lines which could not be explained on the basis of this model.

(ii) The intensities of the emission lines seemed to differ from line to line and Bohr's model had no explanation for that.

(iii) On theoretical side also the model was not entirely satisfactory as it arbitrarily assumed orbits following a particular condition to be stable. There was no theoretical basis for that assumption.

A full quantum mechanical study is required for the complete understanding of the structure of atoms which is beyond the scope of this book. Some reasoning for the third shortcoming (i.e., theoretical basis for the second postulate in Bohr's atomic model) was given by de Broglie which we consider next.

15.6.4 De Broglie's Explanation:

We have seen in Chapter 14 that material particles also have dual nature like that for light and there is a wave associated with every material particle. De Broglie suggested that instead of considering the orbiting electrons inside atoms as particles, we should view them as standing waves. Similar to the case of standing waves on strings or in pipes as studied in Chapter 6, the length of the orbit of an electron has to be an integral multiple of its wavelength. Thus, the length of the first orbit will be equal to one de Broglie wavelength, λ_1 of the electron in that orbit, that of the second orbit will be twice the de Broglie wavelength of the electron in that orbit and so on. This is shown for the 4th orbit in Fig.(15.5)

In general, we can write,

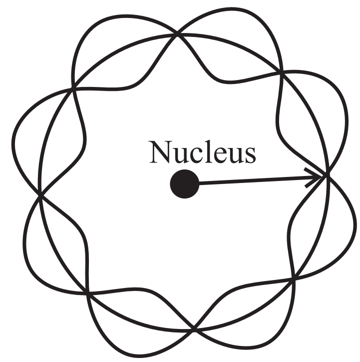

Fig. 15.5: Standing electron wave for the 4th orbit of an electron in an atom.

$2\pi r_n = n\lambda_n$, n = 1,2,3….., giving

$$\lambda_n = \frac{2\pi r_n}{n} \qquad \text{--- (15.12)}$$

The de Broglie wavelength is related to the linear momentum p_n, of the particle by

$$\lambda_n = \frac{h}{p_n} = \frac{h}{m\text{v}_n}.$$

Substituting this in Eq. (15.12) gives,

$$p_n = \frac{hn}{2\pi r_n}.$$

Thus, the angular momentum of the electron in n^{th} orbit, L_n, can be written as

$L_n = p_n r_n = n\dfrac{h}{2\pi}$, which is the second postulate of Bohr's atomic model. Therefore, considering electrons as waves gives some theoretical basis for the second postulate made by Bohr.

15.7. Atomic Nucleus:

15.7.1 Constituents of a Nucleus:

The atomic nucleus is made up of subatomic, meaning smaller than an atom, particles called ***protons*** and ***neutrons***. Together, protons and neutrons are referred to as ***nucleons***. Mass of a proton is about 1836 times that of an electron. Mass of a neutron is nearly same as that of a proton but is slightly higher. The proton is a positively charged particle. The magnitude of its charge is equal to the magnitude of the charge of an electron. The neutron, as the name suggests, is electrically neutral. The number of protons in an atom is called its ***atomic number*** and is designated by ***Z***. The number of electrons in an atom is also equal to Z. Thus, the total positive and total negative charges in an atom are equal in magnitude and the atom as a whole is electrically neutral. The number of neutrons in a nucleus is written as $\boldsymbol{N}$. The total number of nucleons in a nucleus is called the ***mass number*** of the atom and is designated by $\boldsymbol{A} = Z + N$. The mass number determines the mass of a nucleus and of the atom. The atoms of an element X are represented by the symbol for the element and its atomic and mass numbers as ${}_{Z}^{A}X$. For example, symbols for hydrogen, carbon and oxygen atoms are written as ${}_{1}^{1}H$, ${}_{6}^{12}C$ and ${}_{8}^{16}O$. The chemical properties of an atom are decided by the number of electrons present in it, i.e., by Z.

The number of protons and electrons in the atoms of a given element are fixed. For example, hydrogen atom has one proton and one electron, carbon atom has six protons and six electrons. The number of neutrons in the atoms of a given element can vary. For example, hydrogen nucleus can have zero, one or two neutrons. These varieties of hydrogen are referred to as ${}_{1}^{1}H$, ${}_{1}^{2}H$ and ${}_{1}^{3}H$ and are respectively called hydrogen, deuterium and tritium. Atoms having the same number of protons but different number of neutrons are called ***iosotopes***. Thus, deuterium and tritium are isotopes of hydrogen. They have the same chemical properties as those of hydrogen. Similarly, helium nucleus can have one or two neutrons and are referred as ${}_{2}^{3}He$ and ${}_{2}^{4}He$. The atoms having the same mass number A, are called ***isobars***. Thus, ${}_{1}^{3}H$ and ${}_{2}^{3}He$ are isobars. Atoms having the same number of neutrons but different values of atomic number Z, are called ***iosotones***. Thus, ${}_{1}^{3}H$ and ${}_{2}^{4}He$ are isotones.

Units for measuring masses of atoms and subatomic particles

Masses of atoms and subatomic particles are measured in three different units. First unit is the usual unit kg. The masses of electron, proton and neutron, m_e, m_p and m_n respectively, in this unit are:

$m_e = 9.109383 \times 10^{-31}$ kg
$m_p = 1.672623 \times 10^{-27}$ kg
$m_n = 1.674927 \times 10^{-27}$ kg

Obviously, kg is not a convenient unit to measure masses of atoms or subatomic particles which are extremely small compared to one kg. Therefore, another unit called the unified atomic mass unit (u) is used for the purpose. One u is equal to 1/12th of the mass of a neutral carbon atom having atomic number 12, in its lowest electronic state. 1 u = $1.6605402 \times 10^{-27}$ kg. In this unit, the masses of the above three particles are

$m_e = 0.00055$ u
$m_p = 1.007825$ u
$m_n = 1.008665$ u.

The third unit for measuring masses of atoms and subatomic particles is in terms of the amount of energy that their masses are equivalent to. According to Einstein's famous mass-energy relation, a particle having mass m is equivalent to an amount of energy $E = mc^2$. The unit used to measure masses in terms of their energy equivalent is the eV/c^2. One atomic mass unit is equal to 931.5 MeV/c^2. The masses of the three particles in this unit are

$m_e = 0.511$ MeV/c^2
$m_p = 938.28$ MeV/c^2
$m_n = 939.57$ MeV/c^2

15.7.2. Sizes of Nuclei:

The size of an atom is decided by the sizes of the orbits of the electrons in the atom. Larger the number of electrons in an atom, higher are the orbits occupied by them and larger is the size of the atom. Similarly, all nuclei do not have the same size. Obviously, the size of a nucleus depends on the number of nucleons present in it, i.e., on its atomic number A. From experimental observations it has been found that the radius R_X of a nucleus X is related to A as

$$R_X = R_0 A^{\frac{1}{3}} \qquad \text{--- (15.13)}$$

where R_0 = 1.2 x 10^{-15} m.

The density ρ inside a nucleus is given by $\frac{4}{3}\pi R_x^3 \rho = mA$, where, we have assumed m to be the average mass of a nucleon (proton and neutron) as the difference in their masses is rather small. The density is then given by,

$$\rho = \frac{3mA}{4\pi R_X{}^3}$$

Substituting for R_X from Eq.(15.13), we get,

$$\rho = \frac{3m}{4\pi R_0^3} = \text{constant.}$$

Thus, the density of a nucleus does not depend on the atomic number of the nucleus and is the same for all nuclei. Substituting the values of the constants m, π and R_0 the value of the density is obtained as 2.3 x 10^{17} kg m^{-3} which is extremely large. Among all known elements, osmium is known to have the highest density which is only 2.2 x 10^4 kg m^{-3}. This is smaller than the nuclear density by thirteen orders of magnitude.

Example 15.5: Calculate the radius and density of ^{70}Ge nucleus, given its mass to be approximately 69.924 u.

Solution: The radius of a nucleus X with mass number A is given by $R_X = R_0 A^{\frac{1}{3}}$,

where R_0 = 1.2 × 10^{-15} m

Thus, the radius of ^{70}Ge is

R_{Ge} = 1.2 × 10^{-15} × $70^{1/3}$ = 4.945 × 10^{-15} m.

The density is given by $\rho = \frac{3m_{Ge}}{4\pi R_{Ge}}$.

$$\therefore \;\; \rho = 3 \times 69.924 \times 1.66 \times \frac{10^{-27}}{4\pi\left(4.95 \times 10^{-15}\right)^3}$$

$= 2.292 \times 10^{17}$ kg m^{-3}.

15.7.3 Nuclear Forces:

You have learnt about the four fundamental forces that occur in nature. Out of these four, the force that determines the structure of the nucleus is the strong force, also called the nuclear force. This acts between protons and neutrons and is mostly attractive. It is different from the electrostatic and gravitational force in terms of its strength and range, i.e., the distance up to which it is effective. Over short distances of about a few fm, the strength of the nuclear force is much higher than that of the other two forces. Its range is very small and its strength goes to zero when two nucleons are at a distance larger than a few fm. This is in contrast to the ranges of electrostatic and gravitational forces which are infinite.

The protons in the nucleus repel one another due to their similar (positive) charges. The nuclear forces between the nucleons counter the forces of electrostatic repulsion. As nuclear force is much stronger than the electrostatic force for the distances between nucleons in a typical nucleus, it overcomes the repulsive force and keeps the nucleons together, making the nucleus stable.

The nuclear force is not yet well understood. What we know about its properties can be summarized as follows.

1. It is the strongest force among subatomic particles. Its strength is about 50-60 times larger than that of the electrostatic force.
2. Unlike the electromagnetic and gravitational forces which act over large distances (their range is infinity), the nuclear force has a range of about a few fm and the force is negligible when two nucleons are separated by larger distances.
3. The nuclear force is independent of the charge of the nucleons, i.e., the nuclear force between two neutrons with a given separation is the same as that between two protons or between a neutron and a proton at the same separation.

15.8. Nuclear Binding Energy:

We have seen that in a hydrogen atom, the energy with which the electron in its ground state is bound to the nucleus (which is a single proton in this case) is 13.6 eV. This is the amount of energy which is released when a proton and an electron are brought from infinity to form the atom in its ground state. In other words, this is the amount of energy which has to be supplied to the atom to separate the electron and the proton, i.e., to make them free. The protons and the neutrons inside a nucleus are also bound to one another. Energy has to be supplied to the nucleus to make the nucleons free, i.e., separate them and take them to large distances from one another. This energy is the ***binding energy*** of the nucleus. Same amount of energy is released if we bring individual nucleons from infinity to form the nucleus. Where does this released energy come from? It comes from the masses of the nucleons. The mass of a nucleus is smaller than the total mass of its constituent nucleons. Let the mass of a nucleus having atomic number Z and mass number A be M. It is smaller than the sum of masses of Z protons and N (= A-Z) neutrons. We can write,

$$\Delta M = Z\,m_{\mathrm{p}} + N\,m_{\mathrm{n}} - M \qquad \text{---(15.14)}$$

ΔM is called the ***mass defect*** of the nucleus. The binding energy E_B , of the nucleus is given by

$$E_B = \Delta M\,c^2 = (Z\,m_{\mathrm{p}} + N\,m_{\mathrm{n}} - M)c^2 \qquad \text{---(15.15)}$$

On the right hand side of Eq.(15.15), we can add and subtract the mass of Z electrons which will enable us to use atomic masses in the calculation of binding energy. The Eq.(15.15) thus becomes

$$E_B = \left[\left(Z\,m_{\mathrm{p}} + Z\,m_{\mathrm{e}}\right) + N\,m_{\mathrm{n}} - \left(M + Z\,m_{\mathrm{e}}\right)\right]c^2$$

$$E_B = \left[Z\,m_{\mathrm{H}} + N\,m_{\mathrm{n}} - {}^{A}_{Z}M\right]c^2 \qquad \text{---(15.16)}$$

Here, m_{H} is the mass of a hydrogen atom and ${}^{A}_{Z}M$ is the atomic mass of the element being considered. We will be using atomic masses in what follows, unless otherwise specified.

An important quantity in this regard is the binding energy per nucleon (=E_B/A) of a nucleus. This can be considered to be the average energy which has to be supplied to a nucleon to remove it from the nucleus and make it free. This quantity thus, allows us to compare the relative strengths with which nucleons are bound in a nucleus for different species and therefore, compare their stabilities. Nuclei with higher values of E_B/A are more stable as compared to nuclei having smaller values of this quantity. Binding energy per nucleon for different values of A (i.e., for nuclei of different elements) are plotted in Fig.15.6.

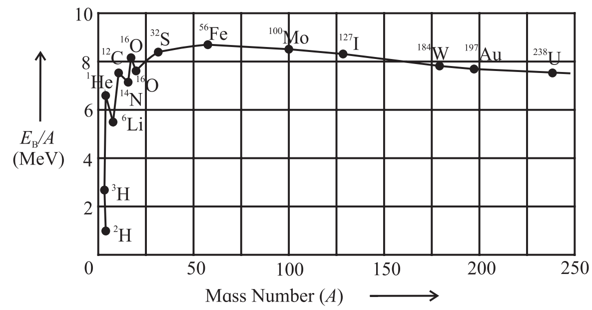

Fig.15.6: Binding energy per nucleon as a function of mass number.

Deuterium nucleus has the minimum value of E_B/A and is therefore, the least stable nucleus. The value of E_B/A increases with increase in atomic number and reaches a plateau for A between 50 to 80. Thus, the nuclei of these elements are the most stable among all the species. The peak occurs around A = 56 corresponding to iron, which is thus one of the most stable nuclei. The value of E_B/A decreases gradually for values of A greater than 80, making the nuclei of those elements slightly less stable. Note that the binding energy of hydrogen nucleus having a single proton is zero.

Example 15.6: Calculate the binding energy of ${}^{7}_{3}\mathrm{Li}$, the masses of hydrogen and lithium atoms being 1.007825 u and 7.016 u respectively.

Solution: The binding energy is given by

$$E_B = (3\,m_{\mathrm{H}} + 4\,m_{\mathrm{n}} - m_{\mathrm{Li}})c^2$$

= (3 × 1.007825 + 4 × 1.00866 - 7.016) × 931.5 = 39.23 MeV

15.9 Radioactive Decays:

Many of the nuclei are stable, i.e., they can remain unchanged for a very long time. These have a particular ratio of the mass number and the atomic number. Other nuclei occurring in nature, are not so stable and undergo changes in their structure by emission of some particles. They change or decay to other nuclei (with different A and Z) in the process. The decaying nucleus is called the ***parent nucleus*** while the nucleus produced after the decay is called the ***daughter nucleus***. The process is called ***radioactive decay*** or ***radioactivity*** and was discovered by Becquerel (1852-1908) in 1876. Radioactive decays occur because the parent nuclei are unstable and get converted to more stable daughter nuclei by the emission of some particles. These decays are of three types as described below.

Alpha Decay: In this type of decay, the parent nucleus emits an alpha particle which is the nucleus of helium atom. The parent nucleus thus loses two protons and two neutrons. The decay can be expressed as

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + \alpha \qquad \text{--- (15.17)}$$

X is the parent nucleus and Y is the daughter nucleus. All nuclei with $A > 210$ undergo alpha decay. The reason is that these nuclei have a large number of protons. The electrostatic repulsion between them is very large and the attractive nuclear forces between the nucleons are not able to cope with it. This makes the nucleus unstable and it tries to reduce the number of its protons by ejecting them in the form of alpha particles. An example of this is the alpha decay of bismuth which is the parent nucleus with $A = 212$ and $Z = 83$. The daughter nucleus has $A = 208$ and $Z = 81$, which is thallium. The reaction is

$$^{212}_{83}\mathrm{Bi} \rightarrow {}^{208}_{81}\mathrm{Tl} + \alpha$$

The total mass of the products of an alpha decay is always less than the mass of the parent atom. The excess mass appears as the kinetic energy of the products. The difference in the energy equivalent of the mass of the parent atom and that of the sum of masses of the products is called the Q-value, Q, of the decay and is equal to the kinetic energy of the products. We can write,

$$Q = [m_X - m_Y - m_{He}]c^2, \qquad \text{--- (15.18)}$$

m_X, m_Y and m_{He} being the masses of the parent atom, the daughter atom and the helium atom. Note that we have used atomic masses to calculate the Q factor.

Do you know?

Becquerel discovered the radioactive decay by chance. He was studying the X-rays emitted by naturally occurring materials when exposed to Sunlight. He kept a photographic plate covered in black paper, separated from the material by a silver foil. When the plates were developed, he found images of the material on them, showing that the X-rays could penetrate the black paper and silver foil. Once while studying uranium-potassium phosphate in a similar way, the Sun was behind the clouds so no exposure to Sunlight was possible. In spite of this, he went ahead and developed the plates and found images to have formed. With further experimentation he concluded that some rays were emitted by uranium itself for which no exposure to Sunlight was necessary. He then passed the rays through magnetic field and found that the rays were affected by the magnetic field. He concluded that the rays must be charged particles and hence were different from the X-rays.

The term radioactivity was coined by Marie Curie who made further studies and later discovered element radium along with her husband. The Nobel Prize for the year 1903 was awarded jointly to Becquerel, Marie Curie and Pierre Curie for their contributions to radioactivity.

Beta Decay: In this type of decay the nucleus emits an electron produced by converting a neutron in the nucleus into a proton. Thus, the basic process which takes place inside the parent nucleus is

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \text{antineutrino} \qquad \text{--- (15.19)}$$

Neutrino and antineutrino are particles

Example 15.7: Calculate the energy released in the alpha decay of ^{238}Pu to ^{234}U, the masses involved being m_{Pu} = 238.04955 u, m_U = 234.04095 u and m_{He} = 4.002603 u.

Solution: The decay can be written as $^{238}Pu \rightarrow {}^{234}U + {}^{4}He$. Its Q value, i.e., the energy released is given by

$$Q = [m_{Pu} - m_U - m_{He}]c^2$$
$$= [238.04955 - 234.04095 - 4.002603]c^2 \text{ u}$$
$$= 0.005997 \times 931.5 \text{ MeV} = 5.5862 \text{ MeV}.$$

which have very little mass and no charge. During beta decay, the number of nucleons i.e., the mass number of the nucleus remains unchanged. The daughter nucleus has one less neutron and one extra proton. Thus, Z increases by one and N decreases by one, A remaining constant. The decay can be written as,

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \text{antineutrino} \quad \text{--- (15.20)}$$

An example is

$$^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + e^- + \text{antineutrino.}$$

There is another type of beta decay called the ***beta plus decay*** in which a proton gets converted to a neutron by emitting a positron and a neutrino. A positron is a particle with the same properties as an electron except that its charge is positive. It is known as the antiparticle of electron. This decay can be written as,

$$p \rightarrow n + e^+ + \text{neutrino} \quad \text{--- (15.21)}$$

The mass number remains unchanged during the decay but Z decreases by one and N increases by one. The decay can be written as

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^+ + \text{neutrino} \quad \text{--- (15.22)}$$

An example is

$$^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + e^+ + \text{neutrino.}$$

An interesting thing about beta plus decay is that the mass of a neutron is higher than the mass of a proton. Thus the decay described by Eq. (15.21) cannot take place for a free proton. However, it can take place when the proton is inside the nucleus as the extra energy needed to produce a neutron can be obtained from the rest of the nucleus.

In beta decay also, the total mass of the products of the decay is less than the mass of the parent atom. The excess mass is converted into kinetic energy of the products. The Q value for the decay can be written as

$$Q = [m_X - m_Y - m_e]c^2 \quad \text{--- (15.23)}$$

Here, we have ignored the mass of the neutrino as it is negligible compared to the masses of the nuclei.

Example 15.8: Calculate the maximum kinetic energy of the beta particle (positron) emitted in the decay of $^{22}_{11}Na$, given the mass of $^{22}_{11}Na$ = 21.994437 u, $^{22}_{10}Ne$ = 21.991385 u and m_e = 0.00055 u.

Solution: The decay can be written as $^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + e^+ +$ neutrino. The energy released is

$$Q = [m_{Na} - m_{Ne} - m_e]c^2$$
$$= [21.994437 - 21.991385 - 0.00055]c^2$$
$$= 0.002502 \times 931.5 \text{ MeV} = 2.3306 \text{ MeV}$$

This is the maximum energy that the beta particle (e^+) can have, the neutrino having zero energy in that case.

Gamma Decay: In this type of decay, gamma rays are emitted by the parent nucleus. As you know, gamma ray is a high energy photon. The daughter nucleus is same as the parent nucleus as no other particle is emitted, but it has less energy as some energy goes out in the form of the emitted gamma ray.

We have seen that the electrons in an atom are arranged in different energy levels (orbits) and an electron from a higher orbit can make a transition to the lower orbit emitting a photon in the process. The situation in a nucleus is similar. The nucleons occupy energy levels with different energies. A nucleon can make a transition from a higher energy level to a lower energy level, emitting a photon in the process. The difference between atomic and nuclear energy levels is in their energies and energy separations. Energies and the differences in the energies of different levels in an atom are of the order of a few eVs, while those in the case of a nucleus are of the order of a few keV to a few MeV. Therefore, whereas the radiations emitted by atoms are in the ultraviolet to radio region, the radiations emitted by nuclei are in the range of gamma rays.

Usually, the nucleons in a nucleus are in the lowest possible energy state. They cannot easily get excited as a large amount of energy (in keVs or MeVs) is required for their excitation. A nucleon however may end up in an excited state as a result of the parent nucleus undergoing alpha or beta decay. Thus, gamma decays usually occur after one of these decays. For example, ^{57}Co undergoes beta plus decay to form the daughter nucleus ^{57}Fe which is in an excited state having energy of 136 keV. There are two ways in which it can make a transition to its ground state. One is by emitting a gamma ray of energy 136 keV and the other is by emitting a gamma ray of energy 122 keV and going to an intermediate state first and then emitting a photon of energy 14 keV to reach the ground state. Both these emissions have been observed experimentally. Which type of decay a nucleus will undergo depends on which of the resulting daughter nucleus is more stable. Often, the daughter nucleus is also not stable and it undergoes further decay. A chain of decays may take place until the final daughter nucleus is stable. An example of such a series decay is that of ^{238}U, which undergoes a series of alpha and beta decays, a total of 14 times, to finally reach a stable daughter nucleus of ^{206}Pb.

Use your brain power

Why don't heavy nuclei decay by emitting a single proton or a single neutron?

15.10. Law of Radioactive Decay:

Materials which undergo alpha, beta or gamma decays are called radioactive materials. The nature of radioactivity is such that if we have one atom of the radioactive material, we can never predict how long it will take to decay. If we have N_0 number of radioactive atoms (parent atoms or nuclei) of a particular kind say uranium, at time $t = 0$, all we can say is that their number will decrease with time as some nuclei (we cannot say which ones) will decay. Let us assume that at time t, number of parent nuclei which are left is $N(t)$. How many of these will decay in the interval between t and $t + dt$? We can guess that the larger the value of $N(t)$, larger will be the number of decays dN in time dt. Thus, we can say that dN will be proportional to $N(t)$. Also, we can guess that the larger the interval dt, larger will be the number of particles decaying in that interval. Thus, we can write,

$$dN \propto -N(t)\,dt,$$

or, $$dN = -\lambda N(t)\,dt \qquad \text{--- (15.24)}$$

where, λ is a constant of proportionality called the ***decay constant***. The negative sign in Eq.(15.24) indicates that the change in the number of parent nuclei dN, is negative, i.e., $N(t)$ is decreasing with time. We can integrate this equation as

$$\int_{N_0}^{N(t)} \frac{dN}{N(t)} = -\lambda \int_0^t dt,$$

Here, N_0 is the number of parent atoms at time $t = 0$. Integration gives,

$$\ln \frac{N(t)}{N_0} = -\lambda t,$$

or, $$N(t) = N_0 e^{-\lambda t} \qquad \text{---(15.25)}$$

This is the ***decay law*** of radioactivity. The rate of decay, i.e., the number of decays per unit time $-\dfrac{dN(t)}{dt}$, also called the ***activity A(t),*** can be written using Eq.(15.24) and (15.25) as,

$$A(t) = -\frac{dN}{dt} = \lambda N(t) = \lambda N_0 e^{-\lambda t} \qquad \text{--- (15.26)}$$

At $t = 0$, the activity is given by $A_0 = \lambda N_0$. Using this, Eq.(15.26) can be written as

$$A(t) = A_0 e^{-\lambda t} \qquad \text{--- (15.27)}$$

Activity is measured in units of becquerel (Bq) in SI units. One becquerel is equal to one decay per second. Another unit to measure activity is curie (Ci). One curie is 3.7 x 10^{10} decays per second. Thus, 1 Ci = 3.7 x 10^{10} Bq.

15.10.1. Half-life of Radioactive Material:

The time taken for the number of parent radioactive nuclei of a particular species to reduce to half its value is called the half-life $T_{1/2}$, of the species. This can be obtained from Eq. (15.25)

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}, \text{ giving}$$

$$e^{\lambda T_{1/2}} = 2,$$

$$\text{or} \quad T_{1/2} = \frac{\ln 2}{\lambda} = 0.693/\lambda \qquad \text{--- (15.28)}$$

The interesting thing about half-life is that even though the number goes down from N_0 to $\frac{N_0}{2}$ in time $T_{1/2}$, after another time interval $T_{1/2}$, the number of parent nuclei will not go to zero. It will go to half of the value at $t = T_{1/2}$, i.e., to $\frac{N_0}{4}$. Thus, in a time interval equal to half-life, the number of parent nuclei reduces by a factor of ½.

15.10.2 Average Life of a Radioactive Species:

We have seen that different nuclei of a given radioactive species decay at different times, i.e., they have different life times. We can calculate the average life time of a nucleus of the material using Eq.(15.25) as described below.

The number of nuclei decaying between time t and $t + dt$ is given by $\lambda N_0 e^{-\lambda t} \mathrm{d}t$. The life time of these nuclei is t. Thus, the average lifetime τ of a nucleus is

$$\tau = \frac{1}{N_0}\int_0^\infty t\,\lambda N_0 e^{-\lambda t} dt = \lambda \int_0^\infty t\, e^{-\lambda t} \mathrm{d}t,$$

Integrating the above we get

$$\tau = 1/\lambda \qquad \text{--- (15.29)}$$

The relation between the average life and half-life can be obtained using Eq.(15.28) as

$$T_{1/2} = \tau \ln 2 = 0.693\tau \qquad \text{--- (15.30)}$$

Example 15.9: The half-life of a nuclear species ${}^{N}X$ is 3.2 days. Calculate its (i) decay constant, (ii) average life and (iii) the activity of its sample of mass 1.5 mg.

Solution: The half-life ($T_{1/2}$) is related to the decay constant (λ) by

$T_{1/2} = 0.693/\lambda$ giving,

$\lambda = 0.693/T_{1/2}$

= 0.693/3.2

= 0.2166 per day

= 0.2166 /(24 × 3600) s^{-1}

= 2.507 × 10^{-6} s^{-1}.

Average life is related to decay constant by $\tau = 1/\lambda$ = 1/0.2166 per day = 4.617 days

The activity is given by $A = \lambda N(t)$, where $N(t)$ is the number of nuclei in the given sample. This is given by

$N(t)$= 6.02 × 10^{23} × 1.5 × $10^{-3}/Y$ = 9.03 × $10^{20}/Y$

Here, Y is the atomic mass of nuclear species X in g per mol.

$\therefore A$ = 9.03 × 10^{20} × 2.5 × $10^{-6}/Y$

= 2.257 × 10^{15} /Y

= 2.257 × 10^{15}/(Y x 3.7 × 10^{10}) Ci

= 6.1 × $10^4/Y$ Ci.

Example 15.10: The activity of a radioactive sample decreased from 350 s^{-1} to 175 s^{-1} in one hour. Determine the half-life of the species.

Solution: The time dependence of activity is given by $A(t) = A_0 e^{-\lambda t}$, where, $A(t)$ and A_0 are the activities at time t and 0 respectively.

175 = 350 $e^{-\lambda 3600}$,

or, 3600 λ = ln (350/175) = ln 2 = 0.6931

λ = 0.6931/3600 = 1.925 × 10^{-4} s^{-1}.

The half-life is given by $T_{1/2} = 0.693/\lambda$.

$$\therefore \quad T_{1/2} = \frac{0.693}{1.925\times10^{-4}} = 3.6\times10^3 s$$

Example 15.11: In an alpha decay, the daughter nucleus produced is itself unstable and undergoes further decay. If the number of parent and daughter nuclei at time t are N_p and N_d respectively and their decay constants are λ_p and λ_d respectively. What condition needs to be satisfied in order for N_d to remain constant?

Solution: The number of parent nuclei decaying between time 0 and *dt*, for small values of *dt* is given by $N_p \lambda_p dt$. This is the number of daughter nuclei produced in time *dt*. The number of daughter nuclei decaying in the same interval is $N_d \lambda_d dt$. For the number of daughter nuclei to remain constant, these two quantities, i.e., the number of daughter nuclei produced in time *dt* and the number decaying in time *dt* have to be equal. Thus, the required condition is given by

$$N_p \lambda_p dt = N_d \lambda_d dt ,$$

$$\text{or, } N_p \lambda_p = N_d \lambda_d$$

15.11. Nuclear Energy:

You are familiar with the naturally occurring, conventional sources of energy. These include the fossil fuels, i.e., coal, petroleum, natural gas, and fire wood. The energy generation from these fuels is through chemical reactions. It takes millions of years for these fuels to form. Naturally, the supply of these conventional sources is limited and with indiscriminate use, they are bound to get over in a couple of hundred years from now. Therefore, we have to use alternative sources of energy. The ones already in use are hydroelectric power, solar energy, wind energy and nuclear energy, nuclear energy being the largest source among these.

Nuclear energy is the energy released when nuclei undergo a ***nuclear reaction***, i.e., when one nucleus or a pair of nuclei, due to their interaction, undergo a change in their structure resulting in new nuclei and generating energy in the process. While the energy generated in chemical reactions is of the order of few eV per reaction, the amount of energy released in a nuclear reaction is of the order of a few MeV. Thus, for the same weight of fuel, the nuclear energy released is about a million times that released through chemical reactions. However, nuclear energy generation is a very complex and expensive process and it can also be extremely harmful. Let us learn more about it.

We have seen in section 15.8 that the mass of a nucleus is smaller than the sum of masses of its constituents. The difference in these two masses is the binding energy of the nucleus. It would be the energy released if the nucleus is formed by bringing together its constituents from infinity. This energy is large (in MeV), and this process can be a good source of energy. In practice, we never form nuclei starting from individual nucleons. However, we can obtain nuclear energy by two other processes (i) nuclear fission in which a heavy nucleus is broken into two nuclei of smaller masses and (ii) nuclear fusion in which two light nuclei undergo nuclear reaction and fuse together to form a heavier nucleus. Both fission and fusion are nuclear reactions. Let us understand how nuclear energy is released in the two processes.

15.11.1. Nuclear Fission:

We have seen in Fig.15.6 that the binding energy per nucleon (E_B/A) depends on the mass number of the nuclei. This quantity is a measure of the stability of the nucleus. As seen from the figure, the middle weight nuclei (mass number ranging from 50 to 80) have highest binding energy per nucleon and are most stable, while nuclei with higher and lower atomic masses have smaller values of E_B/A. The value of E_B/A goes on decreasing till A~238 which is the mass number of the heaviest naturally occurring element which is uranium. Many of the heavy nuclei are unstable and decay into two smaller mass nuclei.

Let us consider a case when a heavy nucleus, say with A ~230, breaks into two nuclei having A between 50 and 150. The E_B/A of the product nuclei will be higher than that of the parent nucleus. This means that the combined masses of the two product nuclei will be smaller than the mass of the parent nucleus. The difference in the mass of the parent nucleus and that of the product nuclei taken together will be released in the form of energy in the process. This process in which a heavy nucleus breaks into two lighter nuclei with the release of energy is called ***nuclear fission*** and is a source of nuclear energy.

One of the nuclei used in nuclear energy generation by fission is ${}^{236}_{92}U$. This has a half-life of 2.3 x 10^7 years and an activity of 6.5 x 10^{-5} Ci/g. However, it being fissionable, most of its nuclei have already decayed and it is not found in nature. More than 99% of natural uranium is in the form of ${}^{238}_{92}U$ and less than 1% is in the form of ${}^{235}_{92}U$. ${}^{238}_{92}U$ also decays, but its half-life is about 10^3 times higher than that of ${}^{236}_{92}U$ and is therefore not very useful for energy generation. The species needed for nuclear energy generation, i.e., ${}^{236}_{92}U$ can be obtained from the naturally occurring ${}^{235}_{92}U$ by bombarding it with slow neutrons. ${}^{235}_{92}U$ absorbs a neutron and yields ${}^{236}_{92}U$. This reaction can be written as ${}^{235}_{92}U + n \rightarrow {}^{236}_{92}U$. ${}^{236}_{92}U$ can undergo fission in several ways producing different pairs of daughter nuclei and generating different amounts of energy in the process. Some of its decays are

$${}^{236}_{92}U \rightarrow {}^{137}_{53}I + {}^{97}_{39}Y + 2n$$
$${}^{236}_{92}U \rightarrow {}^{140}_{56}Ba + {}^{94}_{36}Kr + 2n$$
$${}^{236}_{92}U \rightarrow {}^{133}_{51}Sb + {}^{99}_{41}Nb + 4n \quad \text{--- (15.31)}$$

Some of the daughter nuclei produced are not stable and they further decay to produce more stable nuclei. The energy produced in the fission is in the form of kinetic energy of the products, i.e., in the form of heat which can be collected and converted to other forms of energy as needed.

Uranium Nuclear Reactor:

A nuclear reactor is an apparatus or a device in which nuclear fission is carried out in a controlled manner to produce energy in the form heat which is then converted to electricity. In a uranium reactor, ${}^{235}_{92}U$ is used as the fuel. It is bombarded by slow neutrons to produce ${}^{236}_{92}U$ which undergoes fission.

Example 15.12: Calculate the energy released in the reaction

${}^{236}_{92}U \rightarrow {}^{137}_{53}I + {}^{97}_{39}Y + 2n$.

The masses of ${}^{236}_{92}U$, ${}^{137}_{53}I$ and ${}^{97}_{39}Y$ are 236.04557, 136.91787 and 96.91827 respectively.

Solution: Energy released is given by

$$Q = [m_U - m_I - m_Y - 2m_n]c^2$$
$$= [236.04557 - 136.91787 - 96.91827 - 2 \times 1.00865]\, c^2$$
$$= 0.19011 \times 931.5 \text{ MeV}$$
$$= 177.0875 \text{ MeV}$$

Chain Reaction:

Neutrons are produced in the fission reaction shown in Eq. (15.31). Some reactions produce 2 neutrons while others produce 3 or 4 neutrons. The average number of neutrons per reaction can be shown to be 2.7. These neutrons are in turn absorbed by other ${}^{235}_{92}U$ nuclei to produce ${}^{236}_{92}U$ which undergo fission and produce further 2.7 neutrons per fission. This can have a cascading effect and the number of neutrons produced and therefore the number of ${}^{236}_{92}U$ nuclei produced can increase quickly. This is called a ***chain reaction***. Such a reaction will lead to a fast increase in the number of fissions and thereby in a rapid increase in the amount of energy produced. This will lead to an explosion. In a nuclear reactor, methods are employed to stop a chain reaction from occurring and fission and energy generation is allowed to occur in a controlled fashion. The energy generated, which is in the form of heat, is carried away and converted to electricity by using turbines etc.

More than 15 countries have nuclear reactors and use nuclear power. India is one of them. There are 22 nuclear reactors in India, the largest one being at Kudankulam, Tamil Nadu. Maximum nuclear power is generated by the USA.

15.11.2. Nuclear Fusion:

We have seen that light nuclei (A < 40) have lower E_B/A as compared to heavier ones. If any two of the lighter nuclei come sufficiently close, within about one fm of each other, then they can undergo nuclear reaction and form a heavier nucleus. The heavier nucleus will have higher E_B/A than the reactants. The mass of the product nucleus will therefore be lower than the total mass of the reactants, and energy of the order of MeV will be released in the process. This process wherein two nuclei fuse together to form a heavier nucleus accompanied by a release of nuclear energy is called ***nuclear fusion***.

For a nuclear reaction to take place, it is necessary for two nuclei to come to within about 1 fm of each other so that they can experience the nuclear forces. It is very difficult for two atoms to come that close to each other due to the electrostatic repulsion between the electrons of the two atoms. This problem can be solved by stripping the atoms of their electrons and producing bare nuclei. It is possible to do so by giving the electrons energies larger than the ionization energies of the atoms by heating a gas of atoms. But even after this, the two bare nuclei find it very difficult to go near each other due to the repulsive force between their positive charges. For nuclear fusion to occur, we have to heat the gas to very high temperature thereby providing the nuclei with very high kinetic energies. These high energies can help them to overcome the electrostatic repulsion and come close to one another. As the positive charge of a nucleus goes on increasing with increase in its atomic number, the kinetic energies of the nuclei, i.e., the temperature of the gas necessary for nuclear fusion to occur goes on increasing with increase in Z.

Nuclear fusion is taking place all the time in the universe. It mostly takes place at the centres of stars where the temperatures are high enough for nuclear reactions to take place. There, light nuclei fuse into heavier nuclei generating energy in the process. Nuclear fusion is in fact the source of energy for stars. Most of the elements heavier than boron till iron, that we see around us today have been produced through nuclear fusion inside stars.

Do you know?

Light elements, i.e., deuterium, helium, lithium, beryllium and boron, have not been created inside stars, but are believed to have been created within the first 200 second in the life of the universe, i.e., within 200 seconds of the big bang which marked the beginning of the universe. The temperature at that time was very high and some nuclear reactions could take place. After about 200 s, the temperature decreased and nuclear reactions were no longer possible.

The temperature at the centre of the Sun is about 10^7 K. The nuclear reactions taking place at the centre of the Sun are the fusion of four hydrogen nuclei, i.e., protons to form a helium nucleus. Of course, because of the electrostatic repulsion and the values of densities at the centre of the Sun, it is extremely unlikely that four protons will come sufficiently close to one another at a given time so that they can combine to form helium. Instead, the fusion proceeds in several steps. The effective reaction can be written as

$$4\,\mathrm{p} \rightarrow \alpha + 2e^{+} + \text{neutrinos} + 26.7\ \text{MeV}.$$

These reactions have been going on inside the Sun since past 4.5 billion years and are expected to continue for similar time period in the future. At the centres of other stars where temperatures are higher, nuclei heavier than hydrogen can fuse generating energy.

Do you know?

The fusion inside stars can only take place between nuclei having mass number smaller than that of iron, i.e., 56. The reason for this is that iron has the highest E_B/A value among all elements as seen from Fig.15.6. If an iron nucleus fuses into another nucleus, the atomic number of the resulting nucleus will be higher than that of iron and hence it will have smaller E_B/A. The mass of the resultant nucleus will hence be larger than the sum of masses of the reactants and energy will have to be supplied to the reactants for the reaction to take place. The elements heavier than iron which are present in the universe are produced via other type of nuclear reaction which take place during stellar explosions.

Example 15.13: Calculate the energy released in the fusion reaction taking place inside the Sun, $4\,p \rightarrow \alpha + 2e^{+}$ + neutrinos, neglecting the energy given to the neutrinos. Mass of alpha particle being 4.001506 u.

Solution: The energy released in the process, ignoring the energy taken by the neutrinos is given by

$$Q = \left[4 \times m_p - m_\alpha - 2 \times m_e\right]c^2,$$

$$Q = \left[4 \times 1.00728 - 4.001506 - 2 \times 0.00055\right]c^2$$

$$= 0.026514 \times 931.5 = 24.70 \text{ MeV}$$

The discussion on nuclear energy will not be complete without mentioning its harmful effects. If an uncontrolled chain reaction sets up in a nuclear fuel, an extremely large amount of energy can be generated in a very short time. This fact has been used to produce what are called atom bombs or nuclear devices. Either fission alone or both fission and fusion are used in these bombs. The first such devices were made towards the end of the second world war by America. By now, several countries including India have successfully made and tested such nuclear devices. America remains the only country to have actually used two atom bombs which completely destroyed the cities of Hiroshima and Nagasaki in Japan in early August 1945.

Do you know?

We have seen that the activity of radioactive material decreases exponentially with time. Other examples of exponential decay are

- Amplitude of a simple pendulum decays exponentially as $A = A_0\, e^{-bt}$, where b is damping factor.
- Cooling of an object in an open surrounding is exponential. Temperature $\theta = \theta_0 e^{-kt}$ where k depends upon the object and the surrounding.
- Discharging of a capacitor through a pure resistor is exponential. Charge Q on the capacitor at a given instant is $Q = Q_0\, e^{-[t/RC]}$ where RC is called time constant.
- Charging of a capacitor is also exponential but, it is called exponential growth.

Internet my friend

1. https://www.siyavula.com/read/science/grade-10/the-atom/04-the-atom-02
2. https://en.wikipedia.org/wiki/Bohr_model
3. http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/atomstructcon.html
4. https://en.wikipedia.org/wiki/Atomic_nucleus
5. https://en.wikipedia.org/wiki/Radioactive_decay

Exercises

In solving problems, use m_e = 0.00055 u = 0.5110 MeV/c^2, m_p = 1.00728 u, m_n = 1.00866 u, m_H = 1.007825 u, u = 931.5 MeV, e = 1.602×10^{-19} C, h = 6.626 $\times10^{-34}$ Js, ε_0 = 8.854 $\times10^{-12}$ SI units and m_e = 9.109 $\times10^{-31}$ kg.

1. Choose the correct option.

i) In which of the following systems will the radius of the first orbit of the electron be smallest?
   (A) hydrogen (B) singly ionized helium
   (C) deuteron (D) tritium

ii) The radius of the 4th orbit of the electron will be smaller than its 8th orbit by a factor of
   (A) 2 (B) 4
   (C) 8 (D) 16

iii) In the spectrum of hydrogen atom which transition will yield longest wavelength?
   (A) $n = 2$ to $n = 1$ (B) $n = 5$ to $n = 4$
   (C) $n = 7$ to $n = 6$ (D) $n = 8$ to $n = 7$

iv) Which of the following properties of a nucleus does not depend on its mass number?
   (A) radius (B) mass
   (C) volume (D) density

v) If the number of nuclei in a radioactive sample at a given time is N, what will be the number at the end of two half-lives?
   (A) $N/2$ (B) $N/4$
   (C) $3N/4$ (D) $N/8$

2. Answer in brief.

i) State the postulates of Bohr's atomic model.

ii) State the difficulties faced by Rutherford's atomic model.

iii) What are alpha, beta and gamma decays?

iv) Define excitation energy, binding energy and ionization energy of an electron in an atom.

v) Show that the frequency of the first line in Lyman series is equal to the difference between the limiting frequencies of Lyman and Balmer series.

3. State the postulates of Bohr's atomic model and derive the expression for the energy of an electron in the atom.

4. Starting from the formula for energy of an electron in the n^{th} orbit of hydrogen atom, derive the formula for the wavelengths of Lyman and Balmer series spectral lines and determine the shortest wavelengths of lines in both these series.

5. Determine the maximum angular speed of an electron moving in a stable orbit around the nucleus of hydrogen atom.

6. Determine the series limit of Balmer, Paschen and Bracket series, given the limit for Lyman series is 911.6 Å.
   [Ans: 3646 Å, 8204 Å and 14585 Å]

7. Describe alpha, beta and gamma decays and write down the formulae for the energies generated in each of these decays.

8. Explain what are nuclear fission and fusion giving an example of each. Write down the formulae for energy generated in each of these processes.

9. Describe the principles of a nuclear reactor. What is the difference between a nuclear reactor and a nuclear bomb?

10. Calculate the binding energy of an alpha particle given its mass to be 4.00151 u.
   [Ans: 28.29 MeV]

11. An electron in hydrogen atom stays in its second orbit for 10^{-8} s. How many revolutions will it make around the nucleus in that time?
   [Ans: 8.221 x 10^6]

12. Determine the binding energy per nucleon of the americium isotope $^{244}_{95}\text{Am}$, given the mass of $^{244}_{95}\text{Am}$ to be 244.06428 u.
   [Ans: 7.5 MeV]

13. Calculate the energy released in the nuclear reaction $^{7}_{3}\text{Li} + p \rightarrow 2\alpha$ given mass of $^{7}_{3}\text{Li}$ atom and of helium atom to be 7.016 u and 4.0026 u respectively.
   [Ans: 16.84 MeV]

14. Complete the following equations describing nuclear decays.

(a) ${}^{226}_{86}\text{Ra} \rightarrow \alpha +$ (b) ${}^{19}_{8}\text{O} \rightarrow e^{-} +$

(c) ${}^{228}_{90}\text{Th} \rightarrow \alpha +$ (d) ${}^{12}_{7}\text{N} \rightarrow {}^{12}_{6}\text{C} +$

15. Calculate the energy released in the following reactions, given the masses to be ${}^{223}_{88}\text{Ra}$: 223.0185 u, ${}^{209}_{82}\text{Pb}$: 208.9811, ${}^{14}_{6}\text{C}$: 14.00324, ${}^{236}_{92}\text{U}$: 236.0456, ${}^{140}_{56}\text{Ba}$: 139.9106, ${}^{94}_{36}\text{Kr}$: 93.9341, ${}^{11}_{6}\text{C}$: 11.01143, ${}^{11}_{5}\text{B}$: 11.0093. Ignore neutrino energy.

(a) ${}^{223}_{88}\text{Ra} \rightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C}$

(b) ${}^{236}_{92}\text{U} \rightarrow {}^{140}_{56}\text{Ba} + {}^{94}_{36}\text{Kr} + 2\text{n}$

(c) ${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^{+} +$ neutrino

[Ans: a) 32.096 MeV, b) 172.485 MeV, c) 1.485 MeV]

16. Sample of carbon obtained from any living organism has a decay rate of 15.3 decays per gram per minute. A sample of carbon obtained from very old charcoal shows a disintegration rate of 12.3 disintegrations per gram per minute. Determine the age of the old sample given the decay constant of carbon to be 3.839×10^{-12} per second.

[Ans: 1803 yrs]

17. The half-life of ${}^{90}_{38}\text{Sr}$ is 28 years. Determine the disintegration rate of its 5 mg sample.

[Ans: 2.626×10^{10} s^{-1}]

18. What is the amount of ${}^{60}_{27}\text{Co}$ necessary to provide a radioactive source of strength 10.0 mCi, its half-life being 5.3 years?

[Ans: 8.88×10^{-6} g]

19. Disintegration rate of a sample is 10^{10} per hour at 20 hrs from the start. It reduces to 6.3×10^{9} per hour after 30 hours. Calculate its half life and the initial number of radioactive atoms in the sample.

[Ans: 15 hrs, 5.45×10^{11}]

20. The isotope ^{57}Co decays by electron capture to ^{57}Fe with a half-life of 272 d. The ^{57}Fe nucleus is produced in an excited state, and it almost instantaneously emits gamma rays. (a) Find the mean lifetime and decay constant for ^{57}Co. (b) If the activity of a radiation source ^{57}Co is 2.0 µCi now, how many ^{57}Co nuclei does the source contain? (c) What will be the activity after one year?

[Ans: 3.39×10^{7}, 2.95×10^{-8} s^{-1}, 2.51×10^{12} nuclei, 0.789 µCi]

21. A source contains two species of phosphorous nuclei, ${}^{32}_{15}\text{P}$ ($T_{1/2}$ = 14.3 d) and ${}^{33}_{15}\text{P}$ ($T_{1/2}$ = 25.3 d). At time $t = 0$, 90% of the decays are from ${}^{32}_{15}\text{P}$. How much time has to elapse for only 15% of the decays to be from ${}^{32}_{15}\text{P}$?

[Ans: 186.6 d]

22. Before the year 1900 the activity per unit mass of atmospheric carbon due to the presence of ^{14}C averaged about 0.255 Bq per gram of carbon. (a) What fraction of carbon atoms were ^{14}C? (b) An archaeological specimen containing 500 mg of carbon, shows 174 decays in one hour. What is the age of the specimen, assuming that its activity per unit mass of carbon when the specimen died was equal to the average value of the air? Half-life of ^{14}C is 5730 years?

[Ans: Four atoms in every 3×10^{12} carbon atoms were ^{14}C, 8020 years]

23. How much mass of ^{235}U is required to undergo fission each day to provide 3000 MW of thermal power? Average energy per fission is 202.79 MeV

[Ans: 3.1 kg]

24. In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}^{24}_{12}\text{Mg}$ (23.98504 u), ${}^{25}_{12}\text{Mg}$ (24.98584 u) and ${}^{26}_{12}\text{Mg}$ (25.98259 u). The natural abundance of ${}^{24}_{12}\text{Mg}$ is 78.99% by mass. Calculate the abundances of other two isotopes.

[Ans: 9.3% and 11.7%]

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