Chapter 3 Systems of Linear Equations and Inequalities

Ex 3.8

Answer 1e.

The given equation is of the form $AX = B$, where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants. Thus,

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, \text{ and } B = \begin{bmatrix} 4 \\ -2 \end{bmatrix}.$$

Therefore, the matrix of variables is $\begin{bmatrix} x \\ y \end{bmatrix}$ and the matrix of constants is $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

Answer 1gp.

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Substitute the values of a, b, c, and d.

$$\frac{1}{6(4)-2(1)}\begin{bmatrix} 4 & -1 \\ -2 & 6 \end{bmatrix}$$

Simplify.

$$\frac{1}{6(4)-2(1)}\begin{bmatrix} 4 & -1 \\ -2 & 6 \end{bmatrix} = \frac{1}{24-2}\begin{bmatrix} 4 & -1 \\ -2 & 6 \end{bmatrix}$$

$$= \frac{1}{22}\begin{bmatrix} 4 & -1 \\ -2 & 6 \end{bmatrix}$$

Multiply each element of the matrix by the scalar, $\frac{1}{26}$.

$$\frac{1}{22}\begin{bmatrix} 4 & -1 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} \frac{1}{22}(4) & \frac{1}{22}(-1) \\ \frac{1}{22}(-2) & \frac{1}{22}(6) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{11} & -\frac{1}{22} \\ -\frac{1}{11} & \frac{3}{11} \end{bmatrix}$$

Therefore, the inverse of the given matrix is

$$\begin{bmatrix} \frac{2}{11} & -\frac{1}{22} \\ -\frac{1}{11} & \frac{3}{11} \end{bmatrix}.$$

Answer 1mr.

a. Let the columns of the matrix represent the cost based on the commercial airs, and the rows represent their networks.

There are two networks, Network 1 and Network 2, and three commercial airs D, P, and L.

Use two 2 by 3 matrices A and B to represent the costs in city A and city B.

$$\begin{matrix} & \text{City A}\,(\boldsymbol{A}) & & \text{City B}\,(\boldsymbol{B}) \\ & \begin{matrix} \text{D} & \text{P} & \text{L} \end{matrix} & & \begin{matrix} \text{D} & \text{P} & \text{L} \end{matrix} \\ \begin{matrix} \text{Network 1} \\ \text{Network 2} \end{matrix} & \begin{bmatrix} 4.5 & 6 & 2.5 \\ 5.5 & 8 & 2.5 \end{bmatrix} & & \begin{bmatrix} 4 & 6.5 & 3.25 \\ 5 & 8.5 & 3.25 \end{bmatrix} \end{matrix}$$

b. Substitute the corresponding matrices for B and A.

$$B - A = \begin{bmatrix} 4 & 6.5 & 3.25 \\ 5 & 8.5 & 3.25 \end{bmatrix} - \begin{bmatrix} 4.5 & 6 & 2.5 \\ 5.5 & 8 & 2.5 \end{bmatrix}$$

In order to subtract two matrices, subtract the elements in the second matrix from the corresponding elements in the first. The resultant matrix will have the same dimensions as the matrices that are subtracted.

$$\begin{bmatrix} 4 & 6.5 & 3.25 \\ 5 & 8.5 & 3.25 \end{bmatrix} - \begin{bmatrix} 4.5 & 6 & 2.5 \\ 5.5 & 8 & 2.5 \end{bmatrix} = \begin{bmatrix} 4-4.5 & 6.5-6 & 3.25-2.5 \\ 5-5.5 & 8.5-8 & 3.25-2.5 \end{bmatrix}$$

Subtract.

$$\begin{bmatrix} 4-4.5 & 6.5-6 & 3.25-2.5 \\ 5-5.5 & 6.5-6 & 3.25-2.5 \end{bmatrix} = \begin{bmatrix} -0.5 & 0.5 & 0.75 \\ -0.5 & 0.5 & 0.75 \end{bmatrix}$$

The difference $B - A$ represents the difference in costs of the networks in the two cities. The column elements represent the cost difference in networks during the day time, prime time, and late night. The row elements represent the cost difference of the networks, Network 1 and Network 2, of the two cities.

c. It is given that all the costs increase by 10%. This means that each element in the matrix increases by 1.1. For this, multiply the matrices A and B by a scalar 1.1.

We multiply each element in the matrix by the scalar to multiply a matrix by a scalar. Multiply each element in matrix A by 1.1.

$$1.1\begin{bmatrix} 4.5 & 6 & 2.5 \\ 5.5 & 8 & 2.5 \end{bmatrix} = \begin{bmatrix} 1.1(4.5) & 1.1(6) & 1.1(2.5) \\ 1.1(5.5) & 1.1(8) & 1.1(2.5) \end{bmatrix}$$

Simplify.

$$\begin{bmatrix} 1.1(4.5) & 1.1(6) & 1.1(2.5) \\ 1.1(5.5) & 1.1(8) & 1.1(2.5) \end{bmatrix} = \begin{bmatrix} 4.95 & 6.6 & 2.75 \\ 6.05 & 8.8 & 2.75 \end{bmatrix}$$

Thus, the matrix of next year's costs for City A will be $\begin{bmatrix} 4.95 & 6.6 & 2.75 \\ 6.05 & 8.8 & 2.75 \end{bmatrix}$.

Multiply each element in matrix B by 1.1.

$$1.1\begin{bmatrix} 4 & 6.5 & 3.25 \\ 5 & 8.5 & 3.25 \end{bmatrix} = \begin{bmatrix} 1.1(4) & 1.1(6.5) & 1.1(3.25) \\ 1.1(5) & 1.1(8.5) & 1.1(3.25) \end{bmatrix}$$

Simplify.

$$\begin{bmatrix} 1.1(4) & 1.1(6.5) & 1.1(3.25) \\ 1.1(5) & 1.1(8.5) & 1.1(3.25) \end{bmatrix} = \begin{bmatrix} 4.4 & 7.15 & 3.575 \\ 5.5 & 9.35 & 3.575 \end{bmatrix}$$

Therefore, the matrix of next year's costs for City B will be $\begin{bmatrix} 4.4 & 7.15 & 3.575 \\ 5.5 & 9.35 & 3.575 \end{bmatrix}$.

Answer 1q.

Substitute the matrices for A and B in $2AB$.

$$2AB = 2\begin{bmatrix} 1 & -4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

Multiply the matrices.

$$\begin{bmatrix}1 & -4\\5 & 2\end{bmatrix}\begin{bmatrix}2 & -3\\0 & 1\end{bmatrix} = \begin{bmatrix}1(2)+(-4)(0) & 1(-3)+(-4)(1)\\5(2)+2(0) & 5(-3)+2(1)\end{bmatrix}$$
$$= \begin{bmatrix}2 & -7\\10 & -13\end{bmatrix}$$

Multiply each element in the matrix by 2.

$$2\begin{bmatrix}2 & -7\\10 & -13\end{bmatrix} = \begin{bmatrix}2(2) & 2(-7)\\2(10) & 2(-13)\end{bmatrix}$$
$$= \begin{bmatrix}4 & -14\\20 & -26\end{bmatrix}$$

Therefore, the value of $2AB = \begin{bmatrix}4 & -14\\20 & -26\end{bmatrix}$.

Answer 2e.

The inverse of a 2×2 matrix, $A=\begin{pmatrix}a & b\\c & d\end{pmatrix}$ is

$A^{-1}=\frac{1}{|A|}\begin{pmatrix}d & -b\\-c & a\end{pmatrix}$ provided $|A|\neq 0$.

Answer 2gp.

The inverse of the matrix $A=\begin{pmatrix}a & b\\c & d\end{pmatrix}$ is

$A^{-1}=\frac{1}{|A|}\begin{pmatrix}d & -b\\-c & a\end{pmatrix}$ provided $|A|\neq 0$.

Let $A=\begin{pmatrix}-1 & 5\\-4 & 8\end{pmatrix}$, then determinant of A is

$$|A|=\begin{vmatrix}-1 & 5\\-4 & 8\end{vmatrix}$$
$$=-8-(-20)$$
$$=12$$

Since $A \neq 0$, by definition, the inverse of the matrix A is

$$A^{-1} = \frac{1}{12}\begin{pmatrix} 8 & -5 \\ 4 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} 2/3 & -5/12 \\ 1/3 & -1/12 \end{pmatrix}$$

The inverse of the given matrix is $\boxed{\begin{pmatrix} 2/3 & -5/12 \\ 1/3 & -1/12 \end{pmatrix}}$.

Answer 2mr.

a. Let x, y, and z represent the number of coins in nickels, dimes, and quarters respectively.
Then for the given data, we can construct the following equations

$$x + y + z = 85$$
$$5x + 10y + 25z = 1325$$
$$z = 2y$$

Writing in the standard form, the system of equation becomes,

$$\boxed{\begin{cases} x + y + z = 85 \\ 5x + 10y + 25z = 1325 \\ 2y - z = 0 \end{cases}}$$

b. Let us write this linear system as the matrix equation $AX = B$ where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.
That is,

$$\begin{pmatrix} 1 & 1 & 1 \\ 5 & 10 & 25 \\ 0 & 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 85 \\ 1325 \\ 0 \end{pmatrix}$$

c. Solving for X we get,

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Enter the coefficient matrix A and the matrix of constants B into a graphing calculator. Then find the solution $X = A^{-1}B$

$$A^{-1}B = \begin{pmatrix} 25 \\ 20 \\ 40 \end{pmatrix}$$

Therefore, there were $\boxed{25}$ nickels, $\boxed{20}$ dimes, and $\boxed{40}$ quarters.

Answer 2p.

We have

$$A = \begin{pmatrix} 1 & -4 \\ 5 & 2 \end{pmatrix}, B = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}, C = \begin{pmatrix} -6 & -1 \\ 2 & 4 \end{pmatrix}$$

Let us substitute this in the given expression,

$$\begin{aligned} AB &= \begin{pmatrix} 1 & -4 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1(2)+(-4)(0) & 1(-3)+(-4)(1) \\ 5(2)+2(0) & 5(-3)+2(1) \end{pmatrix} \\ &= \begin{pmatrix} 2+0 & -3-4 \\ 10+0 & -15+2 \end{pmatrix} \\ &= \begin{pmatrix} 2 & -7 \\ 10 & -13 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} AC &= \begin{pmatrix} 1 & -4 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} -6 & -1 \\ 2 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 1(-6)+(-4)(2) & 1(-1)+(-4)(4) \\ 5(-6)+2(2) & 5(-1)+2(4) \end{pmatrix} \\ &= \begin{pmatrix} -6-8 & -1-16 \\ -30+4 & -5+8 \end{pmatrix} \\ &= \begin{pmatrix} -14 & -17 \\ -26 & 3 \end{pmatrix} \end{aligned}$$

$$AB + AC = \begin{pmatrix} 2 & -7 \\ 10 & -13 \end{pmatrix} + \begin{pmatrix} -14 & -17 \\ -26 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2-14 & -7-17 \\ 10-26 & -13+3 \end{pmatrix}$$

$$= \begin{pmatrix} -12 & -24 \\ -16 & -10 \end{pmatrix}$$

Therefore, $\boxed{AB + AC = \begin{bmatrix} -12 & -24 \\ -16 & -10 \end{bmatrix}}$.

Answer 3e.

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Substitute the values of a, b, c, and d.

$$\frac{1}{1(4)-(-1)-5} \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix}$$

Simplify.

$$\frac{1}{1(4)-(-1)-5} \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix} = \frac{1}{4-5} \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix}$$

$$= -1 \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix}$$

Multiply each element of the matrix by the scalar, -1.

$$-1 \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -4 & -5 \\ -1 & -1 \end{bmatrix}$$

Therefore, the inverse of the given matrix is $\begin{bmatrix} -4 & -5 \\ -1 & -1 \end{bmatrix}$.

Answer 3gp.

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Substitute the values of a, b, c, and d.

$$\frac{1}{(-3)(-2)-(-1)(-4)} \begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$$

Simplify.

$$\frac{1}{(-3)(-2)-(-1)(-4)}\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix} = \frac{1}{-6-4}\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$$

$$= -\frac{1}{10}\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$$

Multiply each element of the matrix by the scalar, $\frac{1}{26}$.

$$-\frac{1}{10}\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{10}(-2) & -\frac{1}{10}(4) \\ -\frac{1}{10}(1) & -\frac{1}{10}(-3) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix}$$

Therefore, the inverse of the given matrix is $\begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix}$.

Answer 3mr.

a. In a 2×2 matrix, the difference of the products of the elements on the diagonals is its determinant.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Find the determinant of matrix A.

$$\begin{vmatrix} 3 & 2 \\ 8 & 4 \end{vmatrix} = 3(4) - 8(2)$$

$$= 12 - 16$$

$$= -4$$

The determinant of A will be -4.

Now, we have to determine another matrix, C, such that $C \neq A$ and det $C = -4$.

We have to choose the elements in such a way that the difference of the products of the elements on the diagonals is -4.

Let the new matrix be $C = \begin{bmatrix} 6 & 4 \\ 4 & 2 \end{bmatrix}$.

Find the determinant of the matrix C.

$$\begin{vmatrix} 6 & 4 \\ 4 & 2 \end{vmatrix} = 6(2) - 4(4)$$
$$= 12 - 16$$
$$= -4$$

We note that det $C = -4$. Thus, the matrix can be $C = \begin{bmatrix} 6 & 4 \\ 4 & 2 \end{bmatrix}$.

b. The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

Find the determinant of B.

$$\begin{vmatrix} 7 & 0 & 5 \\ 1 & 4 & 3 \\ 2 & 4 & 6 \end{vmatrix} = [7(4)(6) + 0(3)(2) + 5(1)(4)] - [2(4)(5) + 4(3)(7) + 6(1)(0)]$$

Evaluate.

$$[7(4)(6) + 0(3)(2) + 5(1)(4)] - [2(4)(5) + 4(3)(7) + 6(1)(0)]$$
$$= [168 + 0 + 20] - [40 + 84 + 0]$$
$$= 188 - 124$$
$$= 64$$

The determinant of the matrix B is 64.

We need to find another matrix, D, such that its determinant is also 64. Such a matrix can be

$$D = \begin{bmatrix} 7 & 0 & 5 \\ 2 & 5 & 3 \\ -\frac{1}{5} & 1 & 2 \end{bmatrix}$$

Find the determinant of D.

$$\begin{vmatrix} 7 & 0 & 5 \\ 2 & 5 & 3 \\ -\frac{1}{5} & 1 & 2 \end{vmatrix} = \left[7(5)(2) + 0(3)\left(-\frac{1}{5}\right) + 5(2)(1)\right] - \left[\left(-\frac{1}{5}\right)(5)(5) + 1(3)(7) + 2(2)(0)\right]$$

Evaluate.

$$\begin{aligned} &\left[7(5)(2) + 0(3)\left(-\frac{1}{5}\right) + 5(2)(1)\right] - \left[\left(-\frac{1}{5}\right)(5)(5) + 1(3)(7) + 2(2)(0)\right] \\ &= [70 + 0 + 10] - [(-5) + 21 + 0] \\ &= 80 - 16 \\ &= 64 \end{aligned}$$

We note that $\det D = 64$. Thus, the matrix can be $D = \begin{bmatrix} 7 & 0 & 5 \\ 2 & 5 & 3 \\ -\frac{1}{5} & 1 & 2 \end{bmatrix}$.

Answer 3q.

Substitute the matrices for A, B, and C in $A(B + C)$.

$$A(B + C) = \begin{bmatrix} 1 & -4 \\ 5 & 2 \end{bmatrix} \left(\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -6 & -1 \\ 2 & 4 \end{bmatrix} \right)$$

Add the matrices by adding the elements in the corresponding positions.

$$\begin{bmatrix} 1 & -4 \\ 5 & 2 \end{bmatrix} \left(\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -6 & -1 \\ 2 & 4 \end{bmatrix} \right) = \begin{bmatrix} 1 & -4 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} -4 & -4 \\ 2 & 5 \end{bmatrix}$$

Multiply the matrices.

$$\begin{aligned} \begin{bmatrix} 1 & -4 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} -4 & -4 \\ 2 & 5 \end{bmatrix} &= \begin{bmatrix} 1(-4) + (-4)(2) & 1(-4) + (-4)(5) \\ 5(-4) + 2(2) & 5(-4) + 2(5) \end{bmatrix} \\ &= \begin{bmatrix} -12 & -24 \\ -16 & -10 \end{bmatrix} \end{aligned}$$

Therefore, the value of $A(B + C) = \begin{bmatrix} -12 & -24 \\ -16 & -10 \end{bmatrix}$.

Answer 4e.

The inverse of the matrix $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is

$$A^{-1}=\frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \text{ provided } |A|\neq 0.$$

Let $A=\begin{pmatrix} -2 & 3 \\ -3 & 4 \end{pmatrix}$, then determinant of A is

$$\begin{aligned} |A| &= \begin{vmatrix} -2 & 3 \\ -3 & 4 \end{vmatrix} \\ &= -8-(-9) \\ &= 1 \end{aligned}$$

Since $A\neq 0$, by definition, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{1}\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix} \end{aligned}$$

The inverse of the given matrix is $\boxed{\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}}$.

Answer 4gp.

Let us take the matrix equation $AX=B$ where

$$A=\begin{pmatrix} -4 & 1 \\ 0 & 6 \end{pmatrix} \text{ and } B=\begin{pmatrix} 8 & 9 \\ 24 & 6 \end{pmatrix}$$

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Thus we have to find A^{-1}

$$\begin{aligned} |A| &= \begin{vmatrix} -4 & 1 \\ 0 & 6 \end{vmatrix} \\ &= -24-0 \\ &= -24 \end{aligned}$$

Since $A \neq 0$, the inverse of the matrix A is

$$A^{-1} = \frac{1}{-24}\begin{pmatrix} 6 & -1 \\ 0 & -4 \end{pmatrix}$$
$$= \begin{pmatrix} -1/4 & 1/24 \\ 0 & 1/6 \end{pmatrix}$$

Now we can find X from $X = A^{-1}B$

$$X = A^{-1}B$$
$$= \begin{pmatrix} -1/4 & 1/24 \\ 0 & 1/6 \end{pmatrix}\begin{pmatrix} 8 & 9 \\ 24 & 6 \end{pmatrix}$$
$$= \begin{pmatrix} -2+1 & \frac{-9}{4}+\frac{1}{4} \\ 4 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} -1 & -2 \\ 4 & 1 \end{pmatrix}$$

Therefore, $\boxed{\begin{pmatrix} -1 & -2 \\ 4 & 1 \end{pmatrix}}$.

Answer 4mr.

The product of two matrices A and B is defined provided the number of column in A is equal to the number of rows in B.
If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

For the given data, since A is a 3×2 matrix and B is a 1×3 matrix, the product of AB is not defined. But the product of BA is defined and is a 1×2 matrix.

$$BA = \begin{pmatrix} 0.03 & 0.05 & 0.08 \end{pmatrix}\begin{pmatrix} 175 & 270 \\ 370 & 225 \\ 200 & 255 \end{pmatrix}$$
$$= \begin{pmatrix} 5.25+18.50+16 & 8.10+11.25+20.40 \end{pmatrix}$$
$$= \begin{pmatrix} 39.75 & 39.75 \end{pmatrix}$$

This matrix represents the commission each salespeople have earned.

Answer 4q.

We have

$$A=\begin{pmatrix}1 & -4\\ 5 & 2\end{pmatrix}, B=\begin{pmatrix}2 & -3\\ 0 & 1\end{pmatrix}, C=\begin{pmatrix}-6 & -1\\ 2 & 4\end{pmatrix}$$

Let us substitute this in the given expression,

$$\begin{aligned}B-A&=\begin{pmatrix}2 & -3\\ 0 & 1\end{pmatrix}-\begin{pmatrix}1 & -4\\ 5 & 2\end{pmatrix}\\ &=\begin{pmatrix}2-1 & -3+4\\ 0-5 & 1-2\end{pmatrix}\\ &=\begin{pmatrix}1 & 1\\ -5 & -1\end{pmatrix}\end{aligned}$$

$$\begin{aligned}(B-A)C&=\begin{pmatrix}1 & 1\\ -5 & -1\end{pmatrix}\begin{pmatrix}-6 & -1\\ 2 & 4\end{pmatrix}\\ &=\begin{pmatrix}1(-6)+1(2) & 1(-1)+1(4)\\ (-5)(-6)+(-1)(2) & -5(-1)+(-1)(4)\end{pmatrix}\\ &=\begin{pmatrix}-6+2 & -1+4\\ 30-2 & 5-4\end{pmatrix}\\ &=\begin{pmatrix}-4 & 3\\ 28 & 1\end{pmatrix}\end{aligned}$$

Therefore, $\boxed{(B-A)C=\begin{pmatrix}-4 & 3\\ 28 & 1\end{pmatrix}}$.

Answer 5e.

The inverse of matrix $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is

$$A^{-1}=\frac{1}{ad-cb}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}.$$

Substitute the values of a, b, c, and d.

$$\frac{1}{(6)(2)-(5)(2)}\begin{bmatrix}2 & -2\\ -5 & 6\end{bmatrix}$$

Simplify.

$$\frac{1}{(6)(2)-(5)(2)}\begin{bmatrix} 2 & -2 \\ -5 & 6 \end{bmatrix} = \frac{1}{12-10}\begin{bmatrix} 2 & -2 \\ -5 & 6 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 2 & -2 \\ -5 & 6 \end{bmatrix}$$

Multiply each element of the matrix by the scalar, $\frac{1}{2}$.

$$\frac{1}{2}\begin{bmatrix} 2 & -2 \\ -5 & 6 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(2) & \frac{1}{2}(-2) \\ \frac{1}{2}(-5) & \frac{1}{2}(6) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -1 \\ -\frac{5}{2} & 3 \end{bmatrix}$$

Therefore, the inverse of the given matrix is
$\begin{bmatrix} 1 & -1 \\ -\frac{5}{2} & 3 \end{bmatrix}$.

Answer 5gp.

First enter the given matrix A into the graphing calculator.

$$A = \begin{pmatrix} 2 & -2 & 0 \\ 2 & 0 & -2 \\ 12 & -4 & -6 \end{pmatrix}$$

Now find A^{-1}; we get,

$$A^{-1} = \begin{pmatrix} -1 & -1.5 & 0.5 \\ -1.5 & -1.5 & 0.5 \\ -1 & -2 & 0.5 \end{pmatrix}$$

Then compute AA^{-1} and $A^{-1}A$ to verify that you obtain the 3×3 identity matrix.

$$AA^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad A^{-1}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Therefore, the inverse of the given matrix is $\boxed{\begin{pmatrix} -1 & -1.5 & 0.5 \\ -1.5 & -1.5 & 0.5 \\ -1 & -2 & 0.5 \end{pmatrix}}$.

Answer 5mr.

a. Represent the data using the formula for each compound. Let the atomic weight of hydrogen be H, of nitrogen be N, and of oxygen be O.

From the table, we note that the atomic weight of the nitric acid is 63. Now, frame the equation.

$H + N + 3O = 63$

Similarly, frame equations for the other two compounds. The linear system of equations will be

$$\begin{aligned} H + N + 3O &= 63 \\ 2N + O &= 44 \\ 2H + O &= 18 \end{aligned}$$

b. The coefficient matrix of the new linear system will be $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$. Evaluate the determinant of the coefficient matrix. The determinant of a 3×3 matrix is defined as $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$.

Thus,

$$\begin{vmatrix} 1 & 1 & 3 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{vmatrix} = [1(2)(1) + 1(1)(2) + 3(0)(0)] - [2(2)(3) + 0(1)(1) + 1(0)(1)].$$

Evaluate.

$$\begin{aligned} &[1(2)(1) + 1(1)(2) + 3(0)(0)] - [2(2)(3) + 0(1)(1) + 1(0)(1)] \\ &= (2 + 2 + 0) - (12 + 0 + 0) \\ &= -8 \end{aligned}$$

c. Since the determinant is not 0, we can apply Cramer's rule to solve the system.

According to this rule, If $\det A \neq 0$, then the system has exactly one solution. The solution is

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\det A}, \ y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\det A}, \text{ and } z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\det A}.$$

Substitute the known values in the formula for x and evaluate.

$$x = \frac{\begin{vmatrix} 63 & 1 & 3 \\ 44 & 2 & 1 \\ 18 & 0 & 1 \end{vmatrix}}{-8}$$
$$= \frac{(126 + 18 + 0) - (108 + 0 + 44)}{-8}$$
$$= \frac{144 - 152}{-8}$$
$$= \frac{-8}{-8}$$
$$= 1$$

Substitute the known values in the formula for y and evaluate.

$$y = \frac{\begin{vmatrix} 1 & 63 & 3 \\ 0 & 44 & 1 \\ 2 & 18 & 1 \end{vmatrix}}{-8}$$
$$= \frac{(44 + 126 + 0) - (264 + 18 + 0)}{-8}$$
$$= \frac{170 - 282}{-8}$$
$$= \frac{-112}{-8}$$
$$= 14$$

Substitute the known values in the formula for z and evaluate.

$$z = \frac{\begin{vmatrix} 1 & 1 & 63 \\ 0 & 2 & 44 \\ 2 & 0 & 18 \end{vmatrix}}{-8}$$
$$= \frac{(36 + 88 + 0) - (252 + 0 + 0)}{-8}$$
$$= \frac{124 - 252}{-8}$$
$$= \frac{-128}{-8}$$
$$= 16$$

Therefore, the atomic weight of hydrogen is 1, of nitrogen is 14, and of oxygen is 16.

Answer 5q.

In a 2×2 matrix, the determinant is the difference of the products of the elements on the diagonals.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Thus,

$$\begin{vmatrix} 5 & 4 \\ -2 & -3 \end{vmatrix} = 5(-3) - (-2)(4).$$

Evaluate.

$$\begin{aligned} 5(-3) - (-2)(4) &= -15 + 8 \\ &= -7 \end{aligned}$$

Therefore, the determinant of the given matrix is -7.

Answer 6e.

The inverse of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \text{ provided } |A| \neq 0.$$

Let $A = \begin{pmatrix} -7 & -9 \\ 2 & 3 \end{pmatrix}$, then determinant of A is

$$\begin{aligned} |A| &= \begin{vmatrix} -7 & -9 \\ 2 & 3 \end{vmatrix} \\ &= -21 - (-18) \\ &= -3 \end{aligned}$$

Since $|A| \neq 0$, by definition, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{-3}\begin{pmatrix} 3 & 9 \\ -2 & -7 \end{pmatrix} \\ &= \begin{pmatrix} -1 & -3 \\ 2/3 & 7/3 \end{pmatrix} \end{aligned}$$

The inverse of the given matrix is $\boxed{\begin{pmatrix} -1 & -3 \\ 2/3 & 7/3 \end{pmatrix}}$.

Answer 6gp.

First enter the given matrix A into the graphing calculator.

$$A=\begin{pmatrix}-3 & 4 & 5\\ 1 & 5 & 0\\ 5 & 2 & 2\end{pmatrix}$$

Now find A^{-1}; we get,

$$A^{-1}=\begin{pmatrix}-0.06535 & -0.01307 & 0.16339\\ 0.01307 & 0.20261 & -0.03268\\ 0.15032 & -0.16993 & 0.12418\end{pmatrix}$$

Then compute AA^{-1} and $A^{-1}A$ to verify that you obtain the 3×3 identity matrix.

$$AA^{-1}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \qquad A^{-1}A=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Therefore, the inverse of the given matrix is $\boxed{\begin{pmatrix}-0.06535 & -0.01307 & 0.16339\\ 0.01307 & 0.20261 & -0.03268\\ 0.15032 & -0.16993 & 0.12418\end{pmatrix}}$.

Answer 6mr.

Let x, y, and z represent the number of bushels of corn, soybean, and wheat respectively. Then for the given data, we can construct the following equations

$$x+y+z=1700$$
$$2.35x+5.4y+3.6z=4837$$
$$x=3.25(y+z)$$

Writing in the standard form, the system of equation becomes,

$$x+y+z=1700$$
$$235x+540y+360z=483700$$
$$x-3.25y-3.25z=0$$

Let us write this linear system as the matrix equation $AX = B$ where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.
That is,

$$\begin{pmatrix} 1 & 1 & 1 \\ 235 & 540 & 360 \\ 1 & -3.25 & -3.25 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1700 \\ 483700 \\ 0 \end{pmatrix}$$

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Enter the coefficient matrix A and the matrix of constants B into a graphing calculator. Then find the solution $X = A^{-1}B$

$$A^{-1}B = \begin{pmatrix} 1300 \\ 190 \\ 210 \end{pmatrix}$$

Therefore, there were $\boxed{210}$ bushels of wheat.

Answer 6q.

The determinant of a 3×3 matrix is the difference of the sum of the product of the elements on the diagonal for each column. That is,

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$

$$\begin{aligned} \begin{vmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & 3 & -1 \end{vmatrix}\begin{matrix} 1 & 0 \\ -3 & 1 \\ 2 & 3 \end{matrix} &= (-1+0+18) - (-4+12+0) \\ &= 17 - 8 \\ &= 9 \end{aligned}$$

Therefore, the determinant of the given matrix is $\boxed{9}$.

Answer 7e.

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Substitute the values of a, b, c, and d.

$$\frac{1}{(-4)(7)-(-6)(4)}\begin{bmatrix} 7 & 6 \\ -4 & -4 \end{bmatrix}$$

Simplify.

$$\frac{1}{(-4)(7)-(-6)(4)}\begin{bmatrix} 7 & 6 \\ -4 & -4 \end{bmatrix} = \frac{1}{-28+24}\begin{bmatrix} 7 & 6 \\ -4 & -4 \end{bmatrix}$$
$$= -\frac{1}{4}\begin{bmatrix} 7 & 6 \\ -4 & -4 \end{bmatrix}$$

Multiply each element of the matrix by the scalar, $-\frac{1}{4}$.

$$-\frac{1}{4}\begin{bmatrix} 7 & 6 \\ -4 & -4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{4}(7) & -\frac{1}{4}(6) \\ -\frac{1}{4}(-4) & -\frac{1}{4}(-4) \end{bmatrix}$$
$$= \begin{bmatrix} -\frac{7}{4} & -\frac{3}{2} \\ 1 & 1 \end{bmatrix}$$

Therefore, the inverse of the given matrix is

$$\begin{bmatrix} -\frac{7}{4} & -\frac{3}{2} \\ 1 & 1 \end{bmatrix}.$$

Answer 7gp.

First enter the given matrix A into the graphing calculator.

$$A = \begin{pmatrix} 2 & 1 & -2 \\ 5 & 3 & 0 \\ 4 & 3 & 8 \end{pmatrix}$$

Now find A^{-1}; we get,

$$A^{-1} = \begin{pmatrix} 12 & -7 & 3 \\ -20 & 12 & -5 \\ 1.5 & -1 & 0.5 \end{pmatrix}$$

Then compute AA^{-1} and $A^{-1}A$ to verify that you obtain the 3×3 identity matrix.

$$AA^{-1}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \qquad A^{-1}A=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

Therefore, the inverse of the given matrix is $\boxed{\begin{pmatrix}12&-7&3\\-20&12&-5\\1.5&-1&0.5\end{pmatrix}}$.

Answer 7q.

The determinant of a 3×3 matrix is defined as

$$\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix}=(aei+bfg+cdh)-(gec+hfa+idb).$$

Thus,

$$\begin{vmatrix}2&-1&5\\-3&6&9\\-2&3&1\end{vmatrix}=[2(6)(1)+(-1)(9)(-2)+5(-3)(3)]-[(-2)(6)(5)+3(9)(2)+1(-3)(-1)].$$

Evaluate.

$$\begin{aligned}&[2(6)(1)+(-1)(9)(-2)+5(-3)(3)]-[(-2)(6)(5)+3(9)(2)+1(-3)(-1)]\\&=(12+18-45)-(-60+54+3)\\&=-15-(-3)\\&=-12\end{aligned}$$

Therefore, the determinant of the given matrix is -12.

Answer 8e.

The inverse of the matrix $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is

$$A^{-1}=\frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \text{ if } |A|\neq 0.$$

Let $A=\begin{pmatrix} 6 & -22 \\ -12 & 20 \end{pmatrix}$, then determinant of A is

$$\begin{aligned} |A| &= \begin{vmatrix} 6 & -22 \\ -12 & 20 \end{vmatrix} \\ &= 120-(264) \\ &= 144 \end{aligned}$$

Since $|A|\neq 0$, by definition, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{144}\begin{pmatrix} 20 & 22 \\ 12 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 5/36 & 11/72 \\ 1/12 & 1/24 \end{pmatrix} \end{aligned}$$

The inverse of the given matrix is $\boxed{\begin{pmatrix} 5/36 & 11/72 \\ 1/12 & 1/24 \end{pmatrix}}$.

Answer 8gp.

Let us write the given linear system as the matrix equation $AX = B$ where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.
That is,

$$\begin{pmatrix} 4 & 1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ -1 \end{pmatrix}$$

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Thus we have to find A^{-1}

$$\begin{aligned} |A| &= \begin{vmatrix} 4 & 1 \\ 3 & 5 \end{vmatrix} \\ &= 20 - 3 \\ &= 17 \end{aligned}$$

Since $A \neq 0$, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{17}\begin{pmatrix} 5 & -1 \\ -3 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 5/17 & -1/17 \\ -3/17 & 4/17 \end{pmatrix} \end{aligned}$$

Now we can find X from $X = A^{-1}B$

$$\begin{aligned} X &= A^{-1}B \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 5/17 & -1/17 \\ -3/17 & 4/17 \end{pmatrix}\begin{pmatrix} 10 \\ -1 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} \frac{50}{17} + \frac{1}{17} \\ -\frac{30}{17} - \frac{4}{17} \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 3 \\ -2 \end{pmatrix} \end{aligned}$$

Therefore, the solution of the system is $\boxed{(3, -2)}$.

Answer 8q.

Let us write the given linear system as the matrix equation $AX=B$ where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.
That is,

$$\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -2 \\ -6 \end{pmatrix}$$

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Thus we have to find A^{-1}

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} \\ &= 7-6 \\ &= 1 \end{aligned}$$

Since $A \neq 0$, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{1}\begin{pmatrix} 7 & -3 \\ -2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 7 & -3 \\ -2 & 1 \end{pmatrix} \end{aligned}$$

Now we can find X from $X=A^{-1}B$

$$\begin{aligned} X &= A^{-1}B \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 7 & -3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} -2 \\ -6 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} -14+18 \\ 4-6 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 4 \\ -2 \end{pmatrix} \end{aligned}$$

Therefore, the solution of the system is $\boxed{(4,-2)}$.

Answer 9e.

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Substitute the values of a, b, c, and d.

$$\frac{1}{(-24)(30)-(-6)(60)}\begin{bmatrix} 30 & -60 \\ 6 & -24 \end{bmatrix}$$

Simplify.

$$\frac{1}{(-24)(30)-(-6)(60)}\begin{bmatrix} 30 & -60 \\ 6 & -24 \end{bmatrix} = \frac{1}{-720+360}\begin{bmatrix} 30 & -60 \\ 6 & -24 \end{bmatrix}$$
$$= -\frac{1}{360}\begin{bmatrix} 30 & -60 \\ 6 & -24 \end{bmatrix}$$

Multiply each element of the matrix by the scalar.

$$-\frac{1}{360}\begin{bmatrix} 30 & -60 \\ 6 & -24 \end{bmatrix} = \begin{bmatrix} -\frac{1}{360}(30) & -\frac{1}{360}(-60) \\ -\frac{1}{360}(6) & -\frac{1}{360}(-24) \end{bmatrix}$$
$$= \begin{bmatrix} -\frac{1}{12} & \frac{1}{6} \\ -\frac{1}{60} & \frac{1}{15} \end{bmatrix}$$

Therefore, the inverse of the given matrix is

$$\begin{bmatrix} -\frac{1}{12} & \frac{1}{6} \\ -\frac{1}{60} & \frac{1}{15} \end{bmatrix}.$$

Answer 9gp.

First, number the equations.

$2x - y = -6$ Equation 1

$6x - 3y = -18$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} 2 & -1 \\ 6 & -3 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$=$	$\begin{bmatrix} -6 \\ -18 \end{bmatrix}$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

First find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} \\ &= (2)(-3) - (-1)(6) \\ &= -6 + 6 \\ &= 0 \end{aligned}$$

Since the determinant obtained is 0. The inverse of the A cannot be found.

When Equation 1 is multiplied by 3, we get the second equation. This means that the system has dependent equations. Therefore, it will have infinitely many solutions.

Answer 9q.

Name the equations.

$3x - 4y = 5$ Equation 1

$2x - 3y = 3$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$=$	$\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

For this, first find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 3 & -4 \\ 2 & -3 \end{vmatrix} \\ &= (3)(-3) - (2)(-4) \\ &= -9 + 8 \\ &= -1 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\begin{aligned} A^{-1} &= \frac{1}{-1}\begin{bmatrix} -3 & 4 \\ -2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix} \end{aligned}$$

Step 3 Multiply each side of $AX = B$ by A^{-1}.

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \end{bmatrix} \end{aligned}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{aligned} \begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \end{bmatrix} &= \begin{bmatrix} 3(5) - 4(3) \\ 2(5) - 3(3) \end{bmatrix} \\ &= \begin{bmatrix} 15 - 12 \\ 10 - 9 \end{bmatrix} \\ &= \begin{bmatrix} 3 \\ 1 \end{bmatrix} \end{aligned}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

CHECK

Substitute the values for x and y in Equation 1 and Equation 2 to check whether they satisfy the system.

Equation 1

$$3(3) - 4(1) \stackrel{?}{=} 5$$
$$9 - 4 \stackrel{?}{=} 5$$
$$5 = 5 \checkmark$$

Equation 2

$$2(3) - 3(1) \stackrel{?}{=} 3$$
$$6 - 3 \stackrel{?}{=} 3$$
$$3 = 3 \checkmark$$

Therefore, the solution of the system is (3, 1).

Answer 10e.

The inverse of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \text{ if } |A| \neq 0.$$

Let $A = \begin{pmatrix} \frac{4}{3} & \frac{5}{6} \\ -4 & -1 \end{pmatrix}$, then determinant of A is

$$|A| = \begin{vmatrix} \frac{4}{3} & \frac{5}{6} \\ -4 & -1 \end{vmatrix}$$
$$= -\frac{4}{3} - \left(-\frac{10}{3}\right)$$
$$= 2$$

Since $|A| \neq 0$, by definition, the inverse of the matrix A is

$$A^{-1} = \frac{1}{2}\begin{pmatrix} -1 & -\frac{5}{6} \\ 4 & \frac{4}{3} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{-1}{2} & \frac{-5}{12} \\ 2 & \frac{2}{3} \end{pmatrix}$$

The inverse of the given matrix is $\boxed{\begin{pmatrix} \frac{-1}{2} & \frac{-5}{12} \\ 2 & \frac{2}{3} \end{pmatrix}}$.

Answer 10gp.

Let us write the given linear system as the matrix equation $AX = B$ where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.
That is,

$$\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 \\ 8 \end{pmatrix}$$

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Thus we have to find A^{-1}

$$\begin{aligned} |A| &= \begin{vmatrix} 3 & -1 \\ -4 & 2 \end{vmatrix} \\ &= 5-4 \\ &= 1 \end{aligned}$$

Since $A \neq 0$, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{1}\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} \end{aligned}$$

Now we can find X from $X = A^{-1}B$

$$\begin{aligned} X &= A^{-1}B \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} -5 \\ 8 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} -10+8 \\ -20+24 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} -2 \\ -4 \end{pmatrix} \end{aligned}$$

Therefore, the solution of the system is $\boxed{(-2,-4)}$.

Answer 10q.

Let us write the given linear system as the matrix equation $AX = B$ where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.
That is,

$$\begin{pmatrix} -3 & 2 \\ 6 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -13 \\ 24 \end{pmatrix}$$

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Thus we have to find A^{-1}

$$\begin{aligned} |A| &= \begin{vmatrix} -3 & 2 \\ 6 & -5 \end{vmatrix} \\ &= 15 - 12 \\ &= 3 \end{aligned}$$

Since $A \neq 0$, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{3}\begin{pmatrix} -5 & -2 \\ -6 & -3 \end{pmatrix} \\ &= \begin{pmatrix} -5/3 & -2/3 \\ -2 & -1 \end{pmatrix} \end{aligned}$$

Now we can find X from $X = A^{-1}B$

$$X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5/3 & -2/3 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} -13 \\ 24 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{65}{3} - 16 \\ 26 - 24 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{17}{3} \\ 2 \end{pmatrix}$$

Therefore, the solution of the system is $\boxed{\left(\frac{17}{3}, 2\right)}$.

Answer 11e.

The error is that the scalar to be multiplied is the reciprocal of the determinant. In this case, the determinant is multiplied and not the reciprocal.

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix} = (2)(5) - (1)(4)$$
$$= 10 - 4$$
$$= 6$$

Substitute the required values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\frac{1}{6}\begin{bmatrix} 5 & -4 \\ -1 & 2 \end{bmatrix}$$

Multiply each element of the matrix by $\frac{1}{6}$.

$$\frac{1}{6}\begin{bmatrix} 5 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{6} & -\frac{2}{3} \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix}$$

Therefore, the inverse of the given matrix is

$$\begin{bmatrix} \frac{5}{6} & -\frac{2}{3} \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix}.$$

Answer 11gp.

Let m, p, and d represent the movie pass, popcorn, and DVD respectively.
Then according to the given data, we can construct the following equations.

$$2m + p = 17$$
$$2m + 2p + d = 35$$
$$4m + 3p + 2d = 69$$

These three equations form a system of equations. Let us rewrite the system as a matrix equation.

$$\begin{pmatrix} 2 & 1 & 0 \\ 2 & 2 & 1 \\ 4 & 3 & 2 \end{pmatrix}\begin{pmatrix} m \\ p \\ d \end{pmatrix} = \begin{pmatrix} 17 \\ 35 \\ 69 \end{pmatrix}$$

This is of the form $AX = B$, where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Enter the coefficient matrix A and the matrix of constants B into a graphing calculator. Then find the solution $X = A^{-1}B$

$$A^{-1}B = \begin{pmatrix} 8 \\ 1 \\ 17 \end{pmatrix}$$

Hence a movie pass costs $\boxed{\$8}$, a package of popcorn costs $\boxed{\$1}$, and a DVD costs $\boxed{\$17}$.

Answer 11q.

Name the equations.

$3x - y = -4$ Equation 1

$2x - 2y = -8$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$=$	$\begin{bmatrix} -4 \\ -8 \end{bmatrix}$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

For this, first find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix} \\ &= (3)(-2) - (2)(-1) \\ &= -6 + 2 \\ &= -4 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$A^{-1} = \frac{1}{-4}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix}$$

Step 3 Multiply each side of $AX = B$ by A^{-1}.

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix}\begin{bmatrix} -4 \\ -8 \end{bmatrix}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix}\begin{bmatrix} -4 \\ -8 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(-4) - \frac{1}{4}(-8) \\ \frac{1}{2}(-4) - \frac{3}{4}(-8) \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 2 \\ -2 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}.$$

CHECK

Substitute the values for x and y in Equation 1 and Equation 2 to check whether they satisfy the system.

Equation 1

$$3(0) - 4 \stackrel{?}{=} -4$$
$$0 - 4 \stackrel{?}{=} -4$$
$$-4 = -4 \checkmark$$

Equation 2

$$2(0) - 2(4) \stackrel{?}{=} -8$$
$$0 - 8 \stackrel{?}{=} -8$$
$$-8 = -8 \checkmark$$

Therefore, the solution of the system is (0, 4).

Answer 12e.

The inverse of the matrix $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is

$$A^{-1}=\frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \text{ if } |A|\neq 0.$$

Let $A=\begin{pmatrix} 10 & -3 \\ 3 & -1 \end{pmatrix}$, then determinant of A is

$$\begin{aligned} |A| &= \begin{vmatrix} 10 & -3 \\ 3 & -1 \end{vmatrix} \\ &= -10-(-9) \\ &= -1 \end{aligned}$$

Since $|A|\neq 0$, by definition, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{-1}\begin{pmatrix} -1 & 3 \\ -3 & 10 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -3 \\ 3 & -10 \end{pmatrix} \end{aligned}$$

The inverse of the given matrix is (C) $\boxed{\begin{pmatrix} 1 & -3 \\ 3 & -10 \end{pmatrix}}$.

Answer 12q.

Let us write the given linear system as the matrix equation $AX = B$ where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.
That is,

$$\begin{pmatrix} 7 & 4 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ -25 \end{pmatrix}$$

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Thus we have to find A^{-1}

$$\begin{aligned} |A| &= \begin{vmatrix} 7 & 4 \\ 5 & 3 \end{vmatrix} \\ &= 21 - 20 \\ &= 1 \end{aligned}$$

Since $A \neq 0$, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{1}\begin{pmatrix} 3 & -4 \\ -5 & 7 \end{pmatrix} \\ &= \begin{pmatrix} 3 & -4 \\ -5 & 7 \end{pmatrix} \end{aligned}$$

Now we can find X from $X = A^{-1}B$

$$X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -5 & 7 \end{pmatrix}\begin{pmatrix} 6 \\ -25 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 + 100 \\ -30 - 175 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 118 \\ -205 \end{pmatrix}$$

Therefore, the solution of the system is $\boxed{(118, -205)}$.

Answer 13e.

The given equation has the form $AX = B$. Thus,

$$A = \begin{bmatrix} -1 & 0 \\ 6 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -1 \\ 4 & 5 \end{bmatrix}.$$

In order to solve the matrix equation, first multiply both the sides of the equation $AX = B$ by A^{-1} on the left.

$A^{-1}AX = A^{-1}B$

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Find the determinant of A first. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 1 \\ 4 & 5 \end{vmatrix} \\ &= (1)(5) - (4)(1) \\ &= 5 - 4 \\ &= 1 \end{aligned}$$

Now, find A^{-1}.

$$\begin{aligned} A^{-1} &= \frac{1}{1}\begin{bmatrix} 5 & -1 \\ -4 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 5 & -1 \\ -4 & 1 \end{bmatrix} \end{aligned}$$

Substitute the known values in $A^{-1}AX = A^{-1}B$.

$$\begin{bmatrix} 5 & -1 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}X = \begin{bmatrix} 5 & -1 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 6 \end{bmatrix}$$

Next, multiply the matrices. We know that $AA^{-1} = I$.

To find the element in the ith row and jth column of the product matrix $A^{-1}B$, multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, then add the products.

$$\begin{aligned} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}X &= \begin{bmatrix} 5(2) - 1(-1) & 5(3) - 1(6) \\ -4(2) + 1(-1) & -4(3) + 1(6) \end{bmatrix} \\ &= \begin{bmatrix} 11 & 9 \\ -9 & -6 \end{bmatrix} \end{aligned}$$

The equation is now in the form $IX = A^{-1}B$.

The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix.. We know that $IX = X$.

Thus, $X = A^{-1}B$.

The matrix for X is $\begin{bmatrix} 11 & 9 \\ -9 & -6 \end{bmatrix}$.

Answer 13q.

Name the equations.

$4x + y = -2$ Equation 1

$-6x + y = 18$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} 4 & 1 \\ -6 & 1 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$=$	$\begin{bmatrix} -2 \\ 18 \end{bmatrix}$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

For this, first find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 4 & 1 \\ -6 & 1 \end{vmatrix} \\ &= (4)(1) - (-6)(1) \\ &= 4 + 6 \\ &= 10 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\begin{aligned} A^{-1} &= \frac{1}{10}\begin{bmatrix} 1 & -1 \\ 6 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{10} & -\frac{1}{10} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix} \end{aligned}$$

Step 3 Multiply each side of $AX = B$ by A^{-1}.

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{1}{10} & -\frac{1}{10} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} -2 \\ 18 \end{bmatrix}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{bmatrix} \frac{1}{10} & -\frac{1}{10} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} -2 \\ 18 \end{bmatrix} = \begin{bmatrix} \frac{1}{10}(-2) - \frac{1}{10}(18) \\ \frac{3}{5}(-2) + \frac{2}{5}(18) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2}{10} - \frac{18}{10} \\ -\frac{6}{5} + \frac{36}{5} \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 6 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}.$$

CHECK

Substitute the values for x and y in Equation 1 and Equation 2 to check whether they satisfy the system.

Equation 1

$$4(-2) + 6 \stackrel{?}{=} -2$$
$$-8 + 6 \stackrel{?}{=} -2$$
$$-2 = -2 \ \checkmark$$

Equation 2

$$-6(-2) + 6 \stackrel{?}{=} 18$$
$$12 + 6 \stackrel{?}{=} 18$$
$$18 = 18 \ \checkmark$$

Therefore, the solution of the system is (–2, 6).

Answer 14e.

Let us take the matrix equation $AX = B$ where

$$A = \begin{pmatrix} 6 & 8 \\ 2 & 3 \end{pmatrix} \text{ And } B = \begin{pmatrix} 4 & 3 \\ 0 & -2 \end{pmatrix}$$

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Thus we have to find A^{-1}

$$\begin{aligned} |A| &= \begin{vmatrix} 6 & 8 \\ 2 & 3 \end{vmatrix} \\ &= 18 - 16 \\ &= 2 \end{aligned}$$

Since $|A| \neq 0$, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{2}\begin{pmatrix} 3 & -8 \\ -2 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 3/2 & -4 \\ -1 & 3 \end{pmatrix} \end{aligned}$$

Now we can find X from $X = A^{-1}B$

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{pmatrix} 3/2 & -4 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 0 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 6+0 & \frac{9}{2}+8 \\ -4+0 & -3-6 \end{pmatrix} \\ &= \begin{pmatrix} 6 & \frac{25}{2} \\ -4 & -9 \end{pmatrix} \end{aligned}$$

Therefore, the solution is $\boxed{\begin{pmatrix} 6 & \frac{25}{2} \\ -4 & -9 \end{pmatrix}}$.

Answer 14q.

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive value.

The given coordinates of the vertices of the triangle are $A(0,2)$, $B(12,2)$, and $C(12,26)$. So the area of the triangle is

$$\begin{aligned}
\text{Area} &= \pm\frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 12 & 2 & 1 \\ 12 & 26 & 1 \end{vmatrix} \\
&= \pm\frac{1}{2}\left|\begin{matrix} 0 & 2 & 1 \\ 12 & 2 & 1 \\ 12 & 26 & 1 \end{matrix}\right|\begin{matrix} 0 & 2 \\ 12 & 2 \\ 12 & 26 \end{matrix} \\
&= \pm\frac{1}{2}\left[(0+24+312)-(24+0+24)\right] \\
&= \pm\frac{1}{2}(336-48) \\
&= \pm\frac{1}{2}(288) \\
&= 144
\end{aligned}$$

Therefore, the area of the sail is $\boxed{144 \text{ sq. feet}}$.

Answer 15e.

The given equation has the form $AX = B$. Thus,

$A = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & 6 \end{bmatrix}$.

In order to solve the matrix equation, first multiply both the sides of the equation $AX = B$ by A^{-1} on the left.
$A^{-1}AX = A^{-1}B$

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Find the determinant of A first. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} -1 & 0 \\ 6 & 4 \end{vmatrix} \\ &= (-1)(4) - (0)(6) \\ &= -4 - 0 \\ &= -4 \end{aligned}$$

Now, find A^{-1}.

$$A^{-1} = \frac{1}{-4}\begin{bmatrix} 4 & 0 \\ -6 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ \frac{3}{2} & \frac{1}{4} \end{bmatrix}.$$

Substitute the known values in $A^{-1}AX = A^{-1}B$.

$$\begin{bmatrix} -1 & 0 \\ \frac{3}{2} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 6 & 4 \end{bmatrix}X = \begin{bmatrix} -1 & 0 \\ \frac{3}{2} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 4 & 5 \end{bmatrix}$$

Next, multiply the matrices. We know that $AA^{-1} = I$.
To find the element in the ith row and jth column of the product matrix $A^{-1}B$, multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, then add the products.

$$\begin{aligned} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}X &= \begin{bmatrix} -1(3) - 0(4) & -1(-1) - 0(5) \\ \frac{3}{2}(3) + \frac{1}{4}(4) & \frac{3}{2}(-1) + \frac{1}{4}(5) \end{bmatrix} \\ &= \begin{bmatrix} -3 & 1 \\ \frac{11}{2} & -\frac{1}{4} \end{bmatrix} \end{aligned}$$

The equation is now in the form $IX = A^{-1}B$.

The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix.. We know that $IX = X$.

Thus, $X = A^{-1}B$.

The matrix for X is $\begin{bmatrix} -3 & 1 \\ \frac{11}{2} & -\frac{1}{4} \end{bmatrix}$.

Answer 16e.

Let us take the matrix equation $AX = B$ where

$$A = \begin{pmatrix} -3 & 6 \\ 1 & 2 \end{pmatrix} \text{ And } B = \begin{pmatrix} 5 & -1 \\ 8 & 2 \end{pmatrix}$$

Solving for X we get,

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Thus we have to find A^{-1}

$$|A| = \begin{vmatrix} -3 & 6 \\ 1 & 2 \end{vmatrix}$$
$$= -6 - 6$$
$$= -12$$

Since $|A| \neq 0$, the inverse of the matrix A is

$$A^{-1} = \frac{1}{-12}\begin{pmatrix} 2 & -6 \\ -1 & -3 \end{pmatrix}$$
$$= \begin{pmatrix} -1/6 & 1/2 \\ 1/12 & 1/4 \end{pmatrix}$$

Now we can find X from $X = A^{-1}B$

$$X = A^{-1}B$$
$$= \begin{pmatrix} -1/6 & 1/2 \\ 1/12 & 1/4 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 8 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} -\frac{5}{6}+4 & \frac{1}{6}+1 \\ \frac{5}{12}+2 & -\frac{1}{12}+\frac{1}{2} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{19}{6} & \frac{7}{6} \\ \frac{29}{12} & \frac{5}{12} \end{pmatrix}$$

Therefore, the solution is $\boxed{\begin{pmatrix} \frac{19}{6} & \frac{7}{6} \\ \frac{29}{12} & \frac{5}{12} \end{pmatrix}}$.

Answer 17e.

The given equation has the form $AX = B$. Thus,

$A = \begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 0 \\ 6 & 8 & 4 \end{bmatrix}$.

In order to solve the matrix equation, first multiply both the sides of the equation $AX = B$ by A^{-1} on the left.
$A^{-1}AX = A^{-1}B$

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Find the determinant of A first. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$|A| = \begin{vmatrix} 1 & 5 \\ 0 & -2 \end{vmatrix}$$
$$= (1)(-2) - (0)(5)$$
$$= -2 - 0$$
$$= -2$$

Now, find A^{-1}.

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} -2 & -5 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & -\frac{1}{2} \end{bmatrix}$$

Substitute the known values in $A^{-1}AX = A^{-1}B$.

$$\begin{bmatrix} 1 & \frac{5}{2} \\ 0 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix}X = \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 3 & -1 & 0 \\ 6 & 8 & 4 \end{bmatrix}$$

Next, multiply the matrices. We know that $AA^{-1} = I$.
To find the element in the ith row and jth column of the product matrix $A^{-1}B$, multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, then add the products.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}X = \begin{bmatrix} 1(3) + \frac{5}{2}(6) & 1(-1) + \frac{5}{2}(8) & 1(0) + \frac{5}{2}(4) \\ 0(3) - \frac{1}{2}(6) & 0(-1) - \frac{1}{2}(8) & 0(0) - \frac{1}{2}(4) \end{bmatrix}$$
$$= \begin{bmatrix} 3 + 15 & -1 + 20 & 10 \\ -3 & -4 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 18 & 19 & 10 \\ -3 & -4 & -2 \end{bmatrix}$$

The equation is now in the form $IX = A^{-1}B$.

The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix.. We know that $IX = X$.

Thus, $X = A^{-1}B$.

The matrix for X is $\begin{bmatrix} 18 & 19 & 10 \\ -3 & -4 & -2 \end{bmatrix}$.

Answer 18e.

Let us take the matrix equation $AX = B$ where

$$A = \begin{pmatrix} -5 & 2 \\ -9 & 3 \end{pmatrix} \text{ And } B = \begin{pmatrix} 4 & 5 & 0 \\ 3 & 1 & 6 \end{pmatrix}$$

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Thus we have to find A^{-1}

$$\begin{aligned} |A| &= \begin{vmatrix} -5 & 2 \\ -9 & 3 \end{vmatrix} \\ &= -15 - (-18) \\ &= 3 \end{aligned}$$

Since $|A| \neq 0$, the inverse of the matrix A is

$$\begin{aligned} A^{-1} &= \frac{1}{3}\begin{pmatrix} 3 & -2 \\ 9 & -5 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -2/3 \\ 3 & -5/3 \end{pmatrix} \end{aligned}$$

Now we can find X from $X = A^{-1}B$

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{pmatrix} 1 & -2/3 \\ 3 & -5/3 \end{pmatrix}\begin{pmatrix} 4 & 5 & 0 \\ 3 & 1 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 4-2 & 5-\frac{2}{3} & 0-4 \\ 12-5 & 15-\frac{5}{3} & 0-10 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 13/3 & -4 \\ 7 & 40/3 & -10 \end{pmatrix} \end{aligned}$$

Therefore, the solution is $\boxed{\begin{pmatrix} 2 & 13/3 & -4 \\ 7 & 40/3 & -10 \end{pmatrix}}$.

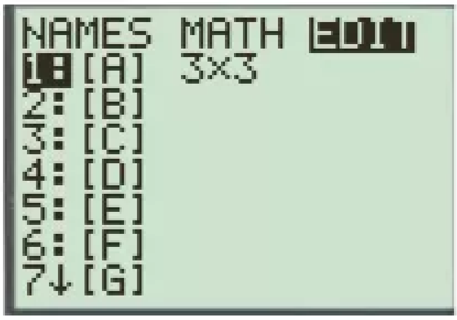
Answer 19e.

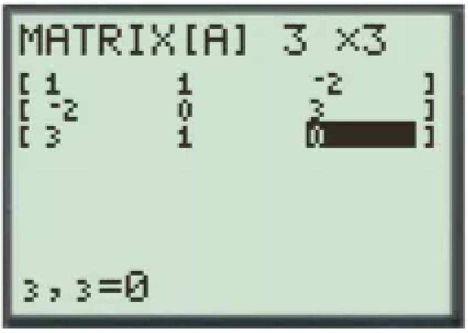
First enter the given matrix A into the graphing calculator.

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -2 & 0 & 3 \\ 3 & 1 & 0 \end{pmatrix}$$

Using the following key strokes for entering the matrix (choose the EDIT option)

2nd *matrix* 1

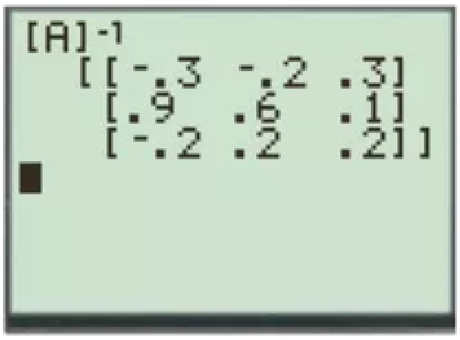




Using the following key strokes for empty window and then find the inverse matrix

2nd MODE *CLEAR*

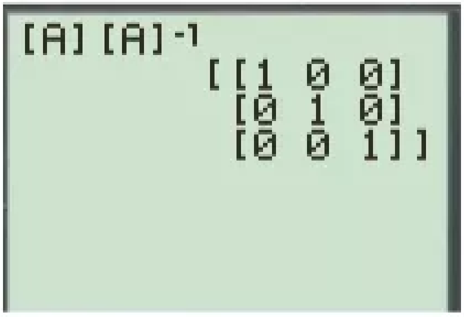
2nd *matrix* 1 x^{-1} *enter*



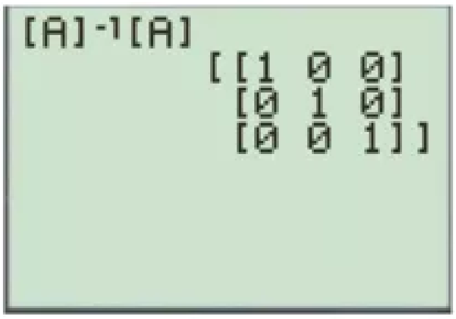
Using the follwing key strokes and then check the equation

$AA^{-1}=I$ AND $A^{-1}A=I$

[2nd] [matrix] [1] [2nd] [matrix] [1] [x^{-1}] [enter]



[2nd] [matrix] [1] [x^{-1}] [2nd] [matrix] [1] [enter]



Therefore, the inverse of the given matrix is $\begin{pmatrix} -0.3 & -0.2 & 0.3 \\ 0.9 & 0.6 & 0.1 \\ -0.2 & 0.2 & 0.2 \end{pmatrix}$.

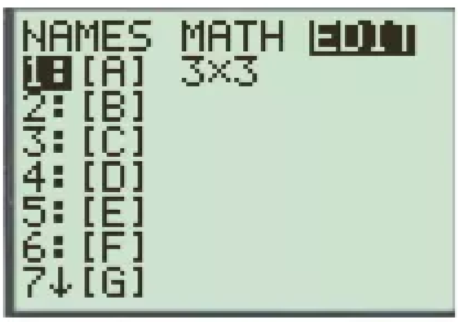
Answer 20e.

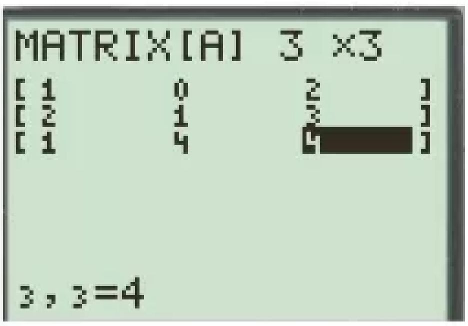
First enter the given matrix A into the graphing calculator.

$$A=\begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 1 & 4 & 4 \end{pmatrix}$$

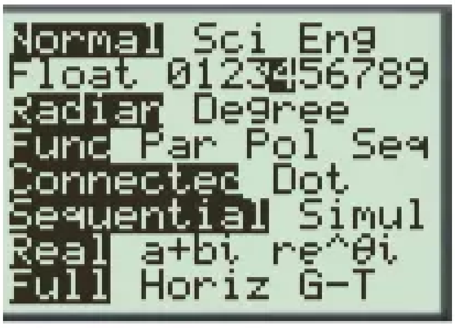
Using the following key strokes for entering the matrix (choose the EDIT option)

[2nd] [matrix] [1]





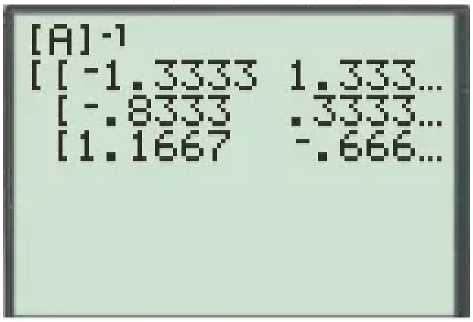
Using the following key strokes for empty window and then find the inverse matrix
Set the 4 digits for decimal number using the key MODE.

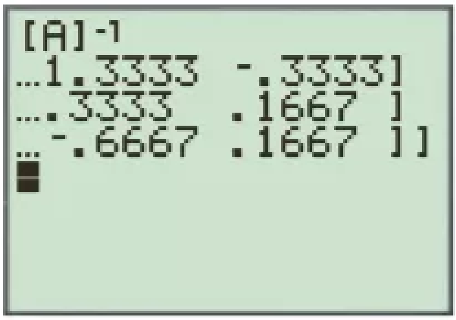


[2nd] [MODE] [CLEAR]

[2nd] [matrix] [1] [x^{-1}] [enter]

FIRST AND 2ND ROW 2ND AND 3RD ROW



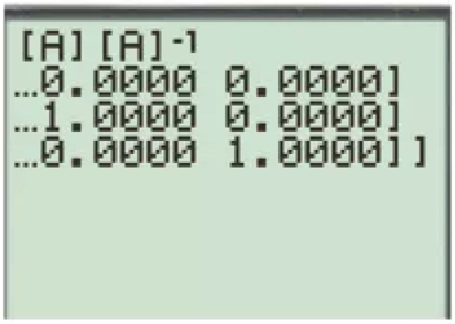


Using the follwing key strokes and then check the equation

$AA^{-1} = I$ AND $A^{-1}A = I$

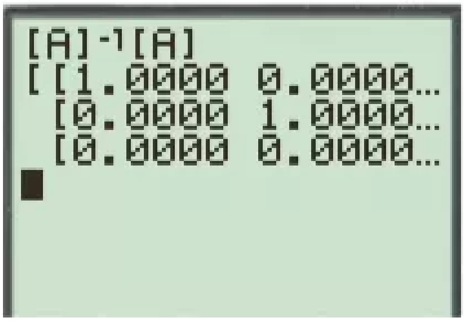
2nd | matrix | 1 | 2nd | matrix | 1 | x^{-1} | enter

FIRST AND 2ND ROW   2ND AND 3RD ROW



2nd | matrix | 1 | x^{-1} | 2nd | matrix | 1 | enter

FIRST AND 2ND ROW   2ND AND 3RD ROW



Therefore, the inverse of the given matrix is $\begin{pmatrix} -1.3333 & 1.3333 & -0.3333 \\ -0.8333 & 0.3333 & 0.1666 \\ 1.1666 & -0.6666 & 0.1666 \end{pmatrix}$.

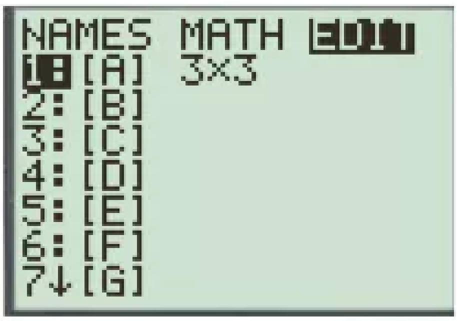
Answer 21e.

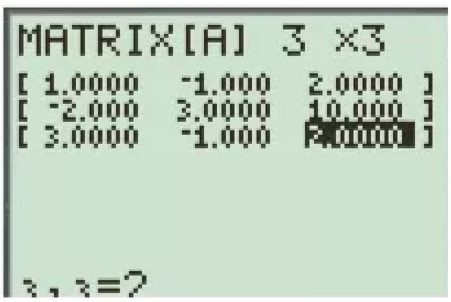
First enter the given matrix A into the graphing calculator.

$$A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 3 & 10 \\ 3 & -1 & 2 \end{pmatrix}$$

Using the following key strokes for entering the matrix (choose the EDIT option)

2nd | matrix | 1





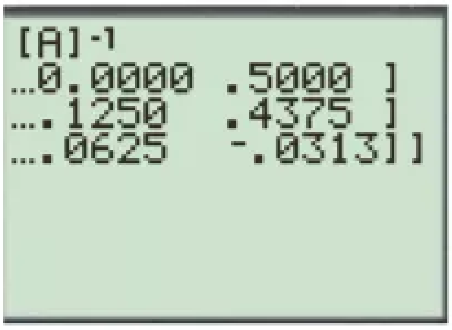
Using the following key strokes for empty window and then find the inverse matrix

2nd | MODE | CLEAR

2nd | matrix | 1 | x^{-1} | enter

FIRST AND 2ND ROW

2ND AND 3RD ROW

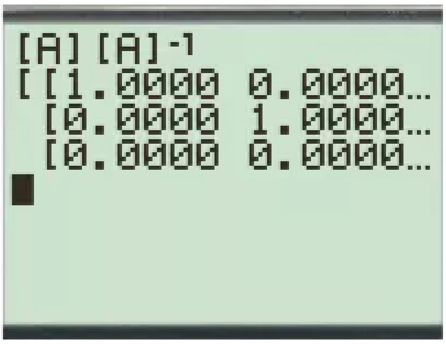


Using the follwing key strokes and then check the equation

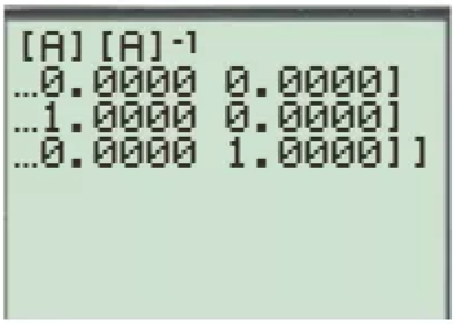
$AA^{-1} = I$ AND $A^{-1}A = I$

2nd | matrix | 1 | 2nd | matrix | 1 | x^{-1} | enter

FIRST AND 2ND ROW

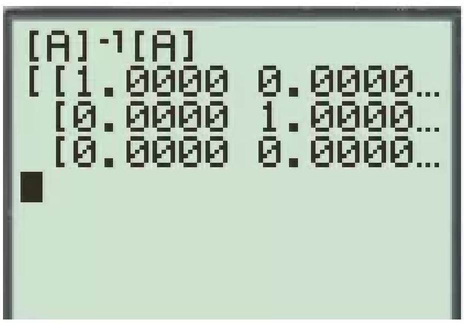


2ND AND 3RD ROW

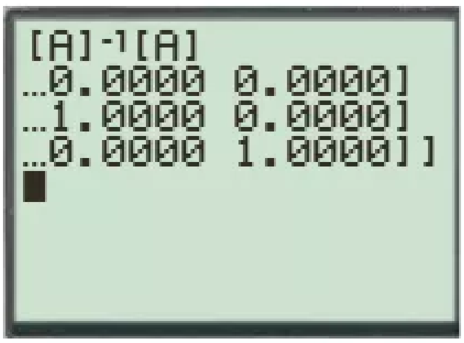


2nd | matrix | 1 | x^{-1} | 2nd | matrix | 1 | enter

FIRST AND 2ND ROW



2ND AND 3RD ROW



Therefore, the inverse of the given matrix is $\begin{pmatrix} -0.5 & 0 & 0.5 \\ -1.0625 & 0.125 & 0.4375 \\ 0.21875 & 0.0625 & -0.03125 \end{pmatrix}$.

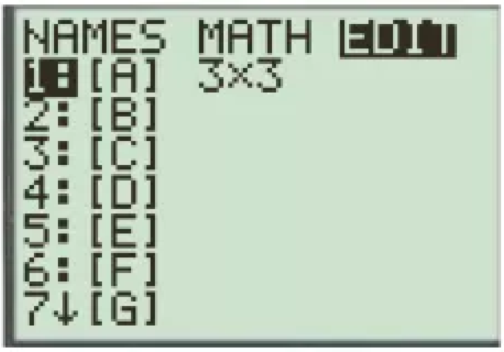
Answer 22e.

First enter the given matrix A into the graphing calculator.

$$A = \begin{pmatrix} -2 & 5 & -1 \\ 0 & 8 & 1 \\ 12 & -5 & 0 \end{pmatrix}$$

Using the following key strokes for entering the matrix (choose the EDIT option)

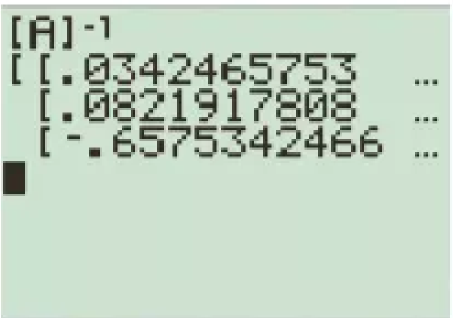
2nd matrix 1

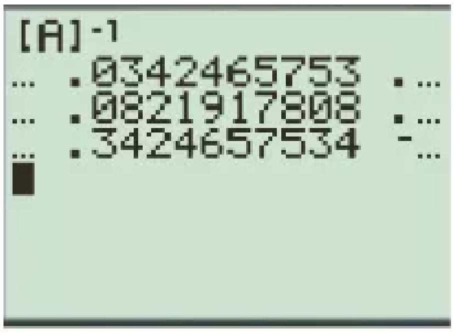


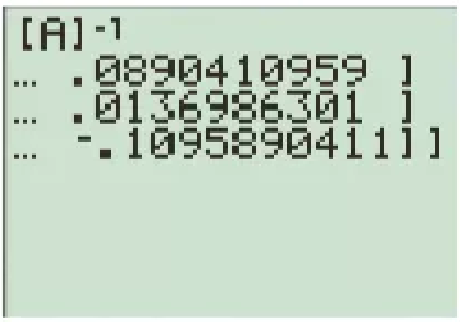
Using the following key strokes for empty window and then find the inverse matrix

2nd MODE CLEAR

2nd matrix 1 x^{-1} enter



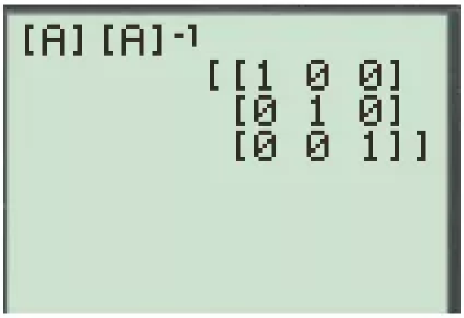




Using the follwing key strokes and then check the equation

$AA^{-1} = I$ AND $A^{-1}A = I$

2nd | matrix | 1 | 2nd | matrix | 1 | x^{-1} | enter



2nd | matrix | 1 | x^{-1} | 2nd | matrix | 1 | enter

Therefore, the inverse of the given matrix is $\begin{pmatrix} 0.03424 & 0.03424 & 0.08904 \\ 0.08219 & 0.08219 & 0.01369 \\ -0.65753 & 0.34246 & -0.10958 \end{pmatrix}$

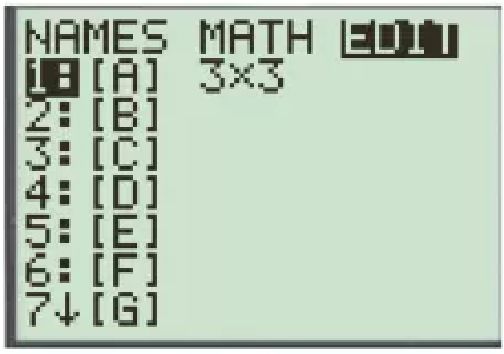
Answer 23e.

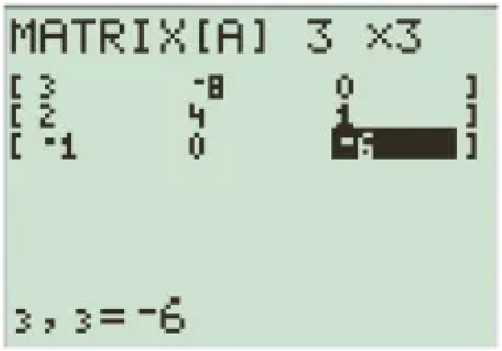
First enter the given matrix A into the graphing calculator.

$$A = \begin{pmatrix} 3 & -8 & 0 \\ 2 & 4 & 1 \\ -1 & 0 & -6 \end{pmatrix}$$

Using the following key strokes for entering the matrix (choose the EDIT option)

2nd | matrix | 1

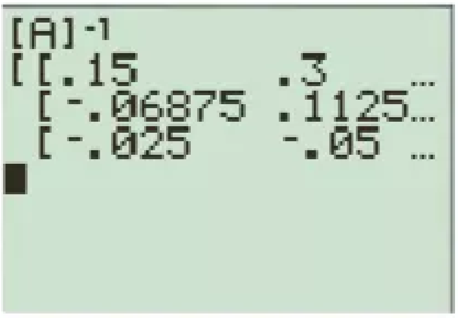


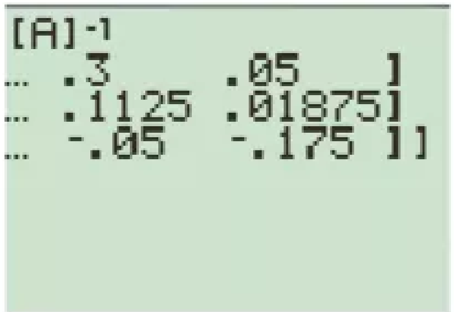


Using the following key strokes for empty window and then find the inverse matrix

$\boxed{2nd}\boxed{\text{MODE}}\boxed{CLEAR}$

$\boxed{2nd}\boxed{matrix}\boxed{1}\boxed{x^{-1}}\boxed{enter}$

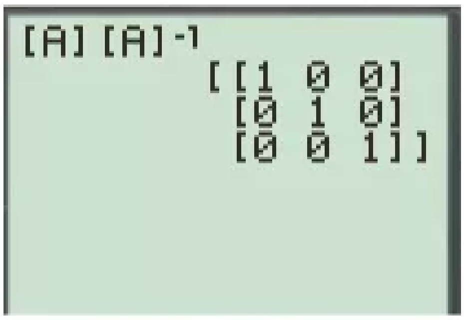




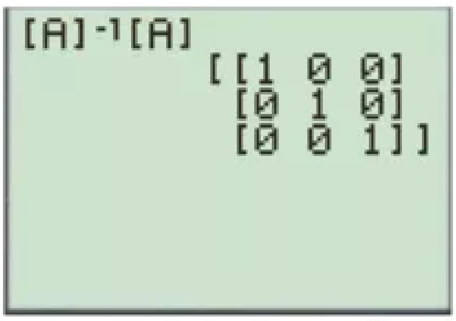
Using the follwing key strokes and then check the equation

$AA^{-1} = I$ AND $A^{-1}A = I$

$\boxed{2nd}\boxed{matrix}\boxed{1}\boxed{2nd}\boxed{matrix}\boxed{1}\boxed{x^{-1}}\boxed{enter}$



$\boxed{2nd}\boxed{matrix}\boxed{1}\boxed{x^{-1}}\boxed{2nd}\boxed{matrix}\boxed{1}\boxed{enter}$



Therefore, the inverse of the given matrix is $\begin{pmatrix} 0.15 & 0.3 & 0.05 \\ -0.06875 & 0.1125 & 0.01875 \\ -0.025 & -0.05 & -0.175 \end{pmatrix}$

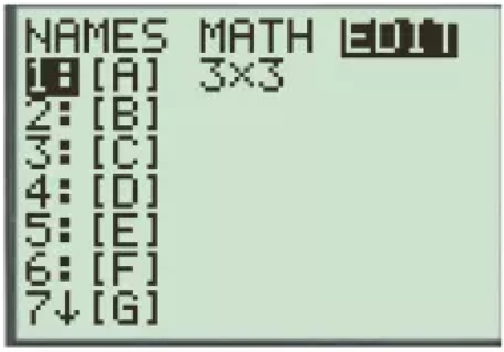
Answer 24e.

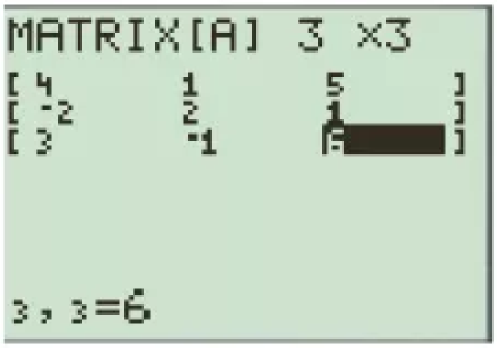
First enter the given matrix A into the graphing calculator.

$$A=\begin{pmatrix} 4 & 1 & 5 \\ -2 & 2 & 1 \\ 3 & -1 & 6 \end{pmatrix}$$

Using the following key strokes for entering the matrix (choose the EDIT option)

| 2nd | *matrix* | 1 |

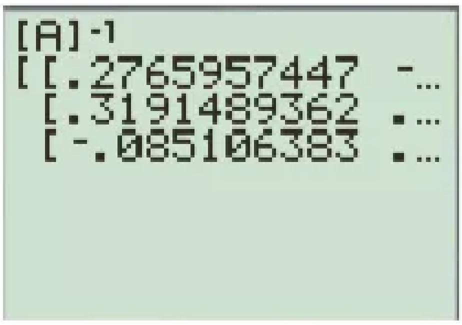


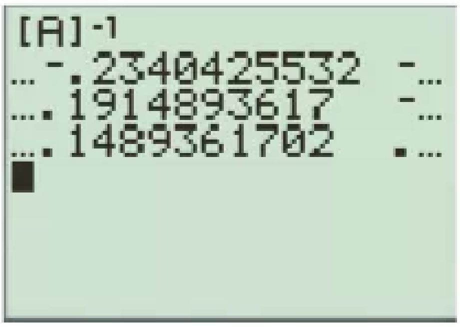


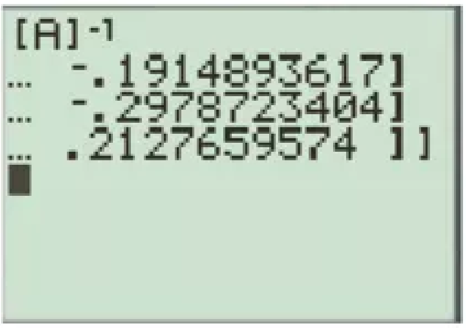
Using the following key strokes for empty window and then find the inverse matrix

| 2nd | MODE | *CLEAR* |

| 2nd | *matrix* | 1 | x^{-1} | *enter* |



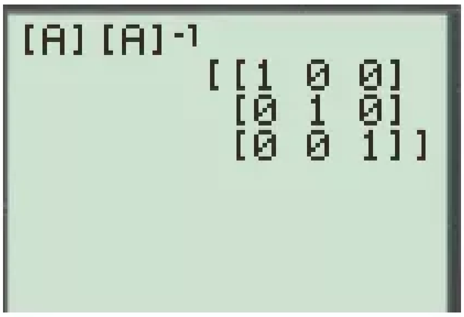




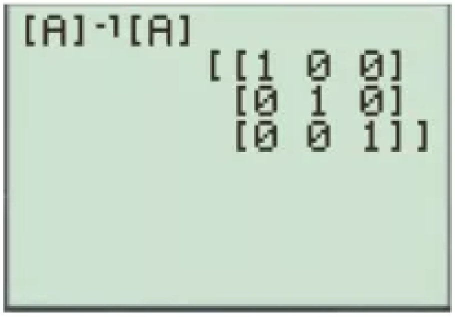
Using the follwing key strokes and then check the equation

$AA^{-1} = I$ AND $A^{-1}A = I$

2nd | matrix | 1 | 2nd | matrix | 1 | x^{-1} | enter



2nd | matrix | 1 | x^{-1} | 2nd | matrix | 1 | enter



Therefore, the inverse of the given matrix is $\begin{pmatrix} 0.27659 & -0.23404 & -0.19148 \\ 0.31914 & 0.19148 & 0.29787 \\ -0.0851 & 0.14893 & 0.21276 \end{pmatrix}$

Answer 25e.

First, name the equations.

$4x - y = 10$ Equation 1

$-7x - 2y = -25$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} 4 & -1 \\ -7 & -2 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$=$	$\begin{bmatrix} 10 \\ -25 \end{bmatrix}$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

First find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 4 & -1 \\ -7 & -2 \end{vmatrix} \\ &= (4)(-2) - (-7)(-1) \\ &= -8 - 7 \\ &= -15 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\begin{aligned} A^{-1} &= \frac{1}{-15}\begin{bmatrix} -2 & 1 \\ 7 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{2}{15} & -\frac{1}{15} \\ -\frac{7}{15} & -\frac{4}{15} \end{bmatrix}. \end{aligned}$$

Step 3 Multiply each side of $AX = B$ by A^{-1}.

$$X = A^{-1}B = \begin{bmatrix} \frac{2}{15} & -\frac{1}{15} \\ -\frac{7}{15} & -\frac{4}{15} \end{bmatrix}\begin{bmatrix} 10 \\ -25 \end{bmatrix}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{aligned} \begin{bmatrix} \frac{2}{15} & -\frac{1}{15} \\ -\frac{7}{15} & -\frac{4}{15} \end{bmatrix}\begin{bmatrix} 10 \\ -25 \end{bmatrix} &= \begin{bmatrix} \frac{2}{15}(10) - \frac{1}{15}(-25) \\ -\frac{7}{15}(10) - \frac{4}{15}(-25) \end{bmatrix} \\ &= \begin{bmatrix} \frac{4}{3} + \frac{5}{3} \\ -\frac{14}{3} + \frac{20}{3} \end{bmatrix} \\ &= \begin{bmatrix} 3 \\ 2 \end{bmatrix} \end{aligned}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

CHECK

Substitute the values for x and y in Equation 1 and Equation 2 to check whether they satisfy the system.

Equation 1	**Equation 2**
$4(3) - 2 \stackrel{?}{=} 10$	$-7(3) - 2(2) \stackrel{?}{=} -25$
$12 - 2 \stackrel{?}{=} 10$	$-21 - 4 \stackrel{?}{=} -25$
$10 = 10$ ✓	$-25 = -25$ ✓

Therefore, the solution of the system is (3, 2).

Answer 26e.

Let us write the given linear system as the matrix equation $AX = B$
Where A is the coefficient matrix, X is the matrix of variables
B is the matrix of constants.
Now

$$\begin{pmatrix} 4 & 7 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -16 \\ -4 \end{pmatrix}$$

Solving for X we get,

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Thus to find A^{-1} frist find det A

$$|A| = \begin{vmatrix} 4 & 7 \\ 2 & 3 \end{vmatrix}$$
$$= 12 - 14$$
$$= -2$$

Since $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, the inverse of the matrix A is

$$= \text{Det } A \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{-2}\begin{pmatrix} 3 & -7 \\ -2 & 4 \end{pmatrix}$$
$$= \begin{pmatrix} -3/2 & 7/2 \\ 1 & -2 \end{pmatrix}$$

Now we can find X from $X = A^{-1}B$

$$X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3/2 & 7/2 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} -16 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 24-14 \\ -16+8 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ -8 \end{pmatrix}$$

Therefore, the solution of the system is $\boxed{(10,-8)}$

Answer 27e.

First, name the equations.

$3x - 2y = 5$ Equation 1

$6x - 5y = 14$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} 3 & -2 \\ 6 & -5 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$=$	$\begin{bmatrix} 5 \\ 14 \end{bmatrix}$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

First find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 3 & -2 \\ 6 & -5 \end{vmatrix} \\ &= (3)(-5) - (-2)(6). \\ &= -15 + 12 \\ &= -3 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\begin{aligned} A^{-1} &= \frac{1}{-3}\begin{bmatrix} -5 & 2 \\ -6 & 3 \end{bmatrix} \\ &= \begin{bmatrix} \frac{5}{3} & -\frac{2}{3} \\ 2 & -1 \end{bmatrix}. \end{aligned}$$

Step 3 Multiply each side of $AX = B$ by A^{-1}.

$$X = A^{-1}B = \begin{bmatrix} \frac{5}{3} & -\frac{2}{3} \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 14 \end{bmatrix}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{bmatrix} \frac{5}{3} & -\frac{2}{3} \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 14 \end{bmatrix} = \begin{bmatrix} \frac{5}{3}(5) - \frac{2}{3}(14) \\ 2(5) - 1(14) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{25}{3} - \frac{28}{3} \\ 10 - 14 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}.$$

CHECK

Substitute the values for x and y in Equation 1 and Equation 2 to check whether they satisfy the system.

Equation 1

$$3(-1) - 2(-4) \stackrel{?}{=} 5$$
$$-3 + 8 \stackrel{?}{=} 5$$
$$5 = 5 \checkmark$$

Equation 2

$$6(-1) - 5(-4) \stackrel{?}{=} 14$$
$$-6 + 20 \stackrel{?}{=} 14$$
$$14 = 14 \checkmark$$

Therefore, the solution of the system is $(-1, -4)$.

Answer 28e.

Let us write the given linear system as the matrix equation $AX = B$

Where A is the coefficient matrix, X is the matrix of variables

B is the matrix of constants.

Then

$$\begin{pmatrix} 1 & -1 \\ 9 & -10 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 45 \end{pmatrix}$$

Solving for X then

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Thus we have to find A^{-1}

$$|A| = \begin{vmatrix} 1 & -1 \\ 9 & -10 \end{vmatrix}$$
$$= -10 - (-9)$$
$$= -1$$

Since $|A| \neq 0$

Inverse of the matrix A is

$$= \text{Det } A \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{-1}\begin{pmatrix} -10 & 1 \\ -9 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 10 & -1 \\ 9 & -1 \end{pmatrix}$$

Now we can find X from $X = A^{-1}B$

$$X = A^{-1}B$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 & -1 \\ 9 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ 45 \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 40-45 \\ 36-45 \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 \\ -9 \end{pmatrix}$$

Therefore,

Solution of the system is $\boxed{(-5,-9)}$

Answer 29e.

First, name the equations.

$-2x - 9y = -2$ Equation 1

$4x + 16y = 8$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} -2 & -9 \\ 4 & 16 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$=$	$\begin{bmatrix} -2 \\ 8 \end{bmatrix}$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

First find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} -2 & -9 \\ 4 & 16 \end{vmatrix} \\ &= (-2)(16) - (4)(-9) \\ &= -32 + 36 \\ &= 4 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\begin{aligned} A^{-1} &= \frac{1}{4}\begin{bmatrix} 16 & 9 \\ -4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 4 & \frac{9}{4} \\ -1 & -\frac{1}{2} \end{bmatrix} \end{aligned}$$

Step 3 Multiply each side of $AX = B$ by A^{-1}.

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} 4 & \frac{9}{4} \\ -1 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} -2 \\ 8 \end{bmatrix} \end{aligned}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{aligned} \begin{bmatrix} 4 & \frac{9}{4} \\ -1 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} -2 \\ 8 \end{bmatrix} &= \begin{bmatrix} 4(-2) + \frac{9}{4}(8) \\ -1(-2) - \frac{1}{2}(8) \end{bmatrix} \\ &= \begin{bmatrix} -8 + 18 \\ 2 - 4 \end{bmatrix} \\ &= \begin{bmatrix} 10 \\ -2 \end{bmatrix} \end{aligned}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \end{bmatrix}.$$

CHECK
Substitute the values for x and y in Equation 1 and Equation 2 to check whether they satisfy the system.

Equation 1

$$-2(10) - 9(-2) \stackrel{?}{=} -2$$
$$-20 + 18 \stackrel{?}{=} -2$$
$$-2 = -2 \checkmark$$

Equation 2

$$4(10) + 16(-2) \stackrel{?}{=} 8$$
$$40 - 32 \stackrel{?}{=} 8$$
$$8 = 8 \checkmark$$

Therefore, the solution of the system is (10, −2).

Answer 30e.

Let us write the given linear system as the matrix equation $AX = B$
Where A is the coefficient matrix, X is the matrix of variables
B is the matrix of constants.
Now

$$\begin{pmatrix} 2 & -7 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$$

Solving for X we get

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Thus we have to find A^{-1}

$$|A| = \begin{vmatrix} 2 & -7 \\ -1 & 5 \end{vmatrix}$$
$$= 10 - 7$$
$$= 3$$

Since $|A| \neq 0$
Inverse of the matrix A is

$$= \text{Det}\, A \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{3}\begin{pmatrix} 5 & 7 \\ 1 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 5/3 & 7/3 \\ 1/3 & 2/3 \end{pmatrix}$$

Now find X from $X = A^{-1}B$

$$X = A^{-1}B$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5/3 & 7/3 \\ 1/3 & 2/3 \end{pmatrix} \begin{pmatrix} -6 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10+7 \\ -2+2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$$

Therefore, the solution of the system is $\boxed{(-3,0)}$

Answer 31e.

First, name the equations.

$6x + y = -2$ Equation 1

$-x + 3y = -25$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} 6 & 1 \\ -1 & 3 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$=$	$\begin{bmatrix} -2 \\ -25 \end{bmatrix}$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

First find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 6 & 1 \\ -1 & 3 \end{vmatrix} \\ &= (6)(3) - (-1)(1) \\ &= 18 + 1 \\ &= 19 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$A^{-1} = \frac{1}{19}\begin{bmatrix} 3 & -1 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{19} & -\frac{1}{19} \\ \frac{1}{19} & \frac{6}{19} \end{bmatrix}$$

Step 3 Multiply each side of $AX = B$ by A^{-1}.

$$X = A^{-1}B = \begin{bmatrix} \frac{3}{19} & -\frac{1}{19} \\ \frac{1}{19} & \frac{6}{19} \end{bmatrix}\begin{bmatrix} -2 \\ -25 \end{bmatrix}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{bmatrix} \frac{3}{19} & -\frac{1}{19} \\ \frac{1}{19} & \frac{6}{19} \end{bmatrix}\begin{bmatrix} -2 \\ -25 \end{bmatrix} = \begin{bmatrix} \frac{3}{19}(-2) - \frac{1}{19}(-25) \\ \frac{1}{19}(-2) + \frac{6}{19}(-25) \end{bmatrix} = \begin{bmatrix} -\frac{6}{19} + \frac{25}{19} \\ -\frac{2}{19} - \frac{150}{19} \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \end{bmatrix}.$$

CHECK

Substitute the values for x and y in Equation 1 and Equation 2 to check whether they satisfy the system.

Equation 1

$$6(1) + (-8) \stackrel{?}{=} -2$$
$$6 - 8 \stackrel{?}{=} -2$$
$$-2 = -2 \checkmark$$

Equation 2

$$-1 + 3(-8) \stackrel{?}{=} -25$$
$$-1 - 24 \stackrel{?}{=} -25$$
$$-25 = -25 \checkmark$$

Therefore, the solution of the system is (1, –8).

Answer 32e.

Let us write the given linear system as the matrix equation $AX = B$
Where A is the coefficient matrix, X is the matrix of variables
B is the matrix of constants.
Now

$$\begin{pmatrix} 2 & 1 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 38 \end{pmatrix}$$

Solving for X we get

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Thus to find A^{-1} frist find det A

$$|A| = \begin{vmatrix} 2 & 1 \\ 2 & 5 \end{vmatrix}$$
$$= 10 - 2$$
$$= 8$$

Since $|A| \neq 0$
Inverse of the matrix A is

$$= \text{Det } A \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$
$$A^{-1} = \frac{1}{8}\begin{pmatrix} 5 & -1 \\ -2 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 5/8 & -1/8 \\ -1/4 & 1/4 \end{pmatrix}$$

Now we can find X from $X = A^{-1}B$

$$X = A^{-1}B$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5/8 & -1/8 \\ -1/4 & 1/4 \end{pmatrix}\begin{pmatrix} -2 \\ 38 \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{5}{4} - \frac{38}{8} \\ \frac{1}{2} + \frac{38}{4} \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 10 \end{pmatrix}$$

Therefore, the solution of the system is $\boxed{(-6, 10)}$

Answer 33e.

First, name the equations.

$5x + 7y = 20$ Equation 1

$3x + 5y = 16$ Equation 2

Step 1 We can rewrite the linear system as a matrix equation $AX = B$.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} 5 & 7 \\ 3 & 5 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} x \\ y \end{bmatrix}$	$=$	$\begin{bmatrix} 20 \\ 16 \end{bmatrix}$

Step 2 Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

First find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 5 & 7 \\ 3 & 5 \end{vmatrix} \\ &= (5)(5) - (3)(7). \\ &= 25 - 21 \\ &= 4 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\begin{aligned} A^{-1} &= \frac{1}{4}\begin{bmatrix} 5 & -7 \\ -3 & 5 \end{bmatrix} \\ &= \begin{bmatrix} \frac{5}{4} & -\frac{7}{4} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix}. \end{aligned}$$

Step 3 Multiply each side of $AX = B$ by A^{-1}.

$$X = A^{-1}B = \begin{bmatrix} \frac{5}{4} & -\frac{7}{4} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix}\begin{bmatrix} 20 \\ 16 \end{bmatrix}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{bmatrix} \frac{5}{4} & -\frac{7}{4} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix}\begin{bmatrix} 20 \\ 16 \end{bmatrix} = \begin{bmatrix} \frac{5}{4}(20) - \frac{7}{4}(16) \\ -\frac{3}{4}(20) + \frac{5}{4}(16) \end{bmatrix}$$

$$= \begin{bmatrix} 25 - 28 \\ -15 + 20 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 5 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}.$$

CHECK

Substitute the values for x and y in Equation 1 and Equation 2 to check whether they satisfy the system.

Equation 1

$$5(-3) + 7(5) \stackrel{?}{=} 20$$
$$-15 + 35 \stackrel{?}{=} 20$$
$$20 = 20 \checkmark$$

Equation 2

$$13(-3) + 5(5) \stackrel{?}{=} 16$$
$$-39 + 25 \stackrel{?}{=} 16$$
$$16 = 16 \checkmark$$

Therefore, the solution of the system is (–3, 5).

Answer 34e.

Let us write the given linear system as the matrix equation $AX = B$

Where A is the coefficient matrix, X is the matrix of variables

B is the matrix of constants.

Now

$$\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -26 \\ 10 \end{pmatrix}$$

Solving for X we get

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Thus to find A^{-1} frist find det A

$$|A| = \begin{vmatrix} 3 & -5 \\ -1 & 2 \end{vmatrix}$$
$$= 6 - 5$$
$$= 1$$

Since $|A| \neq 0$

Inverse of the matrix A is

$$= \text{Det } A \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$
$$A^{-1} = \frac{1}{1}\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

Now we can find X from $X = A^{-1}B$

$$X = A^{-1}B$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -26 \\ 10 \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -52+50 \\ -26+30 \end{pmatrix}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

Therefore,

solution of the system is $\boxed{(-2, 4)}$

Option $\boxed{(c)}$

Answer 35e.

Let us write the given linear system as the matrix equation $AX = B$

Where A is the coefficient matrix, X is the matrix of variables

B is the matrix of constants.

Now

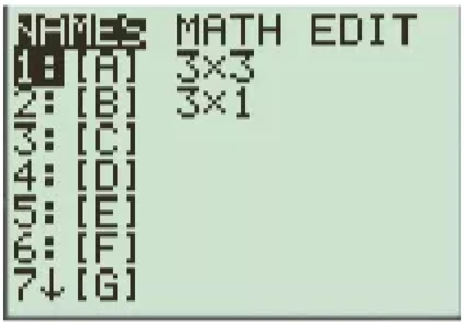
$$\begin{pmatrix} 1 & -1 & -3 \\ 5 & 2 & 1 \\ -3 & -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -17 \\ 8 \end{pmatrix}$$

Solving for X we get

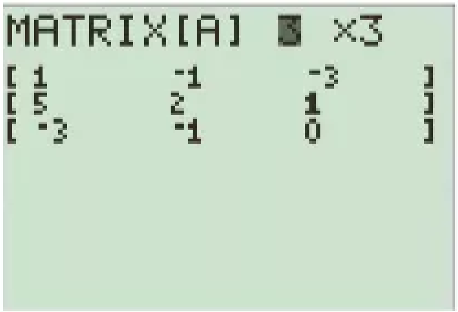
$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Enter the coefficient matrix A and the matrix of constants B into a graphing calculator. Then find the solution $X = A^{-1}B$

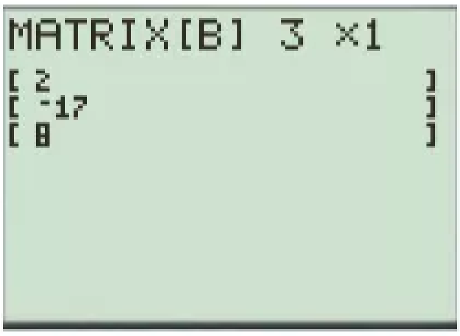
For these key strokes are 2nd matrixes edit



In the edit button enter the dimensions of the required matrix A =3 × 3



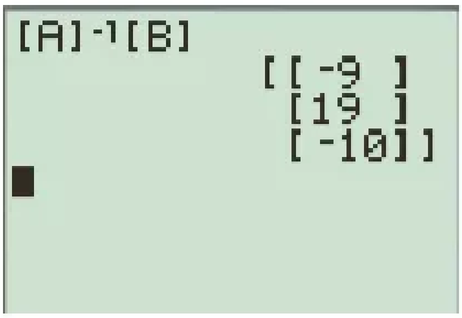
In the edit button enter the dimensions of the required matrix B =3 × 1



After that press 2nd mode clear then

Again enter 2nd matrix 1 x^{-1} and 2nd matrix 2

The result will be as shown



$$A^{-1}B = \begin{pmatrix} -9 \\ 19 \\ -10 \end{pmatrix}$$

The solution set is $\boxed{(-9, 19, -10)}$

Answer 36e.

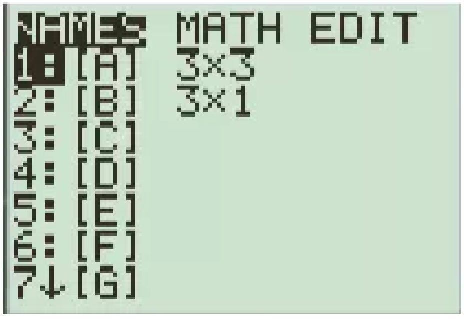
Let us write the given linear system as the matrix equation $AX = B$
Where A is the coefficient matrix, X is the matrix of variables
B is the matrix of constants.
Now

$$\begin{pmatrix} -3 & 1 & -8 \\ 1 & -2 & 1 \\ 2 & -2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 18 \\ -11 \\ -17 \end{pmatrix}$$

Solving for X we get,

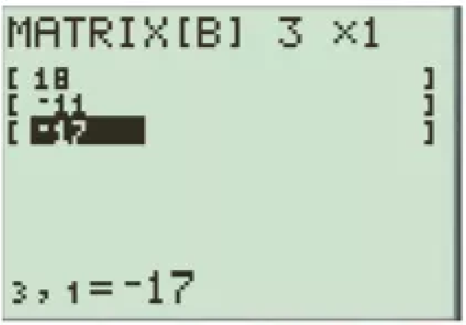
$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Enter the coefficient matrix A and the matrix of constants B into a graphing calculator.
Then find the solution $X = A^{-1}B$
For these key strokes are 2^{nd} matrixes edit

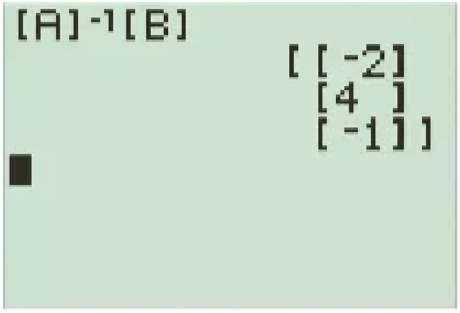


In the edit button enter the dimensions of the required matrix A $= 3 \times 3$

In the edit button enter the dimensions of the required matrix B $= 3 \times 1$



After that press [2nd] [mode] [clear] then again enter [2nd] [matrix 1] [x^{-1}] and [2nd] [matrix 2]
The result will be as shown



$$A^{-1}B = \begin{pmatrix} -2 \\ 4 \\ -1 \end{pmatrix}$$

The solution set is $\boxed{(-2, 4, -1)}$

Answer 37e.

Consider the linear system,

$$2x + 4y + 5z = 5$$
$$x + 2y + 3z = 4$$
$$5x - 4y - 2z = -3$$

Let us write the given linear system as the matrix equation,

$$AX = B$$

Where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.

That is,

$$AX = B$$
$$\begin{pmatrix} 2 & 4 & 5 \\ 1 & 2 & 3 \\ 5 & -4 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}$$

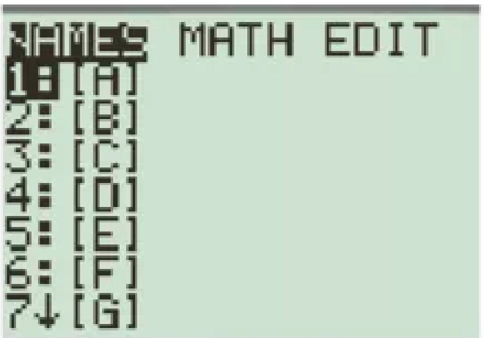
Solve for X we get,

$$AX = B$$

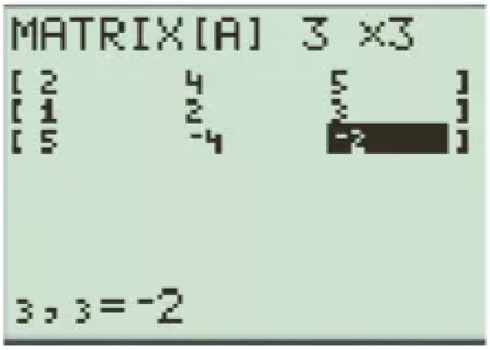
$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

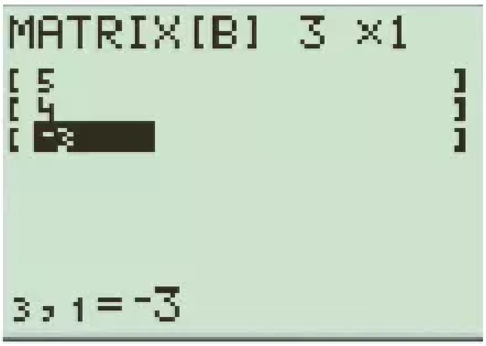
Enter the coefficient matrix A and the matrix of constants B into a graphing calculator. Press 2^{nd} + MATRIX and from the EDIT submenu select the matrix we want to edit.



Set the dimention then enter the new value and press ENTER.



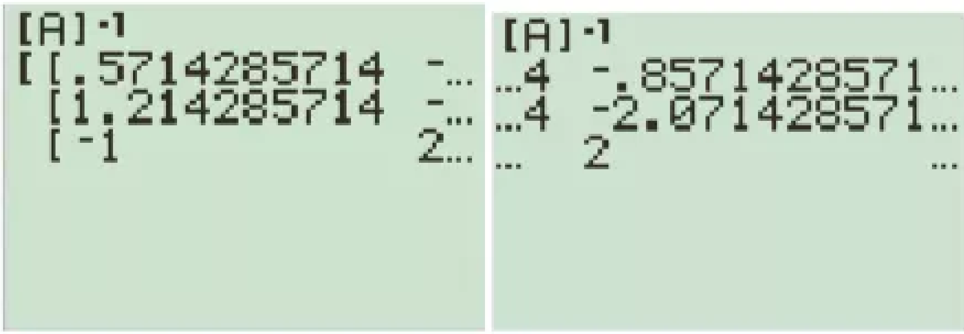
Enter the matrix B as follows.

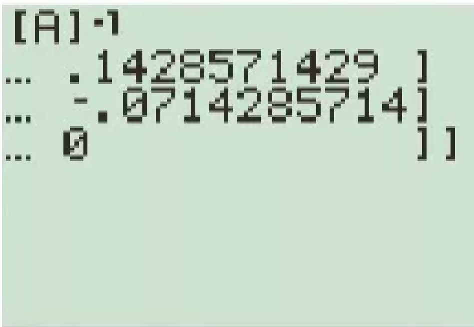


To find the inverse of the matrix A.

Press 2^{nd} + QUIT to returen to the home screen

Press 2^{nd} + MATRIX + NAMES and enter the matrix and enter the inverse button and hit the enter for final answer.

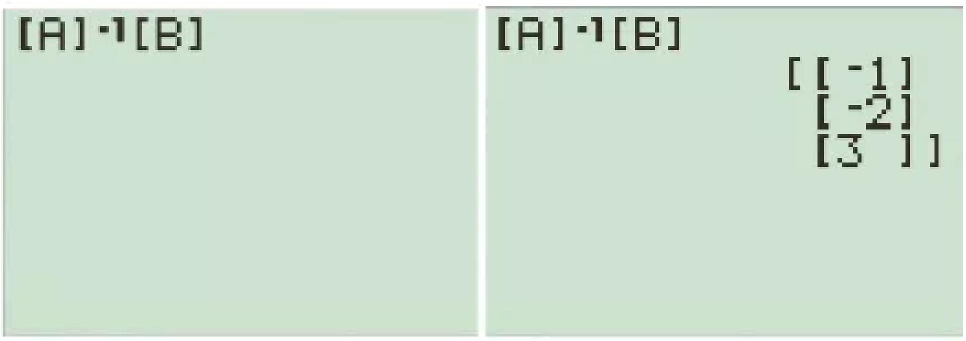




Now, find $A^{-1}B$

Press 2^{nd} + QUIT to returen to the home screen

Press 2^{nd} + MATRIX + NAMES and press1 and enter the inverse button and hit the enter then press 2^{nd} + MATRIX + NAMES press 2 then press enter.



Therefore,

$$A^{-1}B = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$$

Therefore, the solution set is $\boxed{(-1,-2,3)}$

Answer 38e.

Consider the linear system,

$$4x - y - z = -20$$
$$6x - z = -27$$
$$-x + 4y + 5z = 23$$

Let us write the given linear system as the matrix equation,

$$AX = B$$

Where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.

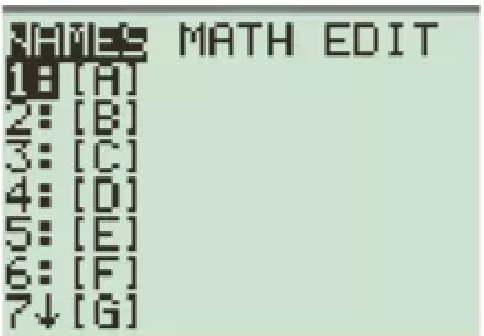
That is,

$$AX = B$$
$$\begin{pmatrix} 4 & -1 & -1 \\ 6 & 0 & -1 \\ -1 & 4 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -20 \\ -27 \\ 23 \end{pmatrix}$$

Solve for X:

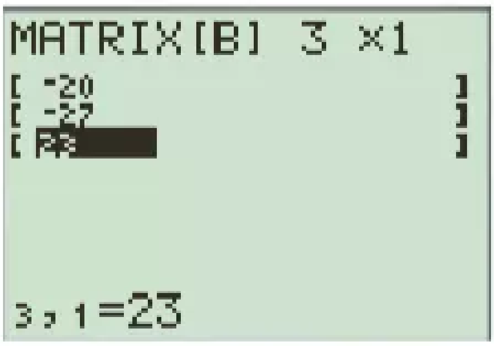
$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Enter the coefficient matrix A and the matrix of constants B into a graphing calculator. Press 2^{nd} + MATRIX and from the EDIT submenu select the matrix we want to edit.

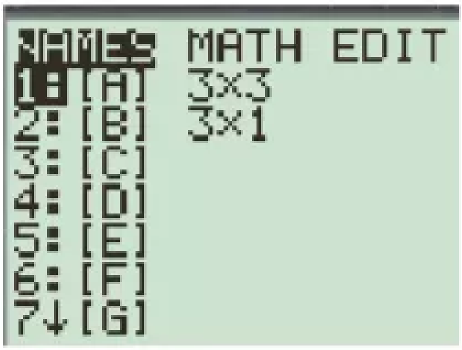


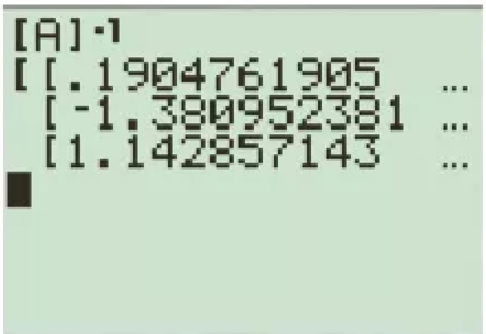
Set the dimention then enter the new value and press ENTER.

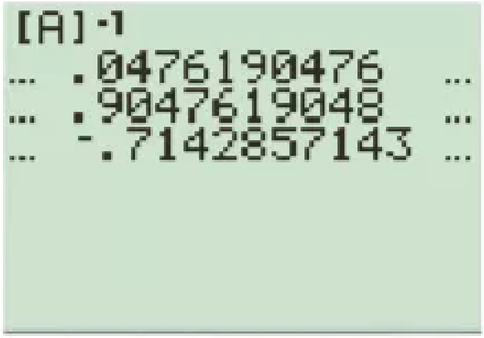
Enter the matrix B as follows.

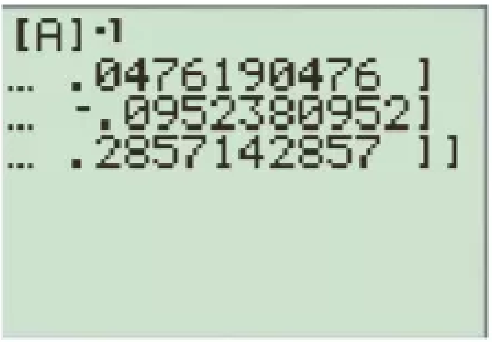


To find the inverse of the matrix A.
Press 2^{nd} + QUIT to returen to the home screen
Press 2^{nd} + MATRIX + NAMES and enter the matrix and enter the inverse button and hit the enter for final answer.



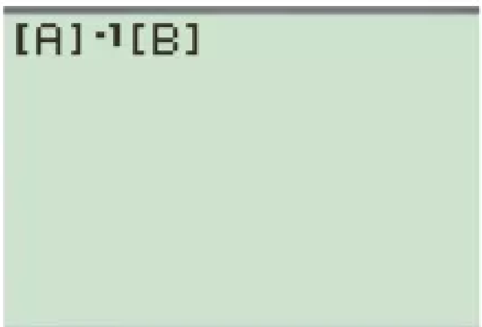


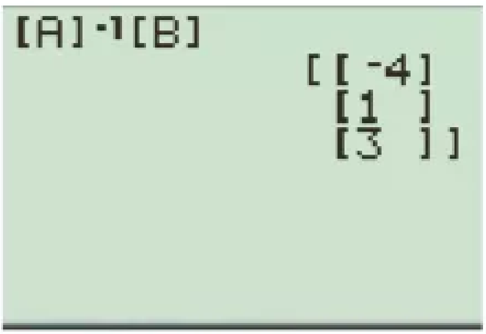




Now, find $A^{-1}B$
Press 2^{nd} + QUIT to returen to the home screen

Press 2^{nd} + MATRIX + NAMES and press1 and enter the inverse button and hit the enter then press 2^{nd} + MATRIX + NAMES press 2 then press enter.





Therefore, $A^{-1}B = \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$

Hence, the solution set is $\boxed{(-4,1,3)}$

Answer 39e.

Consider the linear system,

$$3x+2y-z=14$$
$$-x-5y+4z=-48$$
$$4x+y+z=2$$

Let us write the given linear system as the matrix equation,

$$AX=B$$

Where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.

That is,

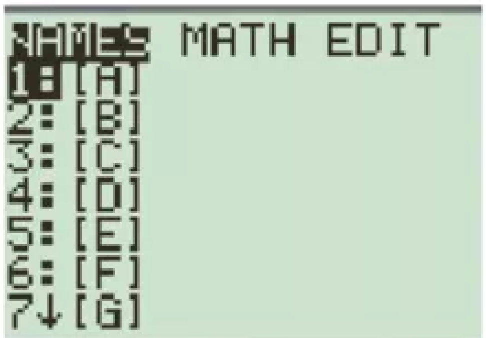
$$AX=B$$

$$\begin{pmatrix} 3 & 2 & -1 \\ -1 & -5 & 4 \\ 4 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 14 \\ -48 \\ 2 \end{pmatrix}$$

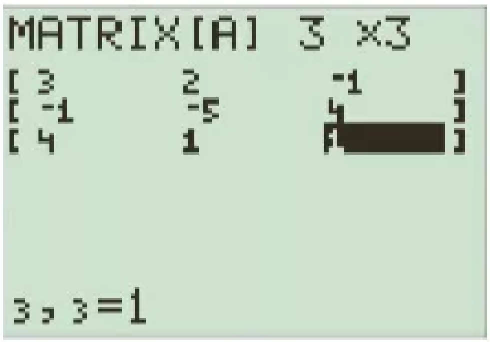
Solve for X:

$$AX=B$$
$$A^{-1}AX=A^{-1}B$$
$$X=A^{-1}B$$

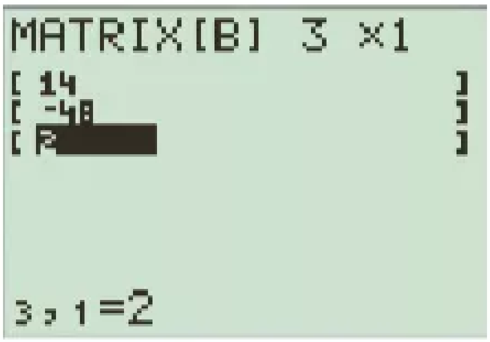
Enter the coefficient matrix A and the matrix of constants B into a graphing calculator.
Press 2^{nd} + MATRIX and from the EDIT submenu select the matrix we want to edit.



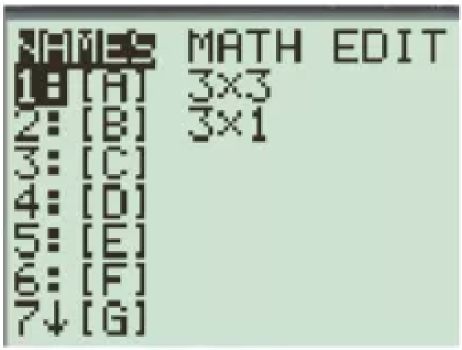
Set the dimention then enter the new value and press ENTER

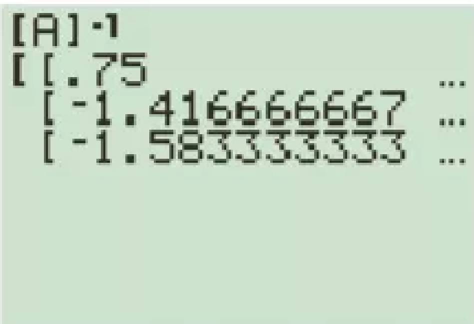


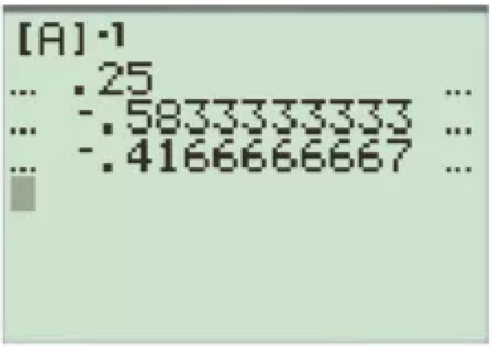
Enter the matrix *B* as follows.

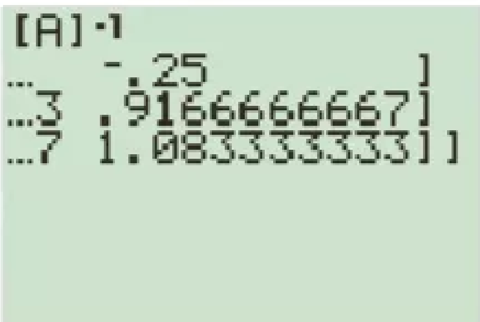


To find the inverse of the matrix *A*.
Press 2nd + QUIT to returen to the home screen
Press 2nd + MATRIX + NAMES and enter the matrix and enter the inverse button and hit the enter for final answer.

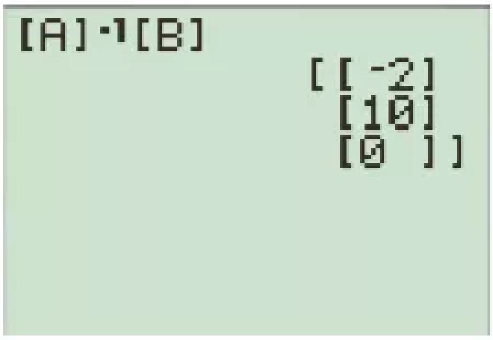








Now, find $A^{-1}B$
Press 2^{nd} + QUIT to returen to the home screen
Press 2^{nd} + MATRIX + NAMES and press1 and enter the inverse button and hit the enter then press 2^{nd} + MATRIX + NAMES press 2 then press enter.



Therefore, $A^{-1}B = X = \begin{pmatrix} -2 \\ 10 \\ 0 \end{pmatrix}$

Hence, the solution set is $\boxed{(-2, 10, 0)}$

Answer 40e.

Consider the linear system,

$$6x + y + 2z = 11$$
$$x - y + z = -5$$
$$-x + 4y - z = 14$$

Let us write the given linear system as the matrix equation,

$$AX = B$$

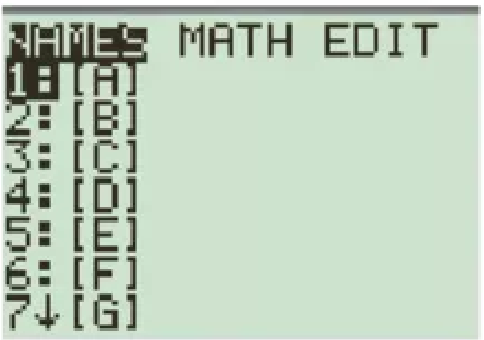
Where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.
That is,

$$AX = B$$
$$\begin{pmatrix} 6 & 1 & 2 \\ 1 & -1 & 1 \\ -1 & 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ -5 \\ 14 \end{pmatrix}$$

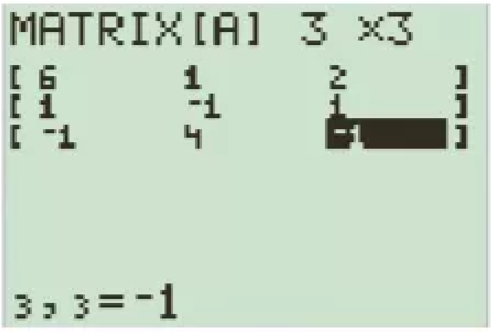
Solve for X:

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

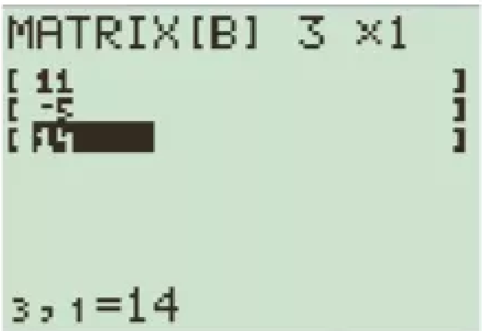
Enter the coefficient matrix A and the matrix of constants B into a graphing calculator. Press 2^{nd} + MATRIX and from the EDIT submenu select the matrix we want to edit.



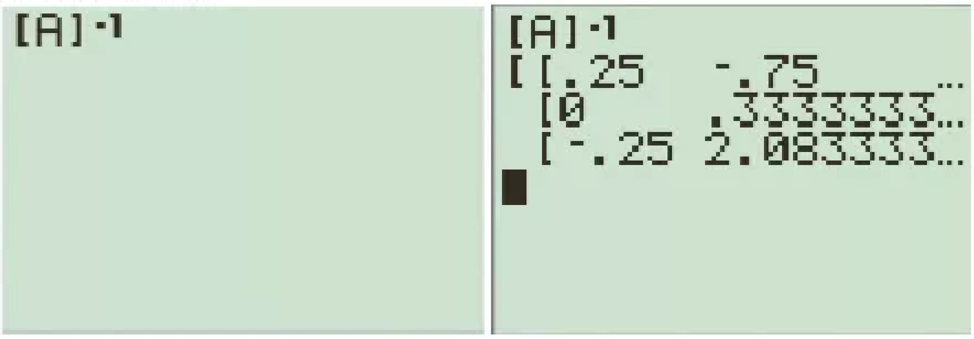
Set the dimention then enter the new value and press ENTER.

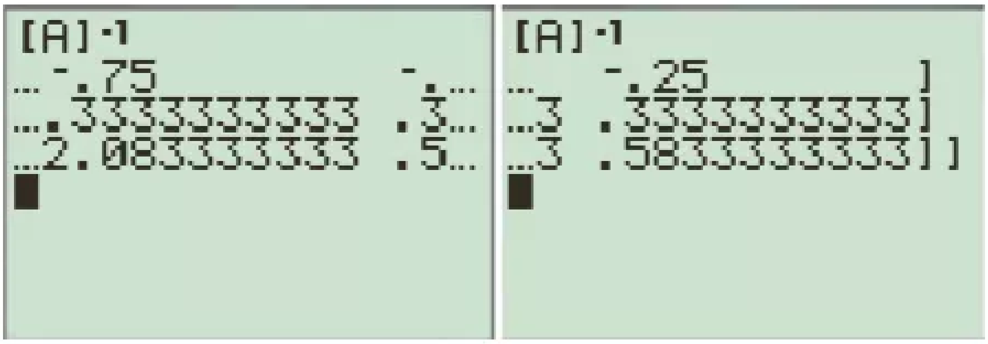


Enter the matrix B as follows.



To find the inverse of the matrix A.
Press 2^{nd} + QUIT to returen to the home screen
Press 2^{nd} + MATRIX + NAMES and enter the matrix and enter the inverse button and hit the enter for final answer.





Now, find $A^{-1}B$
Press 2^{nd} + QUIT to returen to the home screen
Press 2^{nd} + MATRIX + NAMES and press1 and enter the inverse button and hit the enter then press 2^{nd} + MATRIX + NAMES press 2 then press enter.

Therefore, $A^{-1}B = X = \begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix}$

Hence, the solution set is $\boxed{(3,3,-5)}$

Answer 41e.

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

If a determinant of a matrix is 0, then we cannot find the inverse of that matrix.

Such a matrix can be created. The difference of the product of the diagonal elements is the determinant. Thus, we have to create a matrix such that the product of the diagonal elements is equal.

There are an infinite number of 2×2 matrices with the products of the elements along the diagonals are equal. One such matrix is $\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$.

Therefore, a 2×2 matrix that has no inverse is $\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$.

Answer 42e.

Consider the linear system,

$$\begin{aligned} 2w+5x-4y+6z &= 0 \\ 2x+y-7z &= 52 \\ 4w+8x-7y+14z &= -25 \\ 3w+6x-5y+10z &= -16 \end{aligned}$$

And consider the inverse matrix,

$$A^{-1} = \begin{bmatrix} -10 & 4 & 27 & -29 \\ 5 & -2 & -16 & 18 \\ 4 & -2 & -17 & 20 \\ 2 & -1 & -7 & 8 \end{bmatrix}$$

Write the given linear system as the matrix equation,

$$AX = B$$

Where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.

That is,

$$AX = B$$

$$\begin{pmatrix} 2 & 5 & -4 & 6 \\ 0 & 2 & 1 & -7 \\ 4 & 8 & -7 & 14 \\ 3 & 6 & -5 & 10 \end{pmatrix}\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 52 \\ -25 \\ -16 \end{pmatrix}$$

Solving for X we get,

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

Since A^{-1} is given we can directly substitute in the above equation to solve for *X*.

$$X = A^{-1}B$$

$$\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -10 & 4 & 27 & -29 \\ 5 & -2 & -16 & 18 \\ 4 & -2 & -17 & 20 \\ 2 & -1 & -7 & 8 \end{pmatrix} \begin{pmatrix} 0 \\ 52 \\ -25 \\ -16 \end{pmatrix}$$

$$\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -10\cdot 0 + 4\cdot 52 + 27(-25) + (-29)(-16) \\ 5\cdot 0 + (-2)52 + (-16)(-25) + 18(-16) \\ 4\cdot 0 + (-2)52 + (-17)(-25) + 20(-16) \\ 2\cdot 0 + (-1)52 + (-7)52 + 8(-16) \end{pmatrix}$$ Apply matrix multiplication

$$\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 + 208 - 675 + 464 \\ 0 - 104 + 400 - 288 \\ 0 - 104 + 425 - 320 \\ 0 - 52 + 175 - 128 \end{pmatrix}$$ Simplify

$$\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \\ 1 \\ -5 \end{pmatrix}$$ Add like terms

Therefore, the solution of the system is $\boxed{(-3, 8, 1, -5)}$.

Answer 43e.

STEP 1 Write a verbal model to form an equation for the total rent.

Rent for single-engine airplane (dollars/hour)	·	Flight time for single-engine airplane (hours)	+	Rent for twin-engine airplane (dollars/hour)	·	Flight time for twin-engine airplane (hours)	=	Total rent (dollars)

Now, write a verbal model to form an equation for the total flight time.

Flight time for single-engine airplane (hours)	+	Flight time for twin-engine airplane (hours)	=	Total flight time (hours)

STEP 2 Write a system of equations. Let t be the flight time for twin-engine airplanes and s be the flight time for single-engine airplane.

$60s + 240t = 21,000$ Equation 1

$s + t = 200$ Equation 2

STEP 3 Rewrite the system as a matrix equation.

coefficient matrix (A)		matrix of variables (X)		matrix of constants (B)
$\begin{bmatrix} 60 & 240 \\ 1 & 1 \end{bmatrix}$	$\cdot$	$\begin{bmatrix} s \\ t \end{bmatrix}$	$=$	$\begin{bmatrix} 21,000 \\ 200 \end{bmatrix}$

Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. For this, find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$|A| = \begin{vmatrix} 60 & 240 \\ 1 & 1 \end{vmatrix}$$

$$= (60)(1) - (240)(1)$$

$$= 60 - 240$$

$$= -180$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$A^{-1} = \frac{1}{-180}\begin{bmatrix} 1 & -240 \\ -1 & 60 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{180} & \frac{4}{3} \\ \frac{1}{180} & -\frac{1}{3} \end{bmatrix}$$

Multiply each side of $AX = B$ by A^{-1}.

$$X = A^{-1}B$$

$$= \begin{bmatrix} -\frac{1}{180} & \frac{4}{3} \\ \frac{1}{180} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 21,000 \\ 200 \end{bmatrix}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$. Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{bmatrix} -\frac{1}{180} & \frac{4}{3} \\ \frac{1}{180} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 21,000 \\ 200 \end{bmatrix} = \begin{bmatrix} -\frac{1}{180}(21,000) + \frac{4}{3}(200) \\ \frac{1}{180}(21,000) - \frac{1}{3}(200) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{350}{3} + \frac{800}{3} \\ \frac{350}{3} - \frac{200}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 150 \\ 50 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 150 \\ 50 \end{bmatrix}.$$

Therefore, the pilot flew a single-engine airplane for 150 hours and a twin-engine airplane for 50 hours.

Answer 44e.

Consider the statement,

Dallas made total of 976 shots.

And scored 1680 points for 3-point field goals, 2-point field goals, and 1-point free throws and made 135 more 2-point field goals, and 1-point free throws.

Let x, y, and z represent each type of shot, 3-point field goals, 2-point field goals, and 1-point free throws respectively.

Then, from the given data we can construct the following equations.

$$x + y + z = 976$$

$$3x + 2y + z = 1680$$

$$y = z + 135$$

Write this linear system as the matrix equation,

$$AX = B$$

Where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.

That is,

$$AX = B$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 976 \\ 1680 \\ 135 \end{pmatrix}$$

Solving for X we get,

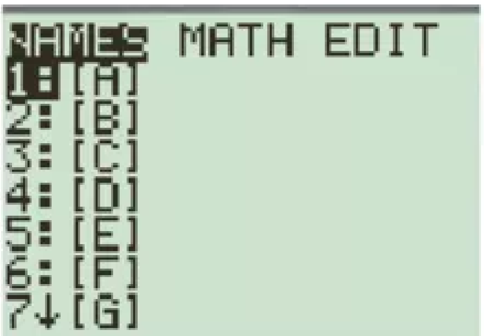
$$AX = B$$

$$A^{-1}AX = A^{-1}B \qquad \text{Multiply with } A^{-1}$$

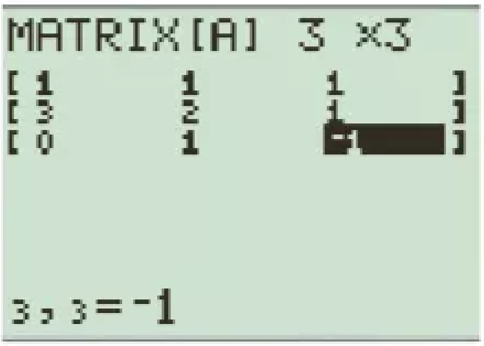
$$X = A^{-1}B$$

Enter the coefficient matrix A and the matrix of constants B into a graphing calculator. Then find the solution $X = A^{-1}B$

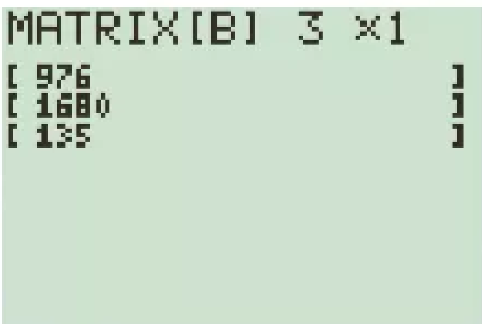
Press 2^{nd} + MATRIX and from the EDIT submenu select the matrix we want to edit.



Set the dimention then enter the new value and press ENTER.



Enter the matrix B as follows.



To find the inverse of the matrix A.

Press 2nd + QUIT to returen to the home screen

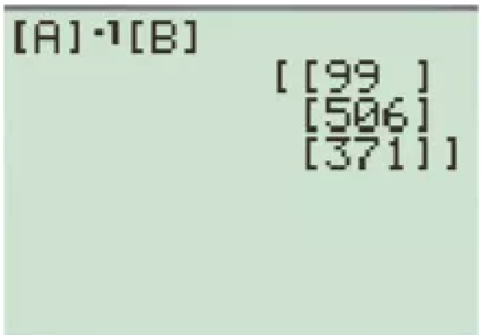
Press 2nd + MATRIX + NAMES and enter the matrix and enter the inverse button and hit the enter for final answer.

Now, find $A^{-1}B$

Press 2nd + QUIT to returen to the home screen

Press 2nd + MATRIX + NAMES and press1 and enter the inverse button and hit the enter then press 2nd + MATRIX + NAMES press 2 then press enter.

Then press enter to get the final answer.



$$A^{-1}B = \begin{pmatrix} 99 \\ 506 \\ 371 \end{pmatrix}$$

Therefore, he made $\boxed{99}$ 3-point field goals, $\boxed{506}$ 2-point field goals, and $\boxed{371}$ 1-point free throws.

Answer 45e.

a. Let x be the number of batches of rolls, and y be the number of batches of muffins. Write a verbal model to form an equation for the total cups of buttermilk.

$$\begin{array}{ccccccccc} \text{Cups of butter milk for a batch of rolls (cups)} & \cdot & \text{Batches of rolls} & + & \text{Cups of butter milk for a batch of muffins (cups)} & \cdot & \text{Batches of muffins} & = & \text{Total cups of butter milk (cups)} \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ 2 & \cdot & x & + & 1 & \cdot & y & = & 8 \end{array}$$

Write a verbal model to form an equation for the total number of eggs.

$$\begin{array}{ccccccccc} \text{Number of eggs for a batch of rolls (eggs)} & \cdot & \text{Batches of rolls} & + & \text{Number of eggs for a batch of muffins (eggs)} & \cdot & \text{Batches of muffins} & = & \text{Total number of eggs (eggs)} \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ 3 & \cdot & x & + & 1 & \cdot & y & = & 11 \end{array}$$

Therefore, a system of equations for the situation is

$2x + y = 8$

$3x + y = 11.$

b. We can rewrite the linear system as a matrix equation $AX = B$.

$$\begin{array}{cccc} \text{coefficient matrix } (A) & & \text{matrix of variables } (X) & & \text{matrix of constants } (B) \\ \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} & \cdot & \begin{bmatrix} x \\ y \end{bmatrix} & = & \begin{bmatrix} 8 \\ 11 \end{bmatrix} \end{array}$$

c. Find the inverse of matrix A. The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. First find the determinant of matrix A. The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - cb$.

$$\begin{aligned} |A| &= \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} \\ &= (2)(1) - (3)(1) \\ &= 2 - 3 \\ &= -1 \end{aligned}$$

Substitute the values in $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}$$

Multiply each side of $AX = B$ by A^{-1}.

$$A^{-1}AX = A^{-1}B$$
$$X = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

Find the element in the ith row and jth column of the product matrix $A^{-1}B$.
Multiply each element in the ith row of A^{-1} by the corresponding element in the jth column of B, and then add the products.

$$\begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 8 \\ 11 \end{bmatrix} = \begin{bmatrix} -1(8)+1(11) \\ 3(8)-2(11) \end{bmatrix}$$
$$= \begin{bmatrix} -8+11 \\ 24-22 \end{bmatrix}$$
$$= \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

Therefore, the class makes 3 batches of rolls and 2 batches of muffins.

Answer 46e.

Consider the statement,
A basic platter with 2 cheeses and 3 meets costs \$18.
A medium platter with 3 cheeses and 5 meets costs \$28.
A super platter with 7 cheeses and 10 meats costs \$60
Now, the information writes in equations.
Let x and y represent the cost of cheese and meat respectively.
Then according to the given data, we can construct the following equations.
Basic platter: $2x+3y=18$
Medium platter: $3x+5y=28$
Super platter: $7x+10y=60$

(a)

The first two equations form the required system of equations.

$$2x+3y=18$$
$$3x+5y=28$$

This can be written in the matrix equation as:

$$Ax=B$$

Where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.

That is,

$$\begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 18 \\ 28 \end{pmatrix}$$

Find the solution $X=A^{-1}B$

Let $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a matrix.

$$A^{-1}=\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$|A|=\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix}$$
$$=10-9$$
$$=1$$

$$A=\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix}$$

$$A^{-1}=\begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix} \qquad \text{Apply } A^{-1}=\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Now, find the matrix for X.

$$X=A^{-1}B$$

$$\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 18 \\ 28 \end{pmatrix} \qquad \text{Substitute the matrixes of } A^{-1} \text{ and } B$$

$$=\begin{pmatrix} 5\cdot 18+(-3)28 \\ (-3)18+2\cdot 28 \end{pmatrix} \qquad \text{Apply matrix multiplication}$$

$$=\begin{pmatrix} 90-84 \\ -54+56 \end{pmatrix} \qquad \text{Simplify}$$

$$=\begin{pmatrix} 6 \\ 2 \end{pmatrix} \qquad \text{Add like terms}$$

Therefore, the cost of cheese is $\boxed{\$6}$ and the cost of meat is $\boxed{\$2}$.

(b)

These two equations form the required system of equations.

$$3x+5y=28$$
$$7x+10y=60$$

This can be written in the matrix equation as:

$$Ax=B$$

Where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.

That is,

$$\begin{pmatrix}3 & 5\\7 & 10\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}28\\60\end{pmatrix}$$

Find the solution $X=A^{-1}B$

$$|A|=\begin{vmatrix}3 & 5\\7 & 10\end{vmatrix}$$
$$=30-35$$
$$=-5$$

Now, find the inverse matrix for A.

$$A=\begin{pmatrix}3 & 5\\7 & 10\end{pmatrix}$$

$$A^{-1}=\frac{1}{-5}\begin{pmatrix}10 & -5\\-7 & 3\end{pmatrix} \quad \text{Apply } A^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d & -b\\-c & a\end{pmatrix}$$

$$=\begin{pmatrix}-2 & 1\\7/5 & -3/5\end{pmatrix}$$

$$X=A^{-1}B$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-2 & 1\\7/5 & -3/5\end{pmatrix}\begin{pmatrix}28\\60\end{pmatrix} \quad \text{Substitute the matrixes of } A^{-1} \text{ and } B$$

$$=\begin{pmatrix}(-2)28+1\cdot 60\\ \left(\frac{7}{5}\right)28+\left(-\frac{3}{5}\right)60\end{pmatrix} \quad \text{Apply matrix multiplication}$$

$$=\begin{pmatrix}-56+60\\ \frac{196}{5}-\frac{180}{5}\end{pmatrix} \quad \text{Simplify}$$

$$=\begin{pmatrix}4\\3.2\end{pmatrix} \quad \text{Add like terms}$$

The cost of cheese is $\boxed{\$4}$ and the cost of meat is $\boxed{\$3.2}$.

(c)

The results from parts (a) and (b) differ. This discrepancy occurs due to the uneven system of linear equations.
Since there are only two variables in each of the equation, we can only make a system of two equations. Hence the system of equations in part (a) and (b) is not the same.
Thus their solution also differs.

Answer 47e.

Consider the statement,

Cereal	Calories	Fat	Carbohydrates
Bren Crunchies	78	1g	22g
Toasted Oats	104	0g	25.5g
Whole Wheat Flakes	198	0.6g	23.8g

Each brand combined to get 500 calories,3 grams of fat, and 100 grams of carbohydrates.
Let x, y, and z represent the number of ounces in each brand; Bran Crunchies, Toasted Oats, and Whole Wheat Flakes respectively.
Then according to the given data, we can construct the following equations.

$$78x + 104y + 198z = 500$$

$$x + 0.6z = 3$$

$$22x + 25.5y + 23.8z = 100$$

Let us write this linear system as the matrix equation,

$$AX = B$$

Where A is the coefficient matrix, X is the matrix of variables, and B is the matrix of constants.
That is,

$$AX = B$$

$$\begin{pmatrix} 78 & 104 & 198 \\ 1 & 0 & 0.6 \\ 22 & 25.2 & 23.8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 500 \\ 3 \\ 100 \end{pmatrix}$$

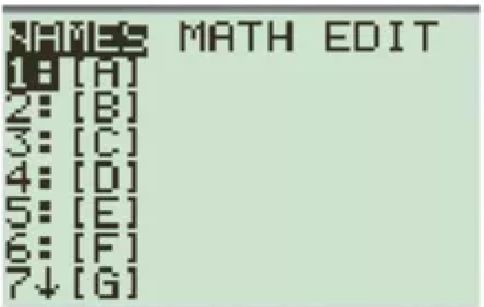
Solving for X we get,

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

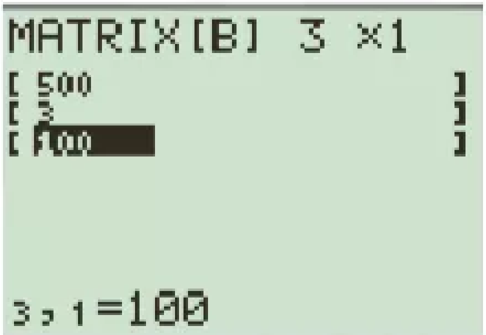
$$X = A^{-1}B$$

Enter the coefficient matrix A and the matrix of constants B into a graphing calculator. Press 2[nd] + MATRIX and from the EDIT submenu select the matrix we want to edit.

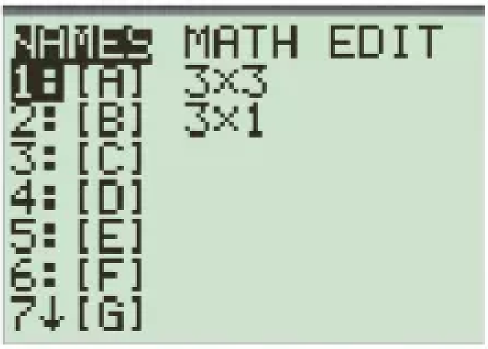


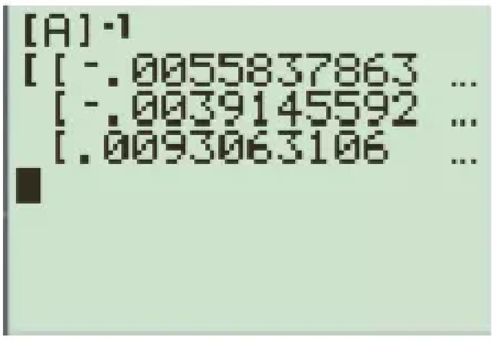
Set the dimention then enter the matrix and press ENTER.

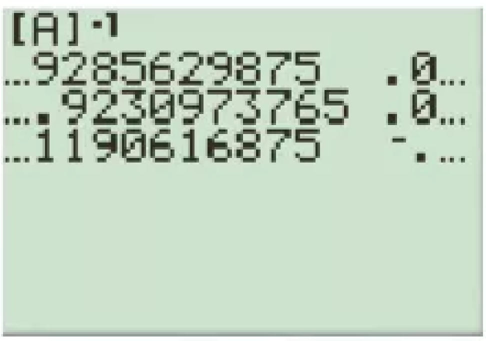
Enter the matrix B as follows.

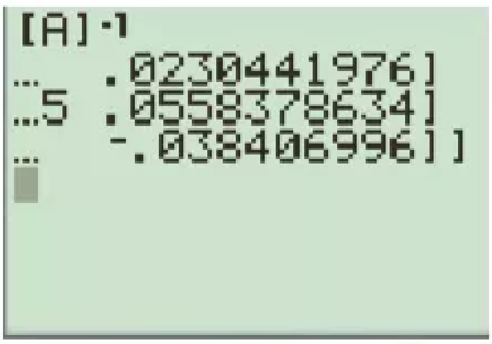


To find the inverse of the matrix A.
Press 2[nd] + QUIT to returen to the home screen
Press 2[nd] + MATRIX + NAMES and enter the matrix and enter the inverse button and hit the enter for final answer.



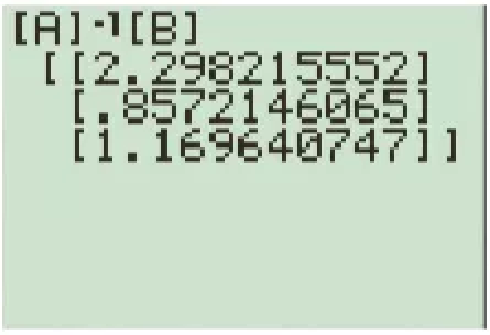






Now, find $A^{-1}B$
Press 2nd + QUIT to returen to the home screen
Press 2nd + MATRIX + NAMES and press1 and enter the inverse button and hit the enter then press 2nd + MATRIX + NAMES press 2 then press enter.

Now, press enter to get the final answer.



Therefore, $A^{-1}B = \begin{pmatrix} 2.298215552 \\ 0.8572146065 \\ 1.169640747 \end{pmatrix}$

Therefore, there must be $\boxed{2.3oz}$ of Bran crunchies, $\boxed{0.8oz}$ of toasted oats, and $\boxed{1.2oz}$ whole wheat flakes.

Answer 49e.

a. First, find AT.
Multiply each element in the ith row of A by the corresponding element in the jth column of T, and then add the products.

$$AT = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 3 & 5 \\ 1 & 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0(1)+1(1) & 0(3)+1(4) & 0(5)+1(2) \\ -1(1)+0(1) & -1(3)+0(4) & -1(5)+0(2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 2 \\ -1 & -3 & -5 \end{bmatrix}$$

The columns of matrix AT give the coordinates of the vertices of the transformed triangle.

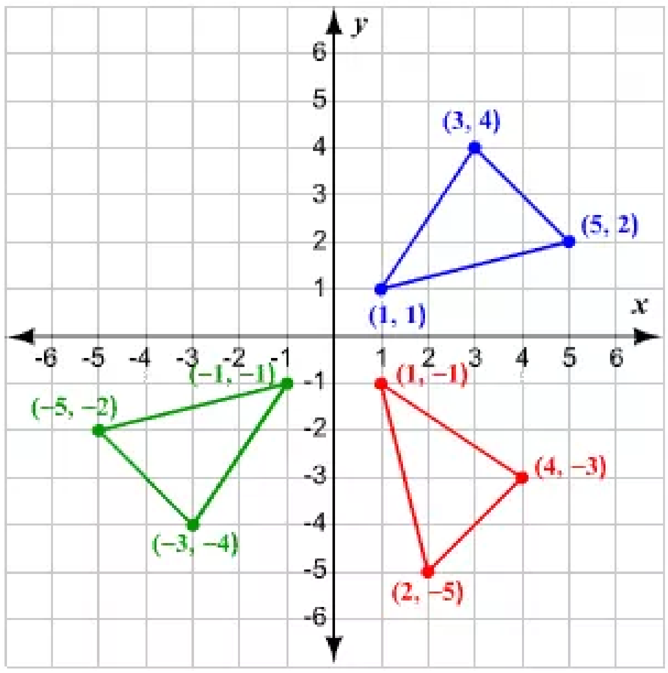
Thus, the coordinates of the vertices of AT are $(1, -1)$, $(4, -3)$, and $(2, -5)$.

Multiply the transformation matrix and Matrix AT to find ATT.

$$ATT = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 4 & 2 \\ -1 & -3 & -5 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -5 \\ -1 & -4 & -2 \end{bmatrix}$$

The coordinates of the vertices of ATT are $(-1, -1)$, $(-3, -4)$ and $(-5, -2)$.

Plot the vertices of the original and transformed triangles and join them.



b. From the figure drawn in part **a.**, we can see that the original triangle is the reflection of the transformed triangle about $y = -x$.

Answer 50e.

Consider the matrixes,

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ And } B = \frac{1}{ad - cb}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

To prove the definition of inverse matrix, show $AB = I = BA$ for $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and

$$B = \frac{1}{ad - cb}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Find the product of the matrixes A and B.

$$AB = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \frac{1}{ad - cb}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$AB = \frac{1}{ad - cb}\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\right)$$

$$= \frac{1}{ad - cb}\begin{pmatrix} ad - cb & -ab + ab \\ cd - cd & -cb + ad \end{pmatrix}$$ Apply the matrix multiplication

$$= \frac{1}{ad - cb}\begin{pmatrix} ad - cb & 0 \\ 0 & -cb + ad \end{pmatrix}$$ Add like terms

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Find the product of the matrixes B and A.

$$BA = \frac{1}{ad-cb}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \frac{1}{ad-cb}\begin{pmatrix} ad-cb & bd-bd \\ -ac+ac & -cb+ad \end{pmatrix} \quad \text{Apply the matrix multiplication}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Therefore, $AB = I = BA$

Hence, $B = A^{-1}$

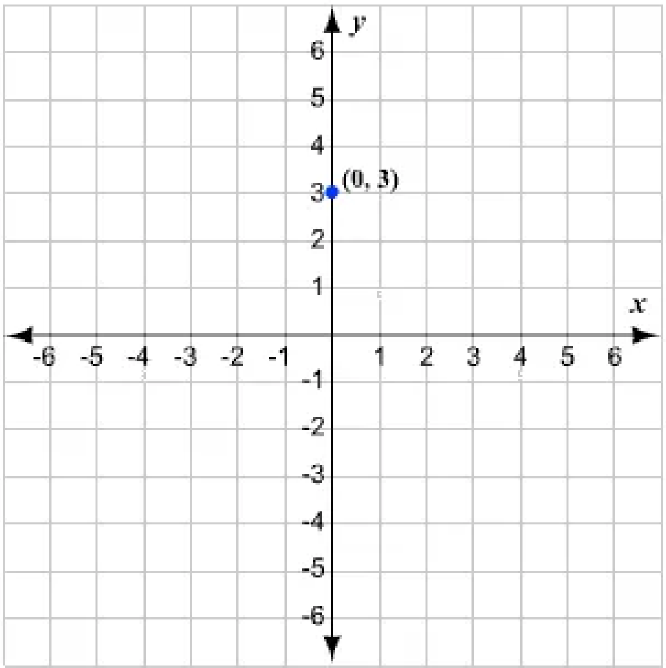
The inverse of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ as $A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ provided $|A| \neq 0$.

Answer 51e.

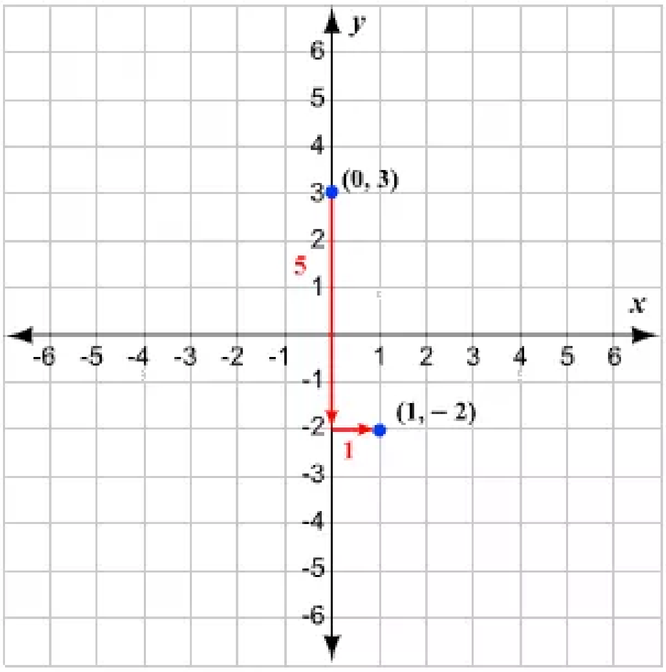
STEP 1 The slope-intercept form of a linear equation is $y = mx + b$. The given equation is already in the slope-intercept form.

On comparing the given equation with $y = mx + b$, we find that m is -5, and b is 3.

STEP 2 The y-intercept is 3. Plot the point (0, 3) on a coordinate plane where the line crosses the y-axis.

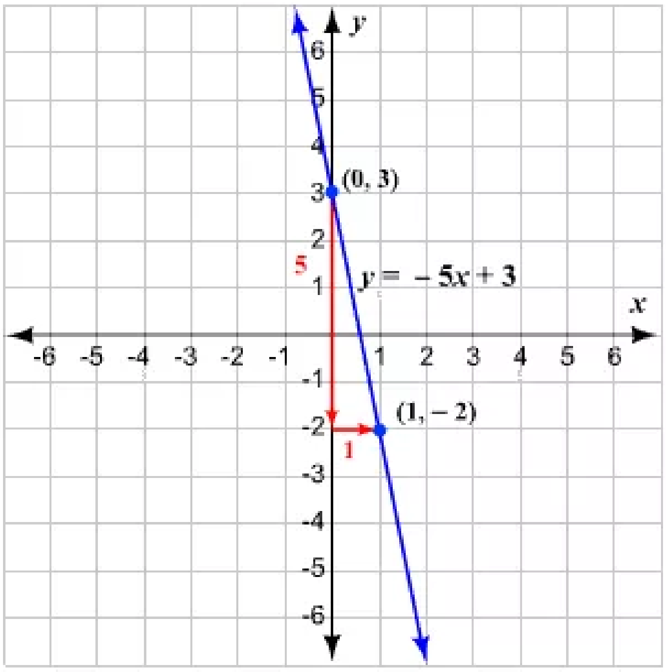


STEP 3 Use the slope to plot a second point on the line. Since the slope is −5, or $\frac{-5}{1}$, start at (0, 3) and then move 5 units down. Now, move 1 unit to the right.



The second point is (1, −2).

STEP 4 Finally, draw a line through the two points.



Answer 52e.

Consider the equation,

$$y = \frac{2}{3}x + 2$$

Step1:

The equation is already in slope-intercept form.

Step2:

Identify the $y-$intercept.

The $y-$intercept is 2, plot the point $(0,2)$ where the line cross the $y-$axis.

Step3:

Identify the slope. The slope of the equation is,

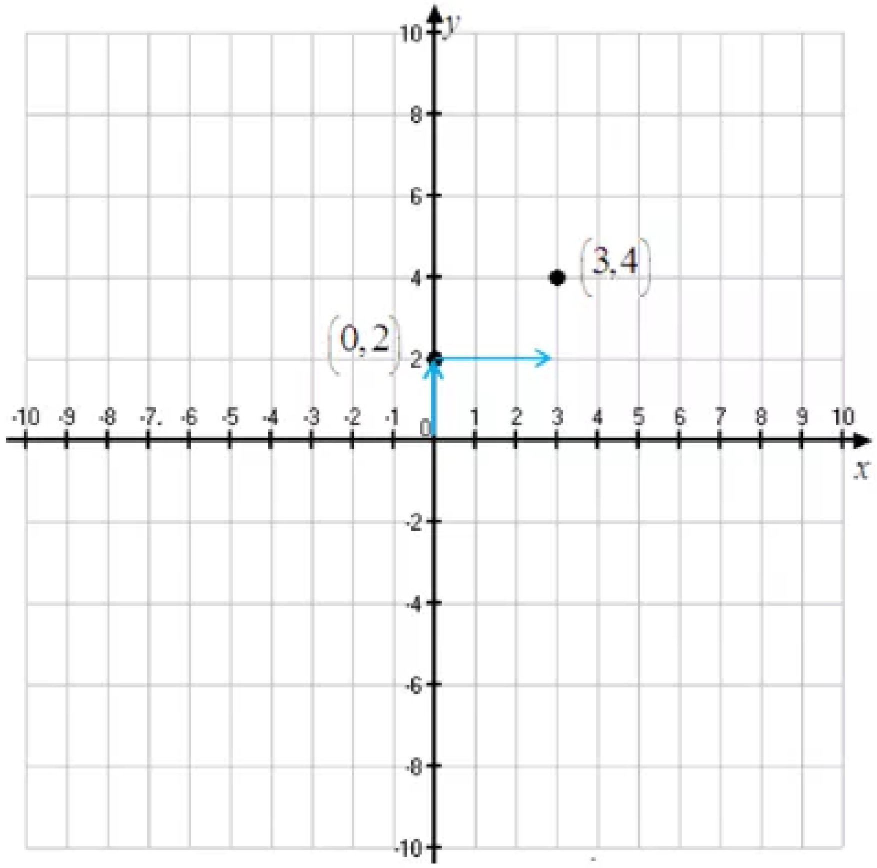
$$m = \frac{2}{3}$$

Plot a second point on the line by starting at $(0,2)$ and then move up 2 units and left 3 units and left 3 units.

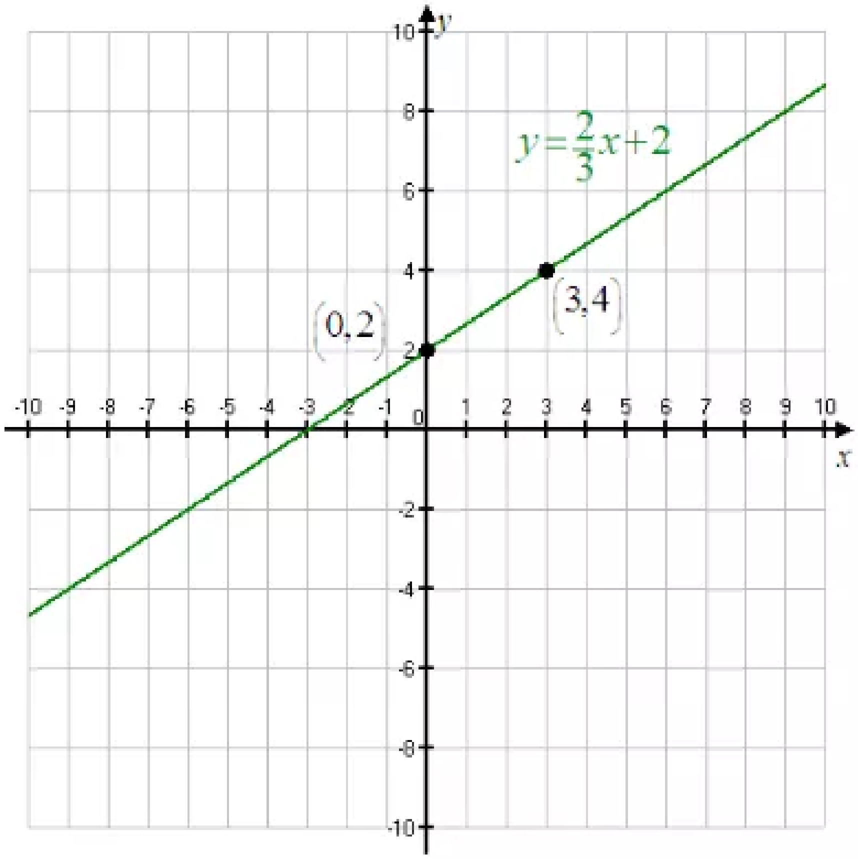
The second point is $(3,4)$

Step4:

Draw a line through the three points.



Sketch the graph of the equation by using

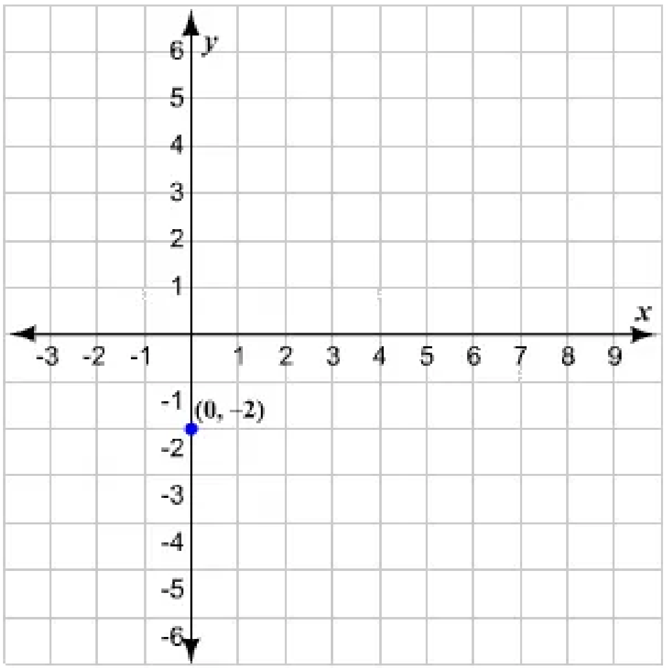


Answer 53e.

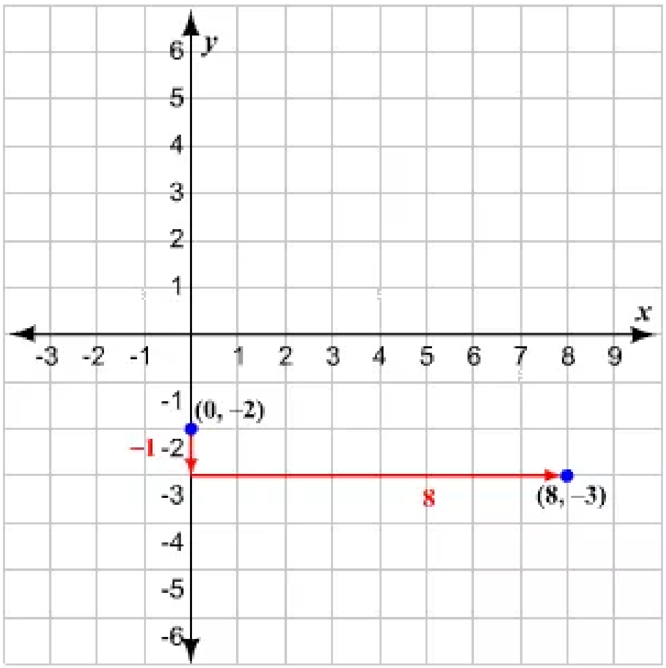
STEP 1 The slope-intercept form of a linear equation is $y = mx + b$. The given equation is already in the slope-intercept form.

On comparing the given equation with $y = mx + b$, we find that m is $-\frac{1}{8}$, and b is -2.

STEP 2 The y-intercept is -2. Plot the point $(0, -2)$ on a coordinate plane where the line crosses the y-axis.

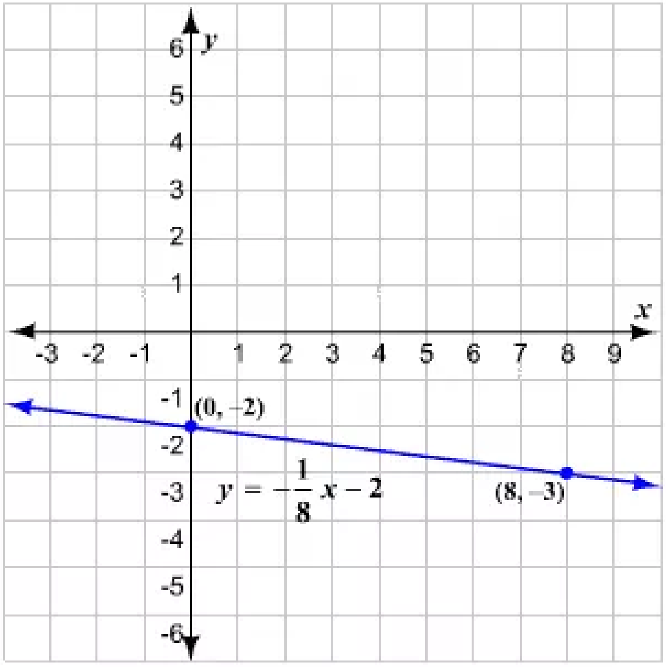


STEP 3 Use the slope to plot a second point on the line. Since the slope is $-\frac{1}{8}$, or $\frac{-1}{8}$, start at (0, −2) and then move 1 unit down. Now, move 8 units to the right.



The second point is (8, −3).

STEP 4 Finally, draw a line through the two points.



Answer 54e.

To graph $y \geq 2|x|$, we begin by graphing the boundary line $y = 2|x|$. Since the inequality contains an $\geq$ symbol, the boundary is a solid line. Because the coordinates of the test point $(0,0)$ satisfy $y \geq 2|x|$, we shade the side of the boundary that contains $(0,0)$.

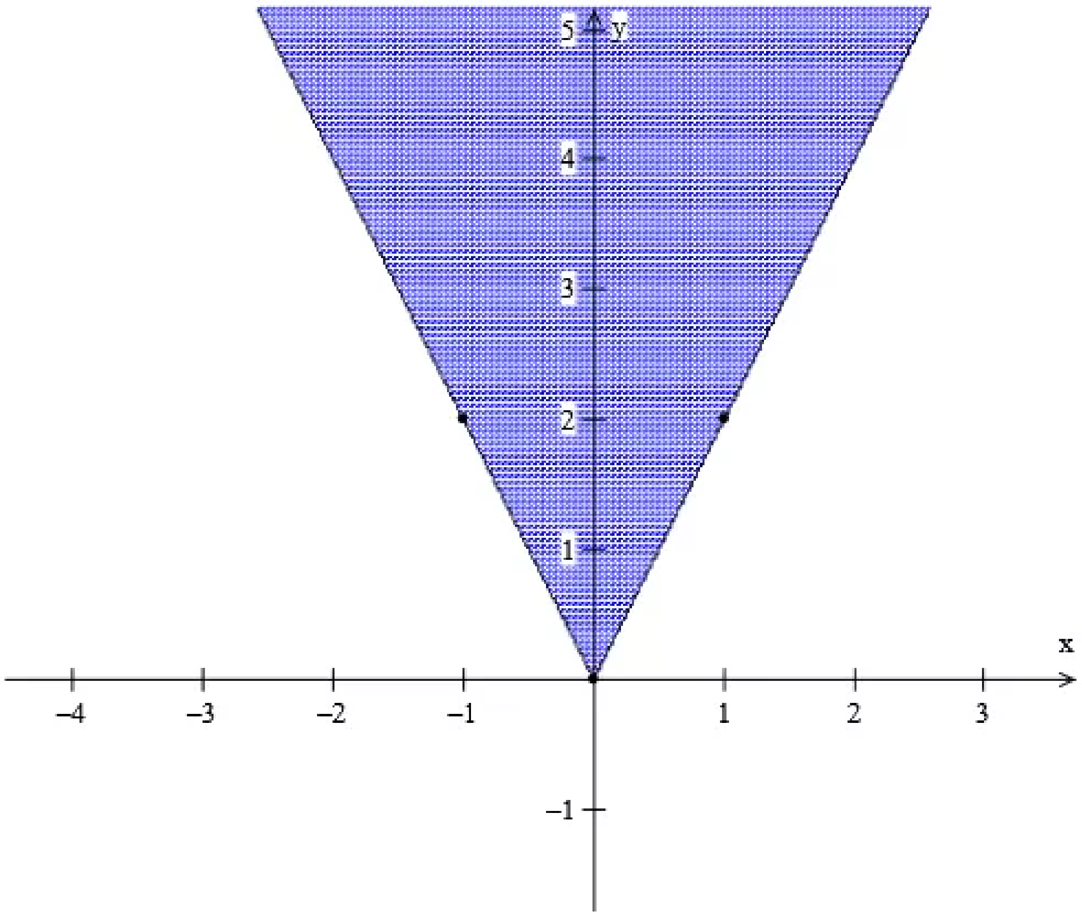
Graph the boundary: $y = 2|x|$

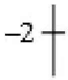
x	y	(x, y)
-1	2	$(-1, 2)$
1	2	$(1, 2)$

Shading: Check the test point $(0,0)$

$$y \geq 2|x|$$
$$0 \geq 2|0|$$
$$0 \geq 0$$

$(0,0)$ is a solution of $y \geq 2|x|$





Answer 55e.

STEP 1 **Graph the boundary line of the inequality.**

In order to obtain the boundary line, replace the inequality sign with "=". Then, we get a function of the form $y = |x - h| + k$, where (h, k) is the vertex of the function's graph.

In this case, we get the value of h as 2 and that of k as 0. Thus, the vertex is (2, 0).

Let us use symmetry to find two more points.
Substitute any value, say, 1 for y in the given function.
$2 = |x - 2|$

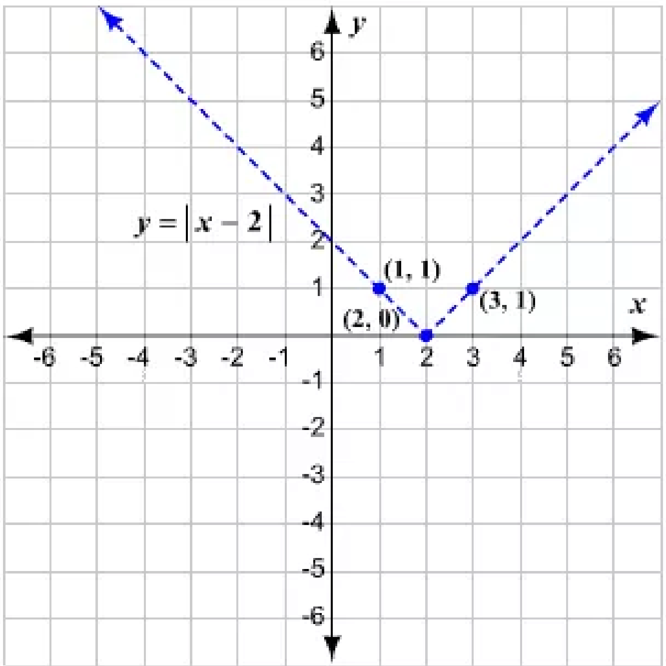
We get $x - 2 = 1$ and $x - 2 = -1$.

Add 2 to both sides of the two equations.

$$x - 2 + 2 = 1 + 2 \quad \text{and} \quad x - 2 + 2 = -1 + 2$$
$$x = 3 \quad \text{and} \quad x = 1$$

The two points are (1, 1) and (3, 1).

Plot (2, 0), (1, 1), and (3, 1) on the graph. Connect these points using straight lines to obtain a V-shaped graph. Since < is the inequality sign used, draw a dashed line.



STEP 2 **Test a point.**

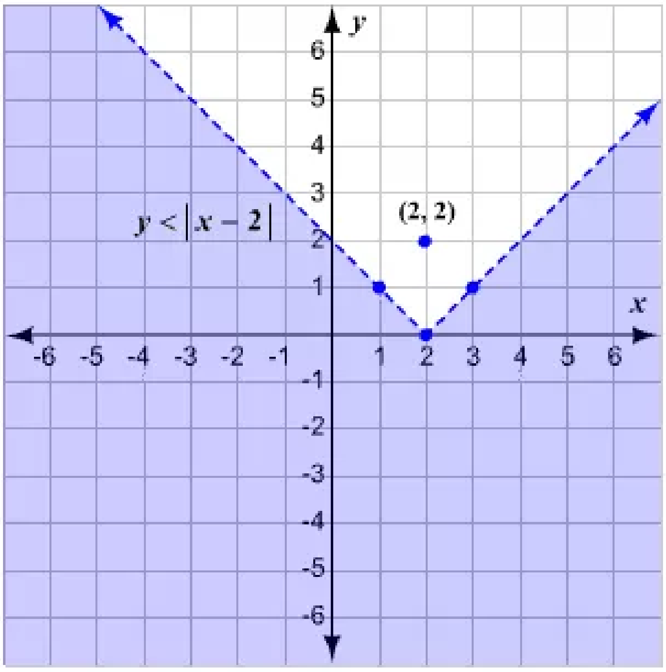
Let us take a test point, that does not lie on the boundary line, say, (2, 2). Substitute 2 for y, and 2 for x in the function. Check if the test point satisfies the given inequality.

$2 \overset{?}{<} |2 - 2|$

$2 \overset{?}{<} |0|$

$2 < 0$ FALSE

The test point is not a solution to the inequality. Shade the half-plane that does not contain (2, 2).



Answer 56e.

To graph $y \le -2|x+1|$, we begin by graphing the boundary line $y = -2|x+1|$. Since the inequality contains an $\le$ symbol, the boundary is a solid line. Because the coordinates of the test point $(0,0)$ does not satisfy $y \le -2|x+1|$, we shade the side of the boundary that does not contain $(0,0)$.

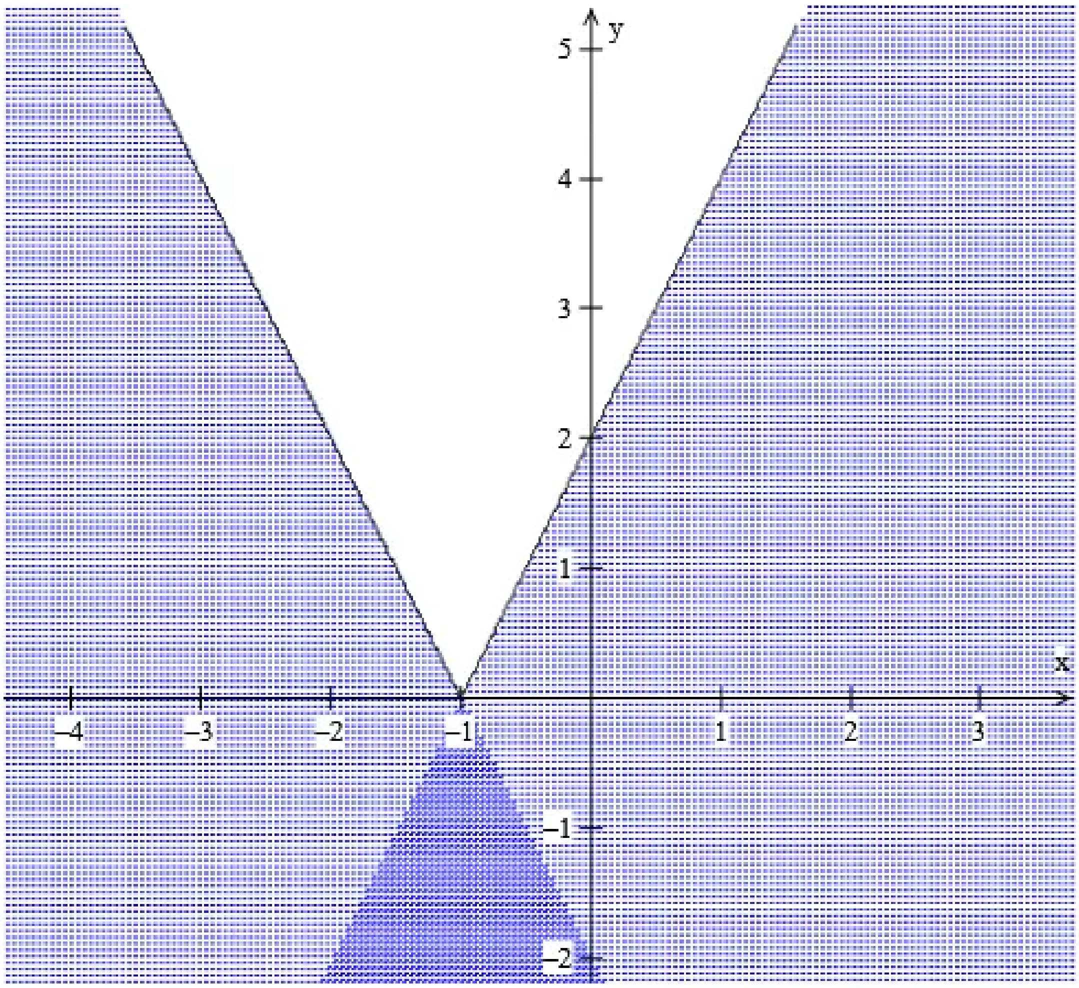
Graph the boundary: $y = -2|x+1|$

x	y	(x, y)
-1	0	$(-1, 0)$
1	-4	$(1, -4)$

Shading: Check the test point $(0,0)$

$$y \le -2|x+1|$$
$$0 \le -2|0+1|$$
$$0 \le -2$$

$(0,0)$ is not a solution of $y \le -2|x+1|$



Answer 57e.

We know that if the correlation coefficient r lies near 1, then the points are close to the line with positive slope. If r lies near -1, then the points are close to the line with negative slope. If r lies near 0, then the points do not lie close to any line.

In the given data, we can see that the scatter plot shows approximately no correlation. That is, the points are not close to any line.

Thus, the value of the correlation coefficient r will be close to 0.

Answer 59e.

We know that in a scatter plot, if y tends to increase as x increases, then the data will have a positive correlation. If y decreases with increase in x, then the data will have a negative correlation. If the points show no specific pattern, then the data will have approximately no correlation.

In the given data, we can see that y is increasing with the decrease in x.

Thus, the data have a negative correlation.

Answer 60e.

The given equation is

$$\begin{pmatrix} 5 & 4x \\ 18 & 6 \end{pmatrix} = \begin{pmatrix} 5 & -20 \\ 3y & 6 \end{pmatrix}$$

Equate the corresponding elements to obtain equations involving x and y

$$4x = -20$$
$$3y = 18$$

Now solve the two resulting equations

$$4x = -20 \qquad\qquad 3y = 18$$
$$x = \frac{-20}{4} \qquad\qquad y = \frac{18}{3}$$
$$x = -5 \qquad\qquad y = 6$$

Therefore, the solution is $\boxed{x = -5, y = 6}$.

Answer 61e.

Add two matrices on the left side of the equation. For this, add the elements in corresponding positions.

$$\begin{bmatrix} -3x + 4 & -9 + 12 \\ 13 + (-5y) & -5 + 16 \end{bmatrix} = \begin{bmatrix} -20 & 3 \\ 18 & 11 \end{bmatrix}$$

$$\begin{bmatrix} -3x + 4 & 3 \\ 13 - 5y & 11 \end{bmatrix} = \begin{bmatrix} -20 & 3 \\ 18 & 11 \end{bmatrix}$$

Equate the corresponding elements.
$-3x + 4 = -20$ and $13 - 5y = 18$

Solve the first equation for x. Subtract 4 from both the sides.
$$-3x + 4 - 4 = -20 - 4$$
$$-3x = -24$$

Divide both the sides by -3.
$$\frac{-3x}{-3} = \frac{-24}{-3}$$
$$x = 8$$

Solve the other equation for y. Subtract 13 from both the sides.
$$13 - 5y - 13 = 18 - 13$$
$$-5y = 5$$

Divide both the sides by -5.
$$\frac{-5}{-5}y = \frac{5}{-5}$$
$$y = -1$$

Therefore, the solution is $x = 8$ and $y = -1$.