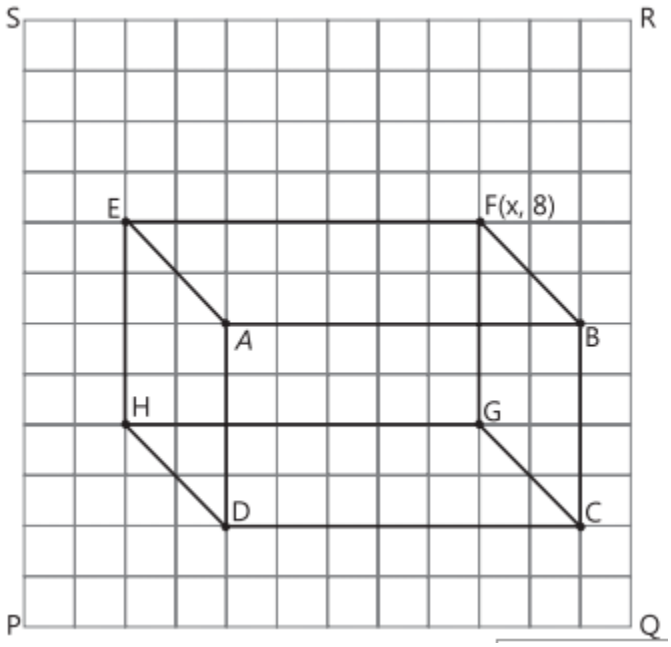
Coordinate Geometry

Case Study Based Questions

Case Study 1

According to medical science and research, keeping an aquarium in the house helps in treating stress, anxiety and health problems associated with blood. It also provides visual stimulation that boost your focus and creativity. A sketch of an aquarium is drawn, which is given in the following figure.



Q1. The coordinates of H are:

a. (4.2)

b. (4,3)

c. (2.4)

d. (4,8)

Q 2. Distance of the point G from the Y-axis is:

a. 3 units

d. 9 units

c. 5 units

b. 4 units

Q3. Length of side HG =

a. 6 units

b. 7 units

c. 8 units

d. 9 units

Q4. The length of diagonal FD and the value of x, respectively are:

a. 8 units, 4

b. √8 units, 5

c. √15 units, 9

d. √61 units, 9

Q5. If Q is considered as origin, then the coordinates of mid-point of BC are:

a. (-1,4)

b. (1,6)

c. (6, 1)

d. (6,-1)

Solutions

1. We are given that P is origin.

.. Coordinates of H are (2, 4).

So, option (c) is correct.

2. Coordinates of G are (9, 4), therefore distance of G from Y-axis = 9 units.

So, option (d) is correct.

3. Coordinates of H are (2, 4) and coordinates of G are (9.4).

Thus, GH $= \sqrt{(9-2)^2 + (4-4)^2} = \sqrt{7^2 + 0} = 7$units

So, option (b) is correct.

4. Coordinates of D are (4, 2) and coordinates of F are (9,8).

⇒x=9

Also, length of diagonal FD $= \sqrt{(4-9)^2 + (2-8)^2}$

$= \sqrt{25+36} = \sqrt{61}$ units

So, option (d) is correct

5. If Q is origin, then

Coordinates of B are (1, 6) and of Care (-1, 2).

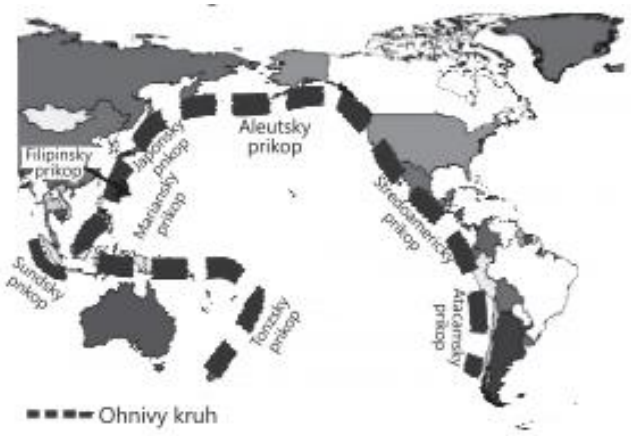
Now, mid-point BC $= \left(\frac{(-1)+(-1)}{2}, \frac{6+2}{2}\right)$ i.e., $(-1, 4)$

So, option (a) is correct

Case Study 2

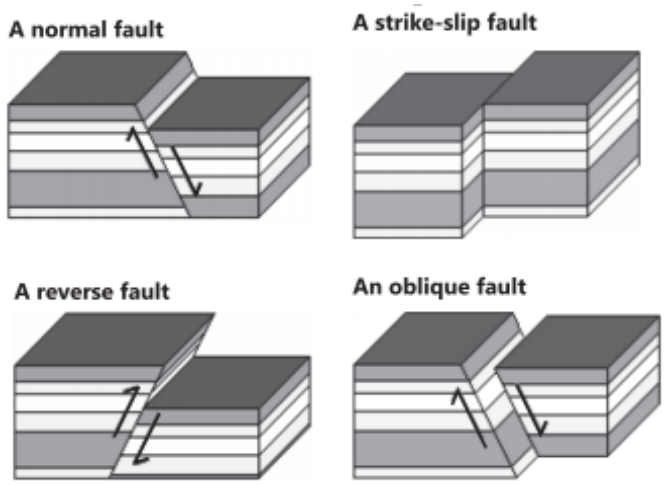
Pacific Ring of Fire

The Pacific Ring of Fire is a major area in the basin of the Pacific Ocean where many earthquakes and volcanic eruptions occur. In a large horseshoe shape, it is associated with a nearly continuous series of oceanic trenches, volcanic arcs, and volcanic belts and plate movements.

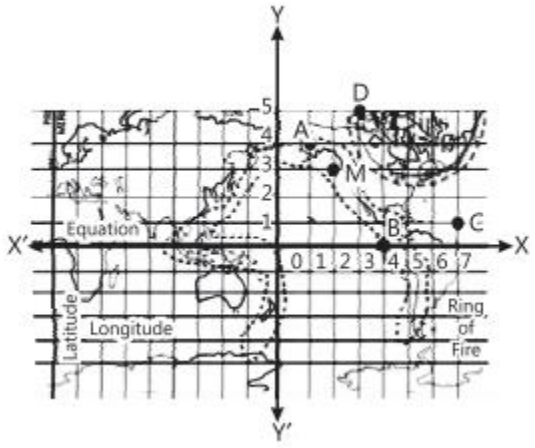


Fault Lines

Large faults within the Earth's crust result from the action of plate tectonic forces, with the largest forming the boundaries between the plates. Energy release associated witrapid movement on active faults is the cause of most earthquakes.



Positions of some countries in the pacific ring of fire is shown in the square grid below.



Based on the above information, solve the following questions: [CBSE SQP 2021 Term-1]

Q1. The distance between the point Country A and Country B is:

a. 4 units

b. 5 units

c. 6 units

d. 7 units

Q2. Find a relation between x and y such that the point (x, y) is equidistant from the Country C and Country D:

a. x-y

b. x+y=2

c. 2x-y=2

d. 2x+y=2

Q3. The fault line 3x+y-9 = 0 divides the line joining the Country P(1, 3) and Country Q(2,7) internally in the ratio:

a. 3:4

b. 3:2

c. 2:3

d. 4:3

Q4. The distance of the Country M from the X-axis is.

a. 1 units

b. 2 units

c. 3 units

d. 5 units

Q5. What are the co-ordinates of the Country lying on the mid-point of Country A and Country D?

a. (1, 3)

b. (2,9/2)

c. (4,5/2)

d. (9/2, 2)

Solutions

1. From the figure,

coordinates of point A = (1, 4)

and coordinate of point B = (4,0)

So, distance between country A and country B

$$= \sqrt{(4-1)^2+(0+4)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

So, option (b) is correct.

2. From the figure,

coordinate of point C = (7.1)

and coordinate of point D = (3, 5)

Given, the point P(x, y) is equidistant from Country C and Country D.

CP = DP

$CP^2 = DP^2$ (By distance formula)

$(x-7)^2+(y-1)^2= (x-3)^2+(y-5)^2$

$x^2+49-14x+y^2+1-2y$

$= x^2+9-6x+y^2+25-10y$

$8x+8y = 16$

$\Rightarrow x+y=2$

So, option (b) is correct.

3. The point which divides the line joining the Country P(1, 3) and Country Q(2, 7) in the ratio k: 1 is,

$$\left[\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right]$$

This point lie on the line $3x + y - 9 = 0$

$$\therefore \quad 3\cdot\frac{2k+1}{k+1}+\frac{7k+3}{k+1}-9=0$$

$$\Rightarrow \quad 6k+3+7k+3-9k-9=0 \qquad [\because k \neq -1]$$

$$\Rightarrow \quad 4k-3=0$$

$$\Rightarrow \quad k=\frac{3}{4}$$

.. Required ratio is 3:4.

So, option (a) is correct.

4. From the figure,

coordinates of the point M = (2, 3)

.. The distance of the Country M from the X-axis

= Perpendicular distance of the point M from the

X-axis = 3 units

So, option (c) is correct.

5. From the figure, Coordinates of the point A = (1, 4)

and coordinates of the point D = (3, 5)

$$\text{Now, mid-point of AD} = \left\{\frac{1+3}{2}, \frac{4+5}{2}\right\} = \left\{\frac{4}{2}, \frac{9}{2}\right\}$$

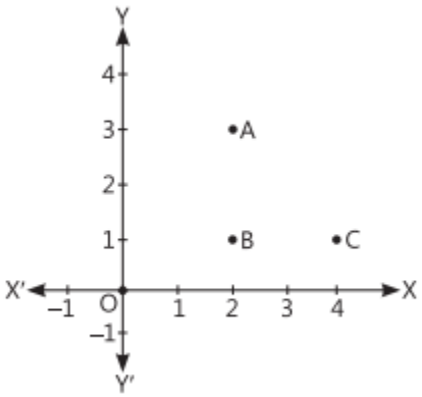
$$= \left(2, \frac{9}{2}\right)$$

$\therefore$ Required coordinates of the country lying on the mid–point of Country A and Country D is $\left(2, \frac{9}{2}\right)$

So, option (b) is correct.

Case Study 3

Alia and Shagun are friends living on the same street in Patel Nagar. Shagun's house is at the intersection of one street with another street on which there is a library. They both study in the same school and that is not far from Shagun's house. Suppose the school is situated at the point O, i.e., the origin, Alia's house is at A. Shagun's house is at B and library is at C.



Based on the above information, solve the following questions: [CBSE SQP 2023-24]

Q1. How far is Alia's house from Shagun's house?

Q2. How far is the library from Shagun's house?

Q3. Show that for Shagun, school is farther compared to Alia's house and library.

Or

Show that Alia's house, Shagun's house and library for an isosceles right triangle.

Solutions

1. The coordinates of Alia's house and Shagun's house are A (2, 3) and B (2, 1) respectively.

∴ Distance of Alia's house from Shagun's house is,

$$BA = \sqrt{(2-2)^2+(3-1)^2} = \sqrt{(0)^2+(2)^2}$$

$$= \sqrt{0+4} = 2 \text{ units.}$$

2. The coordinates of Shagun's house and library are B (2, 1) and (4, 1) respectively.

Distance of library from Shagun's house is,

$$BC = \sqrt{(4-2)^2+(1-1)^2}$$

$$= \sqrt{(2)^2+(0)^2} = \sqrt{4+0} = 2 \text{ units.}$$

3. The coordinates of school, Alia's house, Shagun's house and library are 0(0, 0), A (2, 3), B (2, 1) and C (4,1)

$$\text{Now, } BA = \sqrt{(2-2)^2+(3-1)^2} = \sqrt{(0)^2+(2)^2}$$

$$= \sqrt{0+4} = 2 \text{ units.}$$

$$BC = \sqrt{(4-2)^2+(1-1)^2} = \sqrt{(2)^2+(0)^2}$$

$$= \sqrt{4+0} = 2 \text{ units.}$$

$$\text{and } BO = \sqrt{(0-2)^2+(0-1)^2} = \sqrt{(-2)^2+(-1)^2}$$

$$= \sqrt{4+1} = \sqrt{5} \text{ units.}$$

Here, BO is greater than BA and BC.
For Shagun, School (0) is father than Alia's house (A) and library (C). **Hence proved**.

Or

The coordinates of Alia's house, Shagun's house and library are A (2, 3). B (2. 1) and C (4, 1) respectively using distance formula,

$$AB = \sqrt{(2-2)^2+(1-3)^2} = \sqrt{(0)^2+(-2)^2}$$

$$= \sqrt{0+4} = 2 \text{ units.}$$

$$BC = \sqrt{(4-2)^2+(1-1)^2} = \sqrt{(2)^2+(0)^2}$$

$$= \sqrt{4+0} = 2 \text{ units.}$$

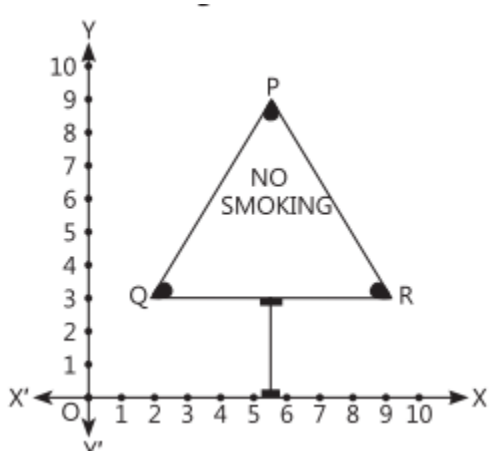
$$\text{and } CA = \sqrt{(2-4)^2+(3-1)^2} = \sqrt{(-2)^2+(2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units.}$$

Here, $AB^2 + BC^2 = (2)2 + (2)^2 = 4 + 4 = 8 = CA^2$ and AB = BC.
Therefore, A, B and C form an isosceles right triangle. **Hence proved.**

Case Study 4

All of the persons know that smoking is injurious to health. So, some college students decided to start a campaign. To raise social awareness about hazards of smoking, they started "NO SMOKING" campaign. Some students were asked to prepare campaign banners in the shape of triangle which is as shown in the figure:



Based on the above information, solve the following questions:

Q1. Find the coordinates of the mid-point of Q and R.

Q2. Find the area of the triangle PQR.

Q3. Find the point on X-axis, which is equidistant from points Q and R.

OR

Find the centroid of the triangle PQR.

Solutions

1. The coordinates of the vertices of Q and R are (2, 3) and (9,3) respectively.

$$\therefore \text{ Mid-point of Q and R } = \left(\frac{2+9}{2}, \frac{3+3}{2}\right) = \left(\frac{11}{2}, 3\right)$$

2. The coordinates of the vertices of P, Q and R are (6, 9), (2, 3) and (9, 3) respectively.

Now, base of the $\Delta PQR = QR = \sqrt{(9-2)^2+(3-3)^2}$

$= \sqrt{49+0} = 7$ units.

and height of the ΔPQR = Perpendicular distance from the vertex × P to the base QR = (9 – 3) = 6 units.

$\therefore$ Area of $\Delta PQR = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 7 \times 6 = 21$ sq. units.

3. Let point on X-axis be P(x, 0)

Then $(QP)^2 = (PR)2$

$(x-2)^2+(0-3)^2=(9-x)^2+(3-0)^2$

$= x^2+4-4x+9=81+x^2-18x+9$

$14x = 77$

$\Rightarrow \quad x = \frac{11}{2}$

$\therefore$ Point on *X*-axis is $\left(\frac{11}{2}, 0\right)$.

Or

$\therefore$ Centroid of the $\Delta PQR = \left(\frac{6+2+9}{3}, \frac{9+3+3}{3}\right)$

$= \left(\frac{17}{3}, \frac{15}{3}\right) = \left(\frac{17}{3}, 5\right)$

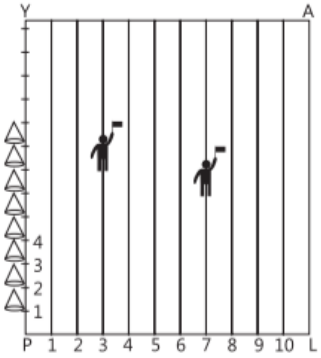
Case Study 5

On Annual Sports Day of a school, parallel lines have been drawn with lime powder at a distance of 1 m from each other in a rectangular shaped school playground. 80 plastic cones have been placed at a distance of 1 m from each other along PY as shown in figure. Pushpendra runs $\frac{1}{4}$th the distance PY on the 3rd line and post a yellow flag.

Pankaj runs $\frac{1}{5}$th the distance PY of the 7th line

and posts a blue flag.

Based on the above information, solve the following questions:



Q1. Find the coordinates of the yellow flag.

Q2. What is the distance between both the flags?

Q3. If Raman has to post a green flag exactly halfway between the line segment joining the two flags, where should he post his flag?

OR

If Raman change his position and post a green flag at a point between the line segment joining the two flags, then find the coordinate of the green flag which divides the line segment internally in the ratio 1 : 2.

Solutions

1. $\frac{1}{4}$th the distance $PY = \frac{80}{4}\,m = 20\,m$

   $\therefore$ Coordinates of yellow flag = (3, 20)

2. $\frac{1}{5}$th the distance $PY = \frac{80}{5}\,m = 16\,m$

   $\therefore$ Coordinates of yellow flag = (7, 16) Distance between both flags

   $$= \sqrt{(7-3)^2 + (16-20)^2} = \sqrt{(4)^2 + (-4)^2}$$

   $$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}\,m$$

3. Position of green flag = Mid-point of yellow and blue flag

   $$= \left(\frac{3+7}{2}, \frac{20+16}{2}\right) = \left(\frac{10}{2}, \frac{36}{2}\right) = (5, 18)$$

Hence, Raman should post his green flag at 18 m on 5th line.

Or

Given points: (3, 20) and (7.16)
Here, x_1 =3, y_1 =20, X2 =7, y_2=16
and ratio m_1 m_2 1:2
Let the point of division be P(x, y):

Then from division formula:

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$= \frac{1\times7 + 2\times3}{1+2} = \frac{13}{3} \text{ and } y = \frac{1\times16 + 2\times20}{1+2} = \frac{56}{3}$$

Therefore, the coordinate of the green flag is $\left(\frac{13}{3}, \frac{56}{3}\right)$.

Solutions for Questions 6 to 20 are Given Below

Case Study 6

Controlling Air Pollution

The Chief Minister of Delhi launched the, 'Switch Delhi', an electric vehicle mass awareness campaign in the National Capital. The government has also issued tenders for setting up 100 charging stations across the city. Each station will have five charging points. For demo charging station is set up along a straight line and has charging points at $A\left(\frac{-7}{3}, 0\right)$, $B\left(0, \frac{7}{4}\right)$, $C(3, 4)$, $D(7, 7)$ and $E(x, y)$. Also, the distance between C and E is 10 units.

Based on the above information, answer the following questions.

(i) The distance DE is

(a) 5 units (b) 10 units (c) 4 units (d) 6 units

(ii) The value of $x + y$ is

(a) 20 (b) 21 (c) 22 (d) 23

(iii) Which of the following is true?

(a) The points C, D and E are vertices of a triangle

(b) The points C, D and E are collinear

(c) The points C, D and E lie on a circle

(d) None of these

(iv) The ratio in which B divides AC is

(a) 9 : 7 (b) 4 : 7 (c) 7 : 4 (d) 7 : 9

(v) Which of the following equations is satisfied by the given points?

(a) $x + y = 0$ (b) $x - y = 0$ (c) $3x - 4y + 7 = 0$ (d) $3x + 4y + 7 = 0$

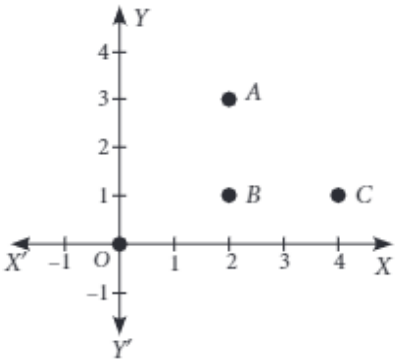
Case Study 7

Analysing Distance

Alia and Shagun are friends living on the same street in Patel Nagar. Shagun's house is at the intersection of one street with another street on which there is a library. They both study in the same school and that is not far from Shagun's house.

Suppose the school is situated at the point O, *i.e.*, the origin, Alia's house is at A. Shagun's house is at B and library is at C.

Based on the above information, answer the following questions.



(i) How far is Alia's house from Shagun's house?

(a) 3 units (b) 4 units (c) 5 units (d) 2 units

(ii) How far is the library from Shagun's house?

(a) 3 units (b) 2 units (c) 5 units (d) 4 units

(iii) How far is the library from Alia's house?

(a) 2 units (b) 3 units (c) 4 units (d) None of these

(iv) Which of the following is true?

(a) ABC forms a scalene triangle (b) ABC forms an isosceles triangle

(c) ABC forms an equilateral triangle (d) None of these

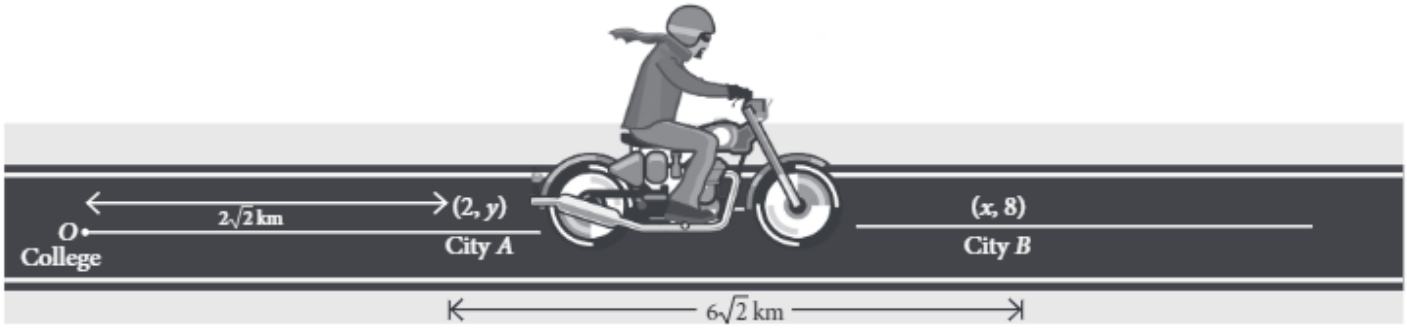
(v) How far is the school from Alia's house than Shagun's house?

(a) $\sqrt{13}$ units (b) $\sqrt{5}$ units (c) $(\sqrt{13} + \sqrt{5})$ units (d) $(\sqrt{13} - \sqrt{5})$ units

Case Study 8

Measuring Fuel output

A person is riding his bike on a straight road towards East from his college to city A and then to city B. At some point in between city A and city B, he suddenly realises that there is not enough petrol for the journey. Also, there is no petrol pump on the road between these two cities.



Based on the above information, answer the following questions.

(i) The value of y is equal to

(a) 2 (b) 3 (c) 4 (d) 5

(ii) The value of x is equal to

(a) 4 (b) 5 (c) 8 (d) 7

(iii) If M is any point exactly in between city A and city B, then coordinates of M are

(a) (3, 3) (b) (4, 4) (c) (5, 5) (d) (6, 6)

(iv) The ratio in which A divides the line segment joining the points O and M is

(a) 1 : 2 (b) 2 : 1 (c) 3 : 2 (d) 2 : 3

(v) If the person analyse the petrol at the point M(the mid point of AB), then what should be his decision?

(a) Should he travel back to college (b) Should try his luck to move towards city B

(c) Should be travel back to city A (d) None of these

Case Study 9

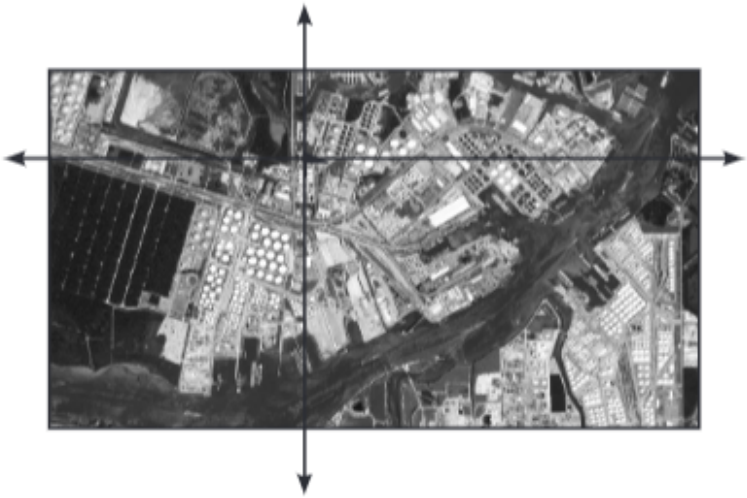
Satellite View

Satellite image of a colony is shown below. In this view, a particular house is pointed out by a flag, which is situated at the point of intersection of x and y-axes. If we go 2 cm east and 3 cm north from the house, then we reach to a Grocery store. If we go 4 cm west and 6 cm south from the house, then we reach to a Electrician's shop. If we go 6 cm east and 8 cm south from the house, then we reach to a food cart. If we go 6 cm west and 8 cm north from the house. then we reach to a bus stand.

Scale

x-axis : 1 cm = 1 unit

y-axis : 1 cm = 1 unit



Based on the above information, answer the following questions.

(i) The distance between grocery store and food cart is

(a) 12 cm (b) 15 cm (c) 18 cm (d) none of these

(ii) The distance of the bus stand from the house is

(a) 5 cm (b) 10 cm (c) 12 cm (d) 15 cm

(iii) If the grocery store and electrician's shop lie on a line, the ratio of distance of house from grocery store to that from electrician's shop, is

(a) 3 : 2 (b) 2 : 3 (c) 1 : 2 (d) 2 : 1

(iv) The ratio of distances of house from bus stand to food cart is

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) none of these

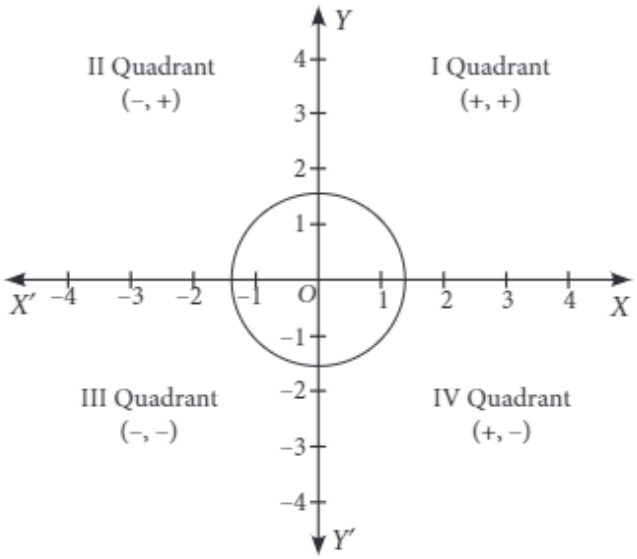
(v) The coordinates of positions of bus stand, grocery store, food cart and electrician's shop form a

(a) rectangle (b) parallelogram (c) square (d) none of these

Case Study 10

Tracing of Clock

A round clock is traced on a graph paper as shown below. The boundary intersect the coordinate axis at a distance of 4/3 units from origin.



Based on the above information, answer the following questions.

(i) Circle intersect the positive y-axis at

(a) $\left(\frac{2}{3}, 0\right)$ (b) $\left(0, \frac{2}{3}\right)$ (c) $\left(0, \frac{4}{3}\right)$ (d) $\left(\frac{4}{3}, 0\right)$

(ii) The centre of circle is the

(a) mid-point of points of intersection with x-axis (b) mid-point of points of intersection with y-axis

(c) both (a) and (b) (d) none of these

(iii) The radius of the circle is

(a) $\frac{4}{3}$ units (b) $\frac{3}{2}$ units (c) $\frac{2}{3}$ units (d) $\frac{3}{4}$ units

(iv) The area of the circle is

(a) $16\pi^2$ sq. units (b) $\frac{16}{9}\pi$ sq. units (c) $\frac{4}{9}\pi^2$ sq. units (d) 4π sq. units

(v) If $\left(1, \frac{\sqrt{7}}{3}\right)$ is one of the ends of a diameter, then its other end is

(a) $\left(-1, \frac{\sqrt{7}}{3}\right)$ (b) $\left(1, -\frac{\sqrt{7}}{3}\right)$ (c) $\left(1, \frac{\sqrt{7}}{3}\right)$ (d) $\left(-1, -\frac{\sqrt{7}}{3}\right)$

Case Study 11

Social Awareness for No-smoking

To raise social awareness about hazards of smoking, a school decided to start 'No smoking' campaign. 10 students are asked to prepare campaign banners in the shape of a triangle. The vertices of one of the triangle are $P(-3, 4)$, $Q(3, 4)$ and $R(-2, -1)$.

Based on the above information, answer the following questions.

(i) The coordinates of centroid of ΔPQR are

(a) $\left(\frac{2}{3}, \frac{7}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{3}\right)$ (c) $\left(\frac{-2}{3}, \frac{7}{3}\right)$ (d) $\left(\frac{7}{3}, \frac{2}{3}\right)$

(ii) If S be the mid-point of line joining P and Q, then coordinates of S are

(a) $(4, 0)$ (b) $(2, 0)$ (c) $(0, 2)$ (d) $(0, 4)$

(iii) If T be the mid-point of line joining R and Q, then coordinates of T are

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (c) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (d) none of these

(iv) If U be the mid-point of line joining R and P, then coordinates of U are

(a) $\left(-\frac{5}{2}, \frac{3}{2}\right)$ (b) $\left(\frac{3}{2}, -\frac{5}{2}\right)$ (c) $\left(\frac{3}{2}, \frac{5}{2}\right)$ (d) $\left(\frac{5}{2}, \frac{3}{2}\right)$

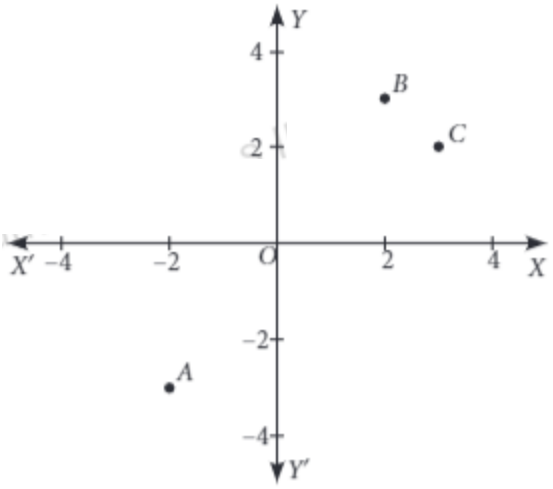
(v) The coordinates of centroid of ΔSTU are

(a) $\left(\frac{2}{3}, \frac{7}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{3}\right)$ (c) $\left(-\frac{2}{3}, \frac{7}{3}\right)$ (d) $\left(\frac{7}{3}, \frac{2}{3}\right)$

Case Study 12

Choosing Best Route

There are two routes to travel from source A to destination B by bus. First bus reaches at B via point C and second bus reaches from A to B directly. The position of A, B and C are represented in the following graph:



Scale : x-axis : 1 unit = 1 km
y-axis : 1 unit = 1 km

Based on the above information, answer the following questions.

(i) The distance between A and B is

(a) 13 km (b) 26 km (c) $\sqrt{13}$ km (d) none of these

(ii) The distance between A and C is

(a) 5 km (b) 2 km (c) $\sqrt{5}$ km (d) $5\sqrt{2}$ km

(iii) If it is assumed that both buses have same speed, then by which bus do you want to travel from A to B?

(a) First bus (b) Second bus (c) Any of them (d) None of these

(iv) If the fare for first bus is ₹ 10/km, then what will be the fare for total journey by that bus?

(a) ₹ 83 (b) ₹ 38 (c) ₹ 45 (d) none of these

(v) If the fare for second bus is ₹ 15/km, then what will be the fare to reach to the destination by this bus?

(a) ₹ 105 (b) ₹ 108 (c) ₹ 110 (d) ₹ 115

Case Study 13

Spreading Awareness on Plastic Use

Students of residential society undertake to work for the campaign "Say no to Plastics". Group A took the region under the coordinates (3, 3), (6, y), (x, 7) and (5, 6) and group B took the region under the coordinates (1, 3), (2, 6), (5, 7) and (4, 4).

Based on the above information, answer the following questions.

(i) If region covered by group A forms a parallelogram, where the coordinates are taken in the given order, then

(a) $x = 8, y = 4$ (b) $x = 4, y = 8$ (c) $x = 2, y = 4$ (d) $x = 4, y = 2$

(ii) Perimeter of the region covered by group A is

(a) $\sqrt{10}$ units (b) $\sqrt{13}$ units (c) $(\sqrt{10}+\sqrt{13})$ units (d) none of these

(iii) If the coordinates of region covered by group B, taken in the same order forms a quadrilateral, then the length of each of its diagonals is

(a) $4\sqrt{2}$ units, $2\sqrt{2}$ units (b) $6\sqrt{2}$ units, $\sqrt{2}$ units

(c) $3\sqrt{2}$ units, $2\sqrt{2}$ units (d) none of these

(iv) If region covered by group B forms a rhombus, where the coordinates are taken in given order, then the perimeter of this region is

(a) $\sqrt{10}$ units (b) $2\sqrt{10}$ units (c) $3\sqrt{10}$ units (d) $4\sqrt{10}$ units

(v) The coordinates of the point which divides the join of points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m : n$ is

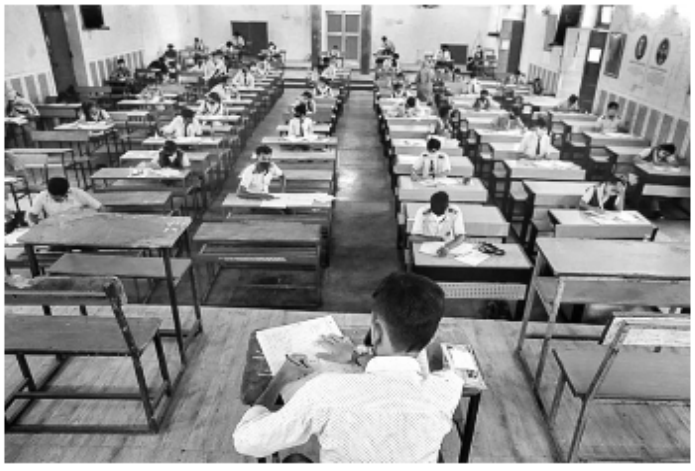
(a) $\left(\frac{mx_2+ny_2}{m+n}, \frac{mx_1+ny_1}{m+n}\right)$ (b) $\left(\frac{mx_1+ny_1}{m+n}, \frac{mx_2+ny_2}{m+n}\right)$

(c) $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$ (d) none of these

Case Study 14

Social Distance in Examination Hall

In an examination hall, students are seated at a distance of 2 m from each other, to maintain the social distance due to CORONA virus pandemic. Let three students sit at points A, B and C whose coordinates are (4, –3), (7, 3) and (8, 5) respectively.



Based on the above information, answer the following questions.

(i) The distance between A and C is

(a) $\sqrt{5}$ units (b) $4\sqrt{5}$ units (c) $3\sqrt{5}$ units (d) none of these

(ii) If an invigilator at the point I, lying on the straight line joining B and C such that it divides the distance between them in the ratio of 1 : 2. Then coordinates of I are

(a) $\left(\frac{22}{3}, \frac{11}{3}\right)$ (b) $\left(\frac{23}{3}, \frac{13}{3}\right)$ (c) (6, 1) (d) (9, 1)

(iii) The mid-point of the line segment joining A and C is

(a) $(1, 6)$ (b) $(6, 1)$ (c) $\left(\frac{11}{2}, 0\right)$ (d) none of these

(iv) The ratio in which B divides the line segment joining A and C is

(a) $2:1$ (b) $3:1$ (c) $1:2$ (d) none of these

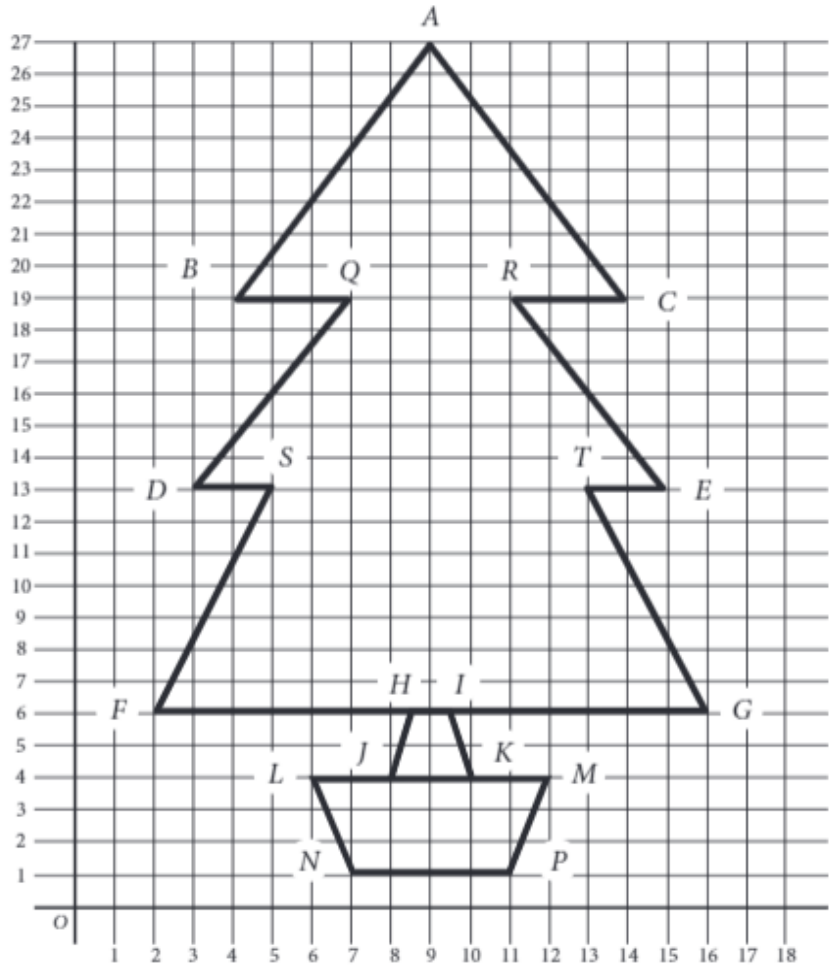
(v) The points A, B and C lie on

(a) a straight line (b) an equilateral triangle
(c) a scalene triangle (d) an isosceles triangle

Case Study 15

Drafting Design of a Christmas Tree

A design of Christmas tree is shown in the following graph:



Now answer the following questions.

(i) The distance of point A from x-axis is

(a) 9 units (b) 26 units (c) 27 units (d) 10 units

(ii) Length of BC is

(a) 12 units (b) 10 units (c) 8 units (d) 6 units

(iii) Length of FG is

(a) 10 units (b) 12 units (c) 14 units (d) 16 units

(iv) The mid-point of FG lies on line represented by

(a) $x = 9$ (b) $x = 10$ (c) $x = 8$ (d) none of these

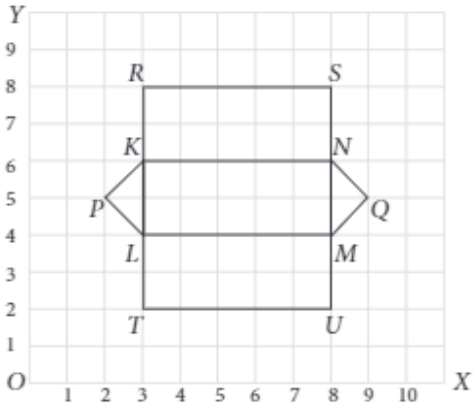
(v) The perimeter of its trunk $LMPN$ is

(a) $\sqrt{10}$ units (b) $2\sqrt{10}$ units (c) 10 units (d) none of these

Case Study 16

Alpine Tents

The camping alpine tent is usually made using high quality canvas and it is water proof. These alpine tents are mostly used in hilly areas, as the snow will not settle on the tent and make it damp. It is easy to lay out and one need not use a manual to set it up. One alpine tent is shown in the figure given below, which has two triangular faces and three rectangular faces. Also, the image of canvas on graph paper is shown in the adjacent figure.



Based on the above information, answer the following questions.

(i) Distance of point Q from y-axis is

(a) 9 units (b) 8 units (c) 4 units (d) 5 units

(ii) What are the coordinates of U?

(a) (2, 8) (b) (8, 2) (c) (6, 9) (d) (9, 6)

(iii) The distance between the points P and Q is

(a) 4 units (b) 5 units (c) 6 units (d) 7 units

(iv) If a point $A(x, y)$ is equidistant from R and T, then

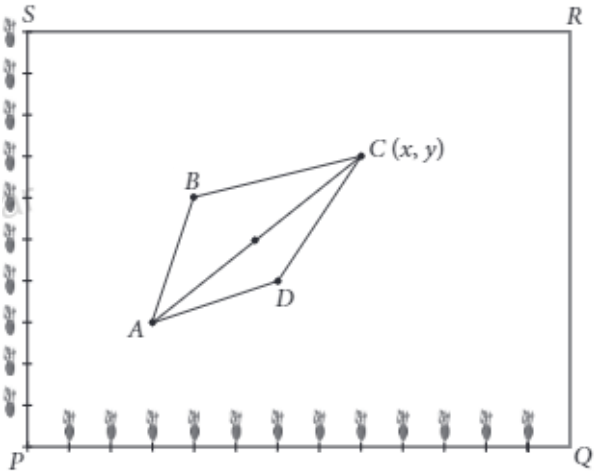
(a) $y - 2 = 0$ (b) $y - 3 = 0$ (c) $y - 5 = 0$ (d) $y - 6 = 0$

(v) Perimeter of image of a rectangular face is

(a) 5 units (b) 8 units (c) 10 units (d) 14 units

Case Study 17

An award function of a Multi National Company (MNC) is arranged in a rectangular shaped meeting room $PQRS$. 200 flower vases placed along two sides of a room, at a distance of 1 m from each other. The employees which are nominated for the position of 'Best Employee of the year' are seated at points A, B, C and D.



If P is considered as origin, then answer the following questions.

(i) The coordinates of A are

(a) (2, 2) (b) (3, 3) (c) (4, 4) (d) (5, 5)

(ii) The mid point of the line segment joining A and C is

(a) (5, 5.5) (b) (5.7, 7) (c) (5.5, 5) (d) None of these

(iii) Which of the following is near to A?

(a) B (b) D (c) C (d) Both (a) and (b)

(iv) Which of the following is equidistant from B and D?

(a) Only A (b) Only C (c) Both A and C (d) Neither A nor C

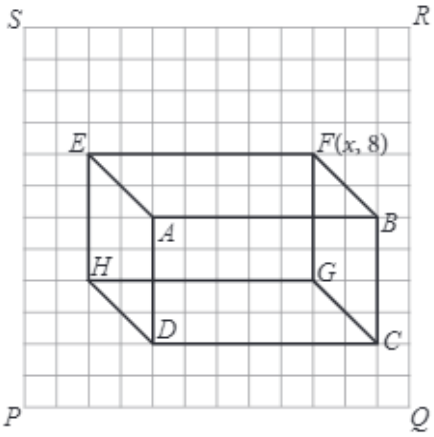
(v) If Q is considered as origin, then the coordinates of D will be

(a) (–7, 4) (b) (6, 4) (c) (–4, 7) (d) (4, 6)

Case Study 18

Advantages of Aquarium

According to medical science and research, keeping an aquarium in the house helps in treating stress, anxiety and health problems associated with blood pressure. It also provides visual stimulation that boost your focus and creativity. A sketch of an aquarium is drawn, which is given in the following figure.



Considering P as origin, answer the following questions.

(i) The coordinates of H are

(a) (4, 2) (b) (4, 3) (c) (2, 4) (d) (4, 8)

(ii) Distance of the point G from the y-axis is

(a) 3 units (b) 4 units (c) 5 units (d) 9 units

(iii) Length of side HG =

(a) 6 units (b) 7 units (c) 8 units (d) 9 units

(iv) The length of diagonal FD and the value of x, respectively are

(a) 8 units, 4 (b) $\sqrt{8}$ units, 5 (c) $\sqrt{15}$ units, 9 (d) $\sqrt{61}$ units, 9

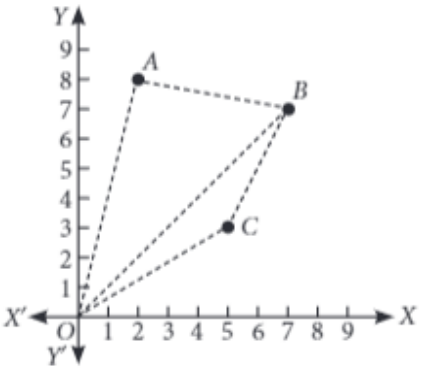
(v) If Q is considered as origin, then the coordinates of mid-point of BC are

(a) (–1, 4) (b) (1, 6) (c) (6, 1) (d) (6, –1)

Case Study 19

Carpool

Three friends Nitin, Alok and Deepika lives in societies represented by the points A, B and C respectively. They all work in the same office located at O. If they decided to share a cab to go to the office, then answer the following questions.



(i) Which society is nearest to the office?

(a) A (b) B (c) C (d) Both A and C

(ii) What is the distance between A and C?

(a) $\sqrt{33}$ units (b) $\sqrt{34}$ units (c) $\sqrt{48}$ units (d) none of these

(iii) Which of the following distance is least?

(a) AB (b) OA (c) BC (d) none of these

(iv) Which of the following is the best route to go to the office?

(a) $CABO$ (b) $CBAO$ (c) $ABCO$ (d) All of these

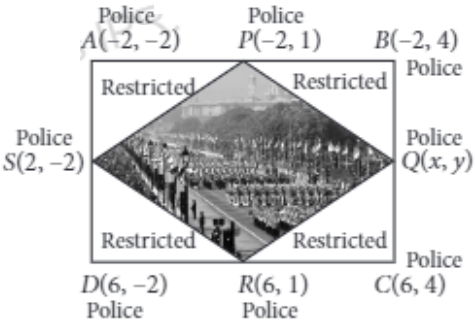
(v) If Alok and Deepika planned to meet at a mall situated at a point D represented by the mid-point of the line joining the points B and C, then find the coordinates of D.

(a) (2, 6) (b) (6, 5) (c) (5, 3) (d) (6, 4)

Case Study 20

Republic Day Parade Programme

In order to facilitate smooth passage of the parade, movement of traffic on certain roads leading to the route of the Parade and Tableaux always restricted. To avoid traffic on the road Delhi Police decided to construct a rectangular route plan, as shown in the figure.
Based on the above information, answer the following questions.



(i) If Q is the mid point of BC, then coordinates of Q are
(a) (2, 4) (b) (2, –4) (c) (1, –1) (d) (–1, 1)

(ii) Quadrilateral $PQRS$ is a
(a) Trapezium (b) Square (c) Rectangle (d) Rhombus

(iii) What is the length of sides of quadrilateral $PQRS$?
(a) 5 units each (b) 3, 4, 5, 6 units (c) 4, 5, 6, 7 units (d) 8 units each

(iv) What is the length of route $PQRS$?
(a) 20 units (b) 25 units (c) 35 units (d) 45 units

(v) What is the length of route $ABCD$?
(a) 26 units (b) 27 units (c) 28 units (d) 29 units

HINTS & EXPLANATIONS

6. **(i)** **(a)**: Here, $CD = \sqrt{(7-3)^2+(7-4)^2}$

$= \sqrt{4^2+3^2} = 5$ units

Also, it is given that $CE = 10$ units
Thus, $DE = CE - CD = 10 - 5 = 5$ units
($\because$ A, B, C, E are a line)

(ii) **(b)**: Since, $CD = DE = 5$ units
$\therefore$ D is the midpoint of CE.

$\therefore$ $\frac{x+3}{2} = 7$ and $\frac{y+4}{2} = 7$

$\Rightarrow$ $x = 11$ and $y = 10$ $\Rightarrow$ $x + y = 21$

(iii) **(b)**

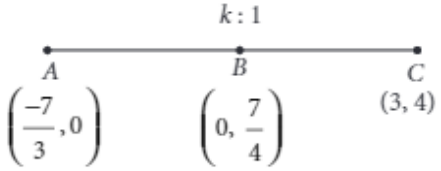
(iv) **(d)**: Let B divides AC in the ratio $k : 1$, then

$\frac{7}{4} = \frac{4k+0}{k+1}$

$\Rightarrow$ $7k + 7 = 16k$

$\Rightarrow$ $7 = 9k$

$\Rightarrow$ $k = \frac{7}{9}$



Thus, the required ratio is 7 : 9

(v) **(c)**: It can be easily verify that all the given points lie on the line represented by $3x - 4y + 7 = 0$.

7. **(i)** **(d)**: Since the coordinates of A and B are (2, 3) and (2, 1) respectively.

$\therefore$ Required distance = AB

$= \sqrt{(2-2)^2+(1-3)^2} = \sqrt{2^2} = 2$ units

(ii) **(b)**: Since, library is situated at $C(4, 1)$

$\therefore$ Required distance = BC

$= \sqrt{(4-2)^2+(1-1)^2} = \sqrt{2^2+0^2} = 2$ units

(iii) **(d)**: Required distance = AC

$= \sqrt{(4-2)^2+(1-3)^2} = \sqrt{2^2+2^2} = 2\sqrt{2}$ units

(iv) **(b)**: Since $AB = BC \neq AC$, therefore ΔABC is an isosceles triangle.

(v) **(d)**: Distance between O and A

$= \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$ units

and distance between O and B =

$\sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}$ units

Thus, required distance $= (\sqrt{13} - \sqrt{5})$ units

8. **(i)** **(a)** : We have, $OA = 2\sqrt{2}$ km

$\Rightarrow \sqrt{2^2 + y^2} = 2\sqrt{2}$

$\Rightarrow 4 + y^2 = 8 \Rightarrow y^2 = 4$

$\Rightarrow y = 2$ ($\because y = -2$ is not possible)

(ii) **(c)** : We have $OB = 8\sqrt{2}$

$\Rightarrow \sqrt{x^2 + 8^2} = 8\sqrt{2}$

$\Rightarrow x^2 + 64 = 128 \Rightarrow x^2 = 64$

$\Rightarrow x = 8$ ($\because x = -8$ is not possible)

(iii) **(c)** : Coordinates of A and B are (2, 2) and (8, 8) respectively, therefore coordinates of point M are $\left(\frac{2+8}{2}, \frac{2+8}{2}\right)$, *i.e.*, (5, 5)

(iv) **(d)** : Let A divides OM in the ratio $k : 1$.

Then, $2 = \frac{5k+0}{k+1} \Rightarrow 2k + 2 = 5k \Rightarrow 3k = 2 \Rightarrow k = \frac{2}{3}$

$\therefore$ Required ratio = 2 : 3

(v) **(b)** : Since M is the mid-point of A and B therefore $AM = MB$. Hence, he should try his luck moving towards B.

9. Consider the house is at origin (0, 0), then coordinates of grocery store, electrician's shop, food cart and bus stand are respectively (2, 3), (–4, –6), (6, – 8) and (–6, 8)

(i) **(d)** : Since, grocery store is at (2, 3) and food cart is at (6, –8)

$\therefore$ Required distance $= \sqrt{(6-2)^2 + (-8-3)^2}$

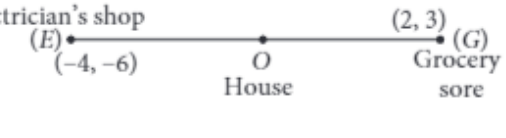
$= \sqrt{4^2 + 11^2} = \sqrt{16+121} = \sqrt{137}$ cm

(ii) **(b)** : Required distance

$= \sqrt{(-6)^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$ cm

(iii) **(c)** : Let O divides EG in the ratio $k : 1$, then

$0 = \frac{2k-4}{k+1}$



$\Rightarrow 2k = 4$

$\Rightarrow k = 2$

Thus, O divides EG in the ratio 2 : 1

Hence, required ratio = $OG : OE$ *i.e.*, 1 : 2

(iv) **(c)** : Since, (0, 0) is the mid-point of (–6, 8) and (6, –8), therefore both bus stand and food cart are at equal distances from the house.

Hence, required ratio is 1 : 1.

(v) **(d)** : Mid-point of grocery store and electrician's shop is $\left(\frac{2-4}{2}, \frac{3-6}{2}\right)$, *i.e.*, $\left(-1, \frac{-3}{2}\right)$

Thus, the diagonals does not bisect each other

[$\because$ Mid-point are not same]

Hence, they form a quadrilateral.

10. **(i)** **(c)** : Required coordinates are $\left(0, \frac{4}{3}\right)$.

(ii) **(c)**

(iii) **(a)** : Radius = Distance between (0, 0) and $\left(\frac{4}{3}, 0\right)$

$= \sqrt{\left(\frac{4}{3}\right)^2 + 0^2} = \frac{4}{3}$ units

(iv) **(b)** : Area of circle = π(radius)^2

$= \pi\left(\frac{4}{3}\right)^2 = \frac{16}{9}\pi$ sq. units

(v) **(d)** : Let the coordinates of the other end be (x, y).

Then (0, 0) will be the mid-point of $\left(1, \frac{\sqrt{7}}{3}\right)$ and (x, y).

$\therefore \left(\frac{1+x}{2}, \frac{\frac{\sqrt{7}}{3}+y}{2}\right) = (0, 0)$

$\Rightarrow \frac{1+x}{2} = 0$ and $\frac{\frac{\sqrt{7}}{3}+y}{2} = 0$

$\Rightarrow x = -1$ and $y = -\frac{\sqrt{7}}{3}$

Thus, the coordinates of other end be $\left(-1, \frac{-\sqrt{7}}{3}\right)$

11. **(i)** **(c)** : We have, $P(-3, 4)$, $Q(3, 4)$ and $R(-2, -1)$.

$\therefore$ Coordinates of centroid of ΔPQR

$= \left(\frac{-3+3-2}{3}, \frac{4+4-1}{3}\right) = \left(\frac{-2}{3}, \frac{7}{3}\right)$

(ii) **(d)** : Coordinates of $S = \left(\frac{-3+3}{2}, \frac{4+4}{2}\right) = (0, 4)$

(iii) **(c)** : Coordinates of $T = \left(\frac{-2+3}{2}, \frac{-1+4}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

(iv) **(a)** : Coordinates of $U = \left(\frac{-2-3}{2}, \frac{-1+4}{2}\right) = \left(\frac{-5}{2}, \frac{3}{2}\right)$

(v) **(c)** : The centroid of triangle formed by joining the mid-points of sides of given triangle is same as that of the given trangle.

So, centroid of $\Delta STU = \left(\frac{-2}{3}, \frac{7}{3}\right)$.

12. Coordinates of A, B and C are $(-2, -3)$, $(2, 3)$ and $(3, 2)$.

(i) **(d)**: Required distance $= \sqrt{(2+2)^2 + (3+3)^2}$

$= \sqrt{4^2 + 6^2} = \sqrt{16+36} = 2\sqrt{13}$ km ≈ 7.2 km

(ii) **(d)**: Required distance $= \sqrt{(3+2)^2 + (2+3)^2}$

$= \sqrt{5^2 + 5^2} = 5\sqrt{2}$ km

(iii) **(b)**: Distance between B and C

$= \sqrt{(3-2)^2 + (2-3)^2} = \sqrt{1+1} = \sqrt{2}$ km

Thus, distance travelled by first bus to reach to B

$= AC + CB = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$ km ≈ 8.48 km

and distance travelled by second bus to reach to B

$= AB = 2\sqrt{13}$ km ≈ 7.2 km

$\therefore$ Distance of first bus is greater than distance of the second bus, therefore second bus should be chosen.

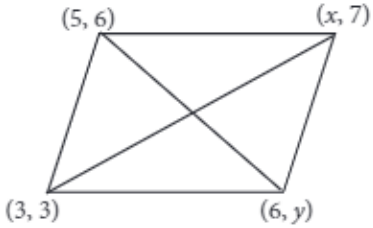
(iv) **(d)**: Distance travelled by first bus = 8.48 km

$\therefore$ Total fare = $8.48 \times 10 =$ ₹ 84.80

(v) **(b)**: Distance travelled by second bus = 7. 2 km

Total fare = $7.2 \times 15 =$ ₹ 108

13. **(i)** **(a)**: Since the diagonals of a parallelogram bisect each other.



$\therefore$ By mid-point formula, we have

$$\left(\frac{x+3}{2}, \frac{3+7}{2}\right) = \left(\frac{5+6}{2}, \frac{6+y}{2}\right)$$

$\Rightarrow$ $x + 3 = 11$ and $y + 6 = 10$ $\Rightarrow$ $x = 8$ and $y = 4$

(ii) **(d)**: Distance between (3, 3) and (6, 4)

$= \sqrt{(6-3)^2 + (4-3)^2} = \sqrt{9+1} = \sqrt{10}$ units

And distance between (6, 4) and (8, 7)

$= \sqrt{(8-6)^2 + (7-4)^2} = \sqrt{4+9} = \sqrt{13}$ units

Now, required perimeter = $2(\sqrt{10} + \sqrt{13})$ units

(iii) **(a)**: Let $A(1, 3)$, $B(2, 6)$, $C(5, 7)$ and $D(4, 4)$ be the given points. Then length of diagonal

$AC = \sqrt{(5-1)^2 + (7-3)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ units

and $BD = \sqrt{(4-2)^2 + (4-6)^2} = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ units

(iv) **(d)**: Length of one of the sides

$= \sqrt{(2-1)^2 + (6-3)^2} = \sqrt{1+9} = \sqrt{10}$ units

$\therefore$ Perimeter = $4\sqrt{10}$ units

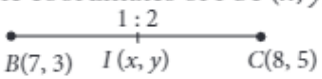
(v) **(c)**

14. **(i)** **(b)**: The distance between A and C

$= \sqrt{(8-4)^2 + (5+3)^2} = \sqrt{4^2 + 8^2}$

$= \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$ units

(ii) **(a)**: Let the coordinates of I be (x, y).



Then, by section formula,

$$x = \frac{1\times 8 + 2\times 7}{1+2} = \frac{8+14}{3} = \frac{22}{3}$$

and $y = \frac{1\times 5 + 2\times 3}{1+2} = \frac{5+6}{3} = \frac{11}{3}$

Thus, the coordinates of I is $\left(\frac{22}{3}, \frac{11}{3}\right)$.

(iii) **(b)**: The mid-point of A and C

$= \left(\frac{8+4}{2}, \frac{5-3}{2}\right) = (6, 1)$

(iv) **(b)**: Let B divides the line segment joining A and C in the ratio $k : 1$. Then, the coordinates of B will be $\left(\frac{8k+4}{k+1}, \frac{5k-3}{k+1}\right)$.

Thus, we have $\left(\frac{8k+4}{k+1}, \frac{5k-3}{k+1}\right) = (7, 3)$

$\Rightarrow$ $\frac{8k+4}{k+1} = 7$ and $\frac{5k-3}{k+1} = 3$

Consider, $\frac{8k+4}{k+1} = 7$ $\Rightarrow$ $8k + 4 = 7k + 7$ $\Rightarrow$ $k = 3$

Hence, the required ratio is 3 : 1.

(v) (a) : $\because$ B divides AC in the ratio 3 : 4.
$\therefore$ A, B, C lie on a straight line.

15. (i) (c) : The coordinates of point A are (9, 27), therefore its distance from x-axis = 27 units.

(ii) (b) : Coordinates of B and C are (4, 19) and (14, 19)

$\therefore$ Required distance $= \sqrt{(14-4)^2+(19-19)^2}$

$= \sqrt{10^2} = 10$ units

(iii) (c) : Coordinates of F and G are (2, 6) and (16, 6) respectively.

$\therefore$ Required distance $= \sqrt{(16-2)^2+(6-6)^2}$

$= \sqrt{14^2} = 14$ units

(iv) (a) : Since the coordinates of F and G are (2, 6) and (16, 6) respectively therefore mid-point of FG is

$$\left(\frac{2+16}{2}, \frac{6+6}{2}\right) = (9, 6)$$

Thus, the mid-point of FG will lie on the line represented by $x = 9$.

(v) (d) : Coordinates of L and N are (6, 4) and (7, 1) respectively.

Length of $LN = \sqrt{(7-6)^2+(1-4)^2}$

$= \sqrt{1+9} = \sqrt{10}$ units

$\Rightarrow$ Length of $MP = \sqrt{10}$ units

Now, perimeter of $LMPN = LN + LM + MP + NP$

$= \sqrt{10}+6+\sqrt{10}+4 = (2\sqrt{10}+10)$ units

[$\because$ $LM = 12 - 6 = 6$ units and $NP = 11 - 7 = 4$ units]

16. (i) (a) : Coordinates of Q are (9, 5).

$\therefore$ Distance of point Q from y-axis = 9 units

(ii) (b) : Coordinates of point U are (8, 2).

(iii) (d) : We have, $P(2, 5)$ and $Q(9, 5)$

$\therefore$ $PQ = \sqrt{(2-9)^2+(5-5)^2} = \sqrt{49+0} = 7$ units

(iv) (c) : Point $A(x, y)$ is equidistant from $R(3, 8)$ and $T(3, 2)$.

$\therefore$ $AR = AT \Rightarrow AR^2 = AT^2$

$\Rightarrow$ $(x - 3)^2 + (y - 8)^2 = (x - 3)^2 + (y - 2)^2$

$\Rightarrow$ $y^2 + 64 - 16y = y^2 + 4 - 4y$

$\Rightarrow$ $16y - 4y = 64 - 4$

$\Rightarrow$ $12y = 60 \Rightarrow y = 5$

(v) (d) : Length of $TU = 5$ units and of $TL = 2$ units

$\therefore$ Perimeter of image of a rectangular face $= 2(5 + 2) = 14$ units

17. (i) (b) : Clearly, coordinates of A are (3, 3).

(ii) (c) : Coordinates of C are (8, 7) therefore mid-point of AC is

$\left(\frac{3+8}{2}, \frac{3+7}{2}\right)$, *i.e.*, (5.5, 5)

(iii) (d) : Coordinates of B and D are (4, 6) and (6, 4), respectively.

Distance between A and $B = \sqrt{(4-3)^2+(6-3)^2}$

$= \sqrt{1^2+3^2} = \sqrt{10}$

and Distance between A and $D = \sqrt{(6-3)^2+(4-3)^2}$

$= \sqrt{3^2+1^2} = \sqrt{10}$

Thus, both B and D are near to A.

(iv) (a) : Distance between B and C

$= \sqrt{(8-4)^2+(7-6)^2} = \sqrt{16+1} = \sqrt{17}$

Distance between D and $C = \sqrt{(8-6)^2+(7-4)^2}$

$= \sqrt{4+9} = \sqrt{13}$

(v) (a) : When Q is taken as origin, then

coordinates of $D = (-7, 4)$

18. (i) (c) : We are given that P is origin.

$\therefore$ Coordinates of H are (2, 4).

(ii) (d) : Coordinates of G are (9, 4), therefore distance of G from y-axis = 9 units.

(iii) (b) : Coordinates of H are (2, 4) and coordinates of G are (9, 4).

Thus, $GH = \sqrt{(9-2)^2+(4-4)^2} = \sqrt{7^2+0} = 7$ units

(iv) (d) : Coordinates of D are (4, 2) and coordinates of F are (9, 8).

$\Rightarrow$ $x = 9$

Also, length of diagonal $FD = \sqrt{(4-9)^2+(2-8)^2}$

$= \sqrt{25+36} = \sqrt{61}$ units

(v) (a) : If Q is origin, then

Coordinates of B are (–1, 6) and of C are (–1, 2).

Now, mid-point $BC = \left(\frac{(-1)+(-1)}{2}, \frac{6+2}{2}\right)$ *i.e.*, (–1, 4)

19. Coordinates of A, B and C are (2, 8), (7, 7) and (5, 3) respectively.

(i) (c) : From the graph, it is clear that C is nearest to the office.

(ii) **(b)**: Distance between A and C

$= \sqrt{(5-2)^2+(3-8)^2} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$ units

(iii) **(c)** : $AB = \sqrt{(7-2)^2+(7-8)^2} = \sqrt{26}$ units

$OA = \sqrt{2^2+8^2} = 2\sqrt{17}$ units

$BC = \sqrt{(7-5)^2+(7-3)^2} = 2\sqrt{5}$ units

$\therefore$ BC is the least.

(iv) **(c)**

(v) **(b)** : Coordinates of $D = \left(\frac{5+7}{2}, \frac{3+7}{2}\right) = (6, 5)$

20. **(i)** **(a)**: $Q(x, y)$ is mid-point of $B(-2, 4)$ and $C(6, 4)$.

$\therefore$ $(x, y) = \left(\frac{-2+6}{2}, \frac{4+4}{2}\right) = \left(\frac{4}{2}, \frac{8}{2}\right) = (2, 4)$

(ii) **(d)**: Since P, Q, R and S are mid-points of sides AB, BC, CD and AD respectively.

$\therefore$ $PQRS$ is a rhombus.

[$\because$ The quadrilateral formed by joining the midpoints of a rectangle is a rhombus]

(iii) **(a)**: Since $PQRS$ is a rhombus, therefore, $PQ = QR = RS = PS$.

$\therefore$ $PQ = \sqrt{(-2-2)^2+(1-4)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units

Thus, length of each side of $PQRS$ is 5 units.

(iv) **(a)**: Length of route $PQRS = 4\ PQ$

$= 4 \times 5 = 20$ units

(v) **(c)**: Length of $CD = 4 + 2 = 6$ units and length of $AD = 6 + 2 = 8$ units

$\therefore$ Length of route $ABCD = 2(6 + 8) = 28$ units