Mathematical Logic

EXERCISE 1.1 [PAGES 2 - 3]

Exercise 1.1 | Q 1 | Page 2

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

A triangle has ‘n’ sides

Solution: It is an open sentence. Hence, it is not a statement.

[**Note:** Answer given in the textbook is ‘it is a statement’. However, we found that ‘It is not a statement’.]

Exercise 1.1 | Q 2 | Page 2

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

The sum of interior angles of a triangle is 180°

Solution: It is a statement which is true. Hence, it’s truth value is T.

Exercise 1.1 | Q 3 | Page 2

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

You are amazing!

Solution: It is an exclamatory sentence. Hence, it is not a statement.

Exercise 1.1 | Q 4 | Page 2

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

Please grant me a loan.

Solution: It is a request. Hence, it is not a statement.

Exercise 1.1 | Q 5 | Page 2

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

$\sqrt{-4}$ is an irrational number.

Solution: It is a statement which is false. Hence, it’s truth value is F.

Exercise 1.1 | Q 6 | Page 2

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

$x^2 - 6x + 8 = 0$ implies $x = -4$ or $x = -2$.

Solution: It is a statement which is false. Hence, it's truth value if F.

Exercise 1.1 | Q 7 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

He is an actor.

Solution: It is an open sentence. Hence, it is not a statement.

Exercise 1.1 | Q 8 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

Did you eat lunch yet?

Solution: It is an interrogative sentence. Hence, it is not a statement.

Exercise 1.1 | Q 9 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

Have a cup of cappuccino.

Solution: It is an interrogative sentence. Hence, it is not a statement.

Exercise 1.1 | Q 10 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

$(x + y)^2 = x^2 + 2xy + y^2$ for all $x, y \in R$.

Solution: It is a statement which is true. Hence, it's truth value is T.

Exercise 1.1 | Q 11 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

Every real number is a complex number.

Solution: It is a statement which is true. Hence, it's truth value is T.

Exercise 1.1 | Q 12 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

1 is a prime number.

Solution: It is a statement which is false. Hence, it's truth value is F.

Exercise 1.1 | Q 13 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

With the sunset the day ends.

Solution: It is a statement which is true. Hence, it's truth value is T.

Exercise 1.1 | Q 14 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

$1 ! = 0$

Solution: It is a statement which is false. Hence, it's truth value is F.

Exercise 1.1 | Q 15 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

$3 + 5 > 11$

Solution: It is a statement which is false. Hence, it's truth value is F.

Exercise 1.1 | Q 16 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

The number π is an irrational number.

Solution: It is a statement which is true. Hence, it's truth value is T.

Exercise 1.1 | Q 17 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

$x^2 - y^2 = (x + y)(x - y)$ for all $x, y \in R$.

Solution: It is a statement which is true. Hence, it's truth value is T.

Exercise 1.1 | Q 18 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

The number 2 is the only even prime number.

Solution: It is a statement which is true. Hence, it's truth value is T.

Exercise 1.1 | Q 19 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

Two co-planar lines are either parallel or intersecting.

Solution: It is a statement which is true. Hence, it's truth value is T.

Exercise 1.1 | Q 20 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

The number of arrangements of 7 girls in a row for a photograph is 7!.

Solution: It is a statement which is true. Hence, it's truth value is T.

Exercise 1.1 | Q 21 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

Give me a compass box.

Solution: It is an imperative sentence. Hence, it is not a statement.

Exercise 1.1 | Q 22 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

Bring the motor car here.

Solution: It is an imperative sentence. Hence, it is not a statement.

Exercise 1.1 | Q 23 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

It may rain today.

Solution: It is an open sentence. Hence, it is not a statement.

Exercise 1.1 | Q 24 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

If $a + b < 7$, where $a \geq 0$ and $b \geq 0$ then $a < 7$ and $b < 7$.

Solution: It is a statement which is true. Hence, it's truth value is T.

Exercise 1.1 | Q 25 | Page 3

State which of the following sentence is a statement. Justify your answer if it is a statement. Write down its truth value.

Can you speak in English?

Solution: It is an interrogative sentence. Hence, it is not a statement.

EXERCISE 1.2 [PAGE 6]

Exercise 1.2 | Q 1.1 | Page 6

Express the following statement in symbolic form.

e is a vowel or $2 + 3 = 5$

Solution: Let p : e is a vowel.

q : $2 + 3 = 5$

The symbolic form is $p \vee q$.

Exercise 1.2 | Q 1.2 | Page 6

Express the following statement in symbolic form.

Mango is a fruit but potato is a vegetable.

Solution: Let p : Mango is a fruit.

q : Potato is a vegetable.

The symbolic form is $p \wedge q$.

Exercise 1.2 | Q 1.3 | Page 6

Express the following statement in symbolic form.

Milk is white or grass is green.

Solution: Let p : Milk is white.

q : Grass is green.

The symbolic form is $p \vee q$.

Exercise 1.2 | Q 1.4 | Page 6

Express the following statement in symbolic form.

I like playing but not singing.

Solution: Let p : I like playing.

q : I do not like singing.

The symbolic form is $p \wedge q$.

Exercise 1.2 | Q 1.5 | Page 6

Express the following statement in symbolic form.

Even though it is cloudy, it is still raining.

Solution: Let p : It is cloudy.

q : It is still raining.

The symbolic form is $p \wedge q$.

Exercise 1.2 | Q 2.1 | Page 6

Write the truth value of the following statement.

Earth is a planet and Moon is a star.

Solution: Let p : Earth is a planet.

q : Moon is a star.

The truth values of p and q are T and F respectively.

The given statement in symbolic form is $p \wedge q$.

$\therefore p \wedge q \equiv T \wedge F \equiv F$

$\therefore$ Truth value of the given statement is F.

Exercise 1.2 | Q 2.2 | Page 6

Write the truth value of the following statement.

16 is an even number and 8 is a perfect square.

Solution: Let p : 16 is an even number.

q : 8 is a perfect square.

The truth values of p and q are T and F respectively.

The given statement in symbolic form is $p \wedge q$.

$\therefore p \wedge q \equiv T \wedge F \equiv F$

∴ Truth value of the given statement is F.

Exercise 1.2 | Q 2.3 | Page 6

Write the truth value of the following statement.

A quadratic equation has two distinct roots or 6 has three prime factors.

Solution: Let p : A quadratic equation has two distinct roots.

q : 6 has three prime factors.

The truth values of p and q are F and F respectively.

The given statement in symbolic form is $p \vee q$.

$\therefore p \vee q \equiv F \vee F \equiv F$

∴ Truth value of the given statement is F.

Exercise 1.2 | Q 2.4 | Page 6

Write the truth value of the following statement.

The Himalayas are the highest mountains but they are part of India in the North East.

Solution: Let p : Himalayas are the highest mountains.

q : Himalayas are the part of India in the north east.

The truth values of p and q are T and T respectively.

The given statement in symbolic form is $p \wedge q$.

$\therefore p \wedge q \equiv T \wedge T \equiv T$

∴ Truth value of the given statement is T.

EXERCISE 1.3 [PAGE 7]

Exercise 1.3 | Q 1.1 | Page 7

Write the negation of the following statement.

All men are animals.

Solution: Some men are not animals.

Exercise 1.3 | Q 1.2 | Page 7

Write the negation of the following statement.

− 3 is a natural number.

Solution: – 3 is not a natural number.

Exercise 1.3 | Q 1.3 | Page 7

Write the negation of the following statement.

It is false that Nagpur is capital of Maharashtra

Solution: Nagpur is capital of Maharashtra.

Exercise 1.3 | Q 1.4 | Page 7

Write the negation of the following statement.

$2 + 3 \neq 5$

Solution: $2 + 3 = 5$

Exercise 1.3 | Q 2.1 | Page 7

Write the truth value of the negation of the following statement.

$\sqrt{5}$ is an irrational number.

Solution: Truth value of the given statement is T.

∴ Truth value of its negation is F.

Exercise 1.3 | Q 2.2 | Page 7

Write the truth value of the negation of the following statement.

London is in England.

Solution: Truth value of the given statement is T.

∴ Truth value of its negation is F.

Exercise 1.3 | Q 2.3 | Page 7

Write the truth value of the negation of the following statement.

For every $x \in N$, $x + 3 < 8$.

Solution: Truth value of the given statement is F.

∴ Truth value of its negation is T.

EXERCISE 1.4 [PAGES 10 - 11]

Exercise 1.4 | Q 1.1 | Page 10

Write the following statement in symbolic form.

If triangle is equilateral then it is equiangular.

Solution: Let p : Triangle is equilateral.

q : Triangle is equiangular.

The symbolic form is $p \rightarrow q$.

Exercise 1.4 | Q 1.2 | Page 10

Write the following statement in symbolic form.

It is not true that “i” is a real number.

Solution: Let p : i is a real number.

The symbolic form is $\sim p$.

Exercise 1.4 | Q 1.3 | Page 10

Write the following statement in symbolic form.

Even though it is not cloudy, it is still raining.

Solution: Let p : It is cloudy.

q : It is raining.

The symbolic form is $\sim p \wedge q$.

Exercise 1.4 | Q 1.4 | Page 10

Write the following statement in symbolic form.

Milk is white if and only if the sky is not blue.

Solution: Let p : Milk is white.

q : Sky is blue.

The symbolic form is $p \leftrightarrow \sim q$.

Exercise 1.4 | Q 1.5 | Page 10

Write the following statement in symbolic form.

Stock prices are high if and only if stocks are rising.

Solution: Let p : Stock prices are high.

q : Stock are rising

The symbolic form is p ↔ q.

Exercise 1.4 | Q 1.6 | Page 10

Write the following statement in symbolic form.

If Kutub-Minar is in Delhi then Taj-Mahal is in Agra.

Solution: Let p : Kutub-Minar is in Delhi.

q : Taj-Mahal Is in Agra.

The symbolic form is p → q.

Exercise 1.4 | Q 2.1 | Page 11

Find the truth value of the following statement.

It is not true that 3 − 7i is a real number.

Solution: Let p : 3 – 7i is a real number.

The truth value of p is F.

The given statement in symbolic form is ~p.

∴ ~ p ≡ ~ F ≡ T

∴ Truth value of the given statement is T.

Exercise 1.4 | Q 2.2 | Page 11

Find the truth value of the following statement.

If a joint venture is a temporary partnership, then discount on purchase is credited to the supplier.

Solution: Let p : A joint venture is a temporary partnership. q : Discount on purchase is credited to the supplier.

The truth value of p and q are T and F respectively.

The given statement in symbolic form is p → q.

∴ p → q ≡ T → F ≡ F

∴ Truth value of the given statement is F.

Exercise 1.4 | Q 2.3 | Page 11

Find the truth value of the following statement.

Every accountant is free to apply his own accounting rules if and only if machinery is an asset.

Solution: Let p : Every accountant is free to apply his own accounting rules.

q : Machinery is an asset.

The truth values of p and q are F and T respectively.

The given statement in symbolic form is $p \leftrightarrow q$.

$\therefore p \leftrightarrow q \equiv F \leftrightarrow T \equiv F$

∴ Truth value of the given statement is F.

Exercise 1.4 | Q 2.4 | Page 11

Find the truth value of the following statement.

Neither 27 is a prime number nor divisible by 4.

Solution: Let p : 27 is a prime number.

q : 27 is divisible by 4.

The truth values of p and q are F and F respectively.

The given statement in symbolic form is $\sim p \wedge \sim q$.

$\therefore \sim p \wedge \sim q \equiv \sim F \wedge \sim F \equiv T \wedge T \equiv T$

∴ Truth value of the given statement is T.

Exercise 1.4 | Q 2.5 | Page 11

Find the truth value of the following statement.

3 is a prime number and an odd number.

Solution: Let p : 3 is a prime number.

q : 3 is an odd number.

The truth values of p and q are T and T respectively.

The given statement in symbolic form is $p \wedge q$.

$\therefore p \wedge q \equiv T \wedge T \equiv T$

∴ Truth value of the given statement is T.

Exercise 1.4 | Q 3.1 | Page 11

If p and q are true and r and s are false, find the truth value of the following compound statement.

$p \wedge (q \wedge r)$

Solution: $p \wedge (q \wedge r) \equiv T \wedge (T \wedge F)$

$\equiv T \wedge F$

≡ F

Hence, truth value if F.

Exercise 1.4 | Q 3.2 | Page 11

If p and q are true and r and s are false, find the truth value of the following compound statement.

(p → q) ∨ (r ∧ s)

Solution: (p → q) ∨ (r ∧ s) ≡ (T → T) ∨ (F ∧ F)

≡ T ∨ F

≡ T
Hence, truth value if T.

Exercise 1.4 | Q 3.3 | Page 11

If p and q are true and r and s are false, find the truth value of the following compound statement.

~ [(~ p ∨ s) ∧ (~ q ∧ r)]

Solution: ~ [(~ p ∨ s) ∧ (~ q ∧ r)] ≡ ~[(~T ∨ F) ∧ (~T ∧ F)]

≡ ~[(F ∨ F) ∧ (F ∧ F)

≡ ~ (F ∧ F)

≡ ~ F

≡ T
Hence, truth value if T.

Exercise 1.4 | Q 3.4 | Page 11

If p and q are true and r and s are false, find the truth value of the following compound statement.

(p → q) ↔ ~(p ∨ q)

Solution: (p → q) ↔ ~(p ∨ q) ≡ (T → T) ↔ (T ∨ T)

≡ T ↔ ~ T

≡ T ↔ F

≡ F

Hence, truth value if F.

Exercise 1.4 | Q 3.5 | Page 11

If p and q are true and r and s are false, find the truth value of the following compound statement.

$[(p \vee s) \rightarrow r] \vee \sim [\sim (p \rightarrow q) \vee s]$

Solution: $[(p \vee s) \rightarrow r] \vee \sim [\sim (p \rightarrow q) \vee s]$

$\equiv [(T \vee F) \rightarrow F] \vee \sim [\sim (T \rightarrow T) \vee F]$

$\equiv (T \rightarrow F) \vee \sim (\sim T \vee F)$

$\equiv F \vee \sim (F \vee F)$

$\equiv F \vee \sim F$

$\equiv F \vee T$

$\equiv T$

Hence, truth value is T.

Exercise 1.4 | Q 3.6 | Page 11

If p and q are true and r and s are false, find the truth value of the following compound statement.

$\sim [p \vee (r \wedge s)] \wedge \sim [(r \wedge \sim s) \wedge q]$

Solution: $\sim [p \vee (r \wedge s)] \wedge \sim [(r \wedge \sim s) \wedge q]$

$\equiv \sim [T \vee (F \wedge F)] \wedge \sim [(F \wedge \sim F) \wedge T]$

$\equiv \sim (T \vee F) \wedge \sim [(F \wedge T) \wedge T]$

$\equiv \sim T \wedge \sim (F \wedge T)$

$\equiv F \wedge \sim F$

$\equiv F \wedge T$

$\equiv F$

Hence, truth value is F.

Exercise 1.4 | Q 4.1 | Page 11

Assuming that the following statement is true,

p : Sunday is holiday,

q : Ram does not study on holiday,

find the truth values of the following statements.

Sunday is not holiday or Ram studies on holiday.

Solution: Symbolic form of the given statement is $\sim p \vee \sim q$

$\therefore \sim p \vee \sim q \equiv \sim T \vee \sim T$

$\equiv F \vee F$

$\equiv F$

Hence, truth value is F.

Exercise 1.4 | Q 4.2 | Page 11

Assuming that the following statement is true,

p : Sunday is holiday,

q : Ram does not study on holiday,

find the truth values of the following statements.

If Sunday is not holiday then Ram studies on holiday.

Solution: Symbolic form of the given statement is

$\sim p \rightarrow \sim q$

$\therefore \sim p \rightarrow \sim q \equiv \sim T \rightarrow \sim T$

$\equiv F \rightarrow F$

$\equiv T$

Hence, truth value is T.

Exercise 1.4 | Q 4.3 | Page 11

Assuming that the following statement is true,

p : Sunday is holiday,

q : Ram does not study on holiday,

find the truth values of the following statements.

Sunday is a holiday and Ram studies on holiday.

Solution: Symbolic form of the given statement is $p \wedge \sim q$

$\therefore p \wedge \sim q \equiv T \wedge \sim T$

$\equiv T \wedge F$

$\equiv F$

Hence, truth value is F.

Exercise 1.4 | Q 5.1 | Page 11

If p : He swims

q : Water is warm

Give the verbal statement for the following symbolic statement.

$p \leftrightarrow \sim q$

Solution: He swims if and only if water is not warm.

Exercise 1.4 | Q 5.2 | Page 11

If p : He swims

q : Water is warm

Give the verbal statement for the following symbolic statement.

$\sim (p \vee q)$

Solution: It is not true that he swims or water is warm.

Exercise 1.4 | Q 5.3 | Page 11

If p : He swims

q : Water is warm

Give the verbal statement for the following symbolic statement.

$q \rightarrow p$

Solution: If water is warm then he swims.

Exercise 1.4 | Q 5.4 | Page 11

If p : He swims

q : Water is warm

Give the verbal statement for the following symbolic statement.

$q \wedge \sim p$

Solution: Water is warm and he does not swim.

EXERCISE 1.5 [PAGE 12]

Exercise 1.5 | Q 1.1 | Page 12

Use quantifiers to convert the following open sentences defined on N, into a true statement.

$x^2 + 3x - 10 = 0$

Solution: ∃ x ∈ N, such that $x^2 + 3x – 10 = 0$

It is true statement, since x = 2 ∈ N satisfies it.

Exercise 1.5 | Q 1.2 | Page 12

Use quantifiers to convert the following open sentences defined on N, into a true statement.

3x - 4 < 9

Solution: ∃ x ∈ N, such that 3x – 4 < 9

It is true statement, since

x = 2, 3, 4 ∈ N satisfies 3x - 4 < 9.

Exercise 1.5 | Q 1.3 | Page 12

Use quantifiers to convert the following open sentences defined on N, into a true statement.

$n^2 \geq 1$

Solution: ∀ n ∈ N, $n^2 \geq 1$

It is true statement, since all n ∈ N satisfy it.

Exercise 1.5 | Q 1.4 | Page 12

Use quantifiers to convert the following open sentences defined on N, into a true statement.

2n - 1 = 5

Solution: ∃ n ∈ N, such that 2n - 1 = 5

It is a true statement since all n = 3 ∈ N satisfy 2n - 1 = 5.

Exercise 1.5 | Q 1.5 | Page 12

Use quantifiers to convert the following open sentences defined on N, into a true statement.

y + 4 > 6

Solution: ∃ y ∈ N, such that y + 4 > 6

It is a true statement since y = 3, 4, ... ∈ N satisfy y + 4 > 6.

Exercise 1.5 | Q 1.6 | Page 12

Use quantifiers to convert the following open sentences defined on N, into a true statement.

$3y - 2 \leq 9$

Solution: $\exists\ y \in N$, such that $3y - 2 \leq 9$

It is a true statement since $y = 1, 2, 3 \in N$ satisfy it.

Exercise 1.5 | Q 2.1 | Page 12

If $B = \{2, 3, 5, 6, 7\}$ determine the truth value of $\forall\ x \in B$ such that x is prime number.

Solution: For $x = 6$, x is not a prime number.

$\therefore$ $x = 6$ does not satisfies the given statement.

$\therefore$ The given statement is false.

$\therefore$ It's truth value is F.

Exercise 1.5 | Q 2.2 | Page 12

If $B = \{2, 3, 5, 6, 7\}$ determine the truth value of

$\exists\ n \in B$, such that $n + 6 > 12$.

Solution: For $n = 7$, $n + 6 = 7 + 6 = 13 > 12$

$\therefore$ $n = 7$ satisfies the equation $n + 6 > 12$.

$\therefore$ The given statement is true.

$\therefore$ It's truth value is T.

Exercise 1.5 | Q 2.3 | Page 12

If $B = \{2, 3, 5, 6, 7\}$ determine the truth value of

$\exists\ n \in B$, such that $2n + 2 < 4$.

Solution: There is no n in B which satisfies $2n + 2 < 4$.

$\therefore$ The given statement is false.

$\therefore$ It's truth value is F.

Exercise 1.5 | Q 2.4 | Page 12

If $B = \{2, 3, 5, 6, 7\}$ determine the truth value of

$\forall\ y \in B$, such that y^2 is negative.

Solution: There is no y in B which satisfies $y^2 < 0$.

$\therefore$ The given statement is false.

$\therefore$ It's truth value is F.

Exercise 1.5 | Q 2.5 | Page 12

If B = {2, 3, 5, 6, 7} determine the truth value of
$\forall y \in B$, such that $(y - 5) \in N$

Solution: For $y = 2$, $y – 5 = 2 – 5 = –3 \notin N$.

$\therefore$ $y = 2$ does not satisfies the equation $(y – 5) \in N$.

$\therefore$ The given statement is false.

$\therefore$ It's truth value is F.

EXERCISE 1.6 [PAGE 16]

Exercise 1.6 | Q 1.1 | Page 16

Prepare truth tables for the following statement pattern.

$p \rightarrow (\sim p \vee q)$

Solution:

$p \rightarrow (\sim p \vee q)$

p	q	~p	~ p ∨ q	p → (~ p ∨ q)
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Exercise 1.6 | Q 1.2 | Page 16

Prepare truth tables for the following statement pattern.

$(\sim p \vee q) \wedge (\sim p \vee \sim q)$

Solution: $(\sim p \vee q) \wedge (\sim p \vee \sim q)$

p	q	~p	~q	~p∨q	~p∨~q	(~p∨q)∧(~p∨~q)
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	T	T	T

F	F	T	T	T	T	T

Exercise 1.6 | Q 1.3 | Page 16

Prepare truth tables for the following statement pattern.

$(p \wedge r) \rightarrow (p \vee \sim q)$

Solution: $(p \wedge r) \rightarrow (p \vee \sim q)$

p	**q**	**r**	**~q**	**p ∧ r**	**p∨~q**	**(p ∧ r) → (p ∨ ~ q)**
T	T	T	F	T	T	T
T	T	F	F	F	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

Exercise 1.6 | Q 1.4 | Page 16

Prepare truth tables for the following statement pattern.

$(p \wedge q) \vee \sim r$

Solution: $(p \wedge q) \vee \sim r$

p	**q**	**r**	**~r**	**p ∧ q**	**(p ∧ q) ∨ ~ r**
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	F	T

F	F	T	F	F	F
F	F	F	T	F	T

Exercise 1.6 | Q 2.1 | Page 16

Examine whether the following statement pattern is a tautology, a contradiction or a contingency.

q ∨ [~ (p ∧ q)]

Solution:

p	q	p ∧ q	~ (p ∧ q)	q ∨ [~ (p ∧ q)]
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

All the truth values in the last column are T. Hence, it is a **tautology**.

Exercise 1.6 | Q 2.2 | Page 16

Examine whether the following statement pattern is a tautology, a contradiction or a contingency.

(~ q ∧ p) ∧ (p ∧ ~ p)

Solution:

p	q	~p	~q	(~q∧p)	(p∧~p)	(~q∧p)∧(p∧~p)
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	F	F	F

All the truth values in the last column are F. Hence, it is a **contradiction**.

Exercise 1.6 | Q 2.3 | Page 16

Examine whether the following statement pattern is a tautology, a contradiction or a contingency.

$(p \wedge \sim q) \rightarrow (\sim p \wedge \sim q)$

Solution:

p	q	~p	~q	p∧~q	~p∧~q	(p∧~q)→(~p∧~q)
T	T	F	F	F	F	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	F	T	T	F	T	T

The truth values in the last column are not identical. Hence, it is **contingency**.

Exercise 1.6 | Q 2.4 | Page 16

Examine whether the following statement pattern is a tautology, a contradiction or a contingency.

$\sim p \rightarrow (p \rightarrow \sim q)$

Solution:

p	q	~p	~q	p→~q	~p→(p→~q)
T	T	F	F	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

All the truth values in the last column are T. Hence, it is **tautology.**

Exercise 1.6 | Q 3.1 | Page 16

Prove that the following statement pattern is a tautology.

$(p \wedge q) \rightarrow q$

Solution:

p	q	p ∧ q	(p∧q)→q
T	T	T	T

T	F	F	T
F	T	F	T
F	F	F	T

All the truth values in the last column are T. Hence, it is **tautology**.

Exercise 1.6 | Q 3.2 | Page 16

Prove that the following statement pattern is a tautology.

$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

Solution:

p	q	~p	~q	p→q	~q→~p	(p→q)↔(~q→~p)
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

All the truth values in the last column are T. Hence, it is a **tautology**.

Exercise 1.6 | Q 3.3 | Page 16

Prove that the following statement pattern is a tautology.

$(\sim p \wedge \sim q) \rightarrow (p \rightarrow q)$

Solution:

p	q	~p	~q	~p∧~q	p→q	(~p∧~q)→(p→q)
T	T	F	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

All the truth values in the last column are T. Hence, it is a **tautology**.

Exercise 1.6 | Q 3.4 | Page 16

Prove that the following statement pattern is a tautology.

(~ p ∨ ~ q) ↔ ~ (p ∧ q)

Solution:

p	q	~p	~q	~p∨~q	p∧q	~p∨~q	(~p∨~q↔~(p ∧ q)
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

All the truth values in the last column are T. Hence, it is a **tautology**.

Exercise 1.6 | Q 4.1 | Page 16

Prove that the following statement pattern is a contradiction.

(p ∨ q) ∧ (~p ∧ ~q)

Solution:

p	q	~p	~q	p∨q	~p∧~q	(p∨q)∧(~p∧~q)
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

All the truth values in the last column are F. Hence, it is a **contradiction**.

Exercise 1.6 | Q 4.2 | Page 16

Prove that the following statement pattern is a contradiction.

(p ∧ q) ∧ ~p

Solution:

p	q	~p	p∧q	(p∧q)∧~p
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F

F	F	T	F	F

All the truth values in the last column are F. Hence, it is a **contradiction**.

Exercise 1.6 | Q 4.3 | Page 16

Prove that the following statement pattern is a contradiction.

$(p \wedge q) \wedge (\sim p \vee \sim q)$

Solution:

p	q	~p	~q	p∧q	~p∨~q	(p∧q)∧(~p∨~q)
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

All the truth values in the last column are F. Hence, it is a **contradiction**.

Exercise 1.6 | Q 4.4 | Page 16

Prove that the following statement pattern is a contradiction.

$(p \rightarrow q) \wedge (p \wedge \sim q)$

Solution:

p	q	~q	p→q	p∧~q	(p→q)∧(p∧~q)
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	F	F

All the truth values in the last column are F. Hence, it is a **contradiction**.

Exercise 1.6 | Q 5.1 | Page 16

Show that the following statement pattern is contingency.

$(p \wedge \sim q) \rightarrow (\sim p \wedge \sim q)$

Solution:

p	q	~p	~q	p∧~q	~p∧~q	(p∧~q)→(~p∧~q)
T	T	F	F	F	F	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	F	T	T	F	T	T

The truth values in the last column are not identical. Hence, it is **contingency**.

Exercise 1.6 | Q 5.2 | Page 16

Show that the following statement pattern is contingency.

$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$

Solution:

p	q	~p	p→q	~p∨q	(p→q)↔(~p∨q)
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

All the truth values in the last column are T. Hence, it is a tautology. Not contingency.

Exercise 1.6 | Q 5.3 | Page 16

Show that the following statement pattern is contingency.

$p \wedge [(p \rightarrow \sim q) \rightarrow q]$

Solution:

p	q	~q	p→~q	(p→~q)→q	p∧[(p→~q)→q]

T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	T	F
F	F	T	T	F	F

Truth values in the last column are not identical. Hence, it is contingency.

Exercise 1.6 | Q 5.4 | Page 16

Show that the following statement pattern is contingency.

$(p \rightarrow q) \wedge (p \rightarrow r)$

Solution:

p	**q**	**r**	**p→q**	**p→r**	**(p→q)∧(p→r)**
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

The truth values in the last column are not identical. Hence, it is contingency.

Exercise 1.6 | Q 6.1 | Page 16

Using the truth table, verify

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Prove that the following pair of statement patterns is equivalent.

$p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$

Solution:

1	2	3	4	5	6	7	8
p	**q**	**r**	**q∧r**	**p∨(q∧r)**	**p∨q**	**p∨r**	**(p∨q)∧(p∨r)**
T	T	T	T	T	T	T	T

T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

The entries in columns 5 and 8 are identical.

∴ p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)

Exercise 1.6 | Q 6.2 | Page 16

Using the truth table, verify

p → (p → q) ≡ ~ q → (p → q)

Solution:

1	2	3	4	5	6
p	**q**	**~q**	**p→q**	**p→(p→q)**	**~q→(p→q)**
T	T	F	T	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

In the above truth table, entries in columns 5 and 6 are identical.

∴ p → (p → q) ≡ ~ q → (p → q)

Exercise 1.6 | Q 6.3 | Page 16

Using the truth table, verify

~(p → ~q) ≡ p ∧ ~ (~ q) ≡ p ∧ q

Solution:

1	2	3	4	5	6	7	8
p	**q**	**~q**	**p→~q**	**~(p→~q)**	**~(~q)**	**p∧~(~q)**	**p∧q**

T	T	F	F	T	T	T	T
T	F	T	T	F	F	F	F
F	T	F	T	F	T	F	F
F	F	T	T	F	F	F	F

In the above table, entries in columns 5, 7, and 8 are identical.

∴ ~(p → ~q) ≡ p ∧ ~ (~ q) ≡ p ∧ q

Exercise 1.6 | Q 6.4 | Page 16

Using the truth table, verify

~(p ∨ q) ∨ (~ p ∧ q) ≡ ~ p

Solution:

1	2	3	4	5	6	7
p	**q**	**~p**	**(p∨q)**	**~(p∨q)**	**~p∧q**	**~(p∨q)∨(~p∧q)**
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

In the above truth table, the entries in columns 3 and 7 are identical.

∴ ~(p ∨ q) ∨ (~ p ∧ q) ≡ ~ p

Exercise 1.6 | Q 7.1 | Page 16

Using the truth table, verify

p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)

Prove that the following pair of statement patterns is equivalent.

p ∨ (q ∧ r) and (p ∨ q) ∧ (p ∨ r)

Solution:

1	2	3	4	5	6	7	8
p	**q**	**r**	**q∧r**	**p∨(q∧r)**	**p∨q**	**p∨r**	**(p∨q)∧(p∨r)**
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T

T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

The entries in columns 5 and 8 are identical.

∴ p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)

Exercise 1.6 | Q 7.2 | Page 16

Prove that the following pair of statement pattern is equivalent.

p ↔ q and (p → q) ∧ (q → p)

Solution:

1	2	3	4	5	6
p	**q**	**p↔q**	**p→q**	**q→p**	**(p→q)∧(q→p)**
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

In the above table, entries in columns 3 and 6 are identical.

∴ Statement p ↔ q and (p → q) ∧ (q → p) are equivalent.

Exercise 1.6 | Q 7.3 | Page 16

Prove that the following pair of statement pattern is equivalent.

p → q and ~ q → ~ p and ~ p ∨ q

Solution:

1	2	3	4	5	6	7
p	**q**	**~p**	**~q**	**p→q**	**~q→~p**	**~p∨q**
T	T	F	F	T	T	T
T	F	F	T	F	F	F

F	T	T	F	T	T	T
F	F	T	T	T	T	T

In the above table, entries in columns 5, 6 and 7 are identical

∴ Statement p → q and ~q → ~p and ~p ∨ q are equivalent.

Exercise 1.6 | Q 7.4 | Page 16

Prove that the following pair of statement pattern is equivalent.

~(p ∧ q) and ~p ∨ ~q

Solution:

1	2	3	4	5	6	7
p	**q**	**~p**	**~q**	**p∧q**	**~(p∧q)**	**~p∨~q**
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

In the above table, entries in columns 6 and 7 are identical.

∴ Statement ~(p ∧ q) and ~p ∨ ~q are equivalent.

EXERCISE 1.7 [PAGE 17]

Exercise 1.7 | Q 1.1 | Page 17

Write the dual of the following:

(p ∨ q) ∨ r

Solution: (p ∧ q) ∧ r

Exercise 1.7 | Q 1.2 | Page 17

Write the dual of the following:

~(p ∨ q) ∧ [p ∨ ~ (q ∧ ~ r)]

Solution: ~(p ∧ q) ∨ [p ∧ ~ (q ∨ ~ r)]

Exercise 1.7 | Q 1.3 | Page 17

Write the dual of the following:

$p \vee (q \vee r) \equiv (p \vee q) \vee r$

Solution: $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

Exercise 1.7 | Q 1.4 | Page 17

Write the dual of the following:

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

Solution: $\sim(p \vee q) \equiv \sim p \wedge \sim q$

Exercise 1.7 | Q 2.1 | Page 17

Write the dual statement of the following compound statement.

13 is a prime number and India is a democratic country.

Solution: 13 is a prime number or India is a democratic country.

Exercise 1.7 | Q 2.2 | Page 17

Write the dual statement of the following compound statement.

Karina is very good or everybody likes her.

Solution: Karina is very good and everybody likes her.

Exercise 1.7 | Q 2.3 | Page 17

Write the dual statement of the following compound statement.

Radha and Sushmita cannot read Urdu.

Solution: Radha or Sushmita cannot read Urdu.

Exercise 1.7 | Q 2.4 | Page 17

Write the dual statement of the following compound statement.

A number is a real number and the square of the number is non-negative.

Solution: A number is a real number or the square of the number is non-negative.

EXERCISE 1.8 [PAGE 21]

Exercise 1.8 | Q 1.1 | Page 21

Write the negation of the following statement.

All the stars are shining if it is night.

Solution: Let q : All stars are shining.

p : It is night.

The given statement in symbolic form is $p \rightarrow q$. It's negation is $\sim (p \rightarrow q) \equiv p \wedge \sim q$

∴ The negation of a given statement is 'It is night and some stars are not shining'.

Exercise 1.8 | Q 1.2 | Page 21

Write the negation of the following statement.

$\forall\ n \in N,\ n + 1 > 0$

Solution: $\exists\ n \in N$ such that $n + 1 \leq 0$.

Exercise 1.8 | Q 1.3 | Page 21

Write the negation of the following statement.

$\exists\ n \in N,\ (n^2 + 2)$ is odd number.

Solution: $\forall\ n \in N,\ (n^2 + 2)$ is not odd number.

Exercise 1.8 | Q 1.4 | Page 21

Write the negation of the following statement.

Some continuous functions are differentiable.

Solution: All continuous functions are not differentiable.

Exercise 1.8 | Q 2.1 | Page 21

Using the rules of negation, write the negation of the following:

$(p \rightarrow r) \wedge q$

Solution: $\sim [(p \rightarrow r) \wedge q] \equiv \sim(p \rightarrow r) \vee \sim q$ ....[Negation of conjunction]

$\equiv (p \wedge \sim r) \vee \sim q$ .....[Negation of implication]

Exercise 1.8 | Q 2.2 | Page 21

Using the rules of negation, write the negation of the following:

$\sim(p \vee q) \rightarrow r$

Solution: $\sim[\sim(p \vee q) \rightarrow r] \equiv \sim(p \vee q) \wedge \sim r$ ....[Negation of implication]

$\equiv (\sim p \wedge \sim q) \wedge \sim r$ .....[Negation of disjunction]

Exercise 1.8 | Q 2.3 | Page 21

Using the rules of negation, write the negation of the following:

$(\sim p \wedge q) \wedge (\sim q \vee \sim r)$

Solution: $\sim[(\sim p \wedge q) \wedge (\sim q \vee \sim r)]$

$\equiv \sim(\sim p \wedge q) \vee \sim (\sim q \vee \sim r)$ ...[Negation of conjunction]

$\equiv [\sim(\sim p) \vee \sim q] \vee [\sim(\sim q) \wedge \sim(\sim r)]$ ...[Negation of conjunction and disjunction]

$\equiv (p \vee \sim q) \vee (q \vee r)$ .....[Negation on negation]

Exercise 1.8 | Q 3.1 | Page 21

Write the converse, inverse, and contrapositive of the following statement.

If it snows, then they do not drive the car.

Solution: Let p : It snows.
q : They do not drive the car.

$\therefore$ The given statement is $p \rightarrow q$.

Its converse is $q \rightarrow p$.
If they do not drive the car then it snows.

Its inverse is $\sim p \rightarrow \sim q$.
If it does not snow then they drive the car.

Its contrapositive is $\sim q \rightarrow \sim p$.
If they drive the car then it does not snow.

Exercise 1.8 | Q 3.2 | Page 21

Write the converse, inverse, and contrapositive of the following statement.

If he studies, then he will go to college.

Solution: Let p : He studies.
q : He will go to college.

$\therefore$ The given statement is $p \rightarrow q$.

Its converse is $q \rightarrow p$.
If he will go to college then he studies.

Its inverse is $\sim p \rightarrow \sim q$.
If he does not study then he will not go to college.

Its contrapositive is $\sim q \rightarrow \sim p$.
If he will not go to college then he does not study.

Exercise 1.8 | Q 4.1 | Page 21

With proper justification, state the negation of the following.

$(p \rightarrow q) \vee (p \rightarrow r)$

Solution: $\sim[(p \rightarrow q) \vee (p \rightarrow r)]$

$\equiv \sim(p \rightarrow q) \wedge \sim(p \rightarrow r)$ ...[Negation of disjunction]

$\equiv (p \wedge \sim q) \wedge (p \wedge \sim r)$ ....[Negation of implication]

Exercise 1.8 | Q 4.2 | Page 21

With proper justification, state the negation of the following.

$(p \leftrightarrow q) \vee (\sim q \rightarrow \sim r)$

Solution: $\sim[(p \leftrightarrow q) \vee (\sim q \rightarrow \sim r)]$

$\equiv \sim(p \leftrightarrow q) \wedge (\sim q \rightarrow \sim r)$ ....[Negation of disjunction]

$\equiv [(p \wedge \sim q) \vee (q \wedge \sim p)] \wedge \sim(\sim q \rightarrow \sim r)$ ....[Negation of double implication]

$\equiv [(p \wedge \sim q) \vee (q \wedge \sim p)] \wedge [\sim q \wedge \sim(\sim r)]$ ....[Negation of implication]

$\equiv [(p \wedge \sim q) \vee (q \wedge \sim p)] \wedge (\sim q \wedge r)$ ....[Negation of negation]

Exercise 1.8 | Q 4.3 | Page 21

With proper justification, state the negation of the following.

$(p \rightarrow q) \wedge r$

Solution: $\sim[(p \rightarrow q) \wedge r]$

$\equiv \sim (p \rightarrow q) \vee \sim r$ ....[Negation of conjunction]

$\equiv (p \wedge \sim q) \vee \sim r$ ....[Negation of implication]

EXERCISE 1.9 [PAGE 22]

Exercise 1.9 | Q 1.1 | Page 22

Without using truth table, show that

$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

Solution: L.H.S.

$\equiv p \leftrightarrow q$

$\equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$

$\equiv [\sim p \wedge (\sim q \vee p)] \vee [q \wedge (\sim q \vee p)]$ ....[Distributive law]

$\equiv [(\sim p \wedge \sim q) \vee (\sim p \wedge p)] \vee [(q \wedge \sim q) \vee (q \wedge p)]$ .....[Distributive Law]

$\equiv [(\sim p \wedge \sim q) \vee F] \vee [F \vee (q \wedge p)]$ ....[Complement Law]

≡ (~ p ∧ ~ q) ∨ (q ∧ p) ....[Identity Law]

≡ (p ∧ q) ∨ (~ p ∧ ~ q) ....[Commutative Law]

≡ R.H.S.

Exercise 1.9 | Q 1.2 | Page 22

Without using truth table, show that

p ∧ [(~ p ∨ q) ∨ ~ q] ≡ p

Solution: L.H.S.

≡ p ∧ [(~ p ∨ q) ∨ ~ q]

≡ p ∧ [(~ p ∨ (q ∨ ~ q)] .....[Associative law]

≡ p ∧ (~ p ∨ T) .....[Complement law]

≡ p ∧ T .....[Identity law]

≡ p .....[Identity law]

≡ R.H.S.

Exercise 1.9 | Q 1.3 | Page 22

Without using truth table, show that

~ [(p ∧ q) → ~ q] ≡ p ∧ q

Solution: L.H.S.

≡ ~ [(p ∧ q) → ~ q]

≡ (p ∧ q) ∧ ~ (~ q) ....[Negation of implication]

≡ (p ∧ q) ∧ q .....[Negation of a negation]

≡ p ∧ (q ∧ q) ....[Associative law]

≡ p ∧ q .....[Identity law]

≡ R.H.S.

Exercise 1.9 | Q 1.4 | Page 22

Without using truth table, show that

~r → ~ (p ∧ q) ≡ [~ (q → r)] → ~ p

Solution: L.H.S.

≡ ~r → ~ (p ∧ q)

≡ ~(~ r) ∨ ~ (p ∧ q) ....[p → q ≡ ~ p ∨ q]

≡ r ∨ ~(p ∧ q) ....[Negation of negation]

≡ r ∨ (~p ∨ ~q) ....[De Morgan's law]

≡ ~p ∨ (~q ∨ r) .....[Commutative and associative law]

≡ ~p ∨ (q → r) ....[p → q ≡ ~ p ∨ q]

≡ (q → r) ∨ ~p .....[Commutative law]

≡ ~[~ (q → r)] ∨ ~ p .....[Negation of negation]

≡ [~ (q → r)] → ~ p .....[p → q ≡ ~ p ∨ q]

= R.H.S.

Exercise 1.9 | Q 1.5 | Page 22

Without using truth table, show that

(p ∨ q) → r ≡ (p → r) ∧ (q → r)

Solution: L.H.S.

≡ (p ∨ q) → r

≡ ~ (p ∨ q) ∨ r ....[p → q → ~ p ∨ q]

≡ (~ p ∧ ~ q) ∨ r ....[De Morgan's law]

≡ (~ p ∨ r) ∧ (~ q ∨ r) .....[Distributive law]

≡ (p → r) ∧ (q → r) .....[p → q → ~ p ∨ q]

= R.H.S.

Exercise 1.9 | Q 2.1 | Page 22

Using the algebra of statement, prove that

[p ∧ (q ∨ r)] ∨ [~ r ∧ ~ q ∧ p] ≡ p

Solution: L.H.S.

= [p ∧ (q ∨ r)] ∨ [~ r ∧ ~ q ∧ p]

≡ [p ∧ (q ∨ r)] ∨ [(~ r ∧ ~ q)∧ p] ...[Associative Law]

≡ [p ∧ (q ∨ r)] ∨ [(~q ∧ ~r) ∧ p] ....[Commutative Law]

≡ [p ∧ (q ∨ r)] ∨ [~ (q ∨ r) ∧ p] ....[De Morgan's Law]

≡ [p ∧ (q ∨ r)] ∨ [p ∧ ~(q ∨ r)] .....[Commutative Law]

≡ p ∧ [(q ∨ r) ∨ ~(q ∨ r)] ....[Distributive Law]

≡ p ∧ t .....[Complement Law]

≡ p .....[Identity Law]

= R.H.S.

Exercise 1.9 | Q 2.2 | Page 22

Using the algebra of statement, prove that

(p ∧ q) ∨ (p ∧ ~ q) ∨ (~ p ∧ ~ q) ≡ (p ∨ ~ q)

Solution: L.H.S.

= (p ∧ q) ∨ (p ∧ ~ q) ∨ (~ p ∧ ~ q)

≡ (p ∧ q) ∨ [(p ∧ ~ q) ∨ (~ p ∧ ~ q)] ....[Associative Law]

≡ (p ∧ q) ∨ [(~q ∧ p) ∨ (~ q ∧ ~ p)] ....[Commutative Law]

≡ (p ∧ q) ∨ [~q ∧ (p ∨ ~ p)] ....[Distributive Law]

≡ (p ∧ q) ∨ (~q ∧ t) .....[Complement Law]

≡ (p ∧ q) ∨ (~q) .....[Identity Law]

≡ (p ∨ ~ q) ∧ (q ∨ ~q) .....[Distributive Law]

≡ (p ∨ ~ q) ∧ t ....[Complement Law]

≡ p ∨ ~ q .....[Identity Law]

= R.H.S.

Exercise 1.9 | Q 2.3 | Page 22

Using the algebra of statement, prove that

(p ∨ q) ∧ (~ p ∨ ~ q) ≡ (p ∧ ~ q) ∨ (~ p ∧ q)

Solution: L.H.S.

= (p ∨ q) ∧ (~ p ∨ ~ q)

≡ [(p ∨ q) ∧ ~ p] ∨ [(p ∨ q) ∧ ~ q] .....[Distributive law]

≡ [(p ∧ ~ p) ∨ (q ∧ ~ p)] ∨ [(p ∧ ~ q) ∨ (q ∧ ~ q)] .....[Distributive law]

≡ [F ∨ (q ∧ ~p)] ∨ [(p ∧ ~ q) ∨ F] .....[Complement law]

≡ (q ∧ ~p) ∨ (p ∧ ~ q) .....[Identity law]

≡ (p ∧ ~ q) ∨ (~ p ∧ q) ....[Commutative law]

= R.H.S.

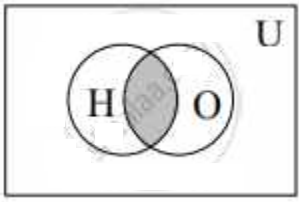
EXERCISE 1.10 [PAGES 22 - 27]

Exercise 1.10 | Q 1.1 | Page 27

Represent the truth of the following statement by the Venn diagram.

Some hardworking students are obedient.

Solution: Let U : The set of all students.
H : The set of all hardworking students.
O : The set of all obedient students.



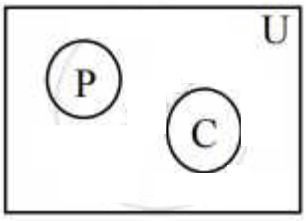
The above Venn diagram represents truth of the given statement, $H \cap O \neq \varphi$

Exercise 1.10 | Q 1.2 | Page 27

Represent the truth of the following statement by the Venn diagram.

No circles are polygons.

Solution: Let U : The set of all closed geometrical figures in plane.
P : The set of all polygons
C : The set of all circles.



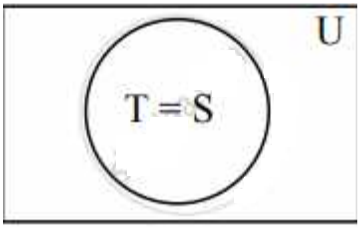
The above Venn diagram represents truth of the given statement, $P \cap C \neq \varphi$

Exercise 1.10 | Q 1.3 | Page 27

Represent the truth of the following statement by the Venn diagram.

All teachers are scholars and scholars are teachers.

Solution: Let U : The set of all human beings.
T : The set of all teachers.
S : The set of all scholars



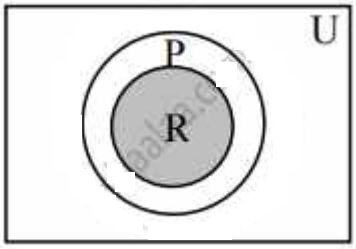
The above Venn diagram represents truth of the given statement, T = S

Exercise 1.10 | Q 1.4 | Page 22

Represent the truth of the following statement by the Venn diagram.

If a quadrilateral is a rhombus, then it is a parallelogram.

Solution: Let U : The set of all quadrilaterals.
P : The set of all parallelograms.
R : The set of all rhombuses.



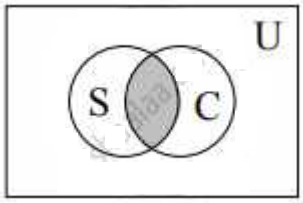
The above Venn diagram represents truth of the given statement, R ⊂ P.

Exercise 1.10 | Q 2.1 | Page 27

Draw a Venn diagram for the truth of the following statement.

Some share brokers are chartered accountants.

Solution: Let U : The set of all human beings.
S : The set of all share brokers.
C : The set of all chartered accountants.



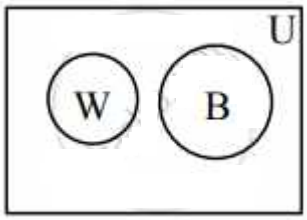
The above Venn diagram represents the truth of the given statement i.e., S ∩ C ≠ φ.

Exercise 1.10 | Q 2.2 | Page 27

Draw a Venn diagram for the truth of the following statement.

No wicket keeper is bowler, in a cricket team.

Solution: Let U : The set of all human beings.
W : The set of all wicket keepers.
B : The set of all bowlers.



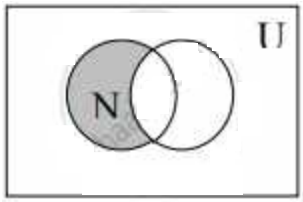
The above Venn diagram represents the truth of the given statement i.e., W ∩ B = φ.

Exercise 1.10 | Q 3.1 | Page 27

Represent the following statement by the Venn diagram.

Some non-resident Indians are not rich.

Solution: Let, U : The set of all human beings.
N : The set of all non-resident Indians.
R : The set of all rich people.



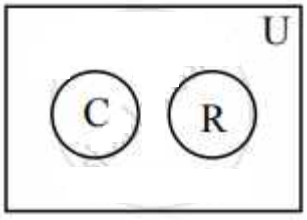
The above Venn diagram represents the truth of the given statement i.e., N - R ≠ φ

Exercise 1.10 | Q 3.2 | Page 27

Represent the following statement by the Venn diagram.

No circle is rectangle.

Solution: Let, U : The set of all geometrical figures.
C : The set of all circles.
R : The set of all rectangles.



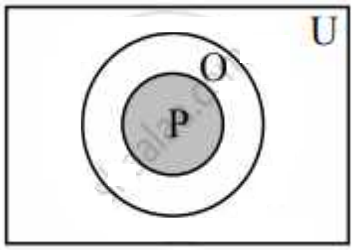
The above Venn diagram represents the truth of the given statement i.e., C ∩ R = φ.

Exercise 1.10 | Q 3.3 | Page 27

Represent the following statement by the Venn diagram.

If n is a prime number and n ≠ 2, then it is odd.

Solution: Let, U : The set of all real numbers.
P : The set of all prime numbers n and n ≠ 2.
O : The set of all odd numbers.



The above Venn diagram represents the truth of the given statement i.e., P ⊂ O.

MISCELLANEOUS EXERCISE 1 [PAGES 29 - 34]

Miscellaneous Exercise 1 | Q 1.01 | Page 29

Choose the correct alternative :

Which of the following is not a statement?

1. Smoking is injuries to health
2. 2 + 2 = 4
3. 2 is the only even prime number.
4. **Come here**

Solution: Come here

Miscellaneous Exercise 1 | Q 1.02 | Page 29

Choose the correct alternative :

Which of the following is an open statement?

1. **x is a natural number.**
2. Give answer a glass of water.
3. WIsh you best of luck.
4. Good morning to all.

Solution: x is a natural number.

Miscellaneous Exercise 1 | Q 1.03 | Page 29

Choose the correct alternative :

Let p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r). Then, this law is known as.

1. commutative law

2. associative law
3. De-Morgan's law
4. **distributive law**

Solution: distributive law.

Miscellaneous Exercise 1 | Q 1.04 | Page 29

Choose the correct alternative :

The false statement in the following is

1. $p \wedge (\sim p)$ is contradiction
2. **$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a contradiction.**
3. $\sim (\sim p) \leftrightarrow p$ is a tautology
4. $p \vee (\sim p) \leftrightarrow p$ is a tautology

Solution: $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a contradiction.

Miscellaneous Exercise 1 | Q 1.05 | Page 29

Choose the correct alternative :

For the following three statements
p : 2 is an even number.
q : 2 is a prime number.
r : Sum of two prime numbers is always even.
Then, the symbolic statement $(p \wedge q) \rightarrow \sim r$ means.

1. 2 is an even and prime number and the sum of two prime numbers is always even.
2. 2 is an even and prime number and the sum of two prime numbers is not always even.
3. **If 2 is an even and prime number, then the sum of two prime numbers is not always even.**
4. If 2 is an even and prime number, then the sum of two prime numbers is also even.

Solution: If 2 is an even and prime number, then the sum of two prime numbers is not always even.

Miscellaneous Exercise 1 | Q 1.06 | Page 30

Choose the correct alternative :

If p : He is intelligent.
q : He is strong
Then, symbolic form of statement “It is wrong that, he is intelligent or strong” is

1. $\sim p \vee \sim p$
2. $\sim (p \wedge q)$
3. **$\sim (p \vee q)$**
4. $p \vee \sim q$

Solution: $\sim (p \vee q)$

Miscellaneous Exercise 1 | Q 1.07 | Page 30

Choose the correct alternative :

The negation of the proposition “If 2 is prime, then 3 is odd”, is

1. If 2 is not prime, then 3 is not odd.
2. **2 is prime and 3 is not odd.**
3. 2 is not prime and 3 is odd.
4. If 2 is not prime, then 3 is odd.

Solution: 2 is prime and 3 is not odd.

Miscellaneous Exercise 1 | Q 1.08 | Page 30

Choose the correct alternative :

The statement $(\sim p \wedge q) \vee \sim q$ is

1. $p \vee q$
2. $p \wedge q$
3. $\sim (p \vee q)$
4. **$\sim (p \wedge q)$**

Solution: $\sim (p \wedge q)$.

Miscellaneous Exercise 1 | Q 1.09 | Page 30

Choose the correct alternative :

Which of the following is always true?

1. $(p \rightarrow q) \equiv \sim q \rightarrow \sim p$
2. $\sim (p \vee q) \equiv \sim p \vee \sim q$
3. **$\sim (p \rightarrow q) \equiv p \wedge \sim q$**

4. $\sim (p \vee q) \equiv \sim p \wedge \sim q$

Solution: $\sim (p \rightarrow q) \equiv p \wedge \sim q$.

Miscellaneous Exercise 1 | Q 1.1 | Page 30

Choose the correct alternative :

$\sim (p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to

1. **$\sim p$**
2. p
3. q
4. $\sim q$

Solution: $\sim p$.

Miscellaneous Exercise 1 | Q 1.11 | Page 30

Choose the correct alternative :

If p and q are two statements then $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is

1. contradiction
2. **tautology**
3. Neither (i) not (ii)
4. None of the these

Solution: tautology.

Miscellaneous Exercise 1 | Q 1.12 | Page 30

Choose the correct alternative :

If p is the sentence 'This statement is false' then

1. truth value of p is T
2. truth value of p is F
3. p is both true and false
4. **p is neither true nor false**

Solution: p is neither true nor false

Miscellaneous Exercise 1 | Q 1.13 | Page 30

Choose the correct alternative :

Conditional $p \rightarrow q$ is equivalent to

1. $p \rightarrow \sim q$
2. **$\sim p \vee q$**
3. $\sim p \rightarrow \sim q$
4. $p \vee \sim q$

Solution: $\sim p \vee q$.

Miscellaneous Exercise 1 | Q 1.14 | Page 30

Choose the correct alternative :

Negation of the statement “This is false or That is true” is

1. That is true or This is false
2. That is true and This is false
3. **That is true and That is false**
4. That is false and That is true

Solution: That is true and That is false.

Miscellaneous Exercise 1 | Q 1.15 | Page 30

Choose the correct alternative :

If p is any statement then $(p \vee \sim p)$ is a

1. contingency
2. contradiction
3. **tautology**
4. None of them

Solution: tautology.

Miscellaneous Exercise 1 | Q 2.1 | Page 30

Fill in the blanks :

The statement $q \rightarrow p$ is called as the ———— of the statement $p \rightarrow q$.

Solution: The statement $q \rightarrow p$ is called as the **Converse** of the statement $p \rightarrow q$.

Miscellaneous Exercise 1 | Q 2.2 | Page 30

Fill in the blanks :

Conjunction of two statement p and q is symbolically written as ————.

Solution: Conjunction of two statement p and q is symbolically written as **$p \wedge q$**.

Miscellaneous Exercise 1 | Q 2.3 | Page 30

Fill in the blanks :

If p ∨ q is true then truth value of ~ p ∨ ~ q is ————————.

Solution: If p ∨ q is true then truth value of ~ p ∨ ~ q is **F**.

Miscellaneous Exercise 1 | Q 2.4 | Page 30

Fill in the blanks :

Negation of “some men are animal” is ————————.

Solution: Negation of “some men are animal” is **No men are animals**.

Miscellaneous Exercise 1 | Q 2.5 | Page 30

Fill in the blanks :

Truth value of if x = 2, then x2 = − 4 is ————————.

Solution: Truth value of if x = 2, then $x^2 = -4$ is **F**.

Miscellaneous Exercise 1 | Q 2.6 | Page 30

Fill in the blanks :

Inverse of statement pattern p ↔ q is given by ————————.

Solution: Inverse of statement pattern p ↔ q is given by **~ p → ~ q**.

Miscellaneous Exercise 1 | Q 2.7 | Page 30

Fill in the blanks :

p ↔ q is false when p and q have ———————— truth values.

Solution: p ↔ q is false when p and q have **different** truth values.

Miscellaneous Exercise 1 | Q 2.8 | Page 31

Fill in the blanks :

Let p : the problem is easy. r : It is not challenging then verbal form of ~ p → r is ————————.

Solution: Let p : the problem is easy. r : It is not challenging then verbal form of ~ p → r is **If the problem is not easy them it is not challenging**.

Miscellaneous Exercise 1 | Q 2.9 | Page 31

Fill in the blanks :

Truth value of 2 + 3 = 5 if and only if − 3 > − 9 is ————————.

Solution: Truth value of 2 + 3 = 5 if and only if − 3 > − 9 is **T**.

Miscellaneous Exercise 1 | Q 3.01 | Page 31

State whether the following statement is True or False :

Truth value of 2 + 3 < 6 is F.

1. True
2. **False**

Solution: False**.**

Miscellaneous Exercise 1 | Q 3.02 | Page 31

State whether the following statement is True or False :

There are 24 months in year is a statement.

1. **True**
2. False

Solution: True

Miscellaneous Exercise 1 | Q 3.03 | Page 31

State whether the following statement is True or False :

p ∨ q has truth value F is both p and q has truth value F.

1. True
2. **False**

Solution: False**.**

Miscellaneous Exercise 1 | Q 3.04 | Page 31

State whether the following statement is True or False :

The negation of 10 + 20 = 30 is, it is false that 10 + 20 ≠ 30.

1. True
2. **False**

Solution: False**.**

Miscellaneous Exercise 1 | Q 3.05 | Page 31

State whether the following statement is True or False :

Dual of $(p \wedge \sim q) \vee t$ is $(p \vee \sim q) \vee C$.

1. True
2. **False**

Solution: False**.**

Miscellaneous Exercise 1 | Q 3.06 | Page 31

State whether the following statement is True or False :

Dual of “John and Ayub went to the forest” is “John and Ayub went to the forest”.

1. **True**
2. False

Solution: True**.**

Miscellaneous Exercise 1 | Q 3.07 | Page 31

State whether the following statement is True or False :

“His birthday is on 29th February” is not a statement.

1. **True**
2. False

Solution: True**.**

Miscellaneous Exercise 1 | Q 3.08 | Page 31

State whether the following statement is True or False :

$x^2 = 25$ is true statement.

1. True
2. **False**

Solution: False**.**

Miscellaneous Exercise 1 | Q 3.09 | Page 31

State whether the following statement is True or False :

Truth value of $\sqrt{5}$ is not an irrational number is T.

1. True

2. **False**

Solution: False**.**

Miscellaneous Exercise 1 | Q 3.1 | Page 31

State whether the following statement is True or False :

$p \wedge t = p$.

1. **True**
2. False

Solution: True**.**

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
Ice cream Sundaes are my favourite.

1. **Is a statement**
2. Is not a statement

Solution: Is a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
$x + 3 = 8$; x is variable.

1. **Is a statement**
2. Is not a statement

Solution: Is a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
Read a lot to improve your writing skill.

1. Is a statement
2. **Is not a statement**

Solution: Is not a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
z is a positive number.

1. **Is a statement**
2. Is not a statement

Solution: Is a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
$(a + b)^2 = a^2 + 2ab + b^2$ for all $a, b \in R$.

1. **Is a statement**
2. Is not a statement

Solution: Is a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
$(2 + 1)^2 = 9$.

1. **Is a statement**
2. Is not a statement

Solution: Is a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
Why are you sad?

1. Is a statement
2. **Is not a statement**

Solution: Is not a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
How beautiful the flower is!

1. Is a statement
2. **Is not a statement**

Solution: Is not a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
The square of any odd number is even.

1. **Is a statement**
2. Is not a statement

Solution: Is a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
All integers are natural numbers.

1. **Is a statement**
2. Is not a statement

Solution: Is a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
If x is real number then $x^2 \geq 0$.

1. **Is a statement**
2. Is not a statement

Solution: Is a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.
Do not come inside the room.

1. Is a statement
2. **Is not a statement**

Solution: Is not a statement

Miscellaneous Exercise 1 | Q 4.01 | Page 31

Solve the following :

State which of the following sentences are statements in logic.

What a horrible sight it was!

1. Is a statement
2. **Is not a statement**

Solution: Is not a statement

Miscellaneous Exercise 1 | Q 4.02 | Page 31

Which of the following sentence is a statement? In case of a statement, write down the truth value.

The square of every real number is positive.

1. **Is a statement**
2. Is not a statement

Solution: It is a statement which is false. Hence, its truth value is F.

Miscellaneous Exercise 1 | Q 4.02 | Page 31

Which of the following sentence is a statement? In case of a statement, write down the truth value.

Every parallelogram is a rhombus.

1. **Is a statement**
2. Is not a statement

Solution: It is a statement which is false. Hence, its truth value is F.

Miscellaneous Exercise 1 | Q 4.02 | Page 31

Which of the following sentence is a statement? In case of a statement, write down the truth value.

$a^2 - b^2 = (a + b)(a - b)$ for all $a, b \in R$.

1. **Is a statement**
2. Is not a statement

Solution: It is a statement which is true. Hence, its truth value is T.

Miscellaneous Exercise 1 | Q 4.02 | Page 31

Which of the following sentence is a statement? In case of a statement, write down the truth value.

Please carry out my instruction.

1. Is a statement
2. **Is not a statement**

Solution: It is an imperative sentence. Hence, it is not a statement.

Miscellaneous Exercise 1 | Q 4.02 | Page 31

Which of the following sentence is a statement? In case of a statement, write down the truth value.

The Himalayas is the highest mountain range.

1. **Is a statement**
2. Is not a statement

Solution: It is a statement which is true. Hence, its truth value is T.

Miscellaneous Exercise 1 | Q 4.02 | Page 31

Which of the following sentence is a statement? In case of a statement, write down the truth value.

$(x - 2)(x - 3) = x^2 - 5x + 6$ for all $x \in R$.

1. **Is a statement**
2. Is not a statement

Solution: It is a statement which is true. Hence, its truth value is T.

Miscellaneous Exercise 1 | Q 4.02 | Page 31

Which of the following sentence is a statement? In case of a statement, write down the truth value.

What are the causes of rural unemployment?

1. Is a statement

2. **Is not a statement**

Solution: It is an interrogative sentence. Hence, it's not a statement.

Miscellaneous Exercise 1 | Q 4.02 | Page 31

Which of the following sentence is a statement? In case of a statement, write down the truth value.

0! = 1

1. **Is a statement**
2. Is not a statement

Solution: It is a statement which is true. Hence, its truth value is T.

Miscellaneous Exercise 1 | Q 4.02 | Page 31

Which of the following sentence is a statement? In case of a statement, write down the truth value.

The quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ always has two real roots.

1. **Is a statement**
2. Is not a statement

Solution: The quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ always has two real roots is a statement.

Hence, its truth value is F.

Miscellaneous Exercise 1 | Q 4.02 | Page 31

Which of the following sentence is a statement? In case of a statement, write down the truth value.

What is happy ending?

1. Is a statement
2. **Is not a statement**

Solution: It is an interrogative sentence. Hence, it's not a statement.

Miscellaneous Exercise 1 | Q 4.03 | Page 31

Assuming the first statement p and second as q. Write the following statement in symbolic form.

The Sun has set and Moon has risen.

Solution: Let p : The sun has set.
q : The moon has risen

The symbolic form is $p \wedge q$.

Miscellaneous Exercise 1 | Q 4.03 | Page 31

Assuming the first statement p and second as q. Write the following statement in symbolic form.

Mona likes Mathematics and Physics.

Solution: Let p : Mona likes Mathematics
q : Mona likes Physics

The symbolic form is $p \wedge q$.

Miscellaneous Exercise 1 | Q 4.03 | Page 31

Assuming the first statement p and second as q. Write the following statement in symbolic form.

3 is prime number if 3 is perfect square number.

Solution: Let p : 3 is a prime number.
q : 3 is a perfect square number.

The symbolic form is $p \leftrightarrow q$.

Miscellaneous Exercise 1 | Q 4.03 | Page 31

Assuming the first statement p and second as q. Write the following statement in symbolic form.

Kavita is brilliant and brave.

Solution: Let p : Kavita is brilliant.
q : Kavita is brave.

The symbolic form is $p \wedge q$.

Miscellaneous Exercise 1 | Q 4.03 | Page 31

Assuming the first statement p and second as q. Write the following statement in symbolic form.

If Kiran drives the car, then Sameer will walk.

Solution: Let p : Kiran drives the car.

q : Sameer will walk.

The symbolic form is $p \rightarrow q$.

Miscellaneous Exercise 1 | Q 4.03 | Page 31

Assuming the first statement p and second as q. Write the following statement in symbolic form.

The necessary condition for existence of a tangent to the curve of the function is continuity.

Solution: The given statement can also be expressed as 'If the function is continuous, then the tangent to the curve exists'.

Let p : The function is continuous
q : The tangent to the curve exists.

∴ $p \rightarrow q$ is the symbolic form of the given statement.

[Note: Answer in the textbook is incorrect.]

Miscellaneous Exercise 1 | Q 4.03 | Page 31

Assuming the first statement p and second as q. Write the following statement in symbolic form.

To be brave is necessary and sufficient condition to climb the Mount Everest.

Solution: Assuming the first statement p and second as q. Write the following statement in symbolic form.

To be brave is necessary and sufficient condition to climb the Mount Everest.

Miscellaneous Exercise 1 | Q 4.03 | Page 31

Assuming the first statement p and second as q. Write the following statement in symbolic form.

$x^3 + y^3 = (x + y)^3$ if $xy = 0$.

Solution: Let p : $x^3 + y^3 = (x + y)^3$

q : $xy = 0$

∴ $p \leftrightarrow q$ is the symbolic form of the given statement.

Miscellaneous Exercise 1 | Q 4.03 | Page 31

Assuming the first statement p and second as q. Write the following statement in symbolic form.

The drug is effective though it has side effects.

Solution: The given statement can also be expressed as "The drug is effective and it has side effects"

Let p : The drug is effective.
q : It has side effects.

∴ $p \wedge q$ is the symbolic form of the given statement.

Miscellaneous Exercise 1 | Q 4.03 | Page 32

Assuming the first statement p and second as q. Write the following statement in symbolic form.

If a real number is not rational, then it must be irrational.

Solution: Let p : A real number is not rational.
q : A real number must be irrational.

The symbolic form is $p \rightarrow q$.

Miscellaneous Exercise 1 | Q 4.03 | Page 32

Assuming the first statement p and second as q. Write the following statement in symbolic form.

It is not true that Ram is tall and handsome.

Solution: Let p : Ram is tall.
q : Ram is handsome.

The symbolic form is $\sim(p \wedge q)$.

Miscellaneous Exercise 1 | Q 4.03 | Page 32

Assuming the first statement p and second as q. Write the following statement in symbolic form.

Even though it is not cloudy, it is still raining.

Solution: Let p : it is cloudy.
q : It is still raining.

The symbolic form is $\sim p \wedge q$.

Miscellaneous Exercise 1 | Q 4.03 | Page 32

Assuming the first statement p and second as q. Write the following statement in symbolic form.

It is not true that intelligent persons are neither polite nor helpful.

Solution: Let p : Intelligent persons are neither polite nor helpful

The symbolic form is $\sim p$.

Alternate method:

Let p : Intelligent persons are polite.
q : Intelligent persons are helpful.
The symbolic form is $\sim(\sim p \wedge \sim q)$.

Miscellaneous Exercise 1 | Q 4.03 | Page 32

Assuming the first statement p and second as q. Write the following statement in symbolic form.

If the question paper is not easy then we shall not pass.

Solution: Let p : The question paper is not easy.
q : We shall not pass.

The symbolic form is $p \rightarrow q$.

Miscellaneous Exercise 1 | Q 4.04 | Page 32

If p : Proof is lengthy.
q : It is interesting.
Express the following statement in symbolic form.

Proof is lengthy and it is not interesting.

Solution: $p \wedge \sim q$

Miscellaneous Exercise 1 | Q 4.04 | Page 32

If p : Proof is lengthy.
q : It is interesting.
Express the following statement in symbolic form.

If proof is lengthy then it is interesting.

Solution: $p \rightarrow q$

Miscellaneous Exercise 1 | Q 4.04 | Page 32

If p : Proof is lengthy.
q : It is interesting.
Express the following statement in symbolic form.

It is not true that the proof is lengthy but it is interesting.

Solution: $\sim(p \wedge q)$

Miscellaneous Exercise 1 | Q 4.04 | Page 32

If p : Proof is lengthy.
q : It is interesting.
Express the following statement in symbolic form.

It is interesting iff the proof is lengthy.

Solution: $q \leftrightarrow p$

Miscellaneous Exercise 1 | Q 4.05 | Page 32

Let p : Sachin wins the match.
q : Sachin is a member of Rajya Sabha.
r : Sachin is happy.
Write the verbal statement of the following.

$(p \wedge q) \vee r$

Solution: Sachin wins the match or he is the member of Rajya Sabha or Sachin is happy.

Miscellaneous Exercise 1 | Q 4.05 | Page 32

Let p : Sachin wins the match.
q : Sachin is a member of Rajya Sabha.
r : Sachin is happy.
Write the verbal statement of the following.

$p \rightarrow r$

Solution: If Sachin wins the match then he is happy.

Miscellaneous Exercise 1 | Q 4.05 | Page 32

Let p : Sachin wins the match.
q : Sachin is a member of Rajya Sabha.
r : Sachin is happy.
Write the verbal statement of the following.

$\sim p \vee q$

Solution: Sachin does not win the match or he is the member of Rajya Sabha.

Miscellaneous Exercise 1 | Q 4.05 | Page 32

Let p : Sachin wins the match.
q : Sachin is a member of Rajya Sabha.
r : Sachin is happy.
Write the verbal statement of the following.

$p \rightarrow (p \wedge r)$

Solution: If sachin wins the match, then he is the member of Rajyasabha or he is happy.

Miscellaneous Exercise 1 | Q 4.05 | Page 32

Let p : Sachin wins the match.
q : Sachin is a member of Rajya Sabha.

r : Sachin is happy.
Write the verbal statement of the following.

$p \rightarrow q$

Solution: If Sachin wins the match then he is a member of Rajyasabha.

Miscellaneous Exercise 1 | Q 4.05 | Page 32

Let p : Sachin wins the match.
q : Sachin is a member of Rajya Sabha.
r : Sachin is happy.
Write the verbal statement of the following.

$(p \wedge q) \wedge \sim r$

Solution: Sachin wins the match and he is the member of Rajyasabha but he is not happy.

Miscellaneous Exercise 1 | Q 4.05 | Page 32

Let p : Sachin wins the match.
q : Sachin is a member of Rajya Sabha.
r : Sachin is happy.
Write the verbal statement of the following.

$\sim (p \vee q) \wedge r$

Solution: It is false that sachin wins the match or he is the member of Rajyasabha but he is happy.

Miscellaneous Exercise 1 | Q 4.06 | Page 32

Determine the truth value of the following statement.

$4 + 5 = 7$ or $9 - 2 = 5$

Solution: Let p : $4 + 5 = 7$
q : $9 – 2 = 5$
The truth values of p and q are F and F respectively. The given statement in symbolic form is $p \vee q$.

$\therefore p \vee q \equiv F \vee F \equiv F$

$\therefore$ Truth value of the given statement is F.

Miscellaneous Exercise 1 | Q 4.06 | Page 32

Determine the truth value of the following statement.

If $9 > 1$ then $x^2 - 2x + 1 = 0$ for $x = 1$

Solution: Let p : 9 > 1
q : $x^2 - 2x + 1 = 0$ for x = 1

The truth values of p and q are T and T respectively. The given statement in symbolic form is $p \rightarrow q$.

$\therefore p \rightarrow q \equiv T \rightarrow T \equiv T$

∴ Truth value of the given statement is T.

Miscellaneous Exercise 1 | Q 4.06 | Page 32

Determine the truth value of the following statement.

x + y = 0 is the equation of a straight line if and only if $y^2 = 4x$ is the equation of the parabola.

Solution: Let p : x + y = 0 is the equation of a straight line.
q : $y^2 = 4x$ is the equation of the parabola.

The truth values of p and q are T and T respectively.
The given statement in symbolic form is $p \leftrightarrow q$.

$\therefore p \leftrightarrow q \equiv T \leftrightarrow T \equiv T$

∴ Truth value of the given statement is T.

Miscellaneous Exercise 1 | Q 4.06 | Page 32

Determine the truth value of the following statement.

It is not true that 2 + 3 = 6 or 12 + 3 =5

Solution: Let p : 2 + 3 = 6

q : 12 + 3 = 5

The truth values of p and q are F and F respectively.

The given statement in symbolic form is $\sim(p \vee q)$.

$\therefore \sim(p \vee q) \equiv \sim(F \vee F) \equiv \sim F \equiv T$

∴ Truth value of the given statement is T.

Miscellaneous Exercise 1 | Q 4.07 | Page 32

Assuming the following statement.
p : Stock prices are high.
q : Stocks are rising.
to be true, find the truth value of the following.

Stock prices are not high or stocks are rising.

Solution: Given that the truth values of both p and q are T.

The symbolic form of the given statement is ~ p ∨ q.

∴ ~ p ∨ q ≡ ~ T ∨ T ≡ F ∨ T

Hence, truth value is T.

Miscellaneous Exercise 1 | Q 4.07 | Page 32

Assuming the following statement.
p : Stock prices are high.
q : Stocks are rising.
to be true, find the truth value of the following.

Stock prices are high and stocks are rising if and only if stock prices are high.

Solution: The symbolic form of the given statement is

(p ∧ q) ↔ p.

∴ (p ∧ q) ↔ p ≡ (T ∧ T) ↔ T

≡ T ↔ T

≡ T

Hence, truth value is T.

Miscellaneous Exercise 1 | Q 4.07 | Page 32

Assuming the following statement.
p : Stock prices are high.
q : Stocks are rising.
to be true, find the truth value of the following.

If stock prices are high then stocks are not rising.

Solution: The Symbolic form of the given statement is p → ~ q.

∴ p → ~ q ≡ T → ~ T ≡ T → F ≡ F

Hence, truth value is F.

Miscellaneous Exercise 1 | Q 4.07 | Page 32

Assuming the following statement.
p : Stock prices are high.
q : Stocks are rising.
to be true, find the truth value of the following.

It is false that stocks are rising and stock prices are high.

Solution: The symbolic form of the given statement is ~(q ∧ p).

∴ ~(q ∧ p) ≡ ~(T ∧ T) ≡ ~T ≡ F

Hence, truth value is F.

Miscellaneous Exercise 1 | Q 4.07 | Page 32

Assuming the following statement.

p : Stock prices are high.

q : Stocks are rising.

to be true, find the truth value of the following.

Stock prices are high or stocks are not rising iff stocks are rising.

Solution: The symbolic form of the given statement is (p ∨ ~q) ↔ q.

∴ (p ∨ ~q) ↔ q ≡ (T ∨ ~T) ↔ T

≡ (T ∨ F) ↔ T

≡ T ↔ T

≡ T

Hence, truth value is T.

Miscellaneous Exercise 1 | Q 4.08 | Page 32

Rewrite the following statement without using conditional –
(Hint : p → q ≡ ~ p ∨ q)

If price increases, then demand falls.

Solution: Let p : Prince increases.
q : demand falls.
The given statement is p → q.
But p → q ≡ ~p ∨ q.

The given statement can be written as ‘Price does not increase or demand falls’.

Miscellaneous Exercise 1 | Q 4.08 | Page 32

Rewrite the following statement without using conditional –
(Hint : p → q ≡ ~ p ∨ q)

If demand falls, then price does not increase.

Solution: Let p : demand falls.

q : Price does not increase.

The given statement is $p \rightarrow q$.

But $p \rightarrow q \equiv \sim p \vee q$.

∴ The given statement can be written as 'Demand does not fall or price does not increase'.

Miscellaneous Exercise 1 | Q 4.09 | Page 32

If p, q, r are statements with truth values T, T, F respectively determine the truth values of the following.

$(p \wedge q) \rightarrow \sim p$.

Solution: $(p \wedge q) \rightarrow \sim p \equiv (T \wedge T) \rightarrow \sim T$

$\equiv T \rightarrow F$

$\equiv F$.

Hence, truth value is F.

Miscellaneous Exercise 1 | Q 4.09 | Page 32

If p, q, r are statements with truth values T, T, F respectively determine the truth values of the following.

$p \leftrightarrow (q \rightarrow \sim p)$

Solution: $p \leftrightarrow (q \rightarrow \sim p) \equiv T \leftrightarrow (T \rightarrow \sim T)$

$\equiv T \leftrightarrow (T \rightarrow F)$

$\equiv T \leftrightarrow F$

$\equiv F$

Hence, truth value is F.

Miscellaneous Exercise 1 | Q 4.09 | Page 32

If p, q, r are statements with truth values T, T, F respectively determine the truth values of the following.

$(p \wedge \sim q) \vee (\sim p \wedge q)$

Solution: $(p \wedge \sim q) \vee (\sim p \wedge q) \equiv (T \wedge \sim T) \vee (\sim T \wedge T)$

$\equiv (T \wedge F) \vee (F \wedge T)$

$\equiv F \vee F$

$\equiv F$

Hence, truth value is F.

Miscellaneous Exercise 1 | Q 4.09 | Page 32

If p, q, r are statements with truth values T, T, F respectively determine the truth values of the following.

$\sim (p \wedge q) \rightarrow \sim (q \wedge p)$

Solution: $\sim (p \wedge q) \rightarrow \sim (q \wedge p) \equiv \sim (T \wedge T) \rightarrow \sim (T \wedge T)$

$\equiv \sim T \rightarrow \sim T$

$\equiv F \rightarrow F$

$\equiv T$

Hence, truth value is T.

Miscellaneous Exercise 1 | Q 4.09 | Page 32

If p, q, r are statements with truth values T, T, F respectively determine the truth values of the following.

$\sim [(p \rightarrow q) \leftrightarrow (p \wedge \sim q)]$

Solution: $\sim[(p \rightarrow q) \leftrightarrow (p \wedge \sim q)] \equiv \sim [(T \rightarrow T) \leftrightarrow (T \wedge \sim T)]$

$\equiv \sim[T \leftrightarrow (T \wedge F)]$

$\equiv \sim(T \leftrightarrow F)$

$\equiv \sim F$

$\equiv T$

Hence, truth value is T.

Miscellaneous Exercise 1 | Q 4.1 | Page 32

Write the negation of the following.

If ΔABC is not equilateral, then it is not equiangular.

Solution: Let p : Δ ABC is not equilateral.
q : Δ ABC is not equiangular.

The given statement is $p \rightarrow q$.

Its negation is $\sim(p \rightarrow q) \equiv p \wedge \sim q$

$\therefore$ The negation of given statement is 'Δ ABC is not equilateral and it is equiangular'.

Miscellaneous Exercise 1 | Q 4.1 | Page 32

Write the negation of the following.

Ramesh is intelligent and he is hard working.

Solution: Let p : Ramesh is intelligent.
q : Ramesh is hard working.
The given statement is $p \wedge q$.

Its negation is $\sim(p \wedge q) \equiv \sim p \vee \sim q$

∴ The negation of the given statement is 'Ramesh is not intelligent or he is not hard-working.'

Miscellaneous Exercise 1 | Q 4.1 | Page 32

Write the negation of the following.

An angle is a right angle if and only if it is of measure 90°.

Solution: Let p : An angle is a right angle.
q : An angle is of measure 90°.

The given statement is $p \leftrightarrow q$.

Its negation is $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

∴ The negation of the given statement is 'An angle is a right angle and it is not of measure 90° or an angle is of measure 90° and it is not a right angle.'

Miscellaneous Exercise 1 | Q 4.1 | Page 32

Write the negation of the following.

Kanchanganga is in India and Everest is in Nepal.

Solution: Let p : Kanchanganga is in India.
q : Everest is in Nepal.
The given statement is $p \wedge q$.

Its negation is $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

The negation of a given statement is 'Kanchanganga is not in India or Everest is not in Nepal'.

Miscellaneous Exercise 1 | Q 4.1 | Page 32

Write the negation of the following.

If $x \in A \cap B$, then $x \in A$ and $x \in B$.

Solution: Let p : $x \in A \cap B$

q : $x \in A$

r : $x \in B$

The given statement is $p \rightarrow (q \wedge r)$.

Its negation is $\sim[p \rightarrow (q \wedge r)]$, and

$\sim[p \rightarrow (q \wedge r)] \equiv p \wedge \sim (q \wedge r) \equiv p \wedge \sim q \vee \sim r$

$\therefore$ The negation of given statement is $x \in A \cap B$ and $x \notin A$ or $x \notin B$.

Miscellaneous Exercise 1 | Q 4.11 | Page 33

Construct the truth table for the following statement pattern.

$(p \wedge \sim q) \leftrightarrow (q \rightarrow p)$

Solution:

p	q	~q	p∧~q	q→p	(p∧~q)↔(q→p)
T	T	F	F	T	F
T	F	T	T	T	T
F	T	F	F	F	T
F	F	T	F	T	F

Miscellaneous Exercise 1 | Q 4.11 | Page 33

Construct the truth table for the following statement pattern.

$(\sim p \vee q) \wedge (\sim p \wedge \sim q)$

Solution:

p	q	~p	~q	~p∨q	~p∧~q	(~p∨q)∧(~p∧~q)
T	T	F	F	T	F	F
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Miscellaneous Exercise 1 | Q 4.11 | Page 33

Construct the truth table for the following statement pattern.

$(p \wedge r) \rightarrow (p \vee \sim q)$

Solution:

p	q	r	~q	p∧r	p∨~q	(p∧r)→(p∨~q)
T	T	T	F	T	T	T
T	T	F	F	F	T	T

T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

Miscellaneous Exercise 1 | Q 4.11 | Page 33

Construct the truth table for the following statement pattern.

$(p \vee r) \rightarrow \sim(q \wedge r)$

Solution:

p	q	r	p∨r	q∧r	~q∧r)	(p∨r)→~(q ∧ r)
T	T	T	T	T	F	F
T	T	F	T	F	T	T
T	F	T	T	F	T	T
T	F	F	T	F	T	T
F	T	T	T	T	F	F
F	T	F	F	F	T	T
F	F	T	T	F	T	T
F	F	F	F	F	T	T

Miscellaneous Exercise 1 | Q 4.11 | Page 33

Construct the truth table for the following statement pattern.

$(p \vee \sim q) \rightarrow (r \wedge p)$

Solution:

p	q	r	~q	p∨~q	r∧p	(p∨~q)→(r∧p)
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T

F	T	F	F	F	F	T
F	F	T	T	T	F	F
F	F	F	T	T	F	F

Miscellaneous Exercise 1 | Q 4.12 | Page 33

What is tautology? What is contradiction?

Show that the negation of a tautology is a contradiction and the negation of a contradiction is a tautology.

Solution:

- **Tautology:**
  A statement pattern having truth value always T, irrespective of the truth values of its component statement is called a tautology.
- **Contradiction:**
  A statement pattern having truth value always F, irrespective of the truth values of its component statement is called a contradiction.

Let Statement p tautology. Consider, truth table

p	~ p
T	F

i.e., negation of tautology is contradiction.
Let statement of contradiction. Consider, truth table

q	~ q
F	T

i.e., negation of contradiction is tautology.

Miscellaneous Exercise 1 | Q 4.13 | Page 33

Determine whether the following statement pattern is a tautology, contradiction, or contingency.

$[(p \wedge q) \vee (\sim p)] \vee [p \wedge (\sim q)]$

Solution:

p	**q**	**~p**	**~q**	**p∧q**	**(p∧q)∨(~p)**	**p∧~q**	**[(p∧q)∨(~p)]∨[p∧(~q)]**
T	T	F	F	T	T	F	T
T	F	F	T	F	F	T	T
F	T	T	F	F	T	F	T
F	F	T	T	F	T	F	T

All the truth values in the last column are T. Hence, it is a **tautology**.

Miscellaneous Exercise 1 | Q 4.13 | Page 33

Determine whether the following statement pattern is a tautology, contradiction, or contingency.

$[(\sim p \wedge q) \wedge (q \wedge r)] \vee (\sim q)$

Solution:

p	q	r	~p	~q	~p∧q	q∧r	(~p∧q)∧(q∧r)	[(~p∧q)∧(q∧r)]∨(~q)
T	T	T	F	F	F	T	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	T	F	F	F	T
T	F	F	F	T	F	F	F	T
F	T	T	T	F	T	T	T	T
F	T	F	T	F	T	F	F	F
F	F	T	T	T	F	F	F	T
F	F	F	T	T	F	F	F	T

Truth values in the last column are not identical. Hence, it is **contingency**.

Miscellaneous Exercise 1 | Q 4.13 | Page 33

Determine whether the following statement pattern is a tautology, contradiction, or contingency.

$[\sim(p \vee q) \rightarrow p] \leftrightarrow [(\sim p) \wedge (\sim q)]$

Solution:

p	q	~p	~q	p∨q	~(p∨q)	~(p∨q)→p	(~p)∧(~q)	[~(p∨q)→p]↔[(~p)∧(~q)]
T	T	F	F	T	F	T	F	F
T	F	F	T	T	F	T	F	F
F	T	T	F	T	F	T	F	F
F	F	T	T	F	T	F	T	F

All the truth values in the last column are F. Hence, it is a **contradiction**.

Miscellaneous Exercise 1 | Q 4.13 | Page 33

Determine whether the following statement pattern is a tautology, contradiction, or contingency.

$[\sim(p \wedge q) \rightarrow p] \leftrightarrow [(\sim p) \wedge (\sim q)]$

Solution:

p	q	~p	~q	p∧q	~(p∧q)	~(p∧q)→p	(~p)∧(~q)	[~(p∧q)→p]↔[(~p)∧(~q)]
T	T	F	F	T	F	T	F	F
T	F	F	T	F	T	T	F	F
F	T	T	F	F	T	F	F	T
F	F	T	T	F	T	F	T	F

Truth values in the last column are not identical. Hence, it is **contingency**.

Miscellaneous Exercise 1 | Q 4.13 | Page 33

Determine whether the following statement pattern is a tautology, contradiction, or contingency.

$[P \rightarrow (\sim q \vee r)] \leftrightarrow \sim[p \rightarrow (q \rightarrow r)]$

Solution:

p	q	r	~q	~q∨r	q→r	p→(q→r)	P→(~q∨r)	~[p→(q→r)]	[P→(~q∨r)]↔~[p → (q → r)]
T	T	T	F	T	T	T	T	F	F
T	T	F	F	F	F	F	F	T	F
T	F	T	T	T	T	T	T	F	F
T	F	F	T	T	T	T	T	F	F
F	T	T	F	T	T	T	T	F	F
F	T	F	F	F	F	T	T	F	F
F	F	T	T	T	T	T	T	F	F
F	F	F	T	T	T	T	T	F	F

All the truth values in the last column are F. Hence, it is **contradiction**.

Miscellaneous Exercise 1 | Q 4.14 | Page 33

Using the truth table, prove the following logical equivalence.

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Solution:

1	2	3	4	5	6	7	8
p	**q**	**r**	**q∨r**	**p∧(q∨r)**	**p∧q**	**p∧r**	**(p∧q)∨(p∧r)**
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

In the above truth table, the entries in columns 5 and 8 are identical.

$\therefore p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Miscellaneous Exercise 1 | Q 4.14 | Page 33

Using the truth table, prove the following logical equivalence.

$[\sim(p \vee q) \vee (p \vee q)] \wedge r \equiv r$

Solution:

1	2	3	4	5	6	7
p	**q**	**r**	**p∨q**	**~(p∨q)**	**[~(p∨q)∨(p∨q)]**	**[~(p∨q)∨(p∨q)]∧r**
T	T	T	T	F	T	T
T	T	F	T	F	T	F
T	F	T	T	F	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F

F	F	T	F	T	T	T
F	F	F	F	T	T	F

In the above truth table, the entries in columns 3 and 7 are identical.

∴ [~(p ∨ q) ∨ (p ∨ q)] ∧ r ≡ r

Miscellaneous Exercise 1 | Q 4.14 | Page 33

Using the truth table, prove the following logical equivalence.

p ∧ (~p ∨ q) ≡ p ∧ q

Solution:

1	2	3	4	5	6
p	**q**	**~p**	**~p∨q**	**p∧(~p∨q)**	**p∧q**
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	T	F	F
F	F	T	T	F	F

In the above truth table, the entries in columns 5 and 6 are identical.

∴ p ∧ (~p ∨ q) ≡ p ∧ q

Miscellaneous Exercise 1 | Q 4.14 | Page 33

Using the truth table, prove the following logical equivalence.

p ↔ q ≡ ~(p ∧ ~q) ∧ ~(q ∧ ~p)

Solution:

1	2	3	4	5	6	7	8	9	10
p	**q**	**~p**	**~q**	**p↔q**	**p∧~q**	**~(p∧~q)**	**(q∧~p)**	**~(q∧~p)**	**~(p∧~q)∧~(q ∧ ~p)**
T	T	F	F	T	F	T	F	T	T
T	F	F	T	F	T	F	F	T	F

F	T	T	F	F	F	T	T	F	F
F	F	T	T	T	F	T	F	T	T

In the above truth table, the entries in columns 5 and 10 are identical.

∴ $p \leftrightarrow q \equiv \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p)$

Miscellaneous Exercise 1 | Q 4.14 | Page 33

Using the truth table, prove the following logical equivalence.

$\sim p \wedge q \equiv [(p \vee q)] \wedge \sim p$

Solution:

1	2	3	4	5	6
p	**q**	**~p**	**~p∧q**	**(p∨q)**	**(p∨q)∧~p**
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	F	F	F

In the above truth table, the entries in columns 4 and 6 are identical.

∴ $\sim p \wedge q \equiv [(p \vee q)] \wedge \sim p$

Miscellaneous Exercise 1 | Q 4.15 | Page 33

Write the converse, inverse, contrapositive of the following statement.

If 2 + 5 = 10, then 4 + 10 = 20.

Solution: Let p : 2 + 5 = 10
q : 4 + 10 = 20

∴ The given statement is $p \rightarrow q$.

Its converse is $q \rightarrow p$.
If 4 + 10 = 20, then 2 + 5 = 10

Its inverse is $\sim p \rightarrow \sim q$.
If 2 + 5 ≠ 10 then 4 + 10 ≠ 20.

Its contrapositive is $\sim q \rightarrow \sim p$.
If 4 + 10 ≠ 20 then 2 + 5 ≠ 10.

Miscellaneous Exercise 1 | Q 4.15 | Page 33

Write the converse, inverse, contrapositive of the following statement.

If a man is bachelor, then he is happy.

Solution: Let p : A man is bachelor.
q : A man is happy.

∴ The given statement is $p \rightarrow q$.

Its converse is $q \rightarrow p$.
If a man is happy then he is bachelor.

Its inverse is $\sim p \rightarrow \sim q$.
If a man is not bachelor then he is not happy.

Its contrapositive is $\sim q \rightarrow \sim p$.
If a man is not happy then he is not bachelor.

Miscellaneous Exercise 1 | Q 4.15 | Page 33

Write the converse, inverse, contrapositive of the following statement.

If I do not work hard, then I do not prosper.

Solution: Let p : I do not work hard.
q : I do not prosper.

∴ The given statement is $p \rightarrow q$.

Its converse is $q \rightarrow p$.
If I do not prosper then I do not work hard.

Its inverse is $\sim p \rightarrow \sim q$.
If I work hard then I prosper.

Its contrapositive is $\sim q \rightarrow \sim p$.
If I prosper then I work hard.

Miscellaneous Exercise 1 | Q 4.16 | Page 33

State the dual of the following statement by applying the principle of duality.

$(p \wedge \sim q) \vee (\sim p \wedge q) \equiv (p \vee q) \wedge \sim(p \wedge q)$

Solution: $(p \vee \sim q) \wedge (\sim p \vee q) \equiv (p \wedge q) \vee \sim(p \vee q)$

Miscellaneous Exercise 1 | Q 4.16 | Page 33

State the dual of the following statement by applying the principle of duality.

$p \vee (q \vee r) \equiv \sim[(p \wedge q) \vee (r \vee s)]$

Solution: $p \wedge (q \wedge r) \equiv \sim[(p \vee q) \wedge (r \wedge s)]$

Miscellaneous Exercise 1 | Q 4.16 | Page 33

State the dual of the following statement by applying the principle of duality.

2 is even number or 9 is a perfect square.

Solution: 2 is even number and 9 is a perfect square.

Miscellaneous Exercise 1 | Q 4.17 | Page 33

Rewrite the following statement without using the connective 'If ... then'.

If a quadrilateral is rhombus then it is not a square.

Solution: Let p : A quadrilateral is rhombus.

q : A quadrilateral is not a square.

The given statement is $p \rightarrow q$.

But $p \rightarrow q \equiv \sim p \vee q$.

∴ The given statement can be written as 'A quadrilateral is not a rhombus or it is not a square'.

Miscellaneous Exercise 1 | Q 4.17 | Page 33

Rewrite the following statement without using the connective 'If ... then'.

If $10 - 3 = 7$ then $10 \times 3 \neq 30$.

Solution: Let p : $10 - 3 = 7$

q : $10 \times 3 \neq 30$

The given statement is $p \rightarrow q$.

But $p \rightarrow q \equiv \sim p \vee q$.

∴ The given statement can be written as
'$10 - 3 \neq 7$ or $10 \times 3 \neq 30$'.

Miscellaneous Exercise 1 | Q 4.17 | Page 33

Rewrite the following statement without using the connective 'If ... then'.

If it rains then the principal declares a holiday.

Solution: Let p : It rains.

q : The principal declares a holiday.

The given statement is $p \rightarrow q$.

But $p \rightarrow q \equiv \sim p \vee q$.

∴ The given statement can be written as 'It does not rain or the principal declares a holiday'.

Miscellaneous Exercise 1 | Q 4.18 | Page 33

Write the dual of the following.

$(\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

Solution: $(\sim p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$

Miscellaneous Exercise 1 | Q 4.18 | Page 33

Write the dual of the following.

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Solution: $(p \vee q) \vee r \equiv p \vee (q \vee r)$

Miscellaneous Exercise 1 | Q 4.18 | Page 33

Write the dual of the following.

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (q \vee r)$

Solution: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (q \wedge r)$

Miscellaneous Exercise 1 | Q 4.18 | Page 33

Write the dual of the following.

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

Solution: $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Miscellaneous Exercise 1 | Q 4.19 | Page 33

Consider the following statements.

i. If D is dog, then D is very good.

ii. If D is very good, then D is dog.

iii. If D is not very good, then D is not a dog.

iv. If D is not a dog, then D is not very good. Identify the pairs of statements having the same meaning. Justify.

Solution: Let p : D is dog.
q : D is very good.
Then the given statement in the symbolic form is i. $p \rightarrow q$

ii. $q \rightarrow p$
iii. $\sim q \rightarrow \sim p$
iv. $\sim p \rightarrow \sim q$

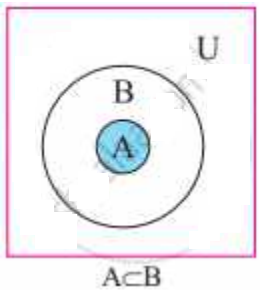
Since a statement and its contrapositive are equivalent, statements (i) and (iii) have the same meaning.
Since converse and inverse of a compound statement are equivalent, statements (ii) and (iv) have same meaning.

Miscellaneous Exercise 1 | Q 4.2 | Page 33

Express the truth of the following statement by the Venn diagram.

All men are mortal.

Solution: U : The set of all human being
A : The set of all men
B : The set of all mortal



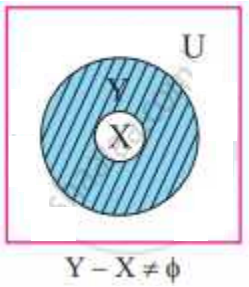
The above Venn diagram represents the truth of the given statement, i.e. $A \subset B$.

Miscellaneous Exercise 1 | Q 4.2 | Page 33

Express the truth of the following statement by the Venn diagram.

Some persons are not politician.

Solution: U : The set of all human beings.
X : The set of all persons.
Y : The set of all politician



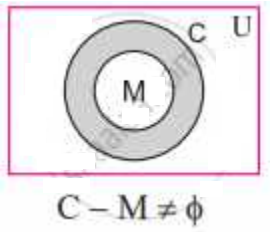
The above Venn diagram represents the truth of the given statement, i.e. $Y - X \neq \Phi$

Miscellaneous Exercise 1 | Q 4.2 | Page 33

Express the truth of the following statement by the Venn diagram.

Some members of the present Indian cricket are not committed.

Solution: U : The set of all human beings.
M : The set of all members of the present Indian cricket.
C : The set of all committed members of the present Indian cricket.



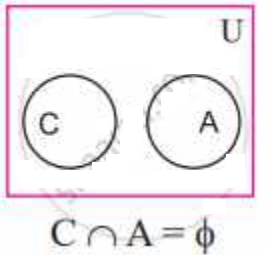
The above Venn diagram represents the truth of the given statement, i.e. C - M = Φ

Miscellaneous Exercise 1 | Q 4.2 | Page 33

Express the truth of the following statement by the Venn diagram.

No child is an adult.

Solution: U : Set of all human beings.
C : Set of all child.
A : Set of all Adult.



The above Venn diagram represents the truth of the given statement, i.e. C ∩ A = Φ

Miscellaneous Exercise 1 | Q 4.21 | Page 34

If A = {2, 3, 4, 5, 6, 7, 8}, determine the truth value of the following statement.

∃ x ∈ A, such that 3x + 2 > 9

Solution: For x = 3, 3x + 2 = 3(3) + 2 = 9 + 2 = 11 > 9

∴ x = 3 satisfies the equation 3x + 2 > 9.

∴ The given statement is true.

∴ Its truth value is T.

Miscellaneous Exercise 1 | Q 4.21 | Page 34

If A = {2, 3, 4, 5, 6, 7, 8}, determine the truth value of the following statement.

$\forall x \in A, x^2 < 18$.

Solution: For $x = 5, x^2 = 5^2 = 25 < 18$

$\therefore$ $x = 5$ does not satisfies the equation $x^2 < 18$.

$\therefore$ The given statement is false.

$\therefore$ Its truth value is F.

Miscellaneous Exercise 1 | Q 4.21 | Page 34

If A = {2, 3, 4, 5, 6, 7, 8}, determine the truth value of the following statement.

$\exists x \in A$, such that $x + 3 < 11$.

Solution: For $x = 2, x + 3 = 2 + 3 = 5 < 11$.

$\therefore$ $x = 2$ satisfies the equation $x + 3 < 11$.

$\therefore$ The given statement is true.

$\therefore$ Its truth value is T.

Miscellaneous Exercise 1 | Q 4.21 | Page 34

If A = {2, 3, 4, 5, 6, 7, 8}, determine the truth value of the following statement.

$\forall x \in A, x^2 + 2 \geq 5$.

Solution: There is no x in A which satisfies $x^2 + 2 \geq 5$.

$\therefore$ The given statement is false.

$\therefore$ Its truth value is F.

Miscellaneous Exercise 1 | Q 4.22 | Page 34

Write the negation of the following statement.

7 is prime number and Tajmahal is in Agra.

Solution: Let p : 7 is prime number.
q : Tajmahal is in Agra.

The given statement in symbolic form is $p \wedge q$.

Its negation is $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

$\therefore$ The negation of given statement is '7 is not prime number or Tajmahal is not in Agra.'

Miscellaneous Exercise 1 | Q 4.22 | Page 34

Write the negation of the following statement.

10 > 5 and 3 < 8

Solution: Let p : 10 > 5.
q : 3 < 8.

The given statement in symbolic form is $p \wedge q$.

Its negation is $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

∴ The negation of given statement is '10 ≤ 5 or 3 ≥ 8.'

Miscellaneous Exercise 1 | Q 4.22 | Page 34

Write the negation of the following statement.

I will have tea or coffee.

Solution: Let p : I will have tea.
q : I will have coffee.

The given statement in symbolic form is $p \vee q$.

Its negation is $\sim(p \vee q) \equiv \sim p \wedge \sim q$.

∴ The negation of given statement is 'I will not have tea and coffee'.

Miscellaneous Exercise 1 | Q 4.22 | Page 34

Write the negation of the following statement.

$\forall\ n \in N, n + 3 > 9$.

Solution: $\exists$ n ∈ N such that $n + 3 \leq 9$.

Miscellaneous Exercise 1 | Q 4.22 | Page 34

Write the negation of the following statement.

$\exists$ n ∈ A, such that $x + 5 < 11$.

Solution: $\forall\ x \in A, x + 5 \leq 11$