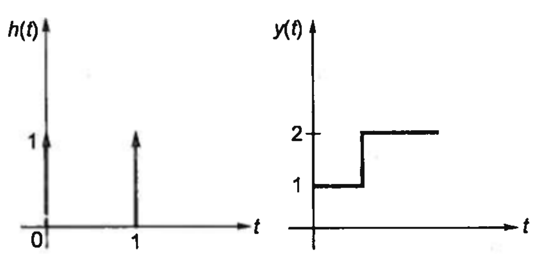
4 Laplace Transform

Multiple Choice Questions

Q.1 Which one of the following is the impulse response of the system whose step response is given as $c(t) = 0.5(1 - e^{-2t})\,u(t)$?

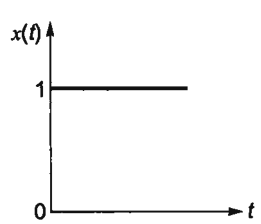
(a) $e^{-2t}u(t)$
(b) $0.5\,\delta(t) + e^{-2t}u(t)$
(c) $0.5\,\delta(t) - 0.5\,e^{-2t}u(t)$
(d) $0.5\,e^{-2t}\,u(t)$

Q.2 Consider for a LTI the impulse response and output are given by

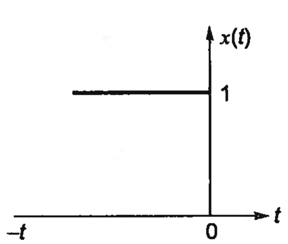


the input signal $x(t)$ should be equal to

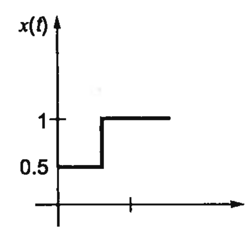
(a)



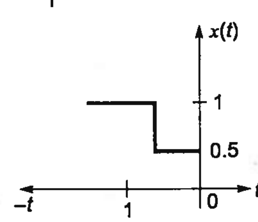
(b)



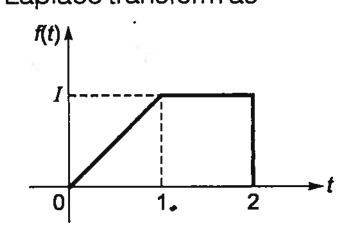
(c)



(d)



Q.3 The function $f(t)$ shown in the given figure will have Laplace transform as



(a) $\frac{1}{s^2} - \frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-2s}$
(b) $\frac{1}{s^2}(1 - e^{-s} - e^{-2s})$
(c) $\frac{1}{s}(1 - e^{-s} - e^{-2s})$
(d) $\frac{1}{s^2}(1 - e^{-s} - se^{-2s})$ **[ESE-1999]**

Q.4 The output of a linear system to a unit step input u(t) is $t^2 e^{-2t}$. The system function $H(s)$ is

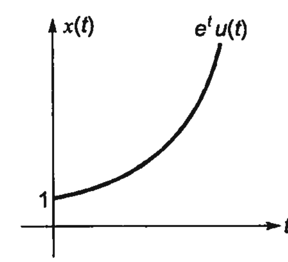
(a) $\frac{2}{s^2(s+2)}$ (b) $\frac{2}{(s+2)^2}$
(c) $\frac{2}{(s+2)^3}$ (d) $\frac{2s}{(s+2)^3}$ **[ESE-2000]**

Q.5 Given that $x_1(t) = e^{k_1 t}u(t)$ and $x_2(t) = e^{-k_2 t}u(t)$. Which one of the following gives their convolution?

(a) $\frac{[e^{k_1 t} - e^{-k_2 t}]}{[k_1 + k_2]}$ (b) $\frac{[e^{k_1 t} - e^{-k_2 t}]}{[k_2 - k_1]}$
(c) $\frac{[e^{k_1 t} + e^{-k_2 t}]}{[k_2 + k_1]}$ (d) $\frac{[e^{k_1 t} + e^{-k_2 t}]}{[k_2 - k_1]}$

[ESE-2004]

Q.6 For the signal shown below.



(a) Only Fourier transform exists
(b) Only Laplace transform exist
(c) Both Laplace and Fourier transforms exist
(d) Neither Laplace nor Fourier transform exists

Q.7 Consider the function $f(t)$ having Laplace transform

$$F(s) = \frac{\omega_0}{s^2 + \omega_0^2} \quad Re[s] > 0$$

The final value of $f(t)$ would be

(a) 0 (b) 1
(c) $-1 \le f(\infty) \le 1$ (d) ∞

[GATE-2006]

Q.8 If the Laplace transform of a signal $y(t)$ is

$Y(s) = \frac{1}{s(s-1)}$, then its final value is

(a) –1 (b) 0
(c) 1 (d) Unbounded

[GATE-2007]

Q.9 Given $X(s) = \frac{2s^2 + 5s + 5}{(s+1)^2(s+2)}$ and $R_e[s] > -1$ then $x(t)$:

(a) $[2te^{-t} + e^{-t} + 3e^{-2t}]\,u(t)$
(b) $[2te^{-t} - e^{-t} - 3e^{-2t}]\,u(t)$
(c) $[2te^{-t} - e^{-t} + 3e^{-2t}]\,u(t)$
(d) None of these

Q.10 If $L[f(t)] = \frac{2(s+1)}{s^2 + 2s + 5}$, then $f(0^+)$ and $f(\infty)$ are given by

(a) 0, 2 respectively
(b) 2, 0 respectively
(c) 0, 1 respectively
(d) 2/5, 0 respectively

Q.11 Consider the following signal:
$x(t) = e^{-2t}\,u(t) + e^{-t}(\cos 3t)\,u(t)$
the laplace transform of above system is

(a) $\frac{2s^2 + 4s + 11}{(s^2 + 2s + 10)(s+1)}$
(b) $\frac{2s^2 + 5s + 12}{(s^2 + 2s + 10)(s+2)}$
(c) $\frac{2s^2 + 5s + 12}{s^2 + 2s + 10}$
(d) $\frac{2s^2 + 5s + 12}{(s^2 + 2s + 10)s}$

Q.12 Consider a signal $x(t)$ having laplace transform given by,

$$X(s) = \log\left(\frac{s+5}{s+6}\right)$$

The time domain signal $x(t)$ is equal to

(a) $\frac{1}{t}[e^{-6t} - e^{-5t}]\,u(t)$
(b) $\frac{1}{t}[e^{-6t} + e^{-5t}]\,u(t)$
(c) $t\,[e^{-6t} - e^{-5t}]\,u(t)$
(d) $\frac{1}{t}[e^{-5t} - e^{-6t}]\,u(t)$

Q.13 The laplace transform of $\left(\frac{1-e^{t}}{t}\right)u(t)$

(a) $\log\left(\frac{s}{s-1}\right)$ (b) $\log\left(\frac{s-1}{s}\right)$

(c) $\log\left(\frac{s-1}{s+1}\right)$ (d) $\log\left(\frac{s+1}{s-1}\right)$

Q.14 The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of $t\,f(t)$ is

(a) $\frac{s}{(s^2+s+1)^2}$ (b) $-\frac{2s+1}{(s^2+s+1)^2}$

(c) $\frac{s}{(s^2+s+1)^2}$ (d) $\frac{2s+1}{(s^2+s+1)^2}$

[GATE-2012]

Q.15 Let the Laplace transform of a function $f(t)$ which exists for $t > 0$ be $F_1(s)$ and the Laplace transform of its delayed version $f(t-\tau)$ be $F_2(s)$. $F_1^*(s)$ be the complex conjugate of $F_1(s)$ with the Laplace variable set as $s = \sigma + j\omega$. If

$G(s) = \frac{F_2(s)F_1^*(s)}{|F_1(s)|^2}$, then the inverse Laplace transform of $G(s)$ is

(a) an ideal impulse $\delta(t)$

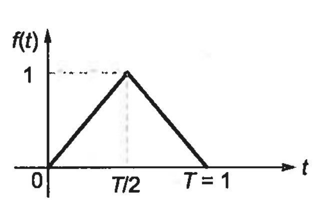
(b) an ideal delayed impulse $\delta(t-\tau)$

(c) an ideal step function $u(t)$

(d) an ideal delayed step function $u(t-\tau)$

[ESE-2011]

Q.16 Laplace transform of the function $f(t)$ shown in the figure is



(a) $\frac{2}{s^2}\left[1-e^{-0.5s}\right]^2$ (b) $\frac{2}{s^2}\left[1+e^{-0.5s}\right]^2$

(c) $\frac{2}{s^2}\left[1-e^{0.5s}\right]^2$ (d) $\frac{2}{s^2}\left[1+e^{0.5s}\right]^2$

[ESE-2011]

Common Data for Questions (17 and 18):

Let $x(t)$ be the sampled signal specified as

$$x(t) = \sum_{n=0}^{\infty} e^{-nT}\,\delta(t-nT),\ T > 0$$

Q.17 The $X(s)$ will be

(a) $\frac{1}{1-e^{-T(s+1)}}$ (b) $\frac{1}{1+e^{-T(s-1)}}$

(c) $\frac{1}{1-e^{T(s+1)}}$ (d) $\frac{1}{1-e^{T(s-1)}}$

Q.18 Location of poles of $X(s)$ are

(a) $S = 1 - j\frac{2\pi K}{T}$, $K = 0, \pm 1, \pm 2, \ldots$

(b) $S = -1 - j\frac{2\pi K}{T}$, $K = 0, \pm 1, \pm 2, \ldots$

(c) $S = 1 - j\frac{\pi K}{T}$, $K = 0, \pm 1, \pm 2, \ldots$

(d) $S = -1 - j\frac{\pi K}{T}$, $K = 0, \pm 1, \pm 2, \ldots$

Q.19 Find ROC of signal.

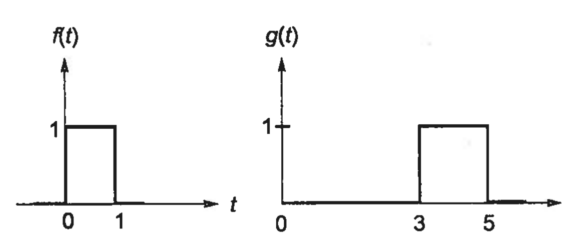
$$x(t) = e^{t\,u(-t)}$$

(a) $\sigma < 1$ (b) $\sigma < 0$

(c) $0 < \sigma < 1$ (d) $\sigma < 0, \sigma > 1$

Common Data for Questions (20 and 21):

Given $f(t)$ and $g(t)$ as shown below:



Q.20 $g(t)$ can be expressed as

(a) $g(t) = f(2t-3)$

(b) $g(t) = f\left(\frac{t}{2}-3\right)$

(c) $g(t) = f\left(2t-\frac{3}{2}\right)$

(d) $g(t) = f\left(\frac{t}{2}-\frac{3}{2}\right)$

[GATE-2010]

Q.21 The Laplace transform of $g(t)$ is

(a) $\frac{1}{s}\left(e^{3s}-e^{5s}\right)$ (b) $\frac{1}{s}\left(e^{-5s}-e^{-3s}\right)$

(c) $\frac{e^{-3s}}{s}\left(1-e^{-2s}\right)$ (d) $\frac{1}{s}\left(e^{5s}-e^{3s}\right)$

[GATE-2010]

Q.22 Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI system?

(a) All the poles of the system must lie on the left side of the $j\omega$ axis.

(b) Zeros of the system can lie anywhere in the s-plane.

(c) All the poles must lie within $|s| = 1$.

(d) All the roots of the characteristic equation must be located on the left side of the $j\omega$ axis.

[GATE-2013]

Q.23 A stable linear time invariant (LTI) system has a transfer function $H(s) = \frac{1}{s^2+s-6}$. To make this system causal it needs to be cascaded with another LTI system having a transfer function $H_1(s)$. A correct choice for $H_1(s)$ among the following options is

(a) $s + 3$ (b) $s - 2$

(c) $s - 6$ (d) $s + 1$

[GATE-2014]

Numerical Data Type Questions

Q.24 Consider the differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t) \text{ with}$$

$$y(t)\Big|_{t=0^-} = -2 \text{ and } \frac{dy}{dt}\Big|_{t=0^-} = 0.$$

The numerical value of $\frac{dy}{dt}\Big|_{t=0^+}$ is ______.

[GATE-2012]

Q.25 If $\left(\frac{27s+97}{s^2+33s}\right)$ is the Laplace transform of $f(t)$, then $f(0^+)$ is ______.

[ESE-2003]

Q.26 The Laplace transform of $i(t)$ is given by

$$I(s) = \frac{2}{s(1+s)}.$$

As $t \to \infty$, the value of $i(t)$ tends to ______.

[GATE-2002]

Q.27 A system with transfer function

$$G(s) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$

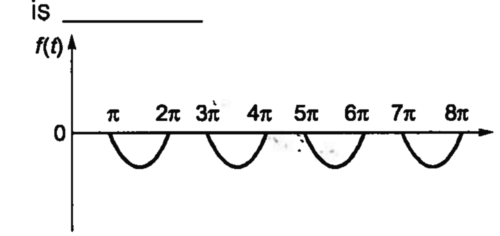
is excited by $\sin \omega t$. The steady-state output of the system is zero at ω = ______rad/s.

[GATE-2012]

Q.28 The Laplace transform of the function f(t) described by the curve below, *i.e.*

$$f(t) = \begin{cases} \sin t & \text{if } (2n-1)\pi \le t \le 2n\pi \quad (n = 1, 2, 3, \cdots) \\ 0 & \text{otherwise} \end{cases}$$

is ______



[GATE-1993]

Q.29 Consider a transfer function, $H(s)$ given by

$$H(s) = \frac{s^2 + as + 2}{s^2 + 2s + 2}$$. If $H(s)$ represents an all pass filter, then the value of 'a' is given by ______.

Try Yourself

T1. Let $x(t)$ be a signal with its Laplace transform $X(s)$. If $x(t)$ is defined as $x(t) = e^{-3t} \cos 2t$. Another function $y(t)$ is defined as

$$y(t) = \int_0^{\tau} x(\tau) d\tau$$

then the Laplace transform of $y(t)$ is

(a) $\frac{-(s+3)}{s[(s+3)^2 + 4]}$ (b) $\frac{s(s+3)}{[(s+3)^2 + 4]}$

(c) $\frac{s+3}{s[(s+3)^2 + 4]}$ (d) $\frac{-s(s+3)}{[(s+3)^2 + 4]}$

[Ans: (c)]

T2. The bilateral Laplace transform of $x(t) = e^{-t} u(t + 1)$ is

(a) $\frac{e^{(s+1)}}{s+1}, Re(s) > -1$

(b) $\frac{s+1}{s}, Re(s) < -1$

(c) $\frac{s}{s+1} \cdot e^{-s}, Re(s) < -1$

(d) $\frac{e^{-(s+1)}}{s+1}, Re(s) > -1$

[Ans: (a)]

T3. If $L[f(t)] = \frac{\omega}{(s^2 + \omega^2)}$, then the value of $\lim_{t \to \infty} f(t)$

(a) cannot be determined
(b) is zero
(c) is unity
(d) is infinite

[Ans: (a)]

T4. The transfer function of a system is given by

$$H(s) = \frac{1}{s^2(s-2)}$$. The impulse response of the system is

(a) $(t^2 * e^{-2t})\, u(t)$ (b) $(t * e^{2t})\, u(t)$
(c) $(t * e^{-2t})\, u(t)$ (d) $(te^{-2t})\, u(t)$
($*$ denotes convolution, and $u(t)$ is unit step function)

[Ans: (b)]

T5. A signal is right sided and has poles such that the system is also causal and stable what can be the set of poles the system

(a) 2, 3, 4 (b) –2, –3, 2
(c) 0, 4, 10 (d) –2, –3, –4

[Ans: (d)]

T6. Specify the filter type if its voltage transfer function $H(s)$ is given by

$$H(s) = \frac{K(s^2 + \omega_0^2)}{s^2 + (\omega_0 / Q)s + \omega_0^2}$$

(a) all pass filter (b) low pass filter
(c) band pass filter (d) notch filter

[Ans: (d)]