

MATHEMATICS PAPER IIA.- MAY 2010.

ALGEBRA AND PROBABILITY.

TIME : 3hrs

Max. Marks.75

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS.

10X2 =20

Noe : Attempt all questions. Each question carries 2 marks.

1. Find the maximum or minimum values of the quadratic equation $2x-7-5x^2=0$.
2. If $-1, 2$ and α are the roots of $2x^3+x^2-7x-6=0$ then find ' α '.
3. Define trace of matrix.
4. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.
5. Find the number of ways of arranging the letters of the word MATHEMATICS
6. If ${}^nC_2 = {}^nC_6$, then find ${}^{13}C_n$
7. Find the middle terms in the expansion of $\left(\frac{3x}{7}-2y\right)^{10}$
8. Show that $\frac{1}{2.3} + \frac{1}{4.5} + \frac{1}{6.7} + \dots = 1 - \log_e 2$.
9. What is the probability that a leap year will contain 53 Mondays?
10. In a binomial distribution random variable X has mean and variance .Find n.

SECTION B

SHORT ANSWER TYPE QUESTIONS.

5X4 =20

Note : Answer any FIVE questions. Each question carries 4 marks.

11. Find the range of the expression $\frac{x^2+x+1}{x^2-x+1}$, $x \in \mathbb{R}$

12. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix then prove that $A^{-1} = \frac{Adj A}{\det A}$

13. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order. Then find the rank of the word MASTER.
14. Find the number of ways selecting a cricket team of 11 players from 7 bats men and 6 bowlers such that there will be at least 5 bowlers.
15. Resolve $\frac{5x^2+2}{x^3+x}$ into partial fractions.
16. Show that $\frac{1}{1!} + \frac{1+5}{2!} + \frac{1+5+5^3}{3!} + \frac{1+5+5^2+5^3}{4!} + \dots = \frac{e^5 - e}{4}$
17. State and prove Multiplication theorem on Probability.

SECTION C

LONG ANSWER TYPE QUESTIONS.

5X7 = 35

Note: Answer any Five of the following. Each question carries 7 marks.

18. Show that $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$.
19. Show that
$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$
20. Solve the following simultaneous linear equation by using Gauss - Jordan method $x + y + z = 9$, $2x + 5y + 7z = 52$, $2x + y - z = 0$.
21. Expand $\frac{x-4}{x^2-5x+6}$ in increasing powers of x and hence find the coefficient of x^n .
22. Find the sum of the infinite series $\frac{3}{4} + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \dots$
23. A speaks truth in 75% of the cases and B in 80% cases. Find the probability that at what percentage of cases, they do not match about an incident.
24. A random variable X has the following probability distribution
- | | | | | | | | | |
|-------------|---|---|----|----|----------------|-----------------|---------------------|---|
| X = x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| P (X = x) | 0 | k | 2k | 3k | k ² | 2k ³ | 7k ² + k | |

Find (i) k (ii) then mean and (iii) $P(0 < X < 5)$