Chapter 11

Mensuration

Introduction to Mensuration

Plane Figures

They have two dimensions like length and breadth. A plane figure occupies the surface of the plane. They are also called a Two- dimensional figure(2-D). Examples: Square,

Triangle, Rectangle, Circle, etc.

For a closed plane figure, the Perimeter is the distance around its boundary and its Area is the region covered by it.

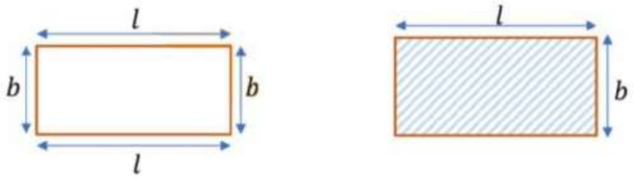
A plane figure is called a rectilinear figure if it is made up of only line segments.

A circle is not a rectilinear figure. The part of the plane which is enclosed by a simple closed figure is called a plane region. The magnitude of a plane region is called its

area.

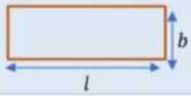
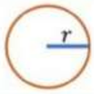
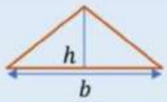
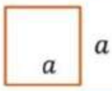
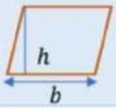
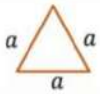
A line segment has one dimension, i.e., length. Hence, its size is measured in terms of its length.

A planer region has two dimensions, i.e., length and breadth. Hence, its size measured in terms of its area.



Perimeter = $l + b + l + b$
$= 2l + 2b$
$= 2(l + b)$

Area = $l \times b$

S. No.	Figure	Shape	Area	Perimeter
1.		Rectangle	$l \times b$	$2(l + b)$
2.		Circle	πr^2	$2\pi r$
3.		Triangle	$\frac{1}{2} \times b \times h$	Sum of all sides
4.		Square	$a \times a$	$4a$
5.		Parallelogram	$b \times h$	$2 \times$ sum of adjacent sides
6.		Equilateral Triangle	$\frac{\sqrt{3}}{4} a^2$	$3a$

The area of a square is 72.25 m^2.

(i) Find the side of the square.

(ii) It the tiles measuring 17 cm × 17 cm are paved on the square area, find how many tiles are used for paving it?

Area of the square = 72.25 m^2

(i) Area of the square = $side^2$

$\Rightarrow$ side = $\sqrt{72.25}$ m

$\Rightarrow$ side = 8. 5 m = 8. 5 × 100 cm = 850 cm

(ii) Number of tiles required = $\frac{Area\ of\ square}{Area\ of\ one\ tile} = \frac{850 \times 850}{17 \times 17} = 2500$

Trapezium

Trapezium

A trapezium is a quadrilateral having a pair of parallel opposite sides.

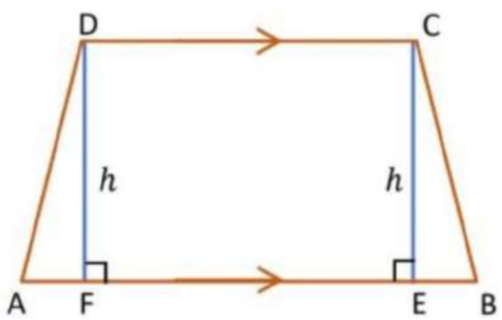
Base: Each of the two parallel sides of a trapezium is called a base of the trapezium.

Height of altitude: The distance between the two bases (parallel sides) is called the height or altitude of the trapezium.

Let ABCD be a trapezium in which AB || DC,

CE ⊥ AB, DF ⊥ AB and CE = DF = h.

Where h is the height of trapezium ABCD.



From the given figure, the area of quadrilateral ABCD

= Area of ΔAFD + Area of rectangle DFEC + Area of ΔCEB

$= (\frac{1}{2} \times AF \times DF) + (FE \times DF) + (\frac{1}{2} \times EB \times CE)$

$= (\frac{1}{2} \times AF \times h) + (FE \times h) + (\frac{1}{2} \times EB \times h)$

$=\frac{1}{2}\times h \times (AF + 2FE + EB)$ $=\frac{1}{2}\times h \times (AF + FE + EB + FE)$

$=\frac{1}{2}\times h \times (AB + FE)$

$=\frac{1}{2}\times h \times (AB + DC)$ [since AF + FE + EB = AB and FE = CD]

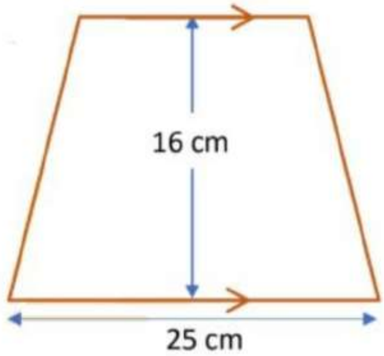
Hence, the area of trapezium $=\frac{1}{2}\times$ (sum of parallel sides) × (height)

The area of a trapezium is 400 cm2 and the distance between its parallel sides is 16 cm. If one of the parallel sides is of length 25 cm^2 find the length of the other.

Let the length of the other side be x cm.

The area of the trapezium

= {12× (sum of parallel sides) × (distance between them)}



Area of the trapezium $= \{\frac{1}{2}\times (x + 25) \times 16\}$

$400 = 8 \times (x + 25)$

$(x + 25) = \frac{400}{8}$

$x + 25 = 50$

$x = 50 - 25$

$x = 25$ cm

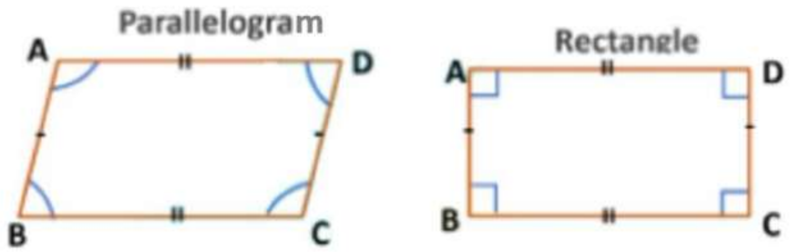
Quadrilateral

Quadrilateral

A quadrilateral is a closed two-dimensional shape formed by joining four straight sides. It has four sides, four angles, and four vertices. The sum of all the interior angles of

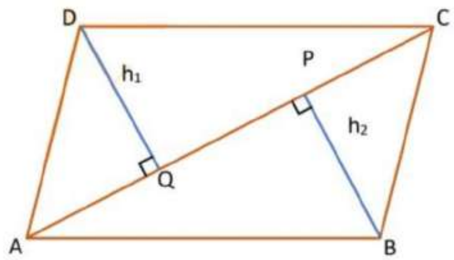
a quadrilateral is 360°.

Parallelogram, Rectangle, Rhombus, and Square are types of Quadrilateral.



Let ABCD be a quadrilateral with AC as one of its diagonals. Let BP and DQ be the perpendiculars drawn from the vertices B and D on diagonal AC.

Area of quadrilateral ABCD = Area of Δ ADC + Area of Δ ABC



Area of $\Delta ADC = \frac{1}{2} \times AC \times DQ$

$= \frac{1}{2} \times AC \times h_1$

Area of $\Delta ABC = \frac{1}{2} \times AC \times BP$

$= \frac{1}{2} \times AC \times h_2$

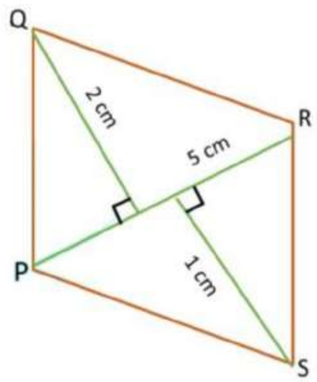
Area of quadrilateral ABCD $= \frac{1}{2} \times AC \times h_1 + \frac{1}{2} \times AC \times h_2$

Area of quadrilateral $= \frac{1}{2} \times AC \times (h_1 + h_2)$

Find the area of quadrilateral PQRS as shown in the figure.

Here, $d = 5$ cm, $h_1 = 2$ cm, $h_2 = 1$ cm

Area of quadrilateral $= \frac{1}{2} \times d \times (h_1 + h_2)$



Area of quadrilateral PQRS

$= \frac{1}{2} \times 5 \times (2 + 1)\ cm^2$

$= \frac{1}{2} \times 5 \times 3\ cm^2$

$= 0.5 \times 15\ cm^2$

Area of quadrilateral PQRS $= 7.5\ cm^2$

Rhombus

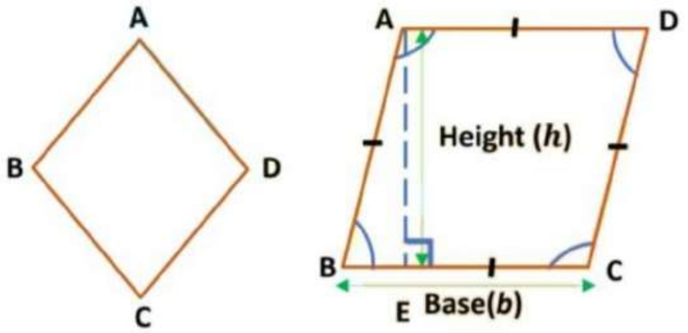
Rhombus

Rhombus is a type of quadrilateral in which both the pairs of opposite sides are parallel and all the sides are having the same length and all the opposite angles are of equal measure. It is also known as diamond.

Any side of a rhombus can be chosen as the base of the rhombus.

The perpendicular dropped on that side from the opposite vertex is known as height (altitude).

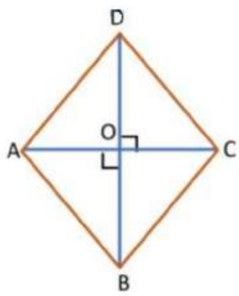
In the rhombus ABCD, AE is perpendicular to BC. Here BC is the base and AE is the height of the rhombus.



Let ABCD be a rhombus and AC and BD be its diagonals which intersect at O. The diagonals of a rhombus bisect each other at right angles.

Area of Rhombus ABCD

$=12\times AC \times BD$



Area of Rhombus$=\frac{1}{2}\times d_1 \times d_2$,

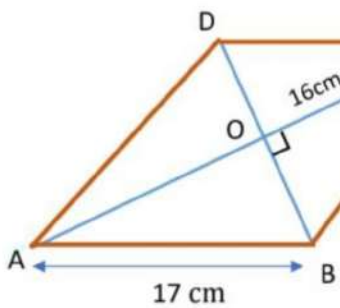
where $AC = d_1$ and $BD = d_2$

Find the area of a Rhombus whose one side measures 17 cm and one diagonal is 16 cm.

Here, $AB = 17$ cm and $AC = 16$ cm

As the diagonals bisect each other at a right angle,

Δ AOB is a right-angled triangle, and $AO = OC = \frac{1}{2}$ AC



$AO = \frac{1}{2} \times 16 = 8$ cm

Using Pythagoras Theorem,

$AB^2 = OB^2 + AO^2$

$17^2 = OB^2 + 8^2$

$289 = OB^2 + 64$

$OB2 = 289 - 64 = 225, \because OB^2 = 15^2$

$OB = 15$ cm,

$BD = 2 \times 15 = 2 \times 15$ cm $= 30$ cm

Area of rhombus $= \frac{1}{2}\times d^1 \times d^2$

$= \frac{1}{2} \times 16 \text{ cm} \times 30 \text{ cm}$

Area of Rhombus ABCD= 240 cm^2

Polygon

A polygon is a simple closed curve made up of line segments only.

Concave and Convex Polygon

- Concave Polygon

A polygon is a concave polygon if a line segment joining two different points does not lie completely inside the polygon.

- Convex Polygon

A polygon is a convex polygon if the line segment joining any two points lies completely inside the polygon

Regular and Irregular Polygon

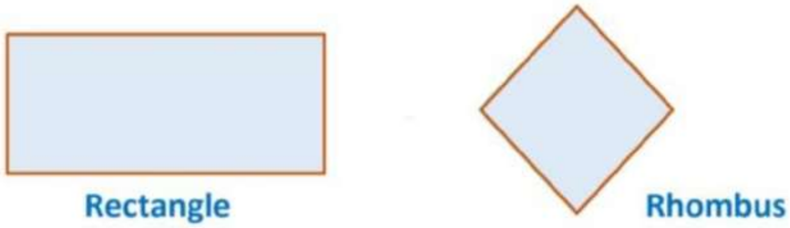
- Regular Polygon

A polygon whose all sides and all angles are equal is called a Regular Polygon. A square has sides of equal length and angles of equal measure, i.e., 90°

Similarly, an Equilateral Triangle has sides of equal length and angle of equal measure i.e., 60°

Therefore, a square and an equilateral triangle are Regular Polygons.

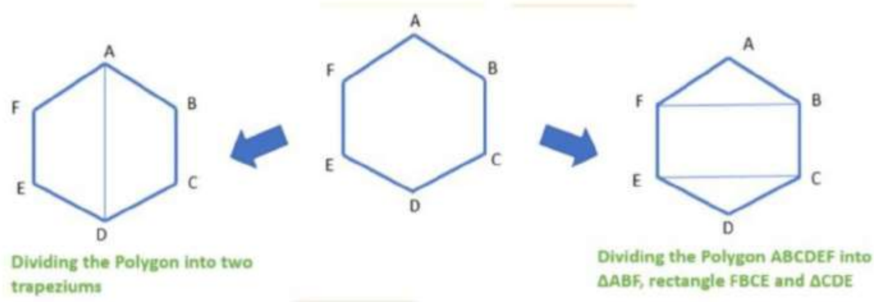
• Irregular Polygons
Polygons which do not have all the sides and angles of equal measure is called an Irregular Polygon. Examples, Rhombus and Rectangle.



Area of Polygon

We can calculate the area of a polygon by dividing them into triangles or quadrilaterals or a combination of the two.

We can find the area of the given polygon ABCDEF by two ways:



Find the area of the pentagon ABCDE as shown in the figure, if AD = 18 cm, AH= 16 cm, AG = 14 cm, AF = 13 cm, BF = 12 cm, CH = 13 cm and EG = 12.5 cm.

Area of Pentagon ABCDE
= Area of Δ AFB + Area of Trapezium BFHC + Area of Δ CHD + Area of Δ ADE

$$=\frac{1}{2}(AF \times BF) + \frac{1}{2}(BF + CH) \times FH + \frac{1}{2} \times DH \times CH + \frac{1}{2} \times AD \times GE$$

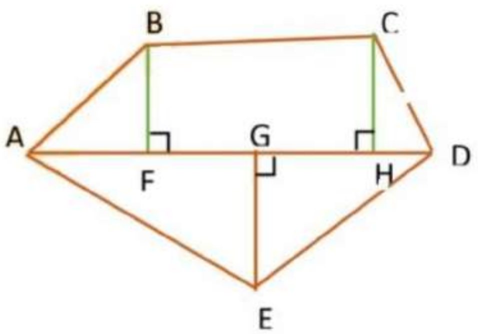
$= \frac{1}{2}$ (AF × BF) $+ \frac{1}{2}$ (BF + CH) × (AH − AF) $+ \frac{1}{2}$ × (AD − AH) × CH $+ \frac{1}{2}$ × AD × GE

$= \frac{1}{2}$ (13 × 12) $+ \frac{1}{2}$ × (12 + 13) × (16 − 13) $+ \frac{1}{2}$ × (18 − 16) × 13 $+ \frac{1}{2}$ × 18 × 12.
5

$= \frac{156}{2} + \frac{1}{2} \times 25 \times 3 + \frac{1}{2} \times 2 \times 13 + (9 \times 12.5)$

$= 78 + \frac{75}{2} + 13 + 112.5$

$= 78 + 37.5 + 13 + 112.5 = 241$
Area of Pentagon ABCDE = 241 cm^2



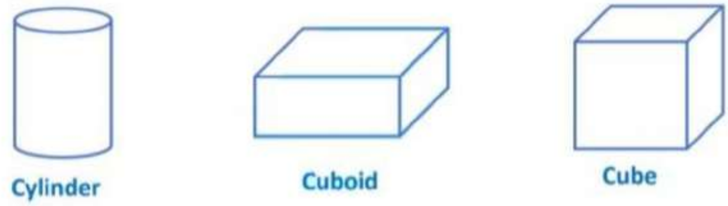
Solid Shapes

Solid Shapes

They have three dimensions like length, breadth and height or depth. They are also called Three dimensional Figures(3-D).

A solid shape is a shape that occupies space. A solid is a three- dimensional figure.

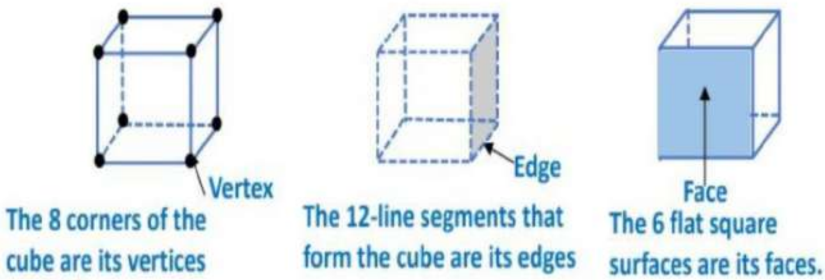
Examples: Cone, Spheres, Cubes, Cylinders, etc.



Faces: Polygons forming a polyhedron are known as their faces. The flat surfaces of any solid called faces.

Edges: Line segments common to intersecting faces of a polyhedron are known as its edges. Line segments that form the solid are called edges.

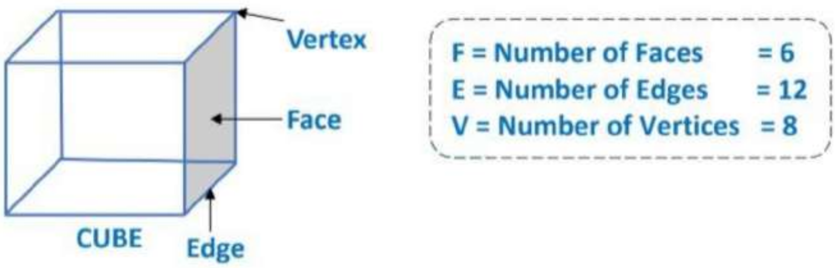
Vertices: Points of intersection of edges of a polyhedron are known as its vertices. Corners of the solid are its vertices.



Cube

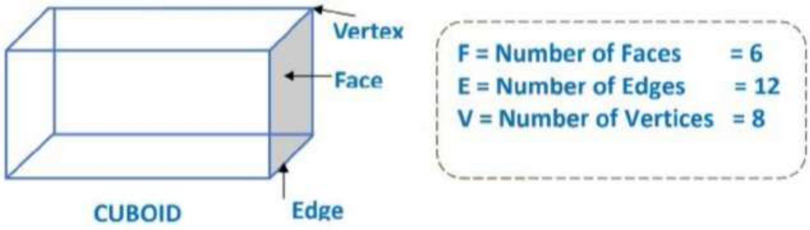
A cube whose length, breadth and height are all equal is called a cube.

For example, ice cubes, dice, sugar cubes.



Cuboid

A solid bounded by six rectangular plane faces with opposite identical faces is called a cuboid. Example a chalk box, a matchbox, a book.



Cylinder

The cylinders have congruent circular faces that are parallel to each other. The line segment joining the center of circular faces is perpendicular to the circular faces. Such

cylinders are known as Right

circular cylinders.

Base: Each of the circular ends on which the cylinder rests is called its base.

Axis: The line segment joining the centers of two circular bases is called the axis of the cylinder.

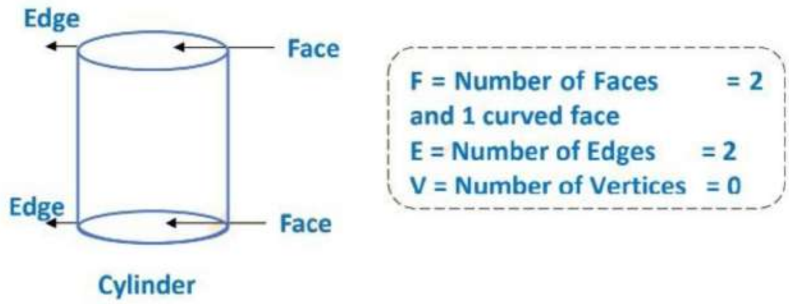
The axis is always perpendicular to the bases of a right circular cylinder.

Radius: The radius of the circular bases is called the radius of the cylinder.

Height: The length of the axis of the cylinder is called the height of the cylinder. In other words, the perpendicular distance between the two parallel circular ends or the

altitude to either base from a point on the other is called the height of the cylinder.

Note: The cylinder has congruent circular faces that are parallel to each other. The line segment joining the center of circular faces is perpendicular to the circular faces. Such cylinders are called right circular cylinders.

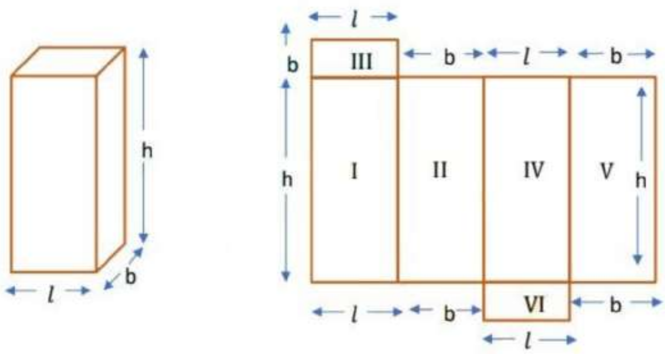


Surface Area of Cuboid, Cube and Cylinder

Surface area

The total surface area of a solid is the sum of the areas of its faces.

The surface area of a cuboid



The Total Surface Area of a Cuboid

= area I + area II + area III + area IV + area V + area VI

$=(l \times h) + (b \times h) + (l \times b) + (l \times h) + (b \times h) + (l \times b)$

$= lh + bh + lb + lh + bh + lb$

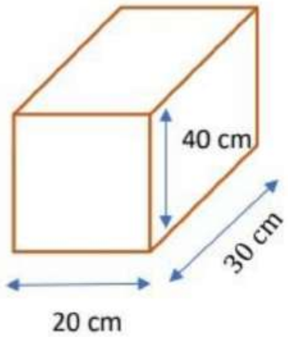
$= 2lb + 2bh + 2lh$

Total surface area $= 2(lb + bh + lh)$

A cuboidal tin is 20 cm × 30 cm × 40 cm. Find the cost of the tin required for making 10 such tins if the cost of the tin sheet is Rs. 10 per square meter.

Here, l = 20 cm, b = 30 cm, h = 40 cm

Surface Area of Cuboid = 2(lb + bh + lh



$= 2(20 \times 30 + 30 \times 40 + 20 \times 40$

$= 2(600 + 1200 + 800)$

$= 2(2600) = 5200$

Surface Area of one tin = 5200 cm^2

Surface Area of 10 tins = 10×5200 cm^2

$= 52000\ cm^2 = \frac{52000}{100 \times 100}\ m^2 = 5.2\ m^2$

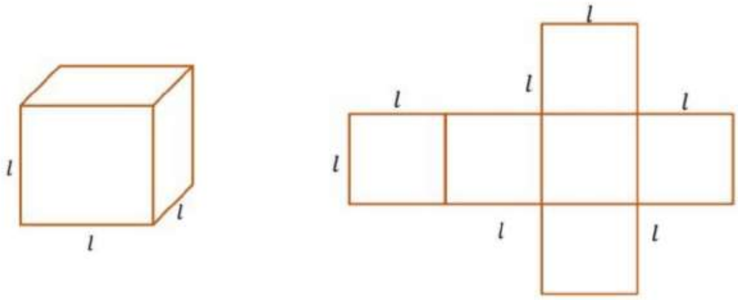
1 m = 100 cm
100 cm = 1 m
$\mathbf{1\ cm = \frac{1}{100}\ m}$
$\mathbf{1 cm^2 = \frac{1}{100 \times 100} m^2}$

Cost of 1 m^2 of tin sheet = ₹10

Cost of 5.2 m^2 of tin sheet = ₹(5.2×10)

Cost of making 10 tins = ₹ 52

The surface area of a cube



Surface area of cube

$= 2(\text{side} \times \text{side} + \text{side} \times \text{side} + \text{side} \times \text{side} + \text{side} \times \text{side})$

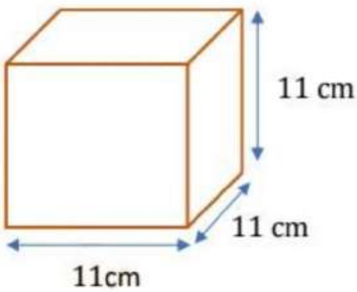
$= 2 \times 3(\text{side})^2 = 6 \times (\text{side})^2$

Surface area of cube $= 6l^2$, where l is the side of the cube.

Find the surface area of a cube whose edge is 11 cm.
The Surface Area of a Cube $= 6l^2$

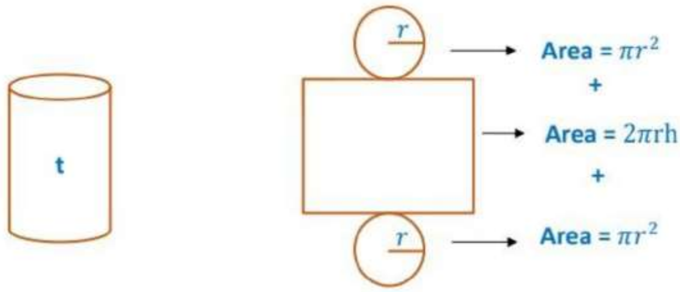
$l = 11\text{cm}$ (given)
Surface Area $= 6 \times (11)^2$

$= 6 \times 11 \times 11$
$= 6 \times 121$
$= 726$
Surface Area of Cube $= 726 \text{ cm}^2$



Surface area of a Cylinder

The total surface area of a right circular cylinder consists of the area of the curved surface and area of the two circular ends.



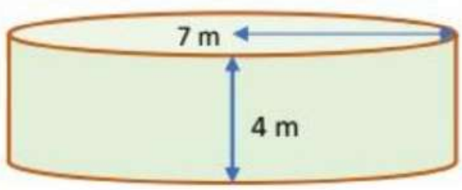
Total surface area
= Curved surface area + 2 × Area of a circular end= $2\pi rh + 2\pi r^2$

Total surface area = $2\pi r(h + r)$

A closed cylindrical tank of radius 7 m and height 4 m is made from a sheet of metal. How many sheets of metal is required? ($\pi = \frac{22}{7}$)

The Surface Area of Cylinder = $2\pi r(r + h)$
r = 7 m, h = 4 m (given)

Area of sheet required = $2 \times \frac{22}{7} \times 7 \times (7 + 4)$



= 44×11 = 484

Area of sheet required = 484 m^2

Volume of Cuboid, Cube and Cylinder

The volume of Solid Figures

The amount of space occupied by a three-dimensional object is called its volume.

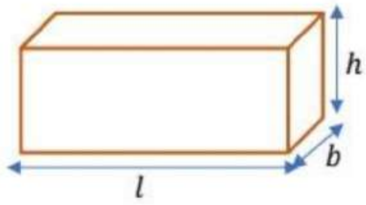
All solid figures are three dimensional i.e., they have length, breadth, and height. These are measured in meters, centimetres, etc

The volume of the solid figure is expressed in cubic meters (m^3), cubic centimetres (cm^3).

One cubic centimetre (1 cm^3) means the space occupied by a cube of side 1 cm and the same for cubic meter.

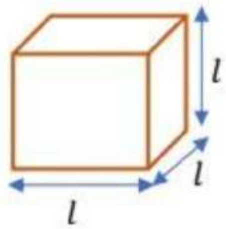
Volume of Cuboid

The volume of a Cuboid = Area of the base × height = $l \times b \times h$



Volume of Cube

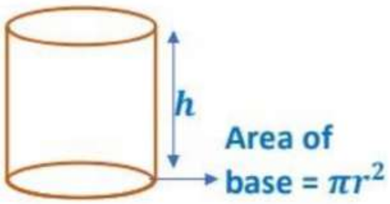
A cube is a special type of cuboid whose length, breadth and height are equal.



Volume of a Cuboid = $l \times l \times l = l^3$

Volume of Cylinder

The volume of a Cylinder = Area of base × height



= $\pi r^2 \times h$

$= \pi r^2 h$

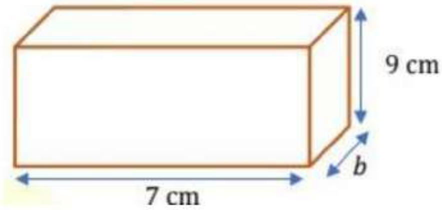
A cuboidal wooden block contains 504 cm^3 wood. If it is 7 cm long and 9 cm high, find its breadth.

The volume of the wooden block = 504 cm^3

Height of the wooden block = 9 cm

Length of the wooden block = 7 cm

Volume of Cuboid = $l \times b \times h$



$504 = 7 \times b \times 9$

$b = \frac{504}{7X9}$ cm

$b = \frac{72}{9} = 8$ cm

Breadth = 8 cm

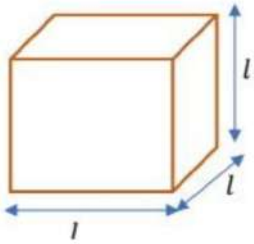
Find the surface area of a Cube whose volume is 512 cm^3.

The volume of Cube = l^3
Volume of Cube = 512cm^3(given)

$512 = l^3$

$8^3 = l^3$

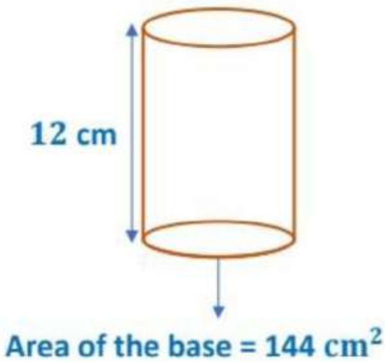
$l = 8$ cm



Surface Area of a Cube = $6l^2$

$= 6(8)^2$
$= 6 \times 64$
The surface area of Cube = 384 cm^2

The area of the base of a right circular cylinder is 144cm^2 and its height is 12 cm. Find the volume of the cylinder.



Area of the base = 144 cm^2

Height = 12 cm

Volume of the Cylinder = Area of the base × height

=144cm^2 × 12 cm

Volume of the Cylinder = 1728 cm^3

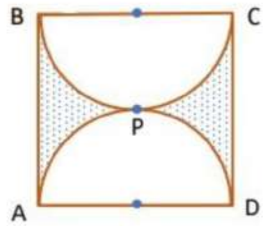
Find the area of the shaded region. If ABCD is a square of side 28 cm and APD and BPC are semi-circles. ($\pi = \frac{22}{7}$)

We have, area of the shaded region = Area of square ABCD – Area of two semi-circles

$$= \left\{(28 \times 28) - 2\left(\frac{1}{2} \times \frac{22}{7} \times 14^2\right)\right\} \text{cm}^2$$

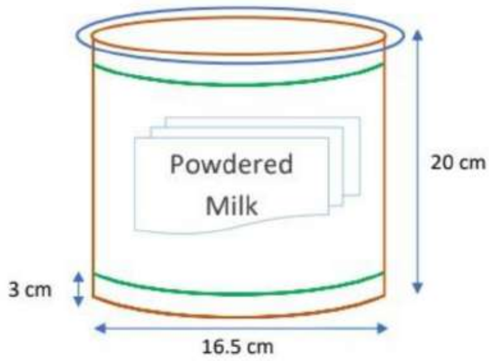
$$= (784 - 616)\ \text{cm}^2$$

$$= 168\ \text{cm}^2$$



A company packages its milk powder in cylindrical containers whose base has a diameter of 16.8 cm and a height of 20 cm. The company places a label around the curved surface of the container. If the label is placed 3 cm from the top and the bottom, what is the surface area of the label?

Surface area of the label is equal to the curved surface area of a cylinder of base radius (r) = $\frac{16.8}{2}$ cm = 8.4 cm and height (h) = (20 – 3 – 3) cm = 14 cm



Surface area of the label = $2\pi rh$

$= 2 \times \frac{22}{7} \times 8.4 \times 14\ \text{cm}^2$

$= 2 \times 22 \times 1.2 \times 14\ \text{cm}^2$

$= 739.2\ \text{cm}^2$