CBSE Board
Class XI Mathematics

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. Find the sum to infinity of the sequence: $\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+...$

2. Write the truth value of the statement p: Intersection of two disjoint sets is an empty set.

3. Find $\cos\left(\frac{\pi}{4}-\alpha\right)\cos\left(\frac{\pi}{4}-\beta\right)-\sin\left(\frac{\pi}{4}-\alpha\right)\sin\left(\frac{\pi}{4}-\beta\right)$.

4. Find the argument of $\frac{1}{1-i}$.

SECTION – B

5. What is the eccentricity of the curve $4x^2 + y^2 = 100$?

6. What is the probability that two friends will have the same birthday?

7. Divide 20 into 4 parts which form an A.P. such that ratio of the product of the I^{st} and the 4^{th} term to the product of the 2^{nd} and the 3^{rd} is 2: 3.

8. If the sum of n terms of an A.P is $(pn + qn^2)$ where p, q are constants, find the common difference**.**

9. Let R be a relation from N to N defined by
R = {(a, b): a, b $\in$ N and a = b^2}.
Then, which of the following statement is true?
(i) (a, a) $\in$ R, for all a $\in$ N
(ii) (a, b) $\in$ R, implies (b, a) $\in$ R

10. Differentiate |2x – 1| w.r.t. x.

11. One end of diameter of the circle $x^2+y^2-3x+5y-4=0$ is (2, 1). Find the co-ordinates of other end.

12. Find the equation of ellipse with e = ¾, foci on y axis, centre at the origin & passing through point (6, 4).

SECTION – C

13. A school gave out medals on its sports day. 38 medals were given for soccer, 15 for basketball, and 20 for cricket. These medals were given to 58 students in all. Only three students got medals in all three sports. How many students received medals in exactly two of the three sports?

14. Show that: $2\cos 6\theta = 64\cos^6\theta - 96\cos^4\theta + 36\cos^2\theta - 2$

OR

Show that: $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$

15. In how many ways can 5 children be arranged in a row such that 2 boys x and y, (i) are always together (ii) are never sit together?

OR

In how many ways can 5 men and 4 women be seated in a row, so that the women occupy even places only?

16. Find the equation of the set of points P, the sum of whose distances from A(4, 0, 0) and B(-4, 0, 0) is equal to 10 ?

17. Prove that 2 cos $\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$

18. Find the domain and range of f(x) = $\sqrt{x-5}$

OR

Find the domain and range of $f(x) = \dfrac{3}{(2-x^2)}$

19. A ladder 12 m long leaning against a wall begins to slide down. Its one end always remains on the wall and the other on the floor. Find the equation of the locus of a point P which is 3 m from the end in contact with the floor. Identify the conic section represented by the equation.

20. Prove that $a^n - b^n$ is a multiple of (a - b), where a and b are natural numbers.

21. Find the equation of a line, perpendicular to the line whose equation is $6x - 7y + 8 = 0$ and which passes through the point of intersection of the two lines whose equations are $2x - 3y - 4 = 0$ and $3x + 4y - 5 = 0$.

22. An administration assistant is given three letters to be mailed to three different people. He is also given three addressed envelopes in which to put them and send to three people X, Y and Z. What is the probability that atleast one person out of X, Y and Z got the letter written to him?

23. If O is the sum of odd terms and E of even terms in the expansion of $(x + a)^n$, prove that:

(i) $O^2 - E^2 = (x^2 - a^2)^n$

(ii) $4OE = (x + a)^{2n} - (x - a)^{2n}$

SECTION – C

24. The sum of n terms of two A.P.s are in the ratio (7n + 1) : (4n + 27). Find the ratio of their 13^{th} term.

25. If in a ΔABC, $\dfrac{b+c}{12} = \dfrac{c+a}{13} = \dfrac{a+b}{15}$,then prove that: $\dfrac{\cos A}{2} = \dfrac{\cos B}{7} = \dfrac{\cos C}{11}$.

26. Show by mathematical induction that the sum to n terms of the series

$1^2 + 2\times 2^2 + 3^2 + 2\times 4^2 + 5^2 + 2\times 6^2 + ...$is

$$S_n = \begin{cases} \dfrac{n(n+1)^2}{2}, & \text{when n is even} \\ \dfrac{n^2(n+1)}{2}, & \text{when n is odd} \end{cases}$$

27. Graph the given inequalities and shade the common solution region.

$2x + y \geq 40,\ x + 2y \geq 50, x + y \geq 35$

28. Given below is the frequency distribution of weekly study hours of a group of class 11 students. Find the mean, variance and standard deviation of the distribution using the short cut method.

Classes	Frequency
0 - 10	5
10 - 20	8
20 - 30	15
30 - 40	16
40 - 50	6

29. (i) Find the derivative of $f(x) = -\frac{1}{x}$, using the first principle.

(ii) Evaluate: $\lim_{x \to 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$

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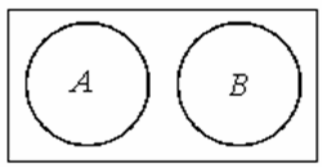
Solution

SECTION – A

1. The sum of the infinite series will be: $\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+...=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{2}$

2. Since A and B are disjoint sets, so their intersection is an empty or a null set. So the statement is true.



$A\cap B=\phi$

3. $\cos\left(\frac{\pi}{4}-\alpha\right)\cos\left(\frac{\pi}{4}-\beta\right)-\sin\left(\frac{\pi}{4}-\alpha\right)\sin\left(\frac{\pi}{4}-\beta\right)$

$=\cos\left[\left(\frac{\pi}{4}-\alpha\right)+\left(\frac{\pi}{4}-\beta\right)\right]$

$=\cos\left[\frac{\pi}{2}-(\alpha+\beta)\right]$

$=\sin(\alpha+\beta)$

4. $\frac{1}{1-i}=\frac{1}{1-i}\times\frac{1+i}{1+i}=\frac{1+i}{(1)^2-(i)^2}=\frac{1+i}{1+1}=\frac{1}{2}+\frac{1}{2}i$

Comparing with $x+iy$, $x=\frac{1}{2},y=\frac{1}{2}$

Argument = $\theta=\tan^{-1}\frac{y}{x}=\tan^{-1}\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right)=\tan^{-1}1=\frac{\pi}{4}$

SECTION – B

5. $4x^2 + y^2 = 100$

$\Rightarrow \frac{x^2}{25} + \frac{y^2}{100} = 1$

$\Rightarrow b^2 = 100, a^2 = 25; c = \sqrt{b^2 - a^2} = \sqrt{100-25} = \sqrt{75} = 5\sqrt{3}$

$\text{eccentricity}, e = \frac{c}{b} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$

6. Probability of both the friends not having the same birthday is $\frac{365}{365} \times \frac{364}{365}$

So the probability of two friends having the same birthday is

$= 1 - \frac{365}{365} \times \frac{364}{365} = 1 - 1 \times \frac{364}{365} = \frac{1}{365}$

7. Let the four parts be a - 2d, a - d, a + d, a + 2d

So a - 2d + a - d + a + d + a + 2d = 20

a = 5

$\frac{[(a-2d)(a+2d)]}{[(a-d)(a+d)]} = \frac{2}{3}$

$\frac{(a^2 - 4d^2)}{(a^2 - d^2)} = \frac{2}{3}$

$3a^2 - 12d^2 = 2a^2 - 2d^2$

$a^2 - 10\,d^2 = 0$

So, $25 = 10\,d^2$

$d = \frac{\pm 5}{\sqrt{10}}$

So A.P. is $5 - \sqrt{10}$, $5 - \frac{5}{\sqrt{10}}$, $5 + \frac{5}{\sqrt{10}}$, $5 + \sqrt{10}$

Or $5 + \sqrt{10}$, $5 + \frac{5}{\sqrt{10}}$, $5 - \frac{5}{\sqrt{10}}$, $5 - \sqrt{10}$

8. $S_n = (pn + qn^2)$

$S_1 = (p.1 + q.1^2) = p + q = a_1$

$S_2 = (p.2 + q.2^2) = 2p + 4q = a_1 + a_2$

$a_2 = S_2 - S_1 = p + 3q$

$d = a_2 - a_1 = (p + 3\,q) - (p + q) = 2q$

9. If we need to prove something false, one counter example is sufficient.

(i) $(a, a) \in R$, for all $a \in N$ is not true

For example take $2 \in N$. we have $2 \neq 2^2$, therefore $(2, 2) \notin R$.

(ii) $(a, b) \in R$, implies $(b, a) \in R$ is also not true for example take a = 9, b = 3.

As $9 = 3^2$, we have $(9, 3) \in R$, but $3 \neq 9^2$, therefore $(3, 9) \notin R$.

10. Let y = |2x – 1|

Differentiating w.r.t. x we get

$$\frac{dy}{dx} = \frac{2x^2-1}{\left|2x^2-1\right|} \times \frac{d}{dx}\left(2x^2-1\right) \ldots\ldots\ldots\ldots\ldots \quad \left\{\frac{d}{dx}|x| = \frac{x}{|x|}\right\}$$

$$= \frac{4x\left(2x^2-1\right)}{\left|2x^2-1\right|} \ldots\ldots\ldots \quad x \neq \pm\frac{1}{\sqrt{2}}$$

11. Equation of the circle is $x^2+y^2-3x+5y-4=0$

∴ Centre is (3/2, -5/2)

Which is the midpoint of diameter having end points (2, 1) & (x, y)……… say

So by mid point formula

$$\frac{x+2}{2} = \frac{3}{2} \quad \& \quad \frac{y+1}{2} = \frac{-5}{2}$$

∴ x = 1, y = -6

12. Since the centre of ellipse is at the origin & foci lie on y-axis, its equation is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \ldots\ldots\ldots\ldots\ldots (1)$$

$$where\ b^2 = a^2\left(1-e^2\right)$$

$$b^2 = a^2\left(1-\left(\frac{3}{4}\right)^2\right)$$

$$b^2 = \frac{7}{16}a^2$$

Sub in (1)

$$\frac{x^2}{\frac{7}{16}a^2} + \frac{y^2}{a^2} = 1 \ldots\ldots\ldots\ldots\ldots (2)$$

Also it passes through (6, 4)

∴ $576 + 112 = 7a^2$

Substituting a^2 and b^2 in (1)

We get equation $16x^2 + 7y^2 = 688$.

SECTION – C

13. Let A, B and C represent the set of students who received medals for Soccer, Basketball and Cricket.

$n(A) = 38, n(B) = 15, n(C) = 20, n(A \cup B \cup C) = 58, n(A \cap B \cap C) = 3$

Using counting theorems,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Substituting the values we get,

$\Rightarrow 58 = 38 + 15 + 20 - n(A \cap B) - n(B \cap C) - n(A \cap C) + 3$

$\Rightarrow n(A \cap B) + n(B \cap C) + n(A \cap C) = 76 - 58 = 18$

Now, each of $n(A \cap B)$, $n(B \cap C)$, $n(A \cap C)$ include the 3 students who received medals for all three sports.

Number of students who received medals in exactly two sports,

$n(A \cap B) - 3 + n(B \cap C) - 3 + n(A \cap C) - 3 = 18 - 3 - 3 - 3 = 9$

14. $\cos 6\theta = \cos 3(2\theta)$

$$= 4\cos^3(2\theta) - 3\cos(2\theta) \qquad \left[\text{Using}, \cos 3\theta = 4\cos^3\theta - 3\cos\theta\right]$$

$$= 4[\cos(2\theta)]^3 - 3\cos(2\theta)$$

$$= 4\left[2\cos^2\theta - 1\right]^3 - 3\left[2\cos^2\theta - 1\right] \qquad \left[\text{Using } \cos 2\theta = 2\cos^2\theta - 1\right]$$

$$= 4\left[8\cos^6\theta - 3\times 4\cos^4\theta \times 1 + 3\times 2\cos^2\theta \times (1)^2 - 1\right] - 3\left[2\cos^2\theta - 1\right]$$

$$= 32\cos^6\theta - 48\cos^4\theta + 24\cos^2\theta - 4 - 6\cos^2\theta + 3$$

$$= 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

$$\therefore 2\cos 6\theta = 2\left(32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1\right)$$

$$= 64\cos^6\theta - 96\cos^4\theta + 36\cos^2\theta - 2$$

OR

Using $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ and $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ in L.H.S.

L.H.S.

$$= \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta}$$

$$= \frac{3\sin\theta - 4\sin^3\theta}{\sin\theta} - \frac{4\cos^3\theta - 3\cos\theta}{\cos\theta}$$

$$= \frac{\sin\theta(3 - 4\sin^2\theta)}{\sin\theta} - \frac{\cos\theta(4\cos^2\theta - 3)}{\cos\theta}$$

$= 3 - 4\sin^2\theta - 4\cos^2\theta + 3$

$= 6 - 4\sin^2\theta - 4\cos^2\theta$

$= 6 - 4(\sin^2\theta + \cos^2\theta)$

$= 6 - 4$

$= 2$

= R.H.S.

15. Five children could be arranged in 5! ways.

(i) If x and y have to sit together, then taking x and y as 1 unit there are 4 ways of arranging them and the two can interchange places, so 2! x 4! = 48 ways

(ii) Number of ways in which the two boys x and y never sit together, Total ways - ways in which x and y are together = 5! - 2! x 4! = 72 ways

OR

There are 9 places in all i.e. 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Out of these, 5 are odd and 4 are even.

The 5 odd places i.e. 1, 3, 5, 7, and 9 have to be occupied by the 5 men as the women do not have to sit on these.

This they can do in ${}^5P_5$ ways = 5!

Now the 4 even places i.e., 2, 4, 6, 8 have to be occupied by the 4 women

This they can do in ${}^4P_4$ ways = 4!

Together the men and women can be seated in 5! x 4! = 120 x 24 = 2880 ways

16. Let P(x, y, z) be the required point.

Given PA + PB =10

i.e. PA = 10 – PB

Squaring both sides,

$PA^2 = (10 - PB)^2 = 100 + PB^2 - 20PB$

$(x - 4)^2 + y^2 + z^2 = 100 + (x + 4)^2 + y^2 + z^2 - 20\sqrt{((x+4)^2 + y^2 + z^2)}$

Simplifying, we get

$-16x - 100 = -20\sqrt{((x+4)^2 + y^2 + z^2)}$

Squaring both sides again and simplifying

$\Rightarrow [16x + 100]^2 = 400[(x+4)^2 + y^2 + z^2)]$

$\Rightarrow 256x^2 + 10000 + 3200x = 400[x^2 + 16 + 8x + y^2 + z^2]$

$\Rightarrow 256x^2 + 10000 + 3200x = 400x^2 + 6400 + 3200x + 400y^2 + 400z^2$

$\Rightarrow 144x^2 + 400y^2 + 400z^2 - 3600 = 0$

$\Rightarrow 18x^2 + 50y^2 + 50z^2 - 450 = 0$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

17. $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=\cos\left(\frac{\pi}{13}+\frac{9\pi}{13}\right)+\cos\left(\frac{\pi}{13}-\frac{9\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=\cos\left(\frac{10\pi}{13}\right)+\cos\left(-\frac{8\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=\cos\left(\frac{10\pi}{13}\right)+\cos\left(\frac{8\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=\cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=-\cos\left(\frac{3\pi}{13}\right)-\cos\left(\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

18. Let $y=f(x)=\sqrt{x-5}$

Square root is defined only for non – negative real numbers so

Domain of f is the set of numbers for which $x-5\geq 0$

i.e $x\geq 5$

$\Rightarrow$ Domain of f=$[5,\infty)$

Now, $\sqrt{x-5}=y$

$\Rightarrow (x-5)=y^2$

$\Rightarrow x=y^2+5$

since x is taking values greater then or equal to 5 so

Range of f = $[0,\infty)$

OR

$f(x)=\frac{3}{2-x^2}$, for f^n to be defined $2-x^2\neq 0$, i.e. $2\neq x^2$

So Domain = {x: x is a real number and $x\neq\pm\sqrt{2}$}

For range,

$\frac{3}{2-x^2}=y$

$2y-x^2y=3$

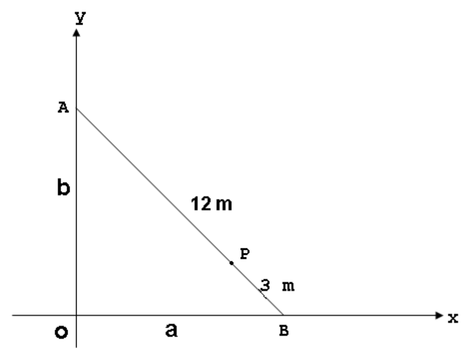
$x^2=\frac{2y-3}{y}$

$x=\pm\sqrt{\frac{2y-3}{y}}$

For x to be defined $\frac{2y-3}{y}$ must be positive, i.e. either both Numerator and Denominator should be positive, in which case y > 3/2 or both should be negative, in which case y < 0.

Range = $(-\infty,0)\cup\left(\frac{3}{2},\infty\right)$

19. Let the ladder be AB, where A is its end on the horizontal axis and B is the end on the vertical axis.



A right angled triangle is formed here: $(12)^2 = a^2 + b^2$

The distance of the point P from the end in contact with the floor axis is 3 m, so from the end where it touches the wall axis is 12 – 3 = 9 m

i.e. P divides AB in the ratio 9 : 3 = 3 : 1

Therefore the co-ordinates of P, using the section formula are:

$$(x,y)=\left(\frac{3\times a+1\times 0}{3+1},\frac{1\times b+3\times 0}{1+3}\right)=\left(\frac{3a}{4},\frac{b}{4}\right)$$

$$\Rightarrow a=\frac{4x}{3},b=4y$$

$$\Rightarrow 144=\left(\frac{4x}{3}\right)^2+(4y)^2$$

$$\Rightarrow 1=\frac{x^2}{9\times 9}+\frac{y^2}{9}$$

$$\Rightarrow \frac{x^2}{81}+\frac{y^2}{9}=1$$

ellipse

20. $a^n - b^n = (a-b+b)^n - b^n = ((a-b)+b)^n - b^n$

$$= {}^nC_0(a-b)^n b^0 + {}^nC_1(a-b)^{n-1}b^1 + {}^nC_2(a-b)^{n-2}b^2$$
$$+ ... + {}^nC_{n-1}(a-b)^{n-(n-1)}b^{n-1} + {}^nC_n(a-b)^{n-n}b^n - b^n$$
$$= {}^nC_0(a-b)^n b^0 + {}^nC_1(a-b)^{n-1}b^1 + {}^nC_2(a-b)^{n-2}b^2 + ... + {}^nC_{n-1}(a-b)^1 b^{n-1}$$
$$+ {}^nC_0(a-b)^0 b^n - b^n$$
$$= {}^nC_0(a-b)^n b^0 + {}^nC_1(a-b)^{n-1}b^1 + {}^nC_2(a-b)^{n-2}b^2 + ... + {}^nC_{n-1}(a-b)b^{n-1}$$
$$+ b^n - b^n$$
$$= {}^nC_0(a-b)^n b^0 + {}^nC_1(a-b)^{n-1}b^1 + {}^nC_2(a-b)^{n-2}b^2 + ... + {}^nC_{n-1}(a-b)b^{n-1}$$
$$= (a-b)\left[{}^nC_0(a-b)^{n-1}b^0 + {}^nC_1(a-b)^{n-2}b^1 + {}^nC_2(a-b)^{n-3}b^2 + ... + {}^nC_{n-1}b^{n-1}\right]$$

Now, ${}^nC_0, {}^nC_1, .., {}^nC_{n-1} \in Z$; Since a and b are natural numbers

$(a-b)^{n-1}, (a-b)^{n-2}, ... \in Z; b^0, b^1, b^2, b^{n-1} \in Z$

$\Rightarrow a^n - b^n = (a-b)[\text{an integer}]$

$\Rightarrow a^n - b^n$ is a multiple of $(a-b)$

21. Consider the equation of the lines

$2x - 3y - 4 = 0$ (i)

$3x + 4y - 5 = 0$ (ii)

Multiplying (i) by 4 and (ii) by 3 we get

$8x - 12y - 16 = 0$

$9x + 12y - 15 = 0$

$17x - 31$

$\Rightarrow x = \frac{31}{17} \Rightarrow 2\left(\frac{31}{17}\right) - 3y - 4 = 0 \Rightarrow y = -\frac{2}{17}$

Now the slope of the given line $6x - 7y + 8 = 0$ is $\frac{6}{7}$

So the slope of the required line = $-\frac{7}{6}$

The equation of the required line is:

$y - y_1 = m(x - x_1)$

$\Rightarrow y - \left(-\frac{2}{17}\right) = \left(-\frac{7}{6}\right)\left(x - \left(\frac{31}{17}\right)\right)$

$\Rightarrow y + \frac{2}{17} = -\frac{7x}{6} + \frac{217}{102}$

$\Rightarrow 119x + 102y = 205$

22. We need to find the probability that atleast one letter was put in the correct envelope.

The total number of ways of putting three letters in 3 envelopes = 3! = 6 ...

The number of ways of putting all three letters in incorrect envelopes is

Letter for X goes into envelope for Y and letter for Y goes into envelope for Z, letter for Z goes into envelope for X or letter for X goes into envelope for Z and letter for Y goes into envelope for X and letter for Z goes into envelope for Y

So there are 2 ways

P(None of the letters is put in the right envelope) $= \frac{2}{6} = \frac{1}{3}$

P(Atleast one letter is put in the right envelope)= 1 − P(None of the letters is put in the right envelope) $= 1 - \frac{1}{3} = \frac{2}{3}$

23. We have,

$$(x+a)^n = {}^nC_0x^na^0 + {}^nC_1x^{n-1}a^1 + {}^nC_2x^{n-2}a^2 + \dots + {}^nC_{n-1}xa^{n-1} + {}^nC_na^n$$

$$\Rightarrow (x+a)^n = \left\{{}^nC_0x^na^0 + {}^nC_2x^{n-2}a^2 + \dots\right\} + \left\{{}^nC_1x^{n-1}a^1 + {}^nC_3x^{n-3}a^3 + \dots\right\}$$

$$\Rightarrow (x+a)^n = O + E \quad \dots.(1)$$

Similarly,

$$(x-a)^n = {}^nC_0x^na^0 - {}^nC_1x^{n-1}a^1 + {}^nC_2x^{n-2}a^2 - \dots + {}^nC_{n-1}(-1)^{n-1}xa^{n-1} + {}^nC_n(-1)^na^n$$

$$\Rightarrow (x-a)^n = \left\{{}^nC_0x^na^0 + {}^nC_2x^{n-2}a^2 + \dots\right\} - \left\{{}^nC_1x^{n-1}a^1 + {}^nC_3x^{n-3}a^3 + \dots\right\}$$

$$\Rightarrow (x-a)^n = O - E \quad \dots.(2)$$

where,

$$O = \left\{{}^nC_0x^na^0 + {}^nC_2x^{n-2}a^2 + \dots\right\};$$

$$E = \left\{{}^nC_1x^{n-1}a^1 + {}^nC_3x^{n-3}a^3 + \dots\right\}$$

(i)

Multiplying equations (1) and (2), we have,

$$(x+a)^n(x-a)^n = (O+E)(O-E)$$

$$\Rightarrow \left(x^2-a^2\right)^n = \left(O^2-E^2\right)$$

(ii)

We know that, $4OE = (O+E)^2 - (O-E)^2$

$$\Rightarrow 4OE = \left((x+a)^n\right)^2 - \left((x-a)^n\right)^2 \quad \text{[from equations (1) and (2)]}$$

$$\Rightarrow 4OE = (x+a)^{2n} - (x-a)^{2n}$$

SECTION – C

24. The sum of n terms of two A.P.s are in the ratio (7n + 1) : (4n + 27)

$$\frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]}=\frac{7n+1}{4n+27}$$

$$\frac{[2a_1+(n-1)d_1]}{[2a_2+(n-1)d_2]}=\frac{7n+1}{4n+27}$$

$$\frac{\left[a_1+\frac{(n-1)}{2}d_1\right]}{\left[a_2+\frac{(n-1)}{2}d_2\right]}=\frac{7n+1}{4n+27}$$

To find the ratio of the 13^{th} terms , we need $\frac{a_1+12d_1}{a_2+12d_2}$

$$\Rightarrow \frac{n-1}{2}=12 \Rightarrow n-1=24 \Rightarrow n=25.$$

$$\therefore \frac{a_1+12d_1}{a_2+12d_2}=\frac{7n+1}{4n+27}=\frac{7\times 25+1}{4\times 25+27}=\frac{176}{127}$$

25. $\frac{b+c}{12}=\frac{c+a}{13}=\frac{a+b}{15}$

$b+c=12k, c+a=13k$ and $a+b=15k$

Therefore

a = 8k, b = 7k and c = 5k

Now

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{(7k)^2+(5k)^2-(8k)^2}{2(7k)(5k)}=\frac{10k^2}{70k^2}=\frac{1}{7}=\frac{2}{14}$$

$$\cos B=\frac{c^2+a^2-b^2}{2ca}=\frac{(5k)^2+(8k)^2-(7k)^2}{2(5k)(8k)}=\frac{40k^2}{80k^2}=\frac{1}{2}=\frac{7}{14}$$

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{(8k)^2+(7k)^2-(5k)^2}{2(8k)(7k)}=\frac{88k^2}{112k^2}=\frac{11}{14}$$

Therefore

$\cos A:\cos B:\cos C=2:7:11$ or $\frac{\cos A}{2}=\frac{\cos B}{7}=\frac{\cos C}{11}$

26.

Let $S_n=1^2+2\times 2^2+3^2+2\times 4^2+5^2+2\times 6^2+..........$

Every odd term is n^2 and every even term is $2\times n^2$

Let $P(n): S_n = \begin{cases} \dfrac{n(n+1)^2}{2}, & \text{when n is even} \\ \dfrac{n^2(n+1)}{2}, & \text{when n is odd} \end{cases}$

when $n = 1, S_1 = \dfrac{n^2(n+1)}{2} = \dfrac{1^2(1+1)}{2} = 1$, which is true

Let the result holds for k i.e P(k) be true to prove P(k+1) to be true.

There are two cases

Case I: If k is odd then $(k + 1)$ is even

To prove: $P(k+1): S_{k+1} = \dfrac{(k+1)((k+1)+1)^2}{2} = \dfrac{(k+1)(k+2)^2}{2}$

$P(k+1): P(k) + (k+1)^{th}$ term $= \dfrac{k^2(k+1)}{2} + 2(k+1)^2$ (since $P(k) = S_k = \dfrac{k^2(k+1)}{2}$)

$$\frac{k^2(k+1)}{2} + 2(k+1)^2 = (k+1)\left[\frac{k^2}{2} + 2(k+1)\right] = \frac{(k+1)}{2}\left[k^2 + 4(k+1)\right]$$

$$= \frac{(k+1)}{2}\left[k^2 + 4k + 4\right] = \frac{(k+1)(k+2)^2}{2}$$

Sp P(k + 1) is true

Case II : k is even then (k+1) is odd

To prove: $P(k+1): S_{k+1} = \dfrac{(k+1)^2((k+1)+1)}{2} = \dfrac{(k+1)^2(k+2)}{2}$

$P(k+1): P(k) + (k+1)^{th}$ term $= \dfrac{k(k+1)^2}{2} + (k+1)^2$ (since, $P(k) = S_k = \dfrac{k(k+1)^2}{2}$)

$$= (k+1)^2\left[\frac{k}{2} + 1\right] = \frac{(k+1)^2(k+2)}{2}$$

$\Rightarrow$ P(k + 1) is true

27. The inequalities are $2x + y \geq 40$, $x + 2y \geq 50$ and $x + y \geq 35$
Converting the inequalities to equations and plotting the corresponding lines,

$2x + y = 40$ or $y = 40 - 2x$

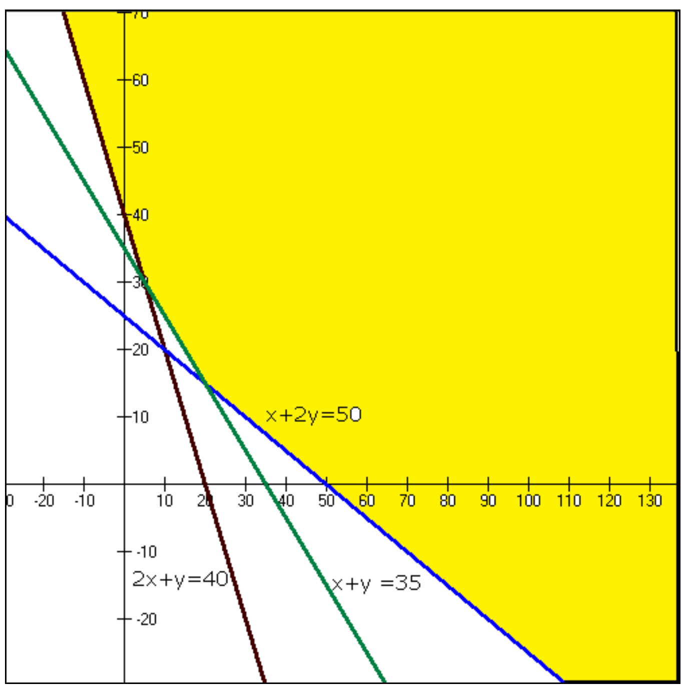
x	0	20	10
y	40	0	20

$x + 2y = 50$ or $x = 50 - 2y$

x	0	50	10
y	25	0	20

$x + y = 35$ or $y = 35 - x$

x	0	35	10
y	35	0	25



28. Let assumed mean be a = 25

Classes	f_i	x_i	$y_i = (x - a)/10$	y_i^2	$f_i y_i$	$f_i y_i^2$
0 - 10	5	5	-2	4	-10	20
10 - 20	8	15	-1	1	-8	8
20 - 30	15	25	0	0	0	0
30 - 40	16	35	1	1	16	16
40 - 50	6	45	2	4	12	24
	50				10	68

$$\sum_{i=1}^{n} f_i y_i = 10, \quad \sum_{i=1}^{n} f_i y_i^2 = 68, \quad \sum_{i=1}^{n} f_i = 50, \quad h = 10$$

$$\bar{x} = a + \frac{\sum_{i=1}^{n} f_i y_i}{\sum_{i=1}^{n} f_i} \times h$$

We get, $\bar{x} = 25 + \frac{10 \times 10}{50} = 27$

$$\sigma_x = \frac{h}{N}\sqrt{N \sum_{i=1}^{n} f_i y_i^2 - \left(\sum_{i=1}^{n} f_i y_i\right)^2}$$

$$\sigma_X = \frac{10}{50}\left[\sqrt{50 \times 68 - (10)^2}\right]$$

$$\sigma_X = \frac{1}{5} \times 10\sqrt{33} = 11.49$$

$$\sigma_X^2 = 132.02$$

So for the given data Mean = 27, Standard Deviation = 11.49 and Variance = 132.02

29.

(i) Derivative of $f(x) = -\frac{1}{x}$, using first principle

$$f(x) = -\frac{1}{x}$$

$$\Rightarrow f(x+\delta x) = -\frac{1}{x+\delta x}$$

$$f(x+\delta x) - f(x) = -\frac{1}{x+\delta x} - \left(-\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{x+\delta x}$$

$$\Rightarrow f(x+\delta x) - f(x) = \frac{(x+\delta x) - x}{x(x+\delta x)} = \frac{\delta x}{x(x+\delta x)}$$

$$\Rightarrow \frac{f(x+\delta x) - f(x)}{\delta x} = \frac{1}{\delta x} \cdot \frac{\delta x}{x(x+\delta x)} = \frac{1}{x(x+\delta x)}$$

$$f'(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} = \lim_{\delta x \to 0} \frac{1}{x(x+\delta x)} = \frac{1}{x(x)}$$

$$\Rightarrow f'(x) = \frac{1}{x^2}$$

$$\text{(ii)} \lim_{x\to 0} \frac{6^x - 3^x - 2^x + 1}{x^2} = \lim_{x\to 0} \frac{2^x(3^x - 1) - 1(3^x - 1)}{x^2}$$

$$= \lim_{x\to 0} \frac{(3^x - 1)(2^x - 1)}{x^2}$$

$$= \lim_{x\to 0} \frac{(3^x - 1)}{x} \lim_{x\to 0} \frac{(2^x - 1)}{x}$$

$$= (\ln 3)(\ln 2)$$