Short Answer Type Questions – II

[3 marks]

Que 1. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

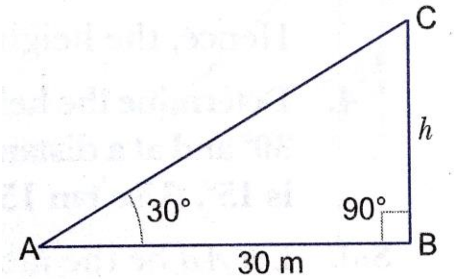

Fig. 11.18

Sol. Let, BC be the tower whose height is h metres and A be the point at a distance of 30 m from the foot of the tower. The angle of elevation of the top of the tower from point A is given to be 30°. Now, in right angleΔCBA, we have,

$$\tan 30° = \frac{BC}{AB} = \frac{h}{30} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\Rightarrow \qquad h = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\, m$$

Hence, the height of the tower is $10\sqrt{3}$ m.

Que 2. A tree breaks due to storm and the broken part bends, so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

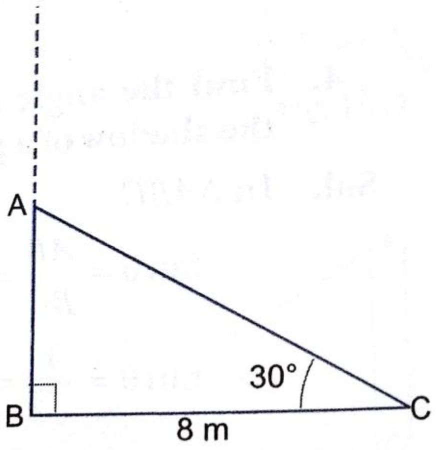

Fig. 11.19

Sol. In right angle $\Delta ABC, AC$ is the broken part of the tree (**Fig. 11.19**). So, the total height of tree = (AB + AC)
Now in right angle ΔABC.

$$\tan 30° = \frac{AB}{BC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{8} \quad \Rightarrow \quad AB = \frac{8}{\sqrt{3}}$$

Again, $\cos 30° = \frac{BC}{AC}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{8}{AC} \quad \Rightarrow \quad AC = \frac{16}{\sqrt{3}}$$

Hence, the height of the tree = AB + AC

$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}\,m$$

Que 3. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m.

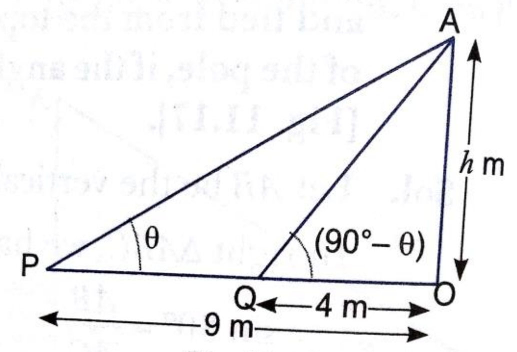

Fig. 11.20

Sol. Let OA be the tower of height h metre and P, Q be the two points at distance of 9 m and 4 m respectively from the base of the tower.
Now, we have OP = 9 m, OQ = 4 m.
Let $\angle APO = \theta, \angle AQO = (90° - \theta)$
and OA = h metre (**Fig. 11.20**)
Now, in ΔPOA, we have

$$\tan\theta = \frac{OA}{OP} = \frac{h}{9} \quad \Rightarrow \quad \tan\theta = \frac{h}{9} \quad \ldots\text{(i)}$$

Again, in ΔAQO, we have

$$\tan(90° - \theta) = \frac{OA}{OQ} = \frac{h}{4} \Rightarrow \quad \cot\theta = \frac{h}{4} \quad \ldots\text{(ii)}$$

Multiplying (i) and (ii), we have

$$\tan\theta \times cot\,\theta = \frac{h}{9} \times \frac{h}{4} \quad \Rightarrow \quad 1 = \frac{h^2}{36} \quad \Rightarrow \quad h^2 = 36$$

$\therefore \qquad h = \underline{+6}$

Height cannot be negative
Hence, the height of the tower is 6 metre.

Que 4. Determine the height of a mountain if the elevation of its top at an unknown distance from the base is 30° and at a distance 10 km further off from the mountain, along the same line, the angle of elevation is 15°. (Use tan 15° = 0.27)

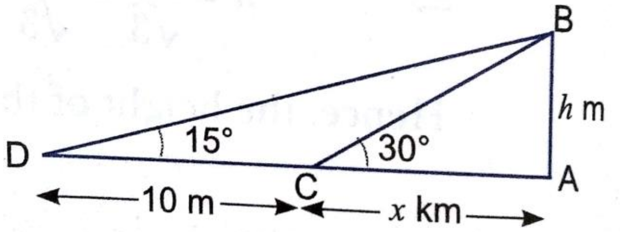

Fig. 11.21

Sol. Let AB be the mountain of height h kilometres. Let C be a point at a distance of x km, from the base of the mountain such that angle of elevation of the top at C is 30°. Let D be a point at a distance of 10 km from C such that angle of elevation at D is of 15°.
In ΔACB (**Fig. 11.21**), we have

$$\tan 30° = \frac{AB}{AC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$\Rightarrow \qquad x = \sqrt{3}h$

In ΔADB, we have

$$\tan 15° = \frac{AB}{AD} \quad \Rightarrow \quad 0.27 = \frac{h}{x+10}$$

$\Rightarrow \qquad (0.27)\,(x + 10) = h$
Substituting x = $\sqrt{3}h$ in equation (i), we get
$\qquad 0.27\,(\sqrt{3}h + 10) = h$
$\Rightarrow \quad 0.27 \times \sqrt{3}h + 0.27 \times 10 = h$
$\Rightarrow \quad 2.7 = h - 0.27 \times \sqrt{3}h \quad \Rightarrow \quad 27 = h(1 - 0.27 \times \sqrt{3})$
$\Rightarrow \quad 27 = h\,(1 - 0.46) \quad \Rightarrow \quad h = \dfrac{2.7}{0.54} = 5$

Hence, the height of the mountain is 5 km.

Que 5. The shadow of a tower standing on a level ground is found to be 40 m longer when the sun's altitude is 30° than when it is 60°. Find the height of the tower.

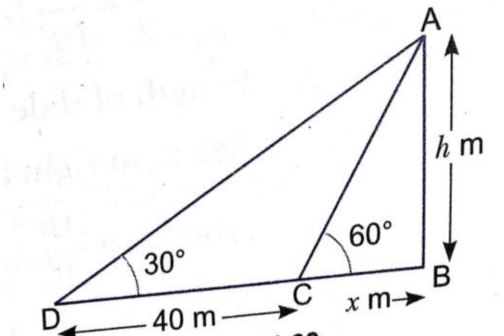

Fig. 11.22

Sol. In **Fig. 11.22**, AB is the tower and BC is the length of the shadow when the sun's altitude is 60°, i.e., the angle of elevation of the top of the tower from the tip of the shadow is 60° and DB is the length of the shadow, when the angle of elevation is 30°.

Now, let AB be h m and BC be x m. According to the question, DB is 40 m longer than BC.

So, BD = (40+x) m

Now, we have two right triangles ABC and ABD.

In $\Delta ABC,$ $\tan 60° = \frac{AB}{BC}$ or $\sqrt{3} = \frac{h}{x}$

$\Rightarrow$ $x\sqrt{3} = h$ …(i)

In $\Delta ABD,$ $\tan 30° = \frac{AB}{BD}$

i.e., $\frac{1}{\sqrt{3}} = \frac{h}{x+40}$ …(ii)

Using (i) in (i), we get $(x\sqrt{3})\ \sqrt{3} = x + 40$ $i.e.,$ $3x = x + 40$

i.e., x = 20

So, h = $20\sqrt{3}$ [From (i)]

Therefore, the height of the tower is $20\sqrt{3}\,m.$

Que 6. From a point P on the ground, the angles of elevation of the top of a 10m tall building is 30°. A flag is hosted at the top of the building and angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (you may take $\sqrt{3} = 1.732$).

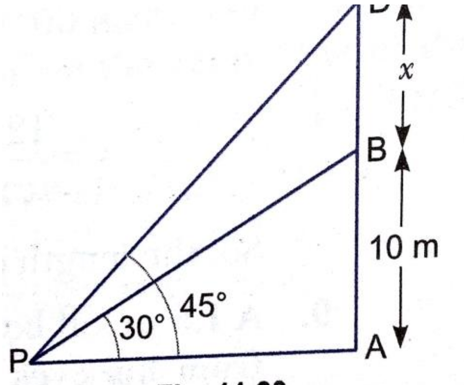

Fig. 11.23

Sol. In **Fig. 11.23,** AB denotes the height of the building, BD the flagstaff and P the given point. Note that there are two right triangles PAB and PAD. We are required to find the length of the flagstaff, i.e., BD and the distance of the building from the point P, i.e., PA. Since, we know the height of the building AB, we will first consider the right ΔPAB.

We have, tan 30° = $\frac{AB}{AP} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{AP}$

$$\Rightarrow \qquad AP = 10\sqrt{3}$$

i.e., the distance of the building from P is $10\sqrt{3}\,m = 10 \times 1.732 = 17.32\,m$.
Next, let us suppose DB = x m. Then, AD = (10+x) m.
Now, in right $\Delta PAD,$

$$\tan 45° = \frac{AD}{AP} = \frac{10+x}{10\sqrt{3}} \quad \Rightarrow \quad 1 = \frac{10+x}{10\sqrt{3}} \quad \Rightarrow \quad 10\sqrt{3} = 10 + x$$

i.e., $\qquad x = 10(\sqrt{3} - 1) = 7.32\,m.$

Que 7. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

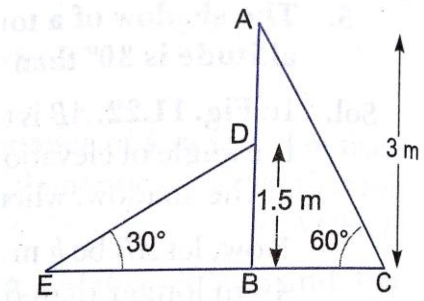

Fig. 11.24

Sol. Let AC be a steep slide for elder children and DE be a slide for younger children. Then AB = 3 m and DB = 1.5 m (**Fig. 11.24**).
Now, in right angleΔDBE, we have

$$\sin 30° = \frac{BD}{DE} = \frac{1.5}{DE}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1.5}{DE} \qquad \therefore \quad DE = 2 \times 15 = 3\,m$$

∴ Length of slide for younger children = 3 m
Again, in right ΔABC, we have

$$\sin 60° = \frac{AB}{AC} \quad \Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3}{AC}$$

$$\Rightarrow \quad AC = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}\,m$$

So, the length of slide for elder children is $2\sqrt{3}\,m$.

Que 8. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the sting with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

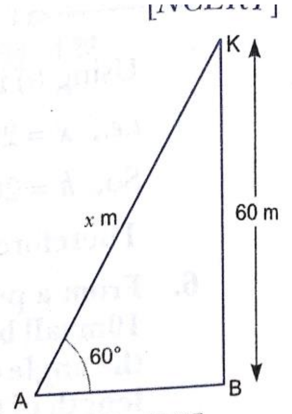

Fig. 11.25

Sol. Let AB be the horizontal ground and k be the position of the kite and its height from the ground is 60 m and length of string AK be x m. **(Fig. 11.25)**

$$\angle KAB = 60°$$

Now, in right angle ΔABK, we have

$$Sin\ 60° = \frac{BK}{AK} = \frac{60}{x} \quad \Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{x} \quad \Rightarrow \quad \sqrt{3}\,x = 120$$

$$\therefore \quad x = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3}\,m$$

So, the length of string is $40\sqrt{3}\,m$.