CHAPTER 07

Impulse and Momentum

7.1 Momentum

Momentum is the property of a moving body. Momentum can be defined as the quantity of motion contained in a body. The product of mass of body and its velocity is called its momentum. It is a vector quantity and denoted by p, so that

$$\vec{p} = m \cdot \vec{v}$$

7.2 Law of Conservation of Linear Momentum

As per Newton second law of motion the external force acting on the body or system is equal to the rate of change of its momentum. Therefore

$$F = \frac{dp}{dt}$$

If no external force acts on the system, i.e., $F = 0$, then

$$\frac{dp}{dt} = 0 \quad \text{or} \quad p = \text{constant}$$

If the resultant external force acting on a system is zero, then the total linear moment in magnitude and direction remains constant. This is called law of conservation of linear momentum.

7.3 Impulse

When a large force is applied on the body for a very short time, then total effect of force is considerable and such type of force is called impulsive force.

Total effect of impulsive force on the motion of body is called impulse. It is measured by the product of the applied force and the time duration during which it acts on the body. It is a vector quantity and is represented by $\vec{J}$.

If a force F acts on the body for a short time dt, the impulse of force is

$$dJ = Fdt$$

If the force acts from time t_1 to t_2, then the total impulse is

$$J = \int_{t_1}^{t_2} Fdt \quad \text{... (i)}$$

if F is constant then

$$J = F\int_{t_1}^{t_2} dt = F(t_2 - t_1) = F\Delta t$$

Hence the impulse of a constant force is equal to the product of the force and the time interval during which force acts.

7.4 Impulse Moment Theorem

If p is the momentum of the body at time t and F is the force applied on the body then from Newton's second law,

$$F = \frac{dp}{dt} \quad \text{or } dp = Fdt$$

If the momentum of a body at time t_1 is p_1 and at time t_2 is p_2,

$$\int_{t_1}^{t_2} Fdt = \int_{p_1}^{p_2} dp$$

or

$$\int_{t_1}^{t_2} Fdt = (p_2 - p_1) = m(v_2 - v_1) \qquad \text{... (ii)}$$

From equation (i) and (ii) we get

$$J = (p_2 - p_1) = \Delta p = m\Delta v$$

Thus the impulse of the force is equal to the change in momentum due to the force applied on the body. This is called impulse-momentum theorem.

7.5 Elastic and Inelastic Impact

When balls of different materials are allowed to fall on a floor, they rebound to different heights. The property of bodies which leads to rebound after impact is called elasticity.

Greater the elasticity of the body, greater will be the rebound. The impact is inelastic if the body does not rebound at all.

7.6 Conservation of Momentum

Consider two bodies of mass m_1 and m_2 moving with respective velocities of u_1 and u_2 before impact and v_1 and v_2 after impact.

During collision, an impulse $F \times t$ exerted by body m_1 on body m_2. This impulse on body B is measured by the change in its momentum. That is

$$F \times t = m_2 v_2 - m_2 u_2 \qquad \text{... (i)}$$

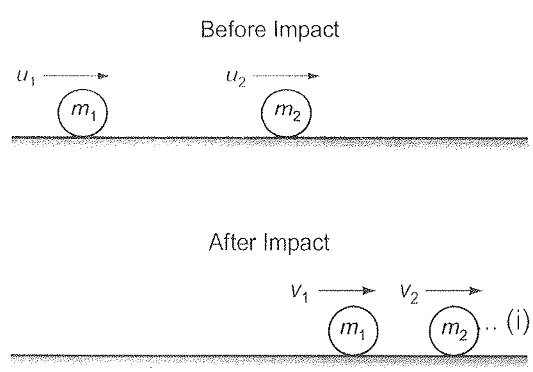


Fig.: 7.1

According to Newton's third law of motion, action and reaction between the colliding bodies is equal in magnitude and opposite in direction, and it acts for the same time. The impulse on body m_1 will be $(-F \times t)$. Accordingly

$$-F \times t = m_1 v_1 - m_1 u_1$$

or

$$F \times t = m_1 u_1 - m_1 v_1 \qquad \text{... (ii)}$$

From equation (i) and (ii) we get

$$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad \text{... (iii)}$$

Thus the momentum of the system before collision is equal to the momentum of the system after collision. If the two bodies moving with velocity u_1 and u_2 before impact get coupled after collision and move together in the same direction with velocity v, then

$$m_1 u_1 + m_2 u_2 = (m_2 + m_2)v \quad \text{... (iv)}$$

7.7 Coefficient of Restitutions

Consider two bodies of mass m_1 and m_2 respectively. Let these bodies be moving with respective velocities u_1 and u_2 before impact. The impact will take place only if $u_1 > u_2$. Thus velocity of approach is $(u_1 - u_2)$.

After a short period of contact, the bodies will separate and will start moving with velocities v_1 and v_2 respectively. The separation will occur only if $v_2 > v_1$ and velocity of separation is $(v_2 - v_1)$.

Newton's law of collision for elastic bodies states that

When two moving bodies collide with each other, their velocity of separation bears constant ratio of their velocity of approach.

or $$v_2 - v_1 = e(u_1 - u_2)$$

where e is a constant of proportionality and called the coefficient of restitution. The coefficient of restitution is a parameter which indicates the energy loss during an impact. The value of e lies between 0 and 1. If $e = 0$, the bodies are inelastic. If $e = 1$, the bodies are prefectly elastic.

7.8 Loss of Kinetic Energy During Impact

Consider two boeis of mass m_1 and m_2 moving with respective velocities of u_1 and u_2 before impact v_1 and v_2 after imapct.

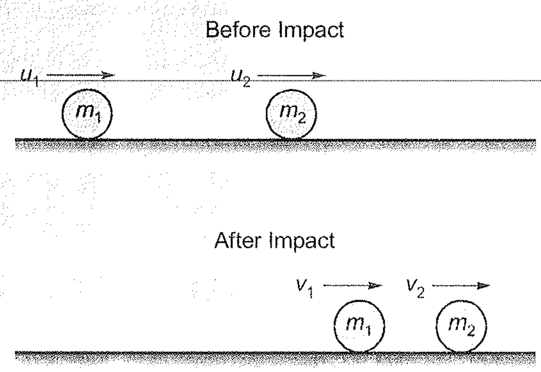


Fig: 7.2

Kinetic energy of the two masses before impact

$$E_1 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

Kinetic energy of the two masses after impact

$$E_2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Loss of kinetic energy during impact

$$E_L = E_1 - E_2$$

or $$E_L = \frac{1}{2}[(m_1u_1^2 + m_2u_2^2) - (m_1v_1^2 + m_2v_2^2)] \quad \text{... (i)}$$

From the law of conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad \text{... (ii)}$$

The relation for coefficient of restitution gives

$$u_2 - u_1 = e(u_1 - u_2) \quad \text{... (iii)}$$

Solving equation (i), (ii) and (iii) gives

$$E_L = \frac{m_1 m_2}{2(m_1 + m_2)}(u_1 - u_2)^2(1 - e^2) \quad \text{... (iv)}$$

Case 1: When the colliding bodies are inelastic or $e = 0$

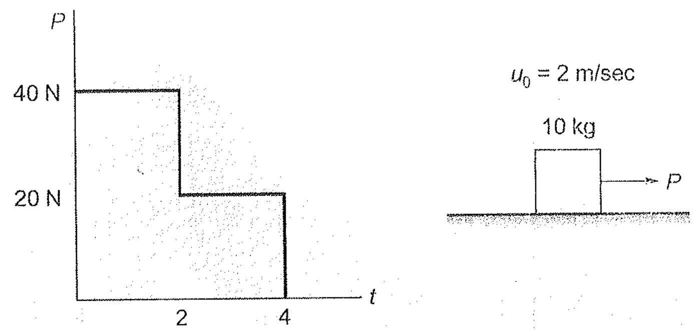
$$E_L = \frac{m_1 m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$$

Case 2: When the colliding bodies are perfectly elastic or $e = 1$

$$E_L = \frac{m_1 + m_2}{2(m_1 + m_2)}(u_1 - u_2)^2 \times (1 - 1) = 0$$

Thus in case of perfectly elastic impact both the momentum and kinetic energy are conserved.

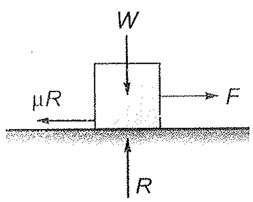
Example 7.1 The 10 kg block is moving to the right with a velocity of 2 m/sec on a horizontal surface when a force P is applied at time $t = 0$ as shown in figure. Calculate the velocity u_B of the block when $t = 4$. The kinetic coefficient of friction is 0.2.



Solution:

Figure show all the force acting on it. Here the horizontal force which acts in the direction of displacement does positive work and the friction force μW does negative work. The normal reaction $R = W$ and the weight of the block do not displace hence do not work do. Net work done is equal to change in momentum.

From the conservation of momentum we know that,



$$\int_{t_1}^{t_2} \Sigma F dt = M\Delta v$$

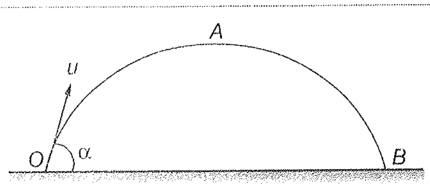
$$\int_0^4 \Sigma P dt - \mu W \cdot 4 = m(v - u)$$

$$0 \times 2 + 20 \times (4 - 2) - 0.2 \times 10 \times 9.8 \times 4 = 10(v - 2)$$

$$41.6 = 10(v - 2)$$

$$v = 4.16 + 2 = 6.16 \text{ m/sec}$$

Example 7.2 A ball of weight W is thrown in the direction shown in figure with an initial speed u.



Determine the time needed for it to reach its highest point A and the speed at which it is travelling at A. Use the principle of impulse and momentum for the solution.

Solution:

Let t be the time that ball take to reach at point A. The acting force on ball is only its weight in vertical downward direction and initial velocity in vertical direction is $u\sin\alpha$. Velocity in vertical direction at point A is zero. Here the change in momentum is equal to the impulse.

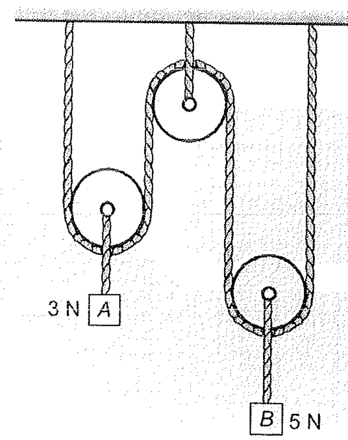
$$\int_{t_1}^{t_2} \Sigma F dt = M\Delta v$$

$$-Wt = \left(\frac{W}{g}\right)0 - \left(\frac{W}{g}\right)u\sin\alpha$$

$$t = \frac{u\sin\alpha}{g}$$

Speed at highest point, $v = v_x = u\cos\alpha$

Example 7.3 Determine the velocities of blocks A and B shown in figure at time $t = 2$ sec after they are released from rest.



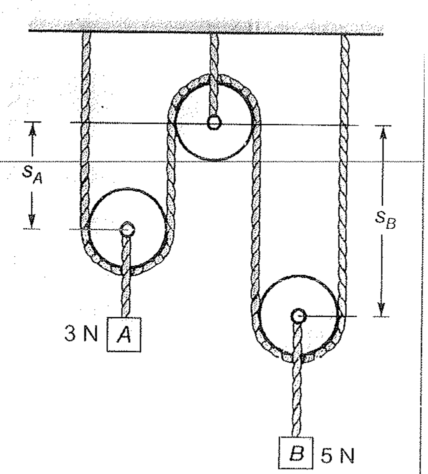
Solution:

From the figure we can write

$$2s_A + 2s_B = L$$

or $$2v_A + 2v_B = 0$$

Thus $$v_A = -v_B$$



writing momentum equation for block A and B we get

$$(2T - W_A)\, t = \frac{W_A}{g}(v_A - 0) = \frac{W_A}{g}v_A \quad \text{... (i)}$$

$$(2T - W_A)\, t = \frac{W_B}{g}(v_B - 0) = \frac{W_B}{g}(-v_A) \quad \text{... (ii)}$$

Subtracting equation (ii) from (i) we get

$$(W_B - W_A)t = \left(\frac{W_B + W_A}{g}\right)v_A$$

or $$v_A = \left(\frac{W_B - W_A}{W_B + W_A}\right)gt$$

Substituting the values we get

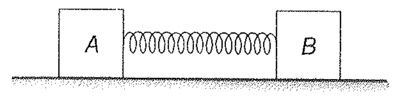
$$v_A = \left(\frac{5-3}{5+3}\right)9.8 \times 2 = 4.9 \text{ m/sec}$$

$$v_B = -v_A = -4.9 \text{ m/sec}$$

Example 7.4 Block A and B masses 40 kg and 60 kg respectively are placed on a smooth surface and the spring connected between them is stretched a distance 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched. Take $k = 150$ N/m

Solution:

Before release the total momentum was zero because of rest condition. From the conservation of momentum we can write



$$0 = m_A v_A + m_B v_B$$

or $$0 = 40\, v_A + 60\, v_B$$

or $$v_A = -1.5\, v_B \quad \text{... (i)}$$

When spring is stretched its has energy and when it is unstretched all its enerty is transferred to both block. Therefore

$$\frac{1}{2}kx^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

or $$\frac{1}{2}\times 150 \times 2^2 = \frac{1}{2}40 \times v_A^2 + \frac{1}{2}60 \times v_B^2$$

or $$30 = 2v_A^2 + 3v_B^2 \quad \text{... (ii)}$$

Substituting the value of v_A from equation (i) we get

$$30 = 2\,(-1.5v_B)^2 + 2v_B^2$$

or $$30 = 7.5\, v_B^2$$

or $$v_B = 2 \text{ m/sec}$$

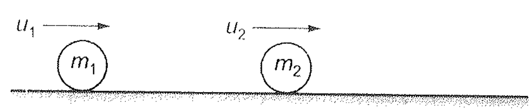
and $$v_A = -3 \text{ m/sec}$$

Example 7.5 A ball mass 8 kg moving with a velocity of 10 m/sec collides with another ball of mass 3 kg moving with 5 m/sec in the same direction and then they form a single body and move. Determine the velocity with which the single body will move.

If ball were moving in opposite direction then what will be the velocity of single body?

Solution:

Before Impact



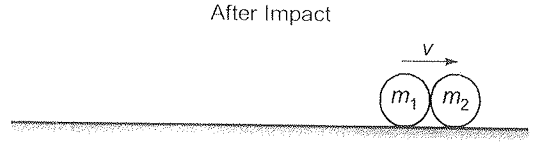
Using the law of conservation of momentum, we can write

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, v$$

or $$8 \times 10 + 3 \times 5 = (8 + 3)v$$

or $$v = \frac{80 + 15}{11} = 8.64 \text{ m/sec}$$

After Impact



The single body moves in opposite direction then direction of both the bodies.

If ball were moving in opposite direction then assume that the formed single body moves along the direction of the ball of 8 kg mass.

We can use conservation of momentum and write

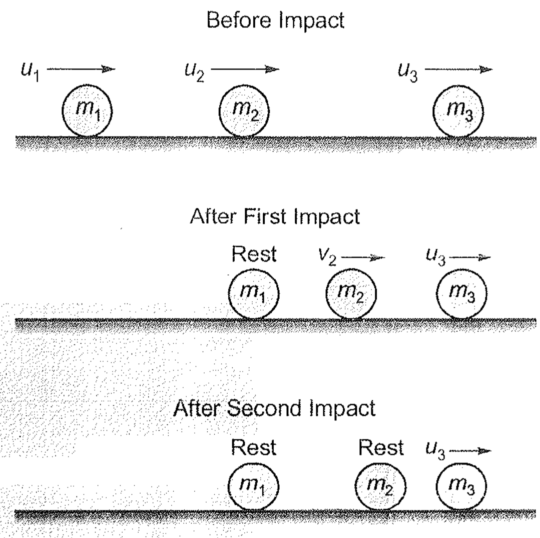
$$m_1 u_1 - m_2 u_2 = mv$$

$$8 \times 10 - 3 \times 5 = 11v$$

$$v = \frac{80 - 15}{11} = \frac{65}{11} = 5.91 \text{ m/sec}$$

Example 7.6 Three perfectly elastic balls A, B and C of masses 1 kg, 2 kg and 4 kg move in the same direction with velocities of 4 m/sec, 1 m/sec and 0.75 m/sec respectively. If the ball A impinges with the ball B, which in turn, impinges with the ball C, prove that the balls A and B will be brought to rest by the impacts.

Solution:



First consider the impact of the first and second ball. From the law of conservation of momentum we can write

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

or $$1 \times 4 + 2 \times 1 = 1 \times v_1 + 2 \times v_2$$

or $$v_1 + 2v_2 = 6 \quad \text{... (i)}$$

From the law of collision of elastic bodies we can write

$$v_2 - v_1 = e(u_1 - u_2)$$
$$= 1(4 - 1) = 3 \quad \text{... (ii)}$$

Adding equation (i) and (ii) we get,

$$3v_2 = 9$$

or $$v_2 = 3 \text{ m/sec}$$

Substituting the value of v_2 in equation (ii)

$$3 - v_1 = 3$$

or $$v_1 = 0$$

Thus the first ball will be brought to rest by the impact of first and second ball. Now consider the impact of second and third ball. In that case $u_2 = 3$ m/sec

From the law of conservation of momentum we can write

$$m_2 u_2 + m_3 u_3 = m_2 v_2 + m_3 v_3$$
$$2 \times 3 + 4 \times 0.75 = 2 \times v_2 + 4 \times v_3$$

or $$2v_2 + 4v_3 = 9 \quad \text{... (iii)}$$

From the law of collision elastic bodies we can write

$$v_3 - v_2 = e(u_2 - u_3)$$

or $$v_3 - v_2 = 1(3 - 0.75) = 2.25$$

or $$4v_3 - 4v_2 = 9 \quad \text{... (iv)}$$

Subtracting equation (iv) from (iii) we get

$$6v_2 = 0$$

or $$v_2 = 0$$

Hence the second ball will also be brought to rest by the impact of second and third ball.

Example 7.7 The masses of two balls are in the ratio of 2 : 1 and their velocities are the ratio of 1 : 2, but in the opposite direction before impact. If the coefficient of restitution be 5/6 then prove that after the impact, each ball will move back with 5/6th of its original velocity.

Solution:

Let mass of ball be $2m$ and m. If initial velocity of first ball is u then initial velocity of second ball will be $-2u$ because of opposite direction.

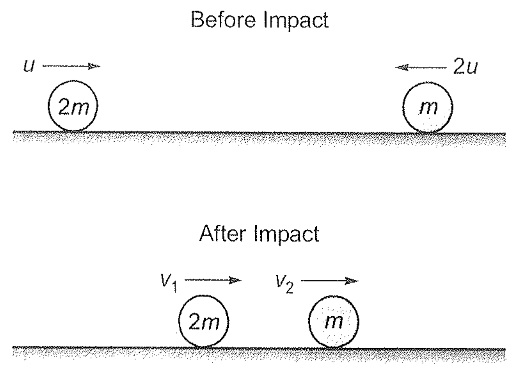
From the law of conservation of momentum we can write

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

or $$2m \times u + m(-2u) = 2mv_1 + mv_2$$

or $$0 = 2mv_1 + mv_2$$

$$v_2 = -2v_1 \quad \text{... (i)}$$



From the law of collision of elastic bodies we can write

$$v_2 - v_1 = e(u_1 - u_2)$$

$$= \frac{5}{6}(u - (-2u)) = \frac{5}{2}u$$

Substituting the value of v_2 from equation (i) we get

$$-2v_1 - v_1 = \frac{5u}{2}$$

or $$v_1 = -\frac{5u}{6}$$

Minus sign indicates that the idrection of v_1 is opposite to that of u. Thus the first ball will move back with 5/6th of its origintal velocity.

Now substituting the value of v_1 in equation (i) we get,

$$v_2 = 2\left(-\frac{5}{6} \times u\right) = +\frac{5}{6} \times 2u$$

Plus sign indicates that the direction of v_2 is the same as that of u of opposite to that of v_1. Thus the second ball will also move back with 5/6th of its original velocity.

Example 7.8 **A ball is dropped form a height of 1 m on a smooth floor. If the height of the first bounce is 81 cm, then determine coefficient of restitution, and expected height aftet the second bounce.**

Solution:

If ball drops from height h, then it impinges on the floor with velocity

$$u = \sqrt{2gh} = \sqrt{2g \times 1} = \sqrt{2g} \quad \text{... (i)}$$

If e is coefficient of restitution then it will bounce with velocity $v_1 = eu$ and will go at height h_1 and we can write

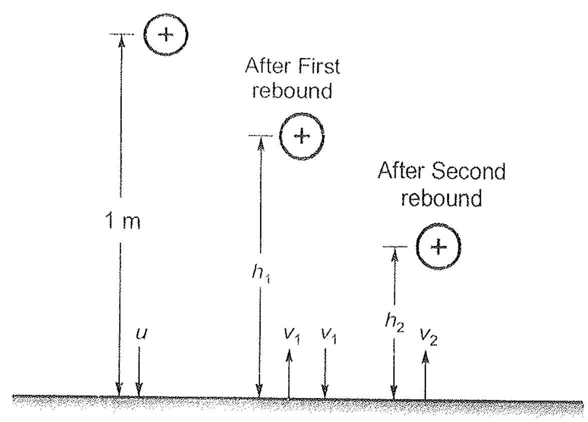
$$v_1 = \sqrt{2gh_1} = \sqrt{2g \times 0.81}$$

$$= 0.9\sqrt{2g}$$

or $$eu = 0.9\sqrt{2g} \quad \text{... (ii)}$$

or $$e\sqrt{2g} = 0.9\sqrt{2g}$$

or $$e = 0.9$$



Let v_2 be the velocity after the second bounce.

Then we can write

$$v_2 = ev_1$$

or $$v_2 = 0.9 \times 0.9\sqrt{2g} \quad \text{... (iii)}$$

Let h_2 expected height after the second bounce, then

$$v_2 = \sqrt{2gh_2} \quad \text{... (iv)}$$

From equation (iii) and (iv) we get

$$0.9 \times 0.9\sqrt{2g} = \sqrt{2gh_2}$$

or $$h_2 = 0.81^2 = 0.656 \text{ m}$$

Example 7.9 A bullet of mass m is shot towards a wooden plate of mass M and penetrates a thickness x when the place is fixed. If the plate is free then prove that thickness of penetration of the bullet x_1 will be

$$\left[\frac{x \cdot m}{M+m}\right]$$

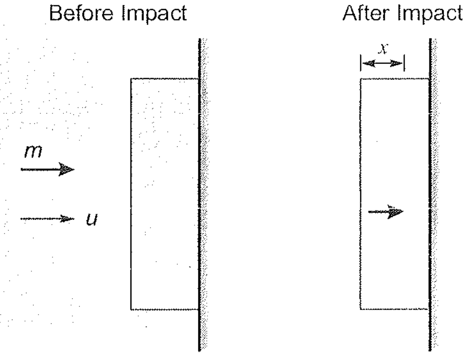


Fig. (a)

Solution:

Let R_f is the resistance of the place per unit thickness of penetration and u be the velocity of the bullet when it touches to the plate. When bullet penetrated the thickness x, its total kinetic energy is lost. Thus total work done in penetration is equal to its kinetics energy.

Thus $$R_f x = \frac{1}{2}mu^2 \quad \text{... (i)}$$

When plate is free, it will move after impact with some velocity with bullet. Let this velocity be v and the thickness through which the bullet penetrated be x_1. Momentum before impact must be momentum aftet impact

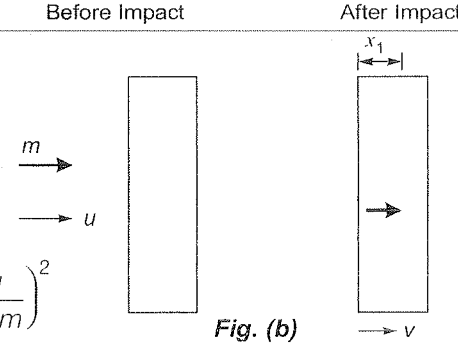


Fig. (b)

Thus $$mu = (M+m)\,v$$

or $$v = \frac{mu}{M+m}$$

Total kinetic energy after impact is

$$= \frac{1}{2}(m+M)v^2 = \frac{1}{2}(M+m)\left(\frac{mu}{M+m}\right)^2$$

Kinetic energy loss due to impact is

$$\Delta KE = \frac{1}{2}mu^2 - \frac{1}{2}(m+M)v^2$$

$$= \frac{1}{2}mu^2 - \frac{1}{2}(M+m)\left(\frac{mu}{M+m}\right)^2$$

$$= \frac{1}{2}mu^2 - \frac{1}{2}\frac{m^2u^2}{M+m} = \frac{1}{2}u^2\left[m - \frac{m^2}{M+m}\right] = \frac{u^2}{2}\frac{Mm}{M+m}$$

This loss in kinetic energy is due to the resistance offered in penetration by plate. Thus

$$R_f x_1 = \frac{1}{2}u^2\left[m - \frac{m^2}{M+m}\right] = \frac{u^2}{2}\frac{Mm}{M+m} \quad \text{... (ii)}$$

Dividing equation (ii) by (i) we get

$$\frac{R_f x_1}{R_f x} = \frac{u^2}{2}\left(\frac{Mm}{M+m}\right)\frac{2}{mu^2}$$

or $$x_1 = x\left(\frac{M}{M+m}\right) \qquad \text{Hence proved.}$$

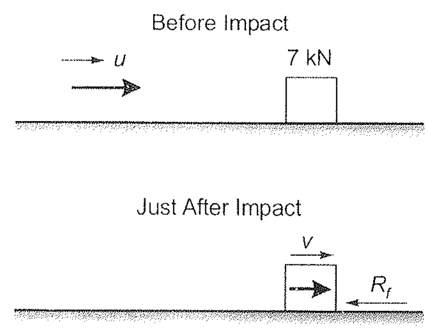
Example 7.10 A bullet of 0.25 N is fired on a wooden block of 7 N which rest on a rough (μ = 0.25) horizontal plane. The velocity of bullet is 150 m/sec and it is embedded into the block. Determine :

(a) The velocity of bullet and block together

(b) The distance travelled by the combined mass along the floor.

Solution:

Here momentum just before the impact must be equal to momentum just after impact. Let v be the common velocity of bullet and block after impact. Then



$$mu = (M + m)\, v$$

or $$\frac{w}{g}u = \frac{(W+w)}{g}v$$

or $$wu = (W + w)v$$

Here $$u = 150 \text{ m/sec}$$

$$w = 0.25 \text{ N}$$

$$W = 7 \text{ N}$$

Thus $$0.25 \times 10 = (7 + 0.25) \times v$$

$$v = \frac{150 \times 0.25}{7.25} = 5.17 \text{ m/sec}$$

At this point combined body has energy due to velocity v. The motion of combined body will retard due to friction force as shown in figure and finally it will come to rest after travelling distance s. All of its energy will go in work against friction.

Thus $$\frac{1}{2}(m+M)v^2 = R_f \cdot s$$

Friction force $$R_f = \mu(W + w) = \mu(M + m)\, g$$

Thus $$\frac{1}{2}(m+M)v^2 = \mu(M + m)\, g \cdot s$$

or $$v^2 = 2\mu g s$$

$$s = \frac{26.73}{2 \times 0.25 \times 9.8}$$

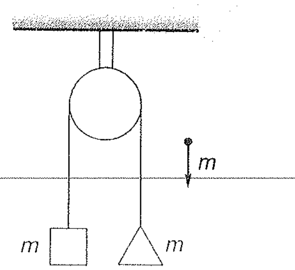
$$= 5.45 \text{ m}$$

Objective Brain Teasers

Q.1 A person standing on the floor of on elevator drops a coin. The coin reaches the floor in time t_1 if the elevator is stationary and in time t_2 if it is moving uniformly. Then

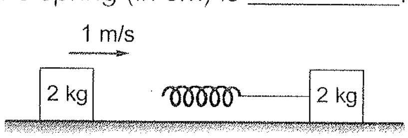
(a) $t_1 = t_2$
(b) $t_1 < t_2$
(c) $t_1 > t_2$
(d) $t_1 < t_2$ or $t_1 > t_2$ depending upon whether the elevator is going up or down

Q.2 A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley as shown in figure. Initially the system is at rest when a particle of mass m falls on the pan and sticks to it. If the particle strikes the pan with a velocity of 1.2 m/s, the speed which the system moves just after the collision is



(a) 0.4 m/s (b) 0.8 m/s
(c) 1 m/s (d) 0.3 m/s

Q.3 A block of mass 2 kg is moving on a horizontal frictionless surface with velocity of 1 m/s, towards another block of equal mass kept at rest. The spring constant of the spring fixed at one end is 100 N/m. The maximum compression in the spring (in cm) is __________.



Q.4 A bullet of mass 0.05 kg moving with a speed of 400 m/s penetrates 15 cm into a fixed block of wood. The average force exerted by the wood on the bullet will be

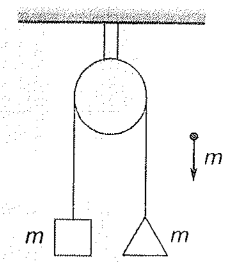
(a) 30 kJ (b) 26.66 kJ
(c) 15 kJ (d) 10 kJ

ANSWERS

1. (d) 2. (a) 4. (b)

Hints & Explanation

2. (a)



Let the N = magnitude of contact force between particle and pan

T = tension in the string

Consider the impulse imparted to the particle. The force N is in upward direction and the impulse is $\int N dt$.

This should be equal to change in momentum.

$$\therefore \quad \int N dt = mv - mV \quad \text{...(1)}$$

For pan impule imported

$$\int (N - T) dT = mV \quad \text{...(2)}$$

for block $\int T dT = mV$ ...(3)

Adding (2) and (3)

$$\Rightarrow \quad \int N dt = 2mV$$

$$\Rightarrow \quad mv - mV = 2\,mV$$

$$\Rightarrow \quad V = \frac{v}{3} = \frac{1.2}{3} = 0.4 \text{ m/s}$$

3. (10)

Mass of each block M_A and M_B is 2 kg.

Initially, $V_A = 1$ m/s, $V_B = 0$

$$K = 100 \text{ N/m}$$

Let final velocity of both block x be the compression in spring be V and the compression is spring be x.

Since no force acting.

Applying conservation of momentum,

$$\Rightarrow M_A V_A + M_B V_B = (M_A + M_B)V$$

$$\Rightarrow \quad 2 \times 1 = 4 \times V$$

$$\Rightarrow \quad V = 0.5 \text{ m/s}$$

Using conservation of energy

$$\Rightarrow \quad \frac{1}{2} M_A V_A^2 = \frac{1}{2}(M_A + M_B)V^2 + \frac{1}{2}Kx^2$$

$$\Rightarrow \quad \frac{1}{2} \times 2 \times (1)^2 = \frac{1}{2} \times 4 \times (0.5)^2 + \frac{1}{2} \times 100x^2$$

$$\Rightarrow \quad x^2 = \frac{1}{100}$$

$$\Rightarrow \quad x = 0.1 \text{ m} = 10 \text{ cm}$$

4. (b)

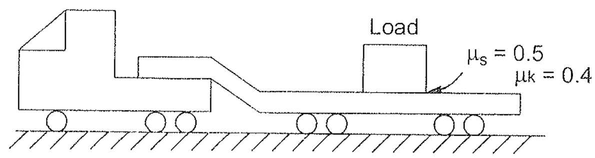
$$\frac{1}{2}mv^2 = F_{avg}S$$

$$\frac{1}{2} \times \frac{0.05 \times (400)^2}{0.15} = F_{avg}$$

$$F_{avg} = 26.66 \text{ kJ}$$

Student's Assignments

Q.1 The coefficient of friction between the load and flat bed trailer shown in figure, are $\mu_s = 0.40$ and $\mu_k = 0.30$. Speed of the rig is 15 m/s, determine the shortest time in which the rig can be brought to a stop if the load is not to shift.



Q.2 A man of mass 70 kg stands in an aluminium canoe of 40 kg. He fires a bullet of 25 gm horizontally over the bow of the canoe to hit a wooden block of mass 2 kg resting on a smooth horizontal surface. If the wooden block and bullet move together with a velocity of 5 m/s, find the velocity of the canoe.

Q.3 A cylinder of radius R, mass moment and moment of inertia I about centre C rolls down the inclined plane as shown in figure. Cylinder starts from rest, what will be its velocity after t seconds if $\theta = 30°$?