

Rotational Mechanics

Exercise Solutions

Question 1: A wheel is making revolutions about its axis with uniform angular acceleration. Starting from rest, it reaches 100 rev/sec in 4 seconds. Find the angular acceleration. Find the angle rotated during these four seconds.

Solution:

we are given that, $\omega_0 = 0$,

Final angular velocity, $\omega' = 100 \text{ revs/s}$.

$t = \text{time} = 4 \text{ s}$.

Let α be angular acceleration.

Now, $\omega' = \omega_0 + \alpha t$

Where ω_0 is initial angular velocity

or $\alpha = 25 \text{ rev/s}^2$

Again, from second equation of kinematics.

$$\theta = \omega_0 t + (1/2) \alpha t^2$$

[angle rotated during 4 seconds, so $t = 4 \text{ sec}$]

$$\theta = 400 \pi \text{ radians}$$

Question 2: A wheel rotating with uniform angular acceleration covers 50 revolutions in the first five seconds after the start. Find the angular acceleration and the angular velocity at the end of five seconds.

Solution:

Given: $\theta = 50$

time = $t = 5 \text{ sec}$.

By equation of kinematics,

$$\theta = \omega t + (1/2) \alpha t^2$$

$$\text{or } \alpha = 4 \text{ rev/s}^2$$

Let After 5 second angular velocity will be ω' .

$$\omega' = \omega + \alpha t$$

$$\Rightarrow \omega = 20 \text{ rev/s}$$

Question 3: A wheel starting from rest is uniformly accelerated at 4 rad/s^2 for 10 seconds. It is allowed to rotate uniformly for the next 10 seconds and is finally brought to rest in the next 10 seconds. Find the total angle rotated by the wheel.

Solution:

Time duration = $t = 10 \text{ sec}$.

angle rotation = θ

Maximum angular velocity = $4 \times 10 = 40 \text{ rad/s}$

Area under the curve will decide the total angle rotated.

Area under the curve = $(1/2) \times 10 \times 40 + 40 \times 10 + (1/2) \times 40 \times 10 = 800 \text{ rad} = \text{total angle rotated.}$

Question 4: A body rotates about a fixed axis with an angular acceleration of one radian/second/ second. Through what angle does it rotate during the time in which its angular velocity increases from 5 rad/s to 15 rad/s .

Solution:

from first equation of kinematics-

$$\omega = \omega_0 + \alpha t \dots (1)$$

Where Initial angular velocity = $\omega_0 = 5 \text{ rad/s}$ and Final angular velocity = $\omega = 15 \text{ rad/s}$ and $\alpha = 1 \text{ rad/s}^2$

$$(1) \Rightarrow t = 10 \text{ sec}$$

Again, from second equation of kinematics.

$$\theta = \omega_0 t + (1/2) \alpha t^2$$

Here θ is the total angle rotated

$$\Rightarrow \theta = 100 \text{ rad.}$$

Question 5: Find the angular velocity of a body rotating with an acceleration of 2 rev/s^2 as it completes the 5th revolution after the start.

Solution:

$$\alpha = 2 \text{ rev/s}^2, \theta = 5 \text{ rev}, \omega_0 = 0 \text{ and } \omega = ?$$

Change in angular velocity

$$\omega^2 = 2\theta\alpha \dots(1)$$

$$\Rightarrow \omega = 2\sqrt{5} \text{ rev/s}$$

$$\text{or } \theta = 10\pi \text{ rad and}$$

$$\alpha = 4\pi \text{ rad/s}^2$$

$$\text{then (1)} \Rightarrow \omega = 2\sqrt{5} \text{ rev/s}$$

Question 6: A disc of radius 10 cm is rotating about its axis at an angular speed of 20 rad/s , Find the linear speed of

- (a) a point on the rim,
- (b) the middle point of a radius.

Solution:

$$\text{Radius of disc} = 10 \text{ cm} = 0.1 \text{ m and Angular velocity} = 20 \text{ rad/s}$$

(a) linear velocity of the rim = $\omega r = 20 \times 0.1 = 2 \text{ m/s}$

(b) Linear velocity at the middle of radius

$$\omega r/2 = (20 \times 0.1)/2 = 1 \text{ m/s}$$

Question 7: A disc rotates about its axis with a constant angular acceleration of 4 rad/s^2 . Find the radial and tangential accelerations of a particle at a distance of 1 cm from the axis at the end of the first second after the disc starts rotating.

Solution:

$$t = 1 \text{ sec and } r = 1 \text{ cm} = 0.01 \text{ m}$$

Tangential acceleration:

$$a_T = r \times a = 0.01 \times 4 \text{ rad/s}^2$$

$$\text{Angular velocity : } \omega = \alpha t = 1 \times 4 = 4 \text{ rad/s}$$

$$\text{Radial acceleration: } a_r = \omega^2 \times r = 0.16 \text{ m/s}^2 \text{ or } 16 \text{ cm/s}^2$$

Question 8: A block hangs from a string wrapped on a disc of radius 20 cm free to rotate about its axis which is fixed in a horizontal position. If the angular speed of the disc is 10 rad/s at some instant, with what speed is the block going down at that instant?

Solution:

Relation between angular speed and linear speed

$$v = r \times \omega$$

Where, Angular speed of the disc = $\omega = 10 \text{ rad/s}$ and Radius of the disc = $r = 20 \text{ cm}$ or 0.20 m

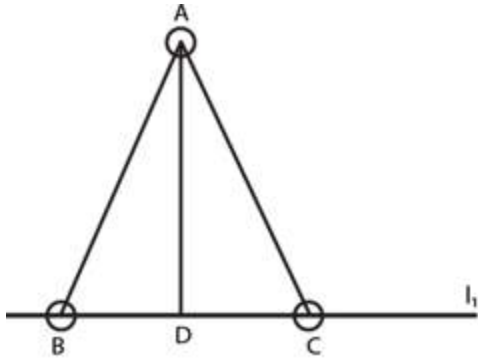
$$v = 10 \times 0.20 = 2 \text{ m/s}$$

Question 9: Three particles, each of mass 200 g, are kept at the corners of an equilateral triangle of side 10 cm. Find the moment of inertia of the system about an axis

(a) joining two of the particles and

(b) passing through one of the particles and perpendicular to the plane of the particles.

Solution:



The perpendicular distance from the axis $AD = \frac{\sqrt{3}}{2} \times 10 = 5\sqrt{3} \text{ cm}$

Moment of inertia about BC = $I = mr^2$

$$= 200 \times (5\sqrt{3})^2 \text{ gm-cm}^2$$

$$= 1.5 \times 10^{-3} \text{ kg-m}^2$$

(b) Let's take AD as perpendicular side.

moment of inertia along BC

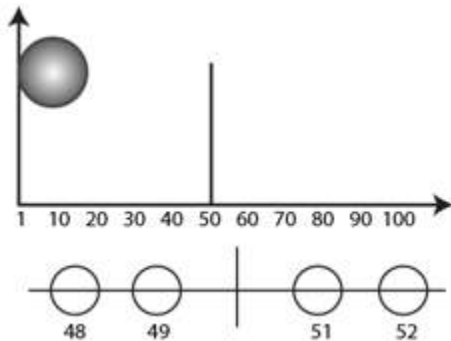
$$I = 2mr^2$$

$$= 2 \times 200 \times 10^2 \text{ gm-cm}^2$$

$$= 4 \times 10^{-3} \text{ kg-m}^2$$

Question 10: Particles of masses 1 g, 2 g, 3 g, ..., 100 g are kept at the marks 1 cm, 2 cm, 3 cm, ..., 100 cm respectively on a meter scale. Find the moment of inertia of the system of particles about a perpendicular bisector of the metre scale.

Solution:



Consider the two particles at the position 49 cm and 51 cm

Moment of inertia due to these 2 particles = $49 \times 1^2 + 51 \times 1^2 = 100 \text{ gm-cm}^2$

Similarly, if consider 48th and 52nd term, we have $100 \times 2^2 \text{ gm-cm}^2$

Thus, we will get 49 such set and one alone particle at 100 cm.

Total Moment of inertia = $100(1^2 + 2^2 + \dots + 49^2) + 100(50)^2$

= 4292500 gm-cm^2

= 0.43 kg-m^2

Question 11: Find the moment of inertia of a pair of spheres, each having a mass m and radius r , kept in contact about the tangent passing through the point of contact.

Solution:

Moment of inertia of the first body and the 2nd body about the respective tangents becomes

$MI_1 = mr^2 + (2/5) mr^2$ and $MI_2 = mr^2 + (2/5) mr^2 = (7/5) mr^2$

Net moment of inertia is:

$$MI_{\text{net}} = (7/5) mr^2 + (7/5) mr^2 = (14/5) mr^2 \text{ units}$$

Question 12: The moment of inertia of a uniform rod of mass 0.50 kg and length 1 m is 0.10 kg-m^2 about a line perpendicular to the rod. Find the distance of this line from the middle point of the rod.

Solution:

Length of the rod $= L = 1 \text{ m}$, and its mass $= 0.5 \text{ kg}$

Let r = distance between the parallel axes.

Let at a distance x from the center the rod is moving

$$I_B = I_A + I_A \cdot r^2$$

[parallel axis theorem]

the moment of inertia at d

$$(mL^2/12) + md^2 = 0.10$$

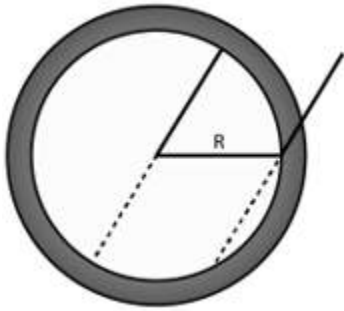
On Putting values, we have

$$d = 0.342 \text{ m}$$

Question 13: Find the radius of gyration of a circular ring of radius r about a line perpendicular to the plane of the ring and passing through one of its particles.

Solution:

First let's take a point on the rim, this point is perpendicular to the ring and moment of Inertia



$$I = mR^2$$

About a point on a rim of the ring and the axis perpendicular to the plane of the ring, the moment of inertia = $mR^2 + mR^2 = 2mR^2$

$$\Rightarrow mK^2 = 2mR^2$$

[Parallel axis theorem]

$$\Rightarrow K = \sqrt{2R^2} = \sqrt{2}R$$

Question 14: The radius of gyration of a uniform disc about a line perpendicular to the disc equals its radius. Find the distance of the line from the centre.

Solution:

Moment of inertia : $I = \frac{mr^2}{2}$

Let us take a line parallel to this axis and at a distance d . Then the radius of gyration becomes r

Moment of inertia = $I' = L + md^2$

$$I' = \frac{mr^2}{2} + md^2$$

Also, $I' = mr^2$ (Given)

Equating above equations, we have

$$d^2 = \frac{r^2}{2}$$

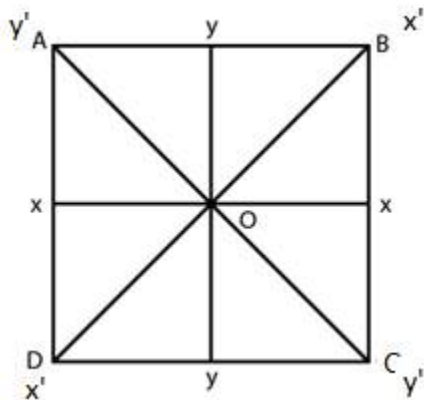
$$\text{or } d = \frac{r}{\sqrt{2}}$$

Question 15: Find the moment of inertia of a uniform square plate of mass m and edge a about one of its diagonals.

Solution:

mass of that cross sectional area = $m/a^2 \times (a \times dx)$

moment of inertia along xx'



Therefore, for two lines

$$I = 2m \times a^2/12$$

$$= ma^2/6$$

Moment of inertia for pair of perpendicular diagonals:

$$I' = 2m \times a^2/12 = ma^2/6$$

Moment of inertia: $I' = 2I$

[perpendicular axis theorem]

$$I = ma^2/12$$

Question 16: The surface density (mass/area) of a circular disc of radius a depends on the distance from the centre as $p(r) = A + Br$. Find its moment of inertia about the line

perpendicular to the plane of the disc through its centre.

Solution:

The inertia of the body for a point of mass is the product of the square of the radius with the mass of the body.

$$I = mAr$$

Where, I = moment of Inertia, A = area of the object and r = radius of the object.

The moment of inertia of a disc:

$$I = \int_0^a (A + Br) 2\pi r dr$$

$$I = \int_0^a 2\pi Ar^3 dr + \int_0^a 2\pi Br^4 dr$$

$$I = 2\pi\alpha^4 \left[\left(\frac{A}{4} \right) + \left(\frac{B\alpha}{5} \right) \right]$$

$$I = 2\pi \left(\frac{A\alpha^4}{4} + \frac{B\alpha^5}{5} \right)$$

Question 17: A particle of mass m is projected with a speed u at an angle θ with the horizontal. Find the torque of the weight of the particle about the point of projection when the particle is at the highest point.

Solution:

Formula used is that of a torque which tells us the mechanics of force which helps the object to rotate.

$$\text{Torque} = F \cdot r \quad \dots\dots(1)$$

F = force applied on the object and r = radius of the object turning.

The force of the object when in motion in linear path = $\tau = mgr/2$

and radius is

$$r = \left(u^2 \cdot \frac{\sin 2\theta}{g} \right)$$

$$r = \left(u^2 \cdot \frac{2\sin\theta \cdot \cos\theta}{g} \right)$$

(1) $\Rightarrow$

$$\tau = F \cdot r$$

$$\tau = \frac{mg}{2} \cdot \left(u^2 \cdot \frac{2\sin\theta \cdot \cos\theta}{g} \right)$$

$$\tau = mu^2 \sin\theta \cos\theta$$

Question 18: A simple pendulum of length l is pulled aside to make an angle θ with the vertical. Find the magnitude torque of the weight w of the bob about the point of suspension. When is the torque zero?

Solution:

$$\text{Torque} = F \cdot r$$

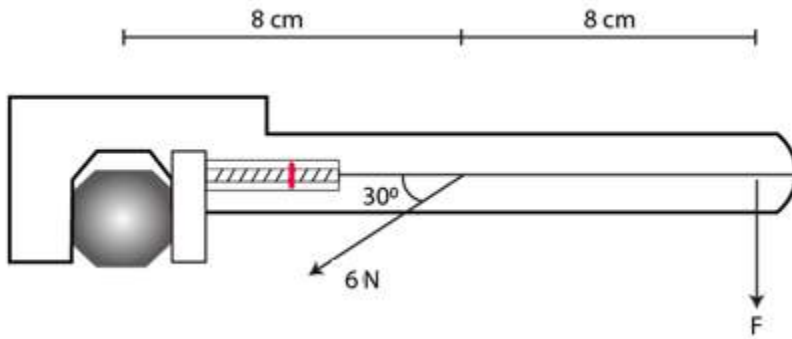
The pendulum bob rotates at a distance of " l " from the center which is also the radius in terms of torque, the force, $F = W$

If angle of turning is θ , the radius in terms of turning angle is $r = l \sin \theta$

$$\text{Torque} = F \cdot l \sin \theta = W \cdot l \sin \theta$$

At lowest point the $\theta = 0$, turning the torque equal to zero at the lowest point.

Question 19: When a force of 6.0 N is exerted at 30° to a wrench at a distance of 8 cm from the nut, it is just able to loosen the nut. What force F would be sufficient to loosen it if it acts perpendicularly to the wrench at 16 cm from the nut?



Solution:

The force exerted into the wrench is $F = 6\text{ N}$, the angle of motion is 30° . The distance from the nut to the wrench end is 16 cm.

Therefore, total torque acting at A about the point O.

$$\text{Torque} = F \cdot r \sin\theta = 6 \times 0.08 \times \sin 30^\circ = 6 \times 0.08 \times (1/2)$$

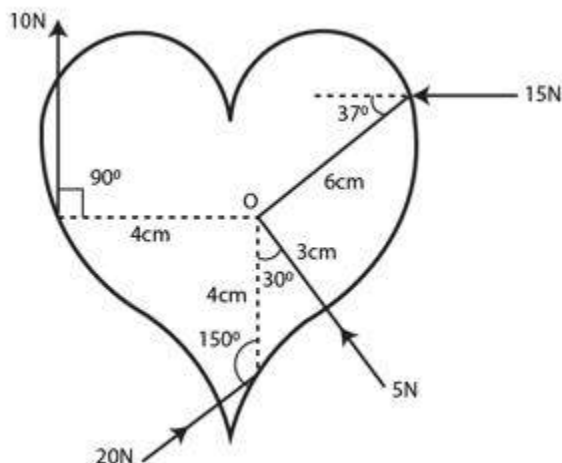
Torque acting on the point B

$$6 \times 0.08 \times (1/2) = F \cdot r \sin\theta$$

$$6 \times 0.08 \times (1/2) = F \times 0.16 \sin\theta$$

$$F = 1.5 \text{ N}$$

Question 20: Calculate the total torque acting on the body shown in figure (below) about the point O.



Solution:

A torque which tells us the mechanics of force which helps the object to rotate.

$$\text{Torque} = F r \sin\theta$$

The torque acting on the point O due to the force of 15N:

$$\tau_{15} = 15 \times 6 \times 10^{-2} \sin 37^\circ = 0.54 \text{ Nm}$$

The torque acting on the point O due to the force of 10N:

$$\tau_{10} = 10 \times 4 \times 10^{-2} = 0.40 \text{ Nm}$$

The torque acting on the point O due to the force of 20N

$$\tau_{20} = 20 \times 4 \times 10^{-2} \sin 30^\circ = 0.40 \text{ Nm}$$

Due to torque negation between τ_{10} and τ_{20} which leaves them to zero, leaving the resultant torque equivalent to 0.54 Nm.

Question 21: A cubical block of mass m and edge α slides down a rough inclined plane of inclination θ with a uniform speed. Find the torque of the normal force acting on the block about its centre.

Solution:

a torque which tells us the mechanics of force which helps the object to rotate.

$$\text{Torque} = F r \sin\theta$$

The block of mass “ m ” moves with a uniform velocity on an inclined plane of angle θ , the force applied on the block.

$$F = mg \sin\theta$$

For the block not to roll the sum of the product of torque and force applied downwards and reactionary force due to the mass of the block should be zero.

$$\tau_F + \tau_{F_N} = 0$$

$$F(a/2) = -F_N$$

$$\text{or } -F_N = (a/2) mg \sin\theta$$

Torque on the sliding object is $(-a/2) mg \sin\theta$

Question 22: A rod of mass m and length L , lying horizontally, is free to rotate about a vertical axis through its centre, A horizontal force of constant magnitude F acts on the rod at a distance of $L/4$ from the centre. The force is always perpendicular to the rod. Find the angle rotated by the rod during the time t after the motion starts.

Solution:

A torque which tells us the mechanics of force which helps the object to rotate.

$$\text{Torque} = Fr \sin\theta \text{ and } I = mL^2/12$$

The mass of rod is given as “ m ” and length “ L ”. So the torque acting on the rod:

$$\tau = F \times L/4$$

And, The moment of Inertia

$$I = mL^2/12$$

The angle of rotation in terms of angular acceleration:

$$\alpha = \tau/I$$

$$= 3F/mL$$

and angle of rotation in term of angular length is

$$\theta = ut + (1/2)\alpha t^2$$

Substituting the value of α and $u = 0$.

$$\Rightarrow \theta = (1/2)(3F/mL) t^2$$

Question 23: A square plate of mass 120 g and edge 5.0 cm rotates about one of the edges. If it has a uniform angular acceleration of 0.2 rad/s^2 , what torque acts on the plate?

Solution:

The mass of the plate is 120g, the edges of the square is 5.0 cm and the angular acceleration is 0.2 rad/sec^2

A torque which tells us the mechanics of force which helps the object to rotate.

$$\text{Torque} = Fr \sin\theta \text{ and } I = mL^2/12$$

The moment of Inertia of the plate:

$$I_{\text{edge}} = I + M \left(\frac{a}{2} \right)^2$$

$$I_{\text{edge}} = \frac{Ma^2}{12} + M \left(\frac{a}{2} \right)^2$$

By substituting the values of variables of mass and edge length

$$\Rightarrow I = (0.12 \times 0.05 \times 0.05)/3$$

Torque produced by the plate: $T = I\alpha$

$$T = 0.0001 \times 0.2 = 2 \times 10^{-5} \text{ Nm}$$

Question 24: Calculate the torque on the square plate of the previous problem if it rotates about a diagonal with the same angular acceleration.

Solution: Moment of inertia of a square plate about its diagonal is $ma^2/12$

Where m = mass of square plate and a = edges of the square

$$\text{Torque produced} = (ma^2/12) \times \alpha$$

$$= [120 \times 10^{-3} \times 5^2 \times 10^{-4}] / [12 \times 0.2]$$

$$= 0.5 \times 10^{-5} \text{ N-m}$$

Question 25: A flywheel of moment of inertia 5.0 kg-m^2 is rotated at a speed of 60 rad/s . Because of the friction at the axle, it comes to rest in 5.0 minutes. Find (a) the average torque of the friction, (b) the total work done by the friction and (c) the angular momentum of the wheel 1 minute before it stops rotating.

Solution:

A flywheel of moment of inertia 5.0 kg-m^2 is rotated at a speed of 60 rad/s .

$$\text{Average torque} = I\alpha$$

Calculation of work done from the torque of the flywheel:

$$W = (1/2) I\omega^2$$

$$\text{The angular momentum of the wheel : } L = I\omega$$

Now, let us calculate the angular acceleration:

$$\alpha = -(60/5 \times 60) \text{ rad/s}^2 = -0.2 \text{ rad/s}^2$$

(a) the average torque of the flywheel

$$\text{average torque} = I\alpha = -\text{Nm}$$

and work done by the torque of the flywheel

$$W = (1/2) I\omega^2$$

$$W = 9 \text{ KJ}$$

(b) angular momentum of the wheel in a time span of 4 minutes

$$\omega = \omega_o + \alpha t$$

$$= 60 - 240/5 = 12 \text{ rad/s}$$

$$\text{So, angular momentum} = L = I\omega = 5 \times 12 = 60 \text{ kg/m}^2$$

Question 26: Because of the friction between the water in oceans with the earth's surface, the rotational kinetic energy of the earth is continuously decreasing. If the earth's angular speed decreases by 0.0016 rad/day in 100 years, find the average torque of the friction on the earth. Radius of the earth is 6400 km and its mass 6.0×10^{24} kg.

Solution:

The earth's angular speed decreases by 0.0016 rad/day in 100 years (Given)

$$\text{and } 1 \text{ year} = 365 \times 24 \times 60 \times 60 \text{ sec}$$

Torque produced by ocean water in decreasing earth's angular velocity is

$$\tau = I\alpha$$

$$= \left(\frac{2}{5}\right) mr^2 \times (\omega - \omega_o)/t$$

$$= \left(\frac{2}{5}\right) \times 6 \times 10^{24} \times (6400 \times 10^3)^2 \times [0.0016 / (24 \times 60 \times 60 \times 100)]$$

$$= 5.678 \times 10^{20} \text{ N-m}$$

Question 27: A wheel rotating at a speed of 600 rpm (revolutions per minute) about its axis is brought to rest by applying a constant torque for 10 seconds. Find the angular deceleration and the angular velocity 5 seconds after the application of the torque.

Solution:

The relationship of velocity and acceleration in basic kinematics.

$$\omega = \omega_o + \alpha t$$

The angular velocity is taken in terms of rad/sec which is 10 and the time taken to rotate is 10 sec

$$0 = 10 + 10\alpha$$

$$\Rightarrow \alpha = -1 \text{ rev/s}^2$$

Angular deceleration after 5 seconds we get

$$\omega_{\text{dec}} = \omega + \alpha t$$

$$\Rightarrow \omega_{\text{dec}} = 5 \text{ rev/s}$$

Question 28 : A wheel of mass 10 kg and radius 20 cm is rotating at an angular speed of 100 rev/mm when the motor is turned off. Neglecting the friction at the axle, calculate the force that must be applied tangentially to the wheel to bring it to rest in 10 revolutions.

Solution:

A torque which tells us the mechanics of force which helps the object to rotate.

$$\text{Torque} = Fr \sin \theta \text{ and } I = (1/2) Mr^2$$

where, F = force applied on the object; r = radius of the object turning.

Let θ be the turning angle and I = moment of Inertia,

Also, L is the length at which the force is applied.

let us find the acceleration using below formula:

$$\omega' = \omega^2 - 2\theta\alpha$$

$$[\omega = 100 \text{ rev/min} = (5/8) \text{ rev/s} = 10\pi/3 \text{ rad/s}]$$

$$[0 = 10 \text{ rev} = 20\pi \text{ rad and } r = 0.2 \text{ m}]$$

$$0 = (3.22)^2 - 2\alpha(10)$$

$$\Rightarrow \alpha = 10\pi/36 \text{ rad/s}^2$$

$$\text{Moment of inertia: } I = (1/2) Mr^2 = (1/2)(10)(0.2)^2 = 0.2 \text{ kg m}^2$$

Therefore, force applied to the wheel = $T = Fa$

$$\Rightarrow T = Fr$$

$$\text{or } I\alpha = Fr$$

$$\Rightarrow F = 2\pi/36 \times (1/0.2)$$

$$F = 0.87 \text{ N}$$

Question 29: A cylinder rotating at an angular speed of 50 rev/s is brought in contact with an identical stationary cylinder. Because of the kinetic friction, torques act on the two cylinders, accelerating the stationary one and decelerating the moving one. If the common magnitude of the acceleration and deceleration be one revolution per second square, how long will it take before the two cylinders have equal angular speed?

Solution:

A cylinder rotating at an angular speed of 50 rev/s is brought in contact with an identical stationary cylinder.

The wheel rotates at a speed angular speed of 50 rev/s, at constant torque both acting in positive and negative acceleration.

$$\text{For the 1st cylinder} = \omega = 50 - \alpha t$$

$$\Rightarrow t = (\omega - 50)/-1 \dots (1)$$

$$\text{For the 2nd cylinder} = \omega = \alpha_2 t$$

$$\Rightarrow t = \omega/\alpha_2 = \omega/1 \dots (2)$$

From (1) and (2),

$$\omega = 25 \text{ rev/s}$$

and (2) $\Rightarrow t = 25/1 = 25 \text{ sec}$

Question 30: A body rotating at 20 rad/s is acted upon by a constant torque providing it a deceleration of 2 rad/s^2 . At what time will the body have kinetic energy same as the initial value if the torque continues to act?

Solution:

Initial angular velocity = 20 rad/s

So, $\alpha = 2 \text{ rad/s}^2$

$$\Rightarrow t_1 = \omega_1 / \alpha_1 = 20/2 = 10 \text{ sec}$$

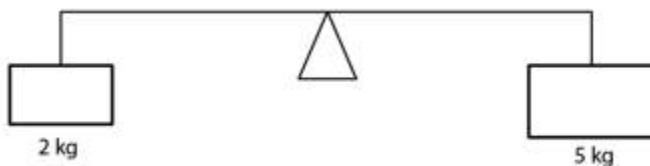
As, the time taken for the torque to be equal to the kinetic energy

Here initial angular velocity = angular velocity at that instant

$$\text{Time require to come to that angular velocity, } t_2 = \omega_2 / \alpha_2 = 20/2 = 10 \text{ sec}$$

$$\text{Total time requires} = t_1 + t_2 = 10 + 10 = 20 \text{ sec}$$

Question 31: A light rod of length 1 m is pivoted at its centre and two masses of 5 kg and 2 kg are hung from the ends as shown in figure (below). Find the initial angular acceleration of the rod assuming that it was horizontal in the beginning.



Solution:

A light rod of length 1 m is pivoted at its centre and two masses of 5 kg and 2 kg are hung from the ends.

Here,

$$\tau = I\alpha$$

Where τ = average frictional torque, I = moment of inertia and α = angular acceleration.

The torque produced on both the ends is:

$$5g \times (l/2) - 2g \times (l/2) = I\alpha$$

$$\alpha = 6/7 g \text{ or } 8.4 \text{ rad/s}^2$$

Question 32: Suppose the rod in the previous problem has a mass of 1 kg distributed uniformly over its length.

(a) Find the initial angular acceleration of the rod.

(b) Find the tension in the supports to the blocks of mass 2 kg and 5 kg.

Solution:

The given is that, the problem the rod has a mass 1 kg.

The angular acceleration is: $\tau_{\text{net}} = I_{\text{net}} \alpha$

The moment of the inertia = $I = ml^2/12$

$$\Rightarrow I = 1/12 \text{ kg m}^2$$

and total moment of inertia = $I = 1.75 + 1/12 = 1.833 \text{ kg m}^2$

(a) The angular acceleration of the rod at initial moment = $\alpha = T/I = 1.5 \times (9.8/1.833) = 8 \text{ rad/s}^2$

(b)

For 2 kg mass

$$T - 2g = 2a$$

$$T = 19.6 + 8 = 27.6 \text{ N}$$

For 5 kg mass

$$T - 5g = 5a$$

$$= 49 - 20$$

$$= 29 \text{ N}$$

Question 33: Figure (below) shows two blocks of masses m and M connected by a string passing over a pulley. The horizontal table over which the mass m slides is smooth. The pulley has a radius r and moment of inertia I about its axis and it can freely rotate about this axis. Find the acceleration of the mass M assuming that the string does not slip on the pulley.

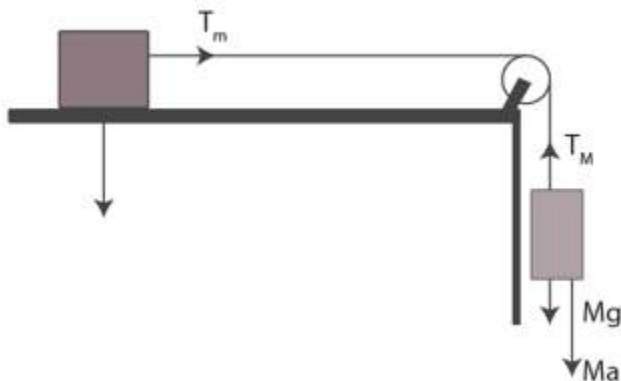


Solution:

The blocks are of “ m ” and “ M ” masses, with radius of r pulley and moment of Inertia “ I ”

Using second law of Newton when the Force/Tension applied is equivalent to the product of mass and acceleration

$$F = ma$$



Tension applied on the first block: $T_1 = M(g-a)$

Tension applied on the 2nd block of mass: $T_2 = ma$

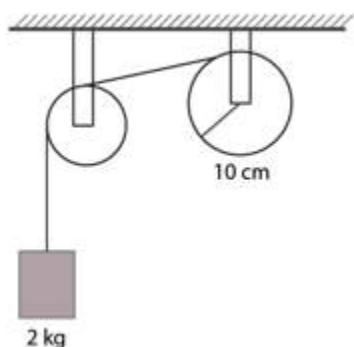
Torque applied on the pulley : $\tau = I\alpha = Ia/r$

Solving above equations, we get

$$Mg = Ma + ma + I \cdot \frac{a}{r^2}$$

$$a = \frac{Mg}{M + m + \frac{I}{r^2}}$$

Question 34: A string is wrapped on a wheel of moment of inertia 0.20 kg-m^2 and radius 10 cm and goes through a light pulley to support a block of mass 2.0 kg as shown in figure (below). Find the acceleration of the block.



Solution:

A string is wrapped on a wheel of moment of inertia $= I = 0.20 \text{ kg-m}^2$ and radius $= r = 10 \text{ cm}$ or 0.1 m .

Mass of the block $= m = 2 \text{ kg}$

We know, $mg - T = ma$

and $T = Ia/r^2$

From above equations,

$$mg = (m + I/r^2)a$$

$$\Rightarrow a = [2 \times 9.8] / [2 + 0.2/0.01] = 0.89 \text{ m/s}^2, \text{ which is the acceleration of the block.}$$

Question 35: Suppose the smaller pulley of the previous problem has its radius 5.0 cm and moment of inertia 0.10 kg-m^2 . Find the tension in the part of the string joining the pulleys.

Solution:

The moment of the inertia of the wheel, $I = 0.20$ and radius, $r = 10$ cm and mass block of block, $m = 2$ kg. (Given)

We know, $mg - T_1 = ma$..(1)

$(T_1 - T_2)r_1 = I_1 \alpha$ (2)

$T_2 r_2 = I_2 \alpha$ (3)

(2) $\Rightarrow$

$(T_1 - I_2 \alpha / r_1)r_2 = I_1 \alpha$

[Using value of T_2]

$\Rightarrow T_1 = [I_1 / r_1^2 + I_2 / r_2^2]a$

on substituting T_1 in (1)

$$a = \frac{mg}{[(I_1 / r_1^2) + (I_2 / r_2^2)] + m}$$

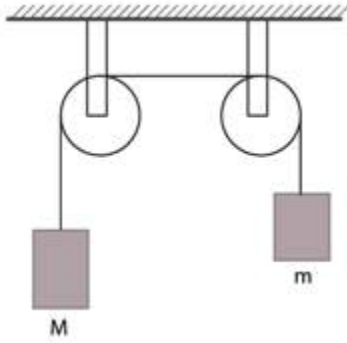
$$a = \frac{2 \times 9.8}{(0.1 / 0.0025) + (0.2 / 0.01) + 2}$$

$$= 0.316 \text{ m/s}^2$$

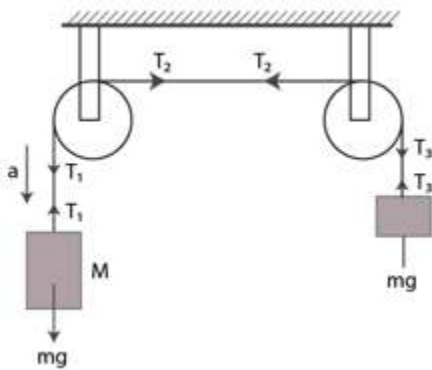
$$T_2 = I_2 a / r_2^2 = \frac{0.20 \times 0.316}{0.01}$$

$$= 6.32 \text{ N}$$

Question 36: The pulleys in figure (below) are identical, each having a radius R and moment of inertia I . Find the acceleration of the block M .



Solution:



From statement given,
 $Mg - T_1 = Ma \dots(1)$

$$(T_2 - T_1)R = Ia/R \dots(2)$$

$$(T_2 - T_3)R = Ia/R^2 \dots(3)$$

$$\text{and } T_3 - mg = ma \dots(4)$$

From (2) and (3)

$$(T_1 - T_3) = 2Ia/R^2 \dots(5)$$

Adding (1) and (4)

$$-mg + Mg + (T_3 - T_1) = Ma + ma \dots(6)$$

Using equation (5), equation (6) implies

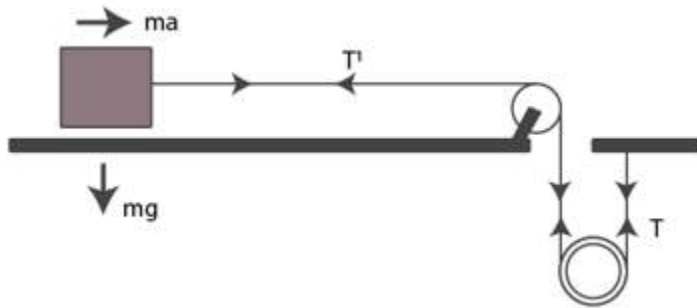
$$Mg - mg = Ma + ma + 2Ia/R^2$$

$$\text{or } a = [(M-m)G]/[(M+m+2I/R^2)]$$

Question 37: The descending pulley shown in figure (below) has a radius 20 cm and moment of inertia 0.20 kg-m^2 . The fixed pulley is light and the horizontal plane frictionless. Find the acceleration of the block if its mass is 1.0 kg .



Solution:



Let T is the tension of plane and T' of the side.

The angular acceleration of the pulley = $\alpha = (a/0.4) \text{ m/s}^2$

So, the torque applied by the pulley due to the mass: $I = (T - T') \times 0.20$

$$= 0.20 \text{ kgm}^2$$

$$\text{Again, } \alpha = T/I$$

$$\Rightarrow (a/0.4) = (T - T') \times 0.20/0.20$$

$$\Rightarrow (T - T') = -25 a \dots(1)$$

Find the value of mass M using the moment of Inertia

$$I = Mr^2/2$$

$$\text{or } M = 10 \text{ kg}$$

$$\text{Now, } Mg - T - T' = Ma/2$$

$$\text{Here } M = 10 \text{ and } g = 9.8$$

$$\Rightarrow T + T' = 98 - 5a \quad \dots(2)$$

Adding (1) and (2) we get

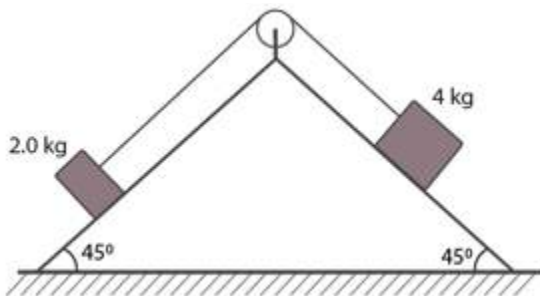
$$2T = 98 - 5a - 2.5a$$

$$[\text{Using formula, } T = ma = 1xa = a]$$

$$2a = 98 - 5a - 2.5a$$

$$\text{or } a = 10.31 \text{ m/s}^2 .$$

Question 38: The pulley shown in figure (below) has a radius 10 cm and moment of inertia 0.5 kg-m^2 about its axis. Assuming the inclined planes to be frictionless, calculate the acceleration of the 4.0 kg block.



Solution:

The radius of the pulley = 10 cm

Inertia = 0.5

Mass of the blocks are given as 2 kg and 4 kg.

The mass and tension relationship of the 4 kg block and the 2kg block :

$$m_1 g \sin \theta - T_1 = m_1 a \dots (1)$$

$$T_1 - T_2 = I a / r^2 \dots (2)$$

$$T_2 - m_2 g \sin \theta = m_2 a \dots (3)$$

$$(1)+(3) \Rightarrow$$

$$m_1 g \sin \theta - m_2 g \sin \theta + (T_2 - T_1) = (m_1 + m_2) a$$

using equation (2) in above, we have

$$a = \frac{(m_1 - m_2) g \sin \theta}{(m_1 + m_2 + I / r^2)}$$

$$= 0.248 \text{ m/s}^2$$

Question 39: Solve the previous problem if the friction coefficient between the 2.0 kg block and the plane below it is 0.5 and the plane below the 4.0 kg block is frictionless.

Solution:

Form the above question's figure,

$$m_1 g \sin \theta - T_1 = m_1 a \dots (1)$$

$$T_1 - T_2 = I a / r^2 \dots (2)$$

$$T_2 - (m_2 g \sin \theta + \mu m_2 g \cos \theta) = m_2 a \dots (3)$$

$$(1)+(3) \Rightarrow$$

$$m_1 g \sin \theta - (m_2 g \sin \theta + \mu m_2 g \cos \theta) + (T_2 - T_1) = m_1 a + m_2 a$$

Given, $m_1 = 4 \text{ kg}$, $m_2 = 2 \text{ kg}$, $g = 9.8$, $\sin \theta = \cos \theta = 1/\sqrt{2}$, $\mu = 0.5$ and $r = 0.1$

Using (2), and substituting above values, we get

$$a = 0.125 \text{ m/s}^2$$

Question 40: A uniform meter stick of mass 200 g is suspended from the ceiling through two vertical strings of equal lengths fixed at the ends. A small object of mass 20 g is placed on the stick at a distance of 70 cm from the left end. Find the tensions in the two strings.

Solution:

Given, $m_1 = 200\text{g}$, $l = 1\text{ m}$, $m_2 = 20\text{g}$

The total tension formed due to the suspended weights: $T_1 + T_2 = 2 + 0.2 = 2.2 \dots(1)$

The rod is kept at an equilibrium when the rod is at rest

$$- T_1 + T_2 = 0.04/0.5 = 0.08 \dots(2)$$

Form (1) and (2)

$$T_2 = 1.14\text{ N}$$

$$\text{and (2)} \Rightarrow T_1 = (1.14 - 0.08) = 1.06\text{N}$$

Question 41: A uniform ladder of length 10.0 m and mass 16.0 kg is resting against a vertical wall making an angle of 37° with it. The vertical wall is frictionless but the ground is rough. An electrician weighing 60.0 kg climbs up the ladder. If he stays on the ladder at a point 8.00 m from the lower end, what will be the normal force and the force of friction on the ladder by the ground? What should be the minimum coefficient of friction for the electrician to work safely?

Solution:

The length of the ladder = 10 m

Mass of the ladder is 16 kg which makes angle of 37° and also given

Weight of the electrician is 60 kg, which stays at height of 8 cm.

Since the ladder should not slip or rotate, the torques expression is

$$mg(8 \sin 37^\circ) + Mg(5 \sin 37^\circ) = F_2 (10 \cos 37^\circ) = 0 \text{ and}$$

$$60 \times 9.8 \times (8 \sin 37^\circ) + 16 \times 9.8 (5 \sin 37^\circ) - F_2 (10 \cos 37^\circ) = 0$$

Where F_1 and F_2 : forces of the masses in terms of reactionary force

$$\Rightarrow F_2 = 412\text{N}$$

let f be the friction force which is equal to the F_2 due to equilibrium of the ladder

$$\Rightarrow f = 412\text{ N}$$

$$\text{Normal force} = F_1 = (m + M)g = (60+16) \times 9.8 = 744.8\text{ N}$$

Now,

The minimum coefficient of friction = μ

$$\Rightarrow \mu = f/F_1$$

[on substituting the values]

$$\mu = 0.553. \text{ Answer!!}$$

Question 42: Suppose the friction coefficient between the ground and the ladder of the previous problem is 0.540. Find the maximum weight of a mechanic who could go up and do the work from the same position of the ladder.

Solution:

The value of the reactionary force = $R_2 = 16g + mg$ and another value of R_2 is R_1/μ

The relationship between R_1 and R_2 is

$$R_1 \times 10 \cos 37^\circ = 16g \times 5 \sin 37^\circ + mg \times 60g \times 8 \times \sin 37^\circ$$

The value of the reactionary forces = R_1 is

$$R_1 = \frac{48g + \frac{24}{5}mg}{8}$$

$$R_2 = \frac{48g + \frac{24}{5}mg}{8 \times 0.54}$$

=>

$$16g + mg = \frac{48g + \frac{24}{5}mg}{8 \times 0.54}$$

Or $m = 44 \text{ kg}$

Which is the mass of the mechanic that can go up the ladder.

Question 43: A 6.5 m long ladder rests against a vertical wall reaching a height of 6.0 m. A 60 kg man stands half way up the ladder. (a) Find the torque of the force exerted by the man on the ladder about the upper end of the ladder. (b) Assuming the weight of the ladder to be negligible as compared to the man and assuming the wall to be smooth, find the force exerted by the ground on the ladder.

Solution:

The length of the ladder = 6.5m

Weight of the man = 60kg and

(a) Torque is exerted at the upper end of the ladder and there is no friction against the wall.

Torque due to weight of body

$$\tau = Fr \sin\theta$$

$$\tau = mg r \sin\theta$$

$$\tau = 60 \times 10 \times 6.5 \times \sin\theta = 750 \text{ N}$$

(b)

force exerted by the man through the ladder on the ground

$$F = mg = 60 \times 10 = 600 \text{ N}$$

Question 44: The door of an Amirah is 6 ft. high, 1.5 ft. wide and weighs 8 kg. The door is supported by two hinges situated at a distance of 1 ft. from the ends. If the magnitudes of the forces exerted by the hinges on the door are equal, find this magnitude.

Solution:

The force exerted by the two hinges: $F_1 + F_2 = mg$

$$\text{or } F_1 + F_2 = 8g$$

$$\text{When } F_1 = F_2$$

$$\Rightarrow F_1 = 40$$

$$[\text{Using } g = 10 \text{ m/s}^2]$$

$$\text{The reactionary forces of the first hinge} = M_1 \times 4 = 8g \times 0.75$$

$$\Rightarrow M_1 = 15 \text{ N}$$

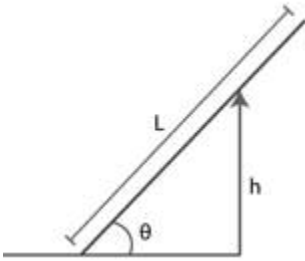
the resultant force due to force and reactionary forces:

$$R = F_1^2 + N_1^2$$

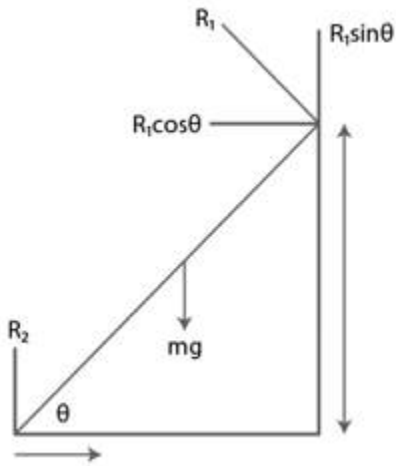
$$= 40^2 + 15^2$$

$$\text{or } R = 43 \text{ N}$$

Question 45: A uniform rod of length L rests against a smooth roller as shown in figure (below). Find the friction coefficient between the ground and the lower end if the minimum angle that the rod can make with the horizontal is θ .



Solution:



The vertical and the horizontal component of the reactionary forces of the rod:

$$R_2 = mg - R_1 \cos \theta \text{ and } R_1 \sin \theta = \mu R_2$$

$$R_1 = \frac{\frac{mgL}{2} \cos \theta}{(\cos^2 \theta / \sin \theta)h + \sin \theta}$$

$$R_1 \cos \theta = \frac{\frac{mgL}{2} \cos^2 \theta \sin \theta}{(\cos^2 \theta / \sin \theta)h + \sin \theta}$$

The coefficient of friction: $\mu = [R_1 \sin \theta] / R_2$

Using above equations, we get

$$\mu = \frac{L \cos \theta \sin^2 \theta}{2h - L \cos^2 \theta \sin \theta}$$

Question 46: A uniform rod of mass 300 g and length 50 cm rotates at a uniform angular speed of 2 rad/s about an axis perpendicular to the rod through an end. Calculate (a) the angular momentum of the rod about the axis of rotation, (b) the speed of the centre of the rod and (c) its kinetic energy.

Solution:

(a) The average momentum

$$L = I\omega$$

We are given, $I = \frac{mr^2}{3}$

$$\Rightarrow L = \frac{mr^2}{3} \times \omega$$

$$\Rightarrow L = \frac{(0.3 \times 0.5^2 \times 2)}{3}$$

$$\Rightarrow L = 0.05 \text{ kgm}^2/\text{s}$$

(b) Speed of the center of rod

$$v = \omega r$$

$$\Rightarrow v = 2 \times (50/2) = 50 \text{ cm/s}$$

(c) K.E. generated

$$\text{K.E.} = \frac{1}{2} \times (0.025^2 = 0.05 \text{ J}$$

Question 47: A uniform square plate of mass 2.0 kg and edge 10 cm rotates about one of its diagonals under the action of a constant torque of 0.10 N-m. Calculate the angular momentum and the kinetic energy of the plate at the end of the fifth second after the start.

Solution:

$$\text{Here } \left(\frac{ma^2}{12}\right) \times \alpha = 0.10 \text{ N - m}$$

and $\omega = 60 \times 5 = 300 \text{ rad/s}$

Therefore, average momentum = $L = I\omega$

$$\Rightarrow L = (0.10/60) \times 300 = 0.5 \text{ kgm}^2/\text{s}$$

Then, the K.E. = $(1/2) I\omega^2$

$$= (0.10/60) \times 300^2$$

$$= 75 \text{ J}$$

Question 48: Calculate the ratio of the angular momentum of the earth about its axis due to its spinning motion to that about the sun due to its orbital motion. Radius of the earth = 6400 km and radius of the orbit of the earth about the sun = $1.5 \times 10^8 \text{ km}$.

Solution:

The angular momentum of earth about its axis:

$$(2/5) mR^2 \times (2\pi \times 35400)$$

And the value of moment of inertia of earth: $I = (2/5) MR^2$

Now the value of the sun's average mass: $mR(2\pi/86400 \times 365)$

Now, the ratio of the mass of sun and the mass of earth :

$$\text{ratio} = \frac{\frac{2}{5} mR^2 \times \left(\frac{2\pi}{36400} \right)}{mR^2 \times \frac{2\pi}{36400 \times 365}}$$

$$= 2.65 \times 10^{-7}$$