Factorization

FUNDAMENTALS

FACTORS: When an algebraic quantity can be expressed as the product of two or more algebraic quantities, then each of these quantities is called a factor of the given algebraic quantity and the process of finding factors, is called FACTORIZATION.

Remarks: Factorization is the opposite process of multiplication,

EXAMPLE Look at the examples given below:

Multiplication	**Factorization (opposite of multiplication)**
(1) $2x(3x-2y)=6x^2-4xy$	$6x^2-4xy=2x(3x-2y)$
(2) $(2a+3)(3a+2)=6a^2+13a+6$	$6a^2+13a+6=(2a+3)(3a+2)$
(3) $(15m+17n)(15m-17n)=225m^2-289n^2$	$225m^2-289n^2=(15m+17n)(15m-17n)$

- **It is advisable that students memorize squares of numbers from 1 to 20. E.g. here, $15^2=225$ and $17^2=289$ are readily used.**

1. Factorization when a Common Monomial Factor Occurs in Each Term.

METHOD: Step 1. Find the HCF of all the terms.

Step2. Divide each term by this HCF.

Step3. Write the given expression= HCF × quotient obtained in step 2.

Conceptual Framework / Idea behind above steps: HCF itself is one of the factors. Hence,

$$\text{other factor will be equal to} \frac{\text{Given pression}}{\text{HCF}}$$

EXAMPLE 1. Factorize (i.e. break into factors) each of the following:

(1) $13n+117$ (2) n^3+2n+n^2

(3) $15x^2y^2z^2+5xy^2z+5xyz$

(4) $6ab-9bc$

(1) $13n+117=13(n+9)$

(2) $n^3+2n+n^2=n(n^2+2+n)$

(3) $15x^2y^2z^2+5xy^2z+5xyz=5xyz(3xyz+y+1)$

2. Factorization when one or more Binomial is Common

METHOD: Step 1. Find the common binomial by intelligent thinking or by trial & error.

Step 2. Divide each term by this common binomial.

Step 3. Write the given expression = this binomial × quotient obtained in **Step 2**

EXAMPLES. Factorize:

(1) $6x(3a-4b)+10y(3a-4b)$

(2) $6(16x-23y)-22(16x-23y)^2$

(3) $mn(ax-2by)^2+mn^2(ax-2by)$

We have,

(1) $6x(3a-4b)+10y(3a-4b)$

$=(6x+10y)(3a-4b)$

(2) $6(16x-23y)-22(16x-23y)^2$

$=(16x-23y)[6-22(16x-23y)]$

$=(16x-23y)\times(-346x+506y)$

$=2\ x(16x-23y)x(-173x+253y)$

(3) $mn(ax-2by)^2+mn^2(ax-2by)$

$=mn(ax-2by)\times(ax-2by+n)$

3. Factorization by Grouping

The terms of the given expression are arranged in suitable groups so that all the groups have a common factor. The key idea is (1) **to identify the common factor** (2) **take out this common factor.**

EXAMPLE 4. Factorize:

(1) $m^2+np+mn+mp$

(2) $ax^2+by^2+ay^2+bx^2$

(3) $1-a-b+ab$

(4) $xy-ny+mn-mx$

(5) $1+y+yz+y^2z$

(6) $xy(m^2+n^2)+mn(x^2+y^2)$

Solution: By suitably rearranging the terms, we have:

(1) $m^2+np+mn+mp=m^2+mn+np+mp$

$=m(m+n)+p(m+n)=(m+n)(m+p).$

(2) $ax^2+by^2+ay^2+bx^2=ax^2+bx^2+by^2+ay^2=x^2(a+b)+y^2(b+a)=(x^2+y^2)(a+b).$

(3) $1-a-b+ab=1-a-b(1-a)$

$=1(1-a)-b(1-a)=(1-a)(1-b)$

(4) $xy-ny+mn-mx=(x-n)y+m(n-x)$

$=(x-n)y-m(x-n)(x-n)(y-m)$

(5) $1+y+yz+y=(1+y)+yz(1+y)$

$=1\times(1+y)+yz\times(1+y)=(1+y)\times(1+yz)$.

(6) $xy(m^2+n^2)+mn(x^2+y^2)$

$=xym^2+xyn^2+mnx^2+mny^2$

Consider factors "mx" common between 1st & 3rd terms. Similarly, consider factors "ny" common between 2nd & 4th terms.

$\Rightarrow mx(my+nx)+ny(nx+my)=(mx+ny)\times(nx+my)$

4. Factorization when given term is a Perfect Square

FORMULA:

(i) $\mathbf{a^2+b^2+2ab=(a+b)^2}$

(ii) $\mathbf{a^2+b^2-2ab=(a-b)^2}$

(1) $x^2+20x+100$

(2) $a^2x^2+2\ abxy+b^2y^2$

(3) $y^2-26xy+169$

(4) $y^2-6my+9m^2$

Let us illustrate through one example. Rest you should try yourself. Let us consider example

(2) $a^2x^2+2abxy+b^2+y^2$

$=(ax)^2+2\times(ax)\times(by)+(by)^2$

It is of the form $a^2+b^2+2ab=(ax+by)^2$

5. Factorization when given term is Difference of Two Squares

FORMULA: $\mathbf{(a^2-b^2)=(a+b)(a-b)}$

(1) $81-x^2$

(2) $m^2y^2-81n^2$

Let us illustrate through one example. Let us consider example: (2) $m^2y^2-81n^2$

$=(my)^2-(9n)^2=(my-9n)^2$

6. Factorization of Quadratic Trinomials

When trinomial is of the form: $\mathbf{(x^2+mx+n)}$

For factorizing (x^2+mx+n), we find two numbers a and b such that $(a+b)=m$ and ab = n. Then,

$x^2 + mx + n = x^2 + (a+b)x + ab = (x+a)(x+b)$.

This is essentially based on concepts discussed under quadratic equations in GMO, Class VII book. Basically , we need to find roots of quadratic equation; a and b are roots which may be found by HIT & TRIAL as discussed above or by exact method. Roots $= \frac{-m \pm \sqrt{m^2 - 4n}}{2}$

E.g. Factorize: $x^2 + 13x + 42$

Solution: In the given expression, sum of roots $= 13$ and product of roots $= 42$.

Clearly, the numbers are 6 and 7.

$\therefore x^2 + 13x + 42 = x^2 + 6x + 7x + 6 \times 7$

$= x(x+6) + 7(x+6) = (x+6)(x+7)$.

When trinomial is of the form: $\mathbf{ax^2 + bx + c}$

For factorizing $\mathbf{ax^2 + bx + c}$ we split b into two parts whose sum is b and product is ac. Then, proceed to factorize.

E.g. $6x^2 + 7x + 2$

Solution: In the given expression, $6x^2 + 7x + 2$, we have to find two numbers whose sum is 7 and product $= ac = 6 \times 2 = 12$.

The numbers are 3 and 4.

$\therefore 6x^2 + 7x + 2 = 6x^2 + 3x + 4x + 2$

$= 3x(2x+1) + 2(2x+1) = (3x+2)(2x+1)$.