

## Chapter 3. Solving Linear Equations

---

### Answer 1VC.

Write the following property of equality;

$$\text{If } a = b, \text{ then } a + c = b + c$$

The objective is to choose the correct term out of addition or multiplication for the property of equality.

The addition property of equality for real numbers states that two numbers,  $a$  and  $b$  are equal then the equality remains unchanged if some real number is added to both the numbers.

Mathematically the property is as follows;

$$\text{If } a = b, \text{ then } a + c = b + c$$

The multiplication property of equality states that for real numbers states that two numbers,  $a$  and  $b$  are equal then the equality remains unchanged if some real number is multiplied to both the numbers.

Mathematically the property is as follows;

$$\text{If } a = b, \text{ then } a \cdot c = b \cdot c$$

The mentioned property is according to the addition property of equality.

So, choose the term **Addition**.

### Answer 2VC.

The objective is to choose the correct term out of the terms means or ratio in relation with the sentence.

When two numbers are compared by division then, it is termed as the ratio of the two numbers.

Means give the average of two or more numbers.

Hence, the mentioned sentence relates to the term ratio.

So, choose the term **ratio**.

### Answer 3VC.

The objective is to choose the correct term out of the terms the same or different related to the definition of rate.

A rate is a term mostly used in uniform motion.

It is the speed at which an object travels.

If  $d$  is the distance,  $r$  is the rate at which a vehicle travels and  $t$  is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

$$r = \frac{d}{t}$$

Thus, the rate is the ratio of distance and time both of which have different units of measure.

Distance is measured in meters, kilometers etc. whereas time is measured in seconds minutes or hours.

In general, in any context, the term rate is the ratio of two quantities that have different units of measure.

So, choose the term different.

### Answer 4VC.

The objective is to choose the correct term out of the terms explore or solve in relation with the four-step problem solving plan.

The four-step problem solving plan involves the following steps;

1. Explore : This step suggests reading and analyzing the problem.
2. Plan : This step involves making a plan to solve a problem.
3. Solve : This step involves using the plan and solving the problem.
4. Examine : This step involves rechecking the solution by various methods.

Hence, the first step on the plan is to explore the problem.

So, choose the term explore.

### Answer 5VC.

Write the following equality;

$$2x + 1 = 2x + 1$$

The objective is to choose the correct term out of identity or formula that describes the equality.

An identity is an equality containing one or more variables that will remain true for any value of the variables in it.

A formula is an equation that expresses one variable in terms of other variables or numbers. It can be used to find the value of any one variable.

The equality  $2x + 1 = 2x + 1$  has same terms on both sides.

Thus, for any value of  $x$ , the equality remains true.

Hence, the equality is an identity.

So, choose the term identity.

### Answer 6VC.

Write the following equality;

$$3x + 5 = 7$$

The objective is to find an equivalent equation for the equality mentioned above.

Simplify the equality such that all the terms containing variables are on one side of the equality and the remaining terms are on the other side.

Subtract 5 from both sides of the equation;

$$3x + 5 = 7$$

$$3x + 5 - 5 = 7 - 5 \text{ Combine the like terms}$$

$$3x = 2$$

The reduced equation is  $3x = 2$  which is equivalent to the original equation.

So, choose the option  $3x = 2$ .

### Answer 7VC.

The original value of the product is 80.

The new value of the product is 90.

The percent of change is 12.5%.

The objective is to choose the correct term out of percent of decrease or increase.

If the new value is greater than the old value then this means that there is an increase in the old value of the quantity. Hence the change in percent is called as the percent of increase.

If the new value is less than the old value then this means that there is a decrease in the original value. Hence the change in percent is called the percent of decrease.

Observe that the new value of the product, 90, is greater than the original value, 80.

Hence, the percent of change is a percent increase.

So, choose the term **Increase**.

### Answer 8VC.

The objective is to choose the correct term out of the terms dimensional analysis or defining the variable.

The process in which the units are carried out throughout the computation is called dimensional analysis.

Defining a variable is the process in which an unknown quantity is assigned a letter or a variable.

Hence, the sentence mentioned in the question relates to the definition of dimensional analysis.

So, choose the term **dimensional analysis**.

### Answer 9VC.

The objective is to choose the correct term out of weighted average or rate.

Weighted average for a set of data points is the ratio of the sum of the products of number of units and value per unit to the sum of the number of units.

A rate is the ratio of two measurements having different units of measure.

Hence, the mentioned sentence is the explanation of the term weighted average.

So, choose the term **Weighted average**.

### Answer 10VC.

Write the following pairs of numbers;

8,9 and 8,10

The objective is to choose the correct pair that represents consecutive integers.

Consecutive integers are a set of integers that are successive to one another.

For example, the integers 2 and 3, 4 and 5 etcetera, are consecutive integers.

Hence, 8 and 9 represent consecutive integers where as 8 and 10 do not represent consecutive integers.

So, choose the integers **8 and 9**.

### Answer 11E.

Consider the number  $n$ .

Three times the number is  $3n$ .

The product is decreased by 21.

So, subtract 21 from the product  $3n$  and write the result as  $3n - 21$ .

The result is equal to 57.

So, equate the result  $3n - 21$  with 57 and write the equation as follows;

$$3n - 21 = 57$$

Therefore, the required equation in mathematical form is  **$3n - 21 = 57$** .

### Answer 12E.

Consider the number  $z$ .

Three times the number  $z$  is  $3z$ .

The product is subtracted from 4.

So, subtract the product  $3z$  from 4 and write the expression as follows;

$$4 - 3z \dots\dots(1)$$

The number  $z$  is decreased by 2 hence; write the expression as follows;

$$z - 2 \dots\dots(2)$$

According to the statement, the two expressions thus formed are equal.

So, equate the expressions **(1) and (2)** and write the equation as follows;

$$4 - 3z = z - 2$$

Therefore, the required equation in mathematical form is  **$4 - 3z = z - 2$** .

**Answer 13E.**

Consider the numbers  $a$  and  $b$ .

Square of the number  $a$ , is  $a^2$ .

Cube of number  $b$  is  $b^3$ .

The sum of the square of the first number and the cube of the second number is 16.

So, write the equation as follows;

$$a^2 + b^3 = 16$$

Therefore, the required equation in mathematical form is  $a^2 + b^3 = 16$ .

**Answer 14E.**

Write the following equation;

$$16 - 9r = r$$

The objective is to translate the mathematical form of the equation into sentence.

The variable in the equation is  $r$ .

The variable  $r$  is multiplied by 9 and the product is subtracted from 16.

The result is then equated with  $r$ .

So, write the sentence form of the mathematical equation as follows;

**“Sixteen minus nine times  $r$  is equal to  $r$ ”.**

**Answer 15E.**

Write the following equation;

$$r - 21 = -37$$

The objective is to solve the equation for  $r$ .

Collect all the terms containing  $r$  on one side of the equality and the remaining terms on the other side.

So, add 21 to both sides of the equation and rewrite the equation as follows;

$$r - 21 = -37$$

$$r - 21 + 21 = -37 + 21$$

Combine the like terms and simplify;

$$r = -16$$

**Check.**

Substitute  $-16$  for  $r$  on the left side of the equation  $r - 21 = -37$ ;

$$\begin{aligned} r - 21 &= -16 - 21 \\ &= -37 \end{aligned}$$

The result is equal to the right hand side.

Hence, the value of  $r = -16$  satisfies the equation.

Therefore, the solution of the equation  $r - 21 = -37$  is  $r = -16$ .

### Answer 16E.

Write the following equation;

$$14 + c = -5$$

The objective is to solve the equation for  $c$ .

Collect all the terms containing  $c$  on one side of the equality and the remaining terms on the other side.

So, subtract 14 from both sides of the equation and rewrite the equation as follows;

$$14 + c = -5$$

$$14 + c - 14 = -5 - 14$$

Combine the like terms and simplify;

$$c = -19$$

**Check.**

Substitute  $-19$  for  $r$  on the left side of the equation  $14 + c = -5$ ;

$$14 + c = 14 + (-19)$$

Multiply the positive and negative sign.

Product of a positive and a negative sign is negative.

So, write the expression as follows and simplify;

$$\begin{aligned} 14 + c &= 14 - 19 \\ &= -5 \end{aligned}$$

The result is equal to the right hand side.

Hence, the value of  $c = -19$  satisfies the equation.

Therefore, the solution of the equation  $14 + c = -5$  is  $\boxed{c = -19}$ .

### Answer 17E.

Write the following equation;

$$27 = 6 + p$$

The objective is to solve the equation for  $p$ .

Collect all the terms containing  $p$  on one side of the equality and the remaining terms on the other side.

So, subtract 6 from both sides of the equation and rewrite the equation as follows;

$$27 = 6 + p$$

$$27 - 6 = 6 + p - 6$$

Combine the like terms and simplify;

$$21 = p$$

Use the symmetric property of real numbers which states that for any two real numbers  $a$  and  $b$ ;

If  $a = b$  then  $b = a$

Hence,  $p = 21$ .

**Check.**

Substitute 21 for  $p$  on the right side of the equation  $27 = 6 + p$ ;

$$\begin{aligned}6 + p &= 6 + 21 \\ &= 27\end{aligned}$$

The result is equal to the left hand side.

Hence, the value of  $p = 21$  satisfies the equation.

Therefore, the solution of the equation  $27 = 6 + p$  is  $\boxed{p = 21}$ .

**Answer 18E.**

Write the following equation;

$$b + (-14) = 6$$

The objective is to solve the equation for  $b$ .

To add a negative number to a number is to subtract the additive inverse of the number from the first number.

The additive inverse of a number is the same number with opposite sign.

Hence, the additive inverse of  $-14$  is 14.

So, rewrite the equation using the additive inverse as follows;

$$\begin{aligned}b + (-14) &= 6 \\ b - 14 &= 6\end{aligned}$$

Collect all the terms containing  $b$  on one side of the equality and the remaining terms on the other side.

So, add 14 on both sides of the equation and rewrite the equation as follows;

$$\begin{aligned}b - 14 &= 6 \\ b - 14 + 14 &= 6 + 14\end{aligned}$$

Combine the like terms and simplify;

$$b = 20$$

**Check.**

Substitute 20 for  $b$  on the left side of the equation  $b + (-14) = 6$ ;

$$b + (-14) = 20 + (-14)$$

Write the expression using additive inverse and simplify;

$$\begin{aligned}b + (-14) &= 20 - 14 \\ &= 6\end{aligned}$$

The result is equal to the right hand side.

Hence, the value of  $b = 20$  satisfies the equation.

Therefore, the solution of the equation  $b + (-14) = 6$  is  $\boxed{b = 20}$ .

**Answer 19E.**

Write the following equation;

$$d - (-1.2) = -7.3$$

The objective is to solve the equation for  $d$ .

To subtract a negative number from a number is to add the additive inverse of the number to the first number.

The additive inverse of a number is the same number with opposite sign.

Hence, the additive inverse of  $-1.2$  is  $1.2$ .

So, rewrite the equation using the additive inverse as follows;

$$d - (-1.2) = -7.3$$

$$d + 1.2 = -7.3$$

Collect all the terms containing  $d$  on one side of the equality and the remaining terms on the other side.

So, subtract  $1.2$  from both sides of the equation and rewrite the equation as follows;

$$d + 1.2 = -7.3$$

$$d + 1.2 - 1.2 = -7.3 - 1.2$$

Combine the like terms and simplify;

$$d = -8.5$$

**Check.**

Substitute  $-8.5$  for  $d$  on the left side of the equation  $d - (-1.2) = -7.3$ ;

$$d - (-1.2) = -8.5 - (-1.2)$$

Write the expression using additive inverse and simplify;

$$\begin{aligned} d - (-1.2) &= -8.5 + 1.2 \\ &= -7.3 \end{aligned}$$

The result is equal to the right hand side.

Hence, the value of  $d = -8.5$  satisfies the equation.

Therefore, the solution of the equation  $d - (-1.2) = -7.3$  is  $\boxed{d = -8.5}$ .

### Answer 20E.

Write the following equation;

$$r + \left(-\frac{1}{2}\right) = -\frac{3}{4}$$

The objective is to solve the equation for  $r$ .

To add a negative number to a number is to subtract the additive inverse of the number from the first number.

The additive inverse of a number is the same number with opposite sign.

Hence, the additive inverse of  $-\frac{1}{2}$  is  $\frac{1}{2}$ .

So, rewrite the equation using the additive inverse as follows;

$$\begin{aligned} r + \left(-\frac{1}{2}\right) &= -\frac{3}{4} \\ r - \frac{1}{2} &= -\frac{3}{4} \end{aligned}$$

Collect all the terms containing  $r$  on one side of the equality and the remaining terms on the other side.

So, add  $\frac{1}{2}$  on both sides of the equation and rewrite the equation as follows;

$$\begin{aligned} r - \frac{1}{2} &= -\frac{3}{4} \\ r - \frac{1}{2} + \frac{1}{2} &= -\frac{3}{4} + \frac{1}{2} \end{aligned}$$

Rationalize the denominator and combine the terms;

$$\begin{aligned} r &= -\frac{3}{4} + \frac{1 \cdot 2}{2 \cdot 2} \\ &= -\frac{3}{4} + \frac{2}{4} \\ &= \frac{-3+2}{4} \\ &= -\frac{1}{4} \end{aligned}$$

**Check.**

Substitute  $-\frac{1}{4}$  for  $r$  on the left side of the equation  $r + \left(-\frac{1}{2}\right) = -\frac{3}{4}$ ;

$$r + \left(-\frac{1}{2}\right) = -\frac{1}{4} + \left(-\frac{1}{2}\right)$$

Write the expression using additive inverse;

$$r + \left(-\frac{1}{2}\right) = -\frac{1}{4} - \frac{1}{2}$$

Rationalize the denominator and simplify;

$$\begin{aligned} r + \left(-\frac{1}{2}\right) &= -\frac{1}{4} - \frac{1}{2} \\ &= -\frac{1}{4} - \frac{1 \cdot 2}{2 \cdot 2} \\ &= -\frac{1}{4} - \frac{2}{4} \\ &= -\frac{3}{4} \end{aligned}$$

The result is equal to the right hand side.

Hence, the value of  $r = -\frac{1}{4}$  satisfies the equation.

Therefore, the solution of the equation  $r + \left(-\frac{1}{2}\right) = -\frac{3}{4}$  is  $\boxed{r = -\frac{1}{4}}$ .

### **Answer 21E.**

Write the following equation;

$$6x = -42$$

The objective is to solve the equation for  $x$ .

Use the property of equality for division.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } \frac{a}{c} = \frac{b}{c}; c \neq 0$$

So, divide both sides of the equation by 6 and simplify the equation as follows;

$$6x = -42$$

$$\frac{6x}{6} = \frac{-42}{6}$$

$$x = -7$$

**Check.**

Substitute  $-7$  for  $x$  on the left side of the equation  $6x = -42$ :

$$\begin{aligned}6x &= 6 \cdot (-7) \\ &= -42\end{aligned}$$

The result is equal to the right hand side.

Hence, the value of  $x = -7$  satisfies the equation.

Therefore, the solution of the equation  $6x = -42$  is  $\boxed{x = -7}$ .

**Answer 22E.**

Write the following equation;

$$-7w = -49$$

The objective is to solve the equation for  $w$

Use the property of equality for division.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } \frac{a}{c} = \frac{b}{c}; c \neq 0$$

So, divide both sides of the equation by  $-7$  and simplify the equation as follows;

$$\begin{aligned}-7w &= -49 \\ \frac{-7w}{-7} &= \frac{-49}{-7} \\ w &= 7\end{aligned}$$

**Check.**

Substitute  $7$  for  $w$  on the left side of the equation  $-7w = -49$ :

$$\begin{aligned}-7w &= -7 \cdot (7) \\ &= -49\end{aligned}$$

The result is equal to the right hand side.

Hence, the value of  $w = 7$  satisfies the equation.

Therefore, the solution of the equation  $-7w = -49$  is  $\boxed{w = 7}$ .

**Answer 23E.**

Write the following equation;

$$\frac{3}{4}n = 30$$

The objective is to solve the equation for  $n$ .

Use the property of equality for multiplication.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } ac = bc$$

So, multiply both sides of the equation by 4 and simplify the equation;

$$\frac{3}{4}n = 30$$

$$4 \cdot \frac{3}{4}n = 4 \cdot 30$$

$$3n = 120$$

Use the property of equality for division.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } \frac{a}{c} = \frac{b}{c}; c \neq 0$$

So, divide both sides of the equation by 3 and simplify the equation as follows;

$$3n = 120$$

$$\frac{3n}{3} = \frac{120}{3}$$

$$n = 40$$

**Check.**

Substitute 40 for  $n$  on the left side of the equation  $\frac{3}{4}n = 30$ ;

$$\frac{3}{4}n = \frac{3}{4}(40)$$

$$= 3 \cdot 10$$

$$= 30$$

The result is equal to the right hand side.

Hence, the value of  $n = 40$  satisfies the equation.

Therefore, the solution of the equation  $\frac{3}{4}n = 30$  is  $\boxed{n = 40}$ .

**Answer 24E.**

Write the following equation;

$$-\frac{3}{5}y = -50$$

The objective is to solve the equation for  $y$ .

Use the property of equality for multiplication.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } ac = bc$$

So, multiply both sides of the equation by 5 and simplify the equation;

$$-\frac{3}{5}y = -50$$

$$5 \cdot \frac{-3}{5}y = 5 \cdot (-50)$$

$$-3y = -250$$

Use the property of equality for division.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } \frac{a}{c} = \frac{b}{c}; c \neq 0$$

So, divide both sides of the equation by  $-3$  and simplify the equation as follows;

$$-3y = -250$$

$$\frac{-3y}{-3} = \frac{-250}{-3}$$

$$y = \frac{250}{3}$$

**Check.**

Substitute  $\frac{250}{3}$  for  $y$  on the left side of the equation  $-\frac{3}{5}y = -50$ ;

$$\begin{aligned} -\frac{3}{5}y &= -\frac{3}{5} \left( \frac{250}{3} \right) \\ &= -50 \end{aligned}$$

The result is equal to the right hand side.

Hence, the value of  $y = \frac{250}{3}$  satisfies the equation.

Therefore, the solution of the equation  $-\frac{3}{5}y = -50$  is  $\boxed{y = \frac{250}{3}}$ .

**Answer 25E.**

Write the following equation;

$$\frac{5}{2}a = -25$$

The objective is to solve the equation for  $a$ .

Use the property of equality for multiplication.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } ac = bc$$

So, multiply both sides of the equation by 2 and simplify the equation;

$$\frac{5}{2}a = -25$$

$$2 \cdot \frac{5}{2}a = 2 \cdot (-25)$$

$$5a = -50$$

Use the property of equality for division.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } \frac{a}{c} = \frac{b}{c}; c \neq 0$$

So, divide both sides of the equation by 5 and simplify the equation as follows;

$$5a = -50$$

$$\frac{5a}{5} = \frac{-50}{5}$$

$$a = -10$$

**Check.**

Substitute  $-10$  for  $a$  on the left side of the equation  $\frac{5}{2}a = -25$ ;

$$\frac{5}{2}a = \frac{5}{2}(-10)$$

$$= 5(-5)$$

$$= -25$$

The result is equal to the right hand side.

Hence, the value of  $a = -10$  satisfies the equation.

Therefore, the solution of the equation  $\frac{5}{2}a = -25$  is  $\boxed{a = -10}$ .

**Answer 26E.**

Write the following equation;

$$5 = \frac{r}{2}$$

The objective is to solve the equation for  $r$ .

Use the property of equality for multiplication.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } ac = bc$$

So, multiply both sides of the equation by 2 and simplify the equation;

$$5 = \frac{r}{2}$$

$$2 \cdot 5 = 2 \cdot \frac{r}{2}$$

$$10 = r$$

Use the symmetric property of real numbers which states that for any two real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } b = a$$

Hence,  $r = 10$ .

**Check.**

Substitute 10 for  $r$  on the right side of the equation  $5 = \frac{r}{2}$ ;

$$\begin{aligned} \frac{r}{2} &= \frac{10}{2} \\ &= 5 \end{aligned}$$

The result is equal to the left hand side.

Hence, the value of  $r = 10$  satisfies the equation.

Therefore, the solution of the equation  $5 = \frac{r}{2}$  is  $\boxed{r = 10}$ .

**Answer 27E.**

Write the following equation;

$$4p - 7 = 5$$

The objective is to solve the equation for  $p$ .

Collect all the terms containing  $p$  on one side of the equality and the remaining terms on the other side.

So, add 7 to both sides of the equation and rewrite the equation as follows;

$$4p - 7 = 5$$

$$4p - 7 + 7 = 5 + 7$$

Combine the like terms and simplify;

$$4p - 7 + 7 = 5 + 7$$

$$4p = 12$$

Use the property of equality for division.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } \frac{a}{c} = \frac{b}{c}; c \neq 0$$

So, divide both sides of the equation by 4 and simplify the equation as follows;

$$4p = 12$$

$$\frac{4p}{4} = \frac{12}{4}$$

$$p = 3$$

**Check.**

Substitute 3 for  $p$  on the left side of the equation  $4p - 7 = 5$ ;

$$\begin{aligned} 4p - 7 &= 4(3) - 7 \\ &= 12 - 7 \\ &= 5 \end{aligned}$$

The result is equal to the right hand side.

Hence, the value of  $p = 3$  satisfies the equation.

Therefore, the solution of the equation  $4p - 7 = 5$  is  $\boxed{p = 3}$ .

### Answer 28E.

Write the following equation;

$$6 = 4v + 2$$

The objective is to solve the equation for  $v$ .

Collect all the terms containing  $v$  on one side of the equality and the remaining terms on the other side.

So, subtract 2 from both sides of the equation and rewrite the equation as follows;

$$6 = 4v + 2$$

$$6 - 2 = 4v + 2 - 2$$

Combine the like terms and simplify;

$$6 - 2 = 4v + 2 - 2$$

$$4 = 4v$$

Use the property of equality for division.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } \frac{a}{c} = \frac{b}{c}; c \neq 0$$

So, divide both sides of the equation by 4 and simplify the equation as follows;

$$4 = 4v$$

$$\frac{4}{4} = \frac{4v}{4}$$

$$1 = v$$

Use the symmetric property of real numbers which states that for any two real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } b = a$$

Hence,  $v = 1$ .

#### Check.

Substitute 1 for  $v$  on the right side of the equation  $6 = 4v + 2$ ;

$$4v + 2 = 4(1) + 2$$

$$= 4 + 2$$

$$= 6$$

The result is equal to the left hand side.

Hence, the value  $v = 1$  satisfies the equation.

Therefore, the solution of the equation  $6 = 4v + 2$  is  $\boxed{v = 1}$ .

**Answer 29E.**

Write the following equation;

$$\frac{y}{3} + 6 = -45$$

The objective is to solve the equation for  $y$ .

Collect all the terms containing  $y$  on one side of the equality and the remaining terms on the other side.

So, subtract 6 from both sides of the equation and rewrite the equation as follows;

$$\frac{y}{3} + 6 = -45$$

$$\frac{y}{3} + 6 - 6 = -45 - 6$$

Combine the like terms and simplify;

$$\frac{y}{3} + 6 - 6 = -45 - 6$$

$$\frac{y}{3} = -51$$

Use the property of equality for multiplication.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } ac = bc$$

So, multiply both sides of the equation by 3 and simplify the equation as follows;

$$3 \cdot \frac{y}{3} = 3 \cdot (-51) \text{ Cancel the common factors}$$

$$y = -153$$

**Check.**

Substitute  $-153$  for  $y$  on the left side of the equation  $\frac{y}{3} + 6 = -45$ ;

$$\begin{aligned} \frac{y}{3} + 6 &= \frac{-153}{3} + 6 \\ &= -51 + 6 \\ &= -45 \end{aligned}$$

The result is equal to the right hand side.

Hence, the value  $y = -153$  satisfies the equation.

Therefore, the solution of the equation  $\frac{y}{3} + 6 = -45$  is  $\boxed{y = -153}$ .

### Answer 30E.

Write the following equation;

$$\frac{c}{-4} - 8 = -42$$

The objective is to solve the equation for  $c$ .

Collect all the terms containing  $c$  on one side of the equality and the remaining terms on the other side.

So, add 8 to both sides of the equation and rewrite the equation as follows;

$$\frac{c}{-4} - 8 = -42$$

$$\frac{c}{-4} - 8 + 8 = -42 + 8$$

Combine the like terms and simplify;

$$\frac{c}{-4} - 8 + 8 = -42 + 8$$

$$\frac{c}{-4} = -34$$

Use the property of equality for multiplication.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } ac = bc$$

So, multiply both sides of the equation by  $-4$  and simplify the equation as follows;

$$(-4) \cdot \frac{c}{-4} = (-4) \cdot (-34) \text{ Cancel the common factors}$$

$$c = 136 \text{ Product of two negative numbers is positive}$$

#### Check.

Substitute 136 for  $c$  on the left side of the equation  $\frac{c}{-4} - 8 = -42$ ;

$$\begin{aligned} \frac{c}{-4} - 8 &= \frac{136}{-4} - 8 \\ &= -34 - 8 \\ &= -42 \end{aligned}$$

The result is equal to the right hand side.

Hence, the value  $c = 136$  satisfies the equation.

Therefore, the solution of the equation  $\frac{c}{-4} - 8 = -42$  is  $\boxed{c = 136}$ .

### Answer 31E.

Write the following equation;

$$\frac{4d+5}{7} = 7$$

The objective is to solve the equation for  $d$ .

Collect all the terms containing  $d$  on one side of the equality and the remaining terms on the other side.

Use the property of equality for multiplication.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } ac = bc$$

So, multiply both sides of the equation by  $7$  and simplify the equation as follows;

$$7 \cdot \frac{4d+5}{7} = 7 \cdot 7$$

$$4d+5 = 49 \text{ Cancel the common factors}$$

Subtract 5 from both sides of the equation and rewrite the equation as follows;

$$4d+5 = 49$$

$$4d+5-5 = 49-5$$

Combine the like terms and simplify;

$$4d+5-5 = 49-5$$

$$4d = 44$$

Use the property of equality for division.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } \frac{a}{c} = \frac{b}{c}; c \neq 0$$

So, divide both sides of the equation by 4 and simplify the equation as follows;

$$4d = 44$$

$$\frac{4d}{4} = \frac{44}{4}$$

$$d = 11$$

**Check.**

Substitute 11 for  $d$  on the left side of the equation  $\frac{4d+5}{7} = 7$  and simplify;

$$\begin{aligned}\frac{4d+5}{7} &= \frac{4(11)+5}{7} \\ &= \frac{44+5}{7} \\ &= \frac{49}{7} \\ &= 7\end{aligned}$$

The result is equal to the right hand side.

Hence, the value  $d = 11$  satisfies the equation.

Therefore, the solution of the equation  $\frac{4d+5}{7} = 7$  is  $\boxed{d = 11}$ .

**Answer 32E.**

Write the following equation;

$$\frac{7n+(-1)}{8} = 8$$

The objective is to solve the equation for  $n$ .

Collect all the terms containing  $n$  on one side of the equality and the remaining terms on the other side.

Use the property of equality for multiplication.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } ac = bc$$

So, multiply both sides of the equation by 8 and simplify the equation as follows;

$$8 \cdot \frac{7n+(-1)}{8} = 8 \cdot 8$$

$$7n+(-1) = 64 \text{ Cancel the common factors}$$

To add a negative number to a number is to subtract the additive inverse of the number from the first number.

The additive inverse of a number is the same number with opposite sign.

Hence, the additive inverse of  $-1$  is  $1$ .

So, rewrite the equation using the additive inverse as follows;

$$7n + (-1) = 64$$

$$7n - 1 = 64$$

Add 1 to both sides of the equation and rewrite the equation as follows;

$$7n - 1 = 64$$

$$7n - 1 + 1 = 64 + 1$$

Combine the like terms and simplify;

$$7n - 1 + 1 = 64 + 1$$

$$7n = 65$$

Use the property of equality for division.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } \frac{a}{c} = \frac{b}{c}; c \neq 0$$

So, divide both sides of the equation by 7 and simplify the equation as follows;

$$7n = 65$$

$$\frac{7n}{7} = \frac{65}{7}$$

$$n = \frac{65}{7}$$

**Check.**

Substitute  $\frac{65}{7}$  for  $n$  on the left side of the equation  $\frac{7n + (-1)}{8} = 8$  and simplify;

$$\begin{aligned} \frac{7n + (-1)}{8} &= \frac{7\left(\frac{65}{7}\right) + (-1)}{8} \\ &= \frac{65 - 1}{8} \\ &= \frac{64}{8} \\ &= 8 \end{aligned}$$

The result is equal to the right hand side.

Hence, the value  $n = \frac{65}{7}$  satisfies the equation.

Therefore, the solution of the equation  $\frac{7n + (-1)}{8} = 8$  is  $n = \frac{65}{7}$ .

**Answer 33E.**

Write the following equation;

$$n - 2 = 4 - 2n$$

The objective is to solve the equation for  $n$ .

Collect all the terms containing  $n$  on one side of the equality and the remaining terms on the other side.

So, add  $2n$  to both sides of the equation and rewrite the equation as follows;

$$n - 2 = 4 - 2n$$

$$n - 2 + 2n = 4 - 2n + 2n$$

Combine the like terms and simplify;

$$n - 2 + 2n = 4 - 2n + 2n$$

$$3n - 2 = 4$$

Add 2 to both sides of the equation and simplify;

$$3n - 2 = 4$$

$$3n - 2 + 2 = 4 + 2 \text{ Combine the like terms}$$

$$3n = 6$$

Use the property of equality for division.

The property states that for any real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } \frac{a}{c} = \frac{b}{c}; c \neq 0$$

So, divide both sides of the equation by 3 and simplify the equation as follows;

$$3n = 6$$

$$\frac{3n}{3} = \frac{6}{3}$$

$$n = 2$$

**Check.**

Substitute 2 for  $n$  on the left side of the equation  $n - 2 = 4 - 2n$  and simplify;

$$\begin{aligned}n - 2 &= 2 - 2 \\ &= 0\end{aligned}$$

Substitute 2 for  $n$  on the right side of the equation  $n - 2 = 4 - 2n$  and simplify;

$$\begin{aligned}4 - 2n &= 4 - 2(2) \\ &= 4 - 4 \\ &= 0\end{aligned}$$

The result on the left hand side and the right hand side are equal.

Hence, the value  $n = 2$  satisfies the equation.

Therefore, the solution of the equation  $n - 2 = 4 - 2n$  is  $\boxed{n = 2}$ .

**Answer 34E.**

Write the following equation;

$$3t - 2(t + 3) = t$$

The objective is to solve the equation for  $t$ .

Use the distributive property to open the parenthesis.

The property states that for any real numbers  $a$ ,  $b$  and  $c$ ;

$$a \cdot (b \pm c) = a \cdot b \pm a \cdot c$$

So, write the equation as follows;

$$3t - 2(t + 3) = t$$

$$3t - 2 \cdot t - 2 \cdot 3 = t$$

$$3t - 2t - 6 = t \text{ Multiply}$$

$$t - 6 = t \text{ Combine the like terms}$$

Collect all the terms containing  $t$  on one side of the equality and the remaining terms on the other side.

Subtract  $t$  from both sides of the equation and rewrite the equation as follows;

$$t - 6 = t$$

$$t - 6 - t = t - t$$

Collect and combine the like terms and simplify;

$$-6 = 0$$

The result is absurd.

Therefore, the equation  $3t - 2(t + 3) = t$  has  $\boxed{\text{no solution}}$ .

### Answer 35E.

Write the following equation;

$$3 - \frac{5}{6}y = 2 + \frac{1}{6}y$$

The objective is to solve the equation for  $y$ .

Collect all the terms containing  $y$  on one side of the equality and the remaining terms on the other side.

So, add  $\frac{5}{6}y$  to both sides of the equation and rewrite the equation as follows;

$$3 - \frac{5}{6}y = 2 + \frac{1}{6}y$$

$$3 - \frac{5}{6}y + \frac{5}{6}y = 2 + \frac{1}{6}y + \frac{5}{6}y$$

Combine the like terms and simplify;

$$3 - \frac{5}{6}y + \frac{5}{6}y = 2 + \frac{1}{6}y + \frac{5}{6}y$$

$$3 = 2 + \frac{1+5}{6}y$$

$$3 = 2 + \frac{6}{6}y$$

$$3 = 2 + y$$

Subtract 2 from both sides of the equation and simplify;

$$3 = 2 + y$$

$$3 - 2 = 2 + y - 2 \quad \text{Combine the like terms}$$

$$1 = y$$

Use the symmetric property of real numbers which states that for any two real numbers  $a$  and  $b$ ;

If  $a = b$  then  $b = a$

Hence,  $y = 1$ .

**Check.**

Substitute 1 for  $y$  on the left side of the equation  $3 - \frac{5}{6}y = 2 + \frac{1}{6}y$  and simplify;

$$3 - \frac{5}{6}y = 3 - \frac{5}{6}(1)$$

$$= 3 - \frac{5}{6}$$

$$3 - \frac{5}{6}y = \frac{18-5}{6} \quad \text{Rationalize the denominator}$$

$$3 - \frac{5}{6}y = \frac{13}{6}$$

Substitute 1 for  $y$  on the right side of the equation  $3 - \frac{5}{6}y = 2 + \frac{1}{6}y$  and simplify;

$$\begin{aligned}2 + \frac{1}{6}y &= 2 + \frac{1}{6}(1) \\ &= 2 + \frac{1}{6}\end{aligned}$$

$$2 + \frac{1}{6}y = \frac{12+1}{6} \text{ Rationalize the denominator}$$

$$2 + \frac{1}{6}y = \frac{13}{6}$$

The result on the left hand side and the right hand side are equal.

Hence, the value  $y = 1$  satisfies the equation.

Therefore, the solution of the equation  $3 - \frac{5}{6}y = 2 + \frac{1}{6}y$  is  $\boxed{y = 1}$ .

### Answer 36E.

Write the following equation;

$$\frac{x-2}{6} = \frac{x}{2}$$

The objective is to solve the equation for  $x$ .

Collect all the terms containing  $x$  on one side of the equality and the remaining terms on the other side.

First, cross multiply the terms and rewrite the equation as follows;

$$\begin{aligned}\frac{x-2}{6} &= \frac{x}{2} \\ 2 \cdot (x-2) &= 6 \cdot x\end{aligned}$$

Use the distributive property to open the parenthesis.

The property states that for any real numbers  $a$ ,  $b$  and  $c$ ;

$$a \cdot (b \pm c) = a \cdot b \pm a \cdot c$$

So, write the equation as follows;

$$\begin{aligned}2 \cdot (x-2) &= 6 \cdot x \\ 2 \cdot x - 2 \cdot 2 &= 6 \cdot x \\ 2x - 4 &= 6x \text{ Multiply}\end{aligned}$$

Subtract  $2x$  from both sides of the equation and rewrite the equation as follows;

$$2x - 4 = 6x$$

$$2x - 4 - 2x = 6x - 2x$$

Combine the like terms and simplify;

$$2x - 4 - 2x = 6x - 2x$$

$$-4 = 4x$$

Divide both sides of the equation by 4;

$$\frac{-4}{4} = \frac{4x}{4} \text{ Cancel the common factors}$$

$$-1 = x$$

Use the symmetric property of real numbers which states that for any two real numbers  $a$  and  $b$ ;

If  $a = b$  then  $b = a$

Hence,  $x = -1$ .

### Check.

Substitute  $-1$  for  $x$  on the left side of the equation  $\frac{x-2}{6} = \frac{x}{2}$  and simplify;

$$\frac{x-2}{6} = \frac{-1-2}{6}$$

$$\frac{x-2}{6} = \frac{-3}{6} \text{ Cancel the common factors}$$

$$\frac{x-2}{6} = \frac{-1}{2}$$

$$= -\frac{1}{2}$$

**Answer 37E.**

Write the following equation;

$$2(b-3) = 3(b-1)$$

The objective is to solve the equation for  $b$ .

Use the distributive property to open the parenthesis.

The property states that for any real numbers  $a$ ,  $b$  and  $c$ ;

$$a \cdot (b \pm c) = a \cdot b \pm a \cdot c$$

So, write the equation as follows;

$$2(b-3) = 3(b-1)$$

$$2 \cdot b - 2 \cdot 3 = 3 \cdot b - 3 \cdot 1$$

$$2b - 6 = 3b - 3 \quad \text{Multiply}$$

Collect all the terms containing  $b$  on one side of the equality and the remaining terms on the other side.

Subtract  $2b$  from both sides of the equation and rewrite the equation as follows;

$$2b - 6 = 3b - 3$$

$$2b - 6 - 2b = 3b - 3 - 2b$$

Combine the like terms and simplify;

$$2b - 6 - 2b = 3b - 3 - 2b$$

$$-6 = b - 3$$

Add 3 to both sides of the equation;

$$-6 = b - 3$$

$$-6 + 3 = b - 3 + 3 \quad \text{Combine the like terms}$$

$$-3 = b$$

Use the symmetric property of real numbers which states that for any two real numbers  $a$  and  $b$ ;

$$\text{If } a = b \text{ then } b = a$$

$$\text{Hence, } b = -3.$$

**Check.**

Substitute  $-3$  for  $b$  on the left side of the equation  $2(b-3) = 3(b-1)$  and simplify;

$$\begin{aligned}2(b-3) &= 2(-3-3) \\ &= 2(-6) \\ &= -12\end{aligned}$$

Substitute  $-3$  for  $b$  on the right side of the equation  $2(b-3) = 3(b-1)$  and simplify;

$$\begin{aligned}3(b-1) &= 3(-3-1) \\ &= 3(-4) \\ &= -12\end{aligned}$$

The result on the left hand side and the right hand side are equal.

Hence, the value  $b = -3$  satisfies the equation.

Therefore, the solution of the equation  $2(b-3) = 3(b-1)$  is  $\boxed{b = -3}$ .

**Answer 38E.**

Write the following equation;

$$8.3h - 2.2 = 6.1h - 8.8$$

The objective is to solve the equation for  $h$ .

Collect all the terms containing  $h$  on one side of the equality and the remaining terms on the other side.

So, subtract  $6.1h$  from both sides of the equation and rewrite the equation as follows;

$$8.3h - 2.2 = 6.1h - 8.8$$

$$8.3h - 2.2 - 6.1h = 6.1h - 8.8 - 6.1h$$

Collect and combine the like terms and simplify;

$$8.3h - 2.2 - 6.1h = 6.1h - 8.8 - 6.1h$$

$$8.3h - 6.1h - 2.2 = 6.1h - 6.1h - 8.8$$

$$2.2h - 2.2 = -8.8$$

Add 2.2 to both sides of the equation;

$$2.2h - 2.2 = -8.8$$

$$2.2h - 2.2 + 2.2 = -8.8 + 2.2$$

Combine the like terms;

$$2.2h - 2.2 + 2.2 = -8.8 + 2.2$$

$$2.2h = -6.6$$

Divide both sides of the equation by 2.2;

$$\frac{2.2h}{2.2} = \frac{-6.6}{2.2} \text{ Cancel the common factors}$$

$$h = -3$$

**Check.**

Substitute  $-3$  for  $h$  on the left side of the equation  $8.3h - 2.2 = 6.1h - 8.8$  and simplify;

$$\begin{aligned}8.3h - 2.2 &= 8.3(-3) - 2.2 \\ &= -24.9 - 2.2 \\ &= -27.1\end{aligned}$$

Substitute  $-3$  for  $h$  on the right side of the equation  $8.3h - 2.2 = 6.1h - 8.8$  and simplify;

$$\begin{aligned}6.1h - 8.8 &= 6.1(-3) - 8.8 \\ &= -18.3 - 8.8 \\ &= -27.1\end{aligned}$$

The result on the left hand side and the right hand side are equal.

Hence, the value  $h = -3$  satisfies the equation.

Therefore, the solution of the equation  $8.3h - 2.2 = 6.1h - 8.8$  is  $\boxed{h = -3}$ .

**Answer 39E.**

Write the following proportion;

$$\frac{6}{15} = \frac{n}{45}$$

The objective is to solve the proportion for  $n$ .

Find the cross product and rewrite the equality as follows;

$$\frac{6}{15} = \frac{n}{45}$$

$$6 \cdot 45 = n \cdot 15 \text{ Multiply}$$

$$270 = 15n$$

Divide both sides of the equation by 15;

$$270 = 15n$$

$$\frac{270}{15} = \frac{15n}{15} \text{ Cancel the common factors}$$

$$18 = n$$

Use the symmetric property of real numbers which states that for any two real numbers  $a$  and  $b$ ;

If  $a = b$  then  $b = a$

Hence,  $n = 18$ .

Therefore, the value of the variable in the proportion is  $\boxed{n = 18}$ .

**Answer 40E.**

Write the following proportion;

$$\frac{x}{11} = \frac{35}{55}$$

The objective is to solve the proportion for  $x$ .

Find the cross product and rewrite the equality as follows;

$$\frac{x}{11} = \frac{35}{55}$$

$$x \cdot 55 = 35 \cdot 11$$

$$55x = 385 \text{ Multiply}$$

Divide both sides of the equation by 55;

$$55x = 385$$

$$\frac{55x}{55} = \frac{385}{55} \text{ Cancel the common factors}$$

$$x = 7$$

Therefore, the value of the variable in the proportion is  $\boxed{x = 7}$ .

**Answer 41E.**

Write the following proportion;

$$\frac{12}{d} = \frac{20}{15}$$

The objective is to solve the proportion for  $d$ .

Find the cross product and rewrite the equality as follows;

$$\frac{12}{d} = \frac{20}{15}$$

$$12 \cdot 15 = 20 \cdot d \text{ Multiply}$$

$$180 = 20d$$

Divide both sides of the equation by 20;

$$180 = 20d$$

$$\frac{180}{20} = \frac{20d}{20} \text{ Cancel the common factors}$$

$$9 = d$$

Use the symmetric property of real numbers which states that for any two real numbers  $a$  and  $b$ ;

If  $a = b$  then  $b = a$

Hence,  $d = 9$ .

Therefore, the value of the variable in the proportion is  $\boxed{d = 9}$ .

**Answer 42E.**

Write the following proportion;

$$\frac{14}{20} = \frac{21}{m}$$

The objective is to solve the proportion for  $m$ .

Find the cross product and rewrite the equality as follows;

$$\frac{14}{20} = \frac{21}{m}$$

$$14 \cdot m = 21 \cdot 20$$

$$14m = 420 \text{ Multiply}$$

Divide both sides of the equation by 14;

$$14m = 420$$

$$\frac{14m}{14} = \frac{420}{14} \text{ Cancel the common factors}$$

$$m = 30$$

Therefore, the value of the variable in the proportion is  $m = 30$ .

**Answer 43E.**

Write the following proportion;

$$\frac{2}{3} = \frac{b+5}{9}$$

The objective is to solve the proportion for  $b$ .

Find the cross product and rewrite the equality as follows;

$$\frac{2}{3} = \frac{b+5}{9}$$

$$2 \cdot 9 = 3 \cdot (b+5)$$

Use the distributive property to open the parenthesis.

The property states that for any real numbers  $a$ ,  $b$  and  $c$ ;

$$a \cdot (b \pm c) = a \cdot b \pm a \cdot c$$

So, write the equation as follows;

$$2 \cdot 9 = 3 \cdot (b+5)$$

$$2 \cdot 9 = 3 \cdot b + 3 \cdot 5$$

$$18 = 3b + 15 \text{ Multiply}$$

Subtract 15 from both sides of the equation;

$$18 = 3b + 15$$

$$18 - 15 = 3b + 15 - 15 \text{ Combine the like terms}$$

$$3 = 3b$$

Divide both sides of the equation by 3;

$$3 = 3b$$

$$\frac{3}{3} = \frac{3b}{3} \text{ Cancel the common factors}$$

$$1 = b$$

Use the symmetric property of real numbers which states that for any two real numbers  $a$  and  $b$ ;

If  $a = b$  then  $b = a$

Hence,  $b = 1$ .

Therefore, the value of the variable in the proportion is  $\boxed{b=1}$ .

### Answer 44E.

Write the following proportion;

$$\frac{6}{8} = \frac{9}{s-4}$$

The objective is to solve the proportion for  $s$ .

Find the cross product and rewrite the equality as follows;

$$\frac{6}{8} = \frac{9}{s-4}$$
$$6 \cdot (s-4) = 8 \cdot 9$$

Use the distributive property to open the parenthesis.

The property states that for any real numbers  $a$ ,  $b$  and  $c$ ;

$$a \cdot (b \pm c) = a \cdot b \pm a \cdot c$$

So, write the equation as follows;

$$6 \cdot (s-4) = 8 \cdot 9$$

$$6 \cdot s - 6 \cdot 4 = 8 \cdot 9$$

$$6s - 24 = 72 \text{ Multiply}$$

Add 24 to both sides of the equation;

$$6s - 24 = 72$$

$$6s - 24 + 24 = 72 + 24 \text{ Combine the like terms}$$

$$6s = 96$$

Divide both sides of the equation by 6;

$$6s = 96$$

$$\frac{6s}{6} = \frac{96}{6} \text{ Cancel the common factors}$$

$$s = 16$$

Therefore, the value of the variable in the proportion is  $s = 16$ .

### Answer 45E.

The original value of the product is 40.

The new value of the product is 32.

The objective is to find the percent of change.

If the new value is greater than the old value then this means that there is an increase in the old value of the quantity. Hence the change in percent is called as the percent of increase.

If the new value is less than the old value then this means that there is a decrease in the original value. Hence the change in percent is called the percent of decrease.

Observe that the new value of the product, 32, is less than the original value, 40.

Hence, the percent of change is a  $\text{percent decrease}$ .

Use the following proportion to find the percent of change;

$$\frac{\text{change in value}}{\text{original value}} = \frac{r}{100}$$

Here  $r$  is the percent of change.

The new value is less than the original value. Hence, find the change in value as follows;

Change in value = Original value – new value.

So, calculate;

$$\begin{aligned} \text{change in value} &= 40 - 32 \\ &= 8 \end{aligned}$$

Substitute 8 for change in value and 40 for original value in the proportion mentioned above and rewrite the proportion as follows;

$$\begin{aligned} \frac{\text{change in value}}{\text{original value}} &= \frac{r}{100} \\ \frac{8}{40} &= \frac{r}{100} \end{aligned}$$

Cross multiply the terms and form the following equation;

$$\frac{8}{40} = \frac{r}{100}$$

$$8 \cdot 100 = r \cdot 40$$

$$800 = 40r$$

$$40r = 800$$

Divide both sides of the equation by 40;

$$\frac{40r}{40} = \frac{800}{40}$$

$$r = 20$$

The percent of change is 20%.

Therefore, the percent of decrease in the original value is 20%.

### Answer 46E.

The original value of the product is 50.

The new value of the product is 88.

The objective is to find the percent of change.

If the new value is greater than the old value then this means that there is an increase in the old value of the quantity. Hence the change in percent is called as the percent of increase.

If the new value is less than the old value then this means that there is a decrease in the original value. Hence the change in percent is called the percent of decrease.

Observe that the new value of the product, 88, is greater than the original value, 50.

Hence, the percent of change is a percent increase.

Use the following proportion to find the percent of change;

$$\frac{\text{change in value}}{\text{original value}} = \frac{r}{100}$$

Here  $r$  is the percent of change.

The new value is greater than the original value. Hence, find the change in value as follows;

Change in value = new value – original value.

So, calculate;

$$\begin{aligned} \text{change in value} &= 88 - 50 \\ &= 38 \end{aligned}$$

Substitute 38 for change in value and 50 for original value in the proportion mentioned above and rewrite the proportion as follows;

$$\begin{aligned} \frac{\text{change in value}}{\text{original value}} &= \frac{r}{100} \\ \frac{38}{50} &= \frac{r}{100} \end{aligned}$$

Cross multiply the terms and form the following equation;

$$\frac{38}{50} = \frac{r}{100}$$

$$38 \cdot 100 = r \cdot 50$$

$$3800 = 50r$$

$$50r = 3800$$

Divide both sides of the equation by 50;

$$\frac{50r}{50} = \frac{3800}{50}$$

$$r = 76$$

The percent of change is 76%.

Therefore, the percent of increase in the original value is 76%.

**Answer 47E.**

The original value of the product is 35.

The new value of the product is 37.1.

The objective is to find the percent of change.

If the new value is greater than the old value then this means that there is an increase in the old value of the quantity. Hence the change in percent is called as the percent of increase.

If the new value is less than the old value then this means that there is a decrease in the original value. Hence the change in percent is called the percent of decrease.

Observe that the new value of the product, 37.1, is greater than the original value, 35.

Hence, the percent of change is a percent increase.

Use the following proportion to find the percent of change;

$$\frac{\text{change in value}}{\text{original value}} = \frac{r}{100}$$

Here  $r$  is the percent of change.

The new value is greater than the original value. Hence, find the change in value as follows;

$$\text{Change in value} = \text{new value} - \text{original value.}$$

So, calculate;

$$\begin{aligned} \text{change in value} &= 37.1 - 35 \\ &= 2.1 \end{aligned}$$

Substitute 2.1 for change in value and 35 for original value in the proportion mentioned above and rewrite the proportion as follows;

$$\begin{aligned} \frac{\text{change in value}}{\text{original value}} &= \frac{r}{100} \\ \frac{2.1}{35} &= \frac{r}{100} \end{aligned}$$

Cross multiply the terms and form the following equation;

$$\frac{2.1}{35} = \frac{r}{100}$$

$$(2.1) \cdot 100 = r \cdot 35$$

$$210 = 35r$$

$$35r = 210$$

Divide both sides of the equation by 35;

$$\frac{35r}{35} = \frac{210}{35}$$

$$r = 6$$

The percent of change is 6%.

Therefore, the percent of increase in the original value is  $\boxed{6\%}$ .

### **Answer 48E.**

The price of the book is \$14.95

The sales tax on the book price is 6.25%.

The objective is to find the total price of the book.

The total price of the book is original price with the tax added to it.

So find the tax to be added.

The tax is 6.25% of the original price of the book.

So calculate 6.25% of \$14.95.

Do the following calculation for tax;

$$\begin{aligned} 6.25\% \text{ of } 14.95 &= \frac{6.25}{100} \times 14.95 \\ &= 0.934375 \end{aligned}$$

Round up the number to two decimal places.

Thus, the sales tax on the original price of the book is \$0.93

Add the sales tax to the original price of the book.

Total price of the book equals original price plus sales tax.

So, calculate the total price as follows;

$$\begin{aligned} \text{Total price} &= 14.95 + 0.93 \\ &= 15.88 \end{aligned}$$

Therefore, the total price of the book is  $\boxed{\$15.88}$ .

**Answer 49E.**

The price of the T- shirt is \$12.99.

The discount on the original price is 20%.

The objective is to find the price of the T- shirt after discount.

The price of the T- shirt after discount is the original price minus the discount price.

So find the discount to be subtracted.

The discount is 20% of the original price of the T- shirt.

So calculate 20% of \$12.99.

Do the following calculation for discount;

$$\begin{aligned} 20\% \text{ of } 12.99 &= \frac{20}{100} \times 12.99 \\ &= 2.598 \end{aligned}$$

Round up the number to two decimal places.

Thus, the discount on the original price of the T- shirt is \$2.60

Subtract the discount price from the original price of the T- shirt.

The new price equals the original price minus the discount.

So, calculate the new price as follows;

$$\begin{aligned} \text{New price} &= 12.99 - 2.60 \\ &= 10.39 \end{aligned}$$

Therefore, the price of the T- shirt after discount is  $\boxed{\$10.39}$ .

**Answer 50E.**

Write the following original equation;

$$5x = y$$

The objective is to solve the equation for the variable  $x$ .

So, collect the terms containing the variable  $x$  on one side of the equation.

Divide both sides of the equation by 5;

$$5x = y$$

$$\frac{5x}{5} = \frac{y}{5}$$

$$x = \frac{y}{5} \text{ Cancel the common factors}$$

Therefore, the solution of the equation in terms of the variable  $x$  is  $\boxed{x = \frac{y}{5}}$ .

**Answer 51E.**

Write the following original equation;

$$ay - b = c$$

The objective is to solve the equation for the variable  $y$ .

So, collect the terms containing the variable  $y$  on one side of the equation.

Add  $b$  on both sides of the equation;

$$ay - b = c$$

$$ay - b + b = c + b \text{ Combine the like terms}$$

$$ay = c + b$$

Divide both sides of the equation by  $a$ ;

$$ay = c + b$$

$$\frac{ay}{a} = \frac{c+b}{a} \text{ Cancel the common factors}$$

$$y = \frac{c+b}{a}$$

Therefore, the solution of the equation in terms of the variable  $y$  is  $\boxed{y = \frac{c+b}{a}}$ .

**Answer 52E.**

Write the following original equation;

$$yx - a = cx$$

The objective is to solve the equation for the variable  $x$ .

So, collect the terms containing the variable  $x$  on one side of the equation.

Subtract  $cx$  from both sides of the equation and rewrite the equation as follows;

$$yx - a = cx$$

$$yx - a - cx = cx - cx$$

$$yx - cx - a = cx - cx \text{ Collect the like terms}$$

$$(y - c)x - a = 0 \text{ Combine the like terms}$$

Add  $a$  to both sides of the equation;

$$(y - c)x - a = 0$$

$$(y - c)x - a + a = 0 + a$$

$$(y - c)x = a \text{ Combine the like terms}$$

Divide both sides of the equation by  $y - c$ ;

$$(y - c)x = a$$

$$\frac{(y - c)x}{y - c} = \frac{a}{y - c}$$

$$x = \frac{a}{y - c} \text{ Cancel the common factors}$$

Therefore, the solution of the equation in terms of the variable  $x$  is  $x = \frac{a}{y - c}$ .

### Answer 53E.

Write the following original equation;

$$\frac{2y - a}{3} = \frac{a + 3b}{4}$$

The objective is to solve the equation for the variable  $y$ .

Find the cross product of the terms and rewrite the equality as follows;

$$\frac{2y - a}{3} = \frac{a + 3b}{4}$$

$$4 \cdot (2y - a) = 3 \cdot (a + 3b)$$

Use the distributive property to open the parenthesis.

The property states that for any real numbers  $a$ ,  $b$  and  $c$ ;

$$a \cdot (b \pm c) = a \cdot b \pm a \cdot c$$

So, write the equation as follows;

$$4 \cdot (2y - a) = 3 \cdot (a + 3b)$$

$$4 \cdot 2y - 4 \cdot a = 3 \cdot a + 3 \cdot 3b$$

$$8y - 4a = 3a + 9b \text{ Multiply}$$

Collect the terms containing the variable  $y$  on one side of the equation.

Add  $4a$  to both sides of the equation and rewrite the equation as follows;

$$8y - 4a = 3a + 9b$$

$$8y - 4a + 4a = 3a + 9b + 4a$$

$$8y - 4a + 4a = 3a + 4a + 9b \text{ Collect the like terms}$$

$$8y = 7a + 9b \text{ Combine the like terms}$$

Divide both sides of the equation by  $8$ ;

$$8y = 7a + 9b$$

$$\frac{8y}{8} = \frac{7a + 9b}{8}$$

$$y = \frac{7a + 9b}{8} \text{ Cancel the common factors}$$

Therefore, the solution of the equation in terms of the variable  $y$  is  $y = \frac{7a + 9b}{8}$ .

## Answer 54E.

Analyze the problem.

Ms. A sells type I coffee at a price of \$8.40 per pound.

The type II coffee is sold at a price of \$7.28 per pound.

Type II coffee is mixed with 9 pounds of type I coffee to form a mixture that is sold at a price of \$7.95 per pound.

The objective is to find the amount of type II coffee in the mixture.

Suppose the amount of type II coffee in the mixture is  $x$  pounds.

The price of the type II coffee is \$7.28 per pound.

Hence, the total sale price of the type II coffee is  $7.28x$ .

The amount of type I coffee in the mixture is 9 pounds.

The price of the type I coffee is \$8.40 per pound.

Hence, the total sale price of the type I coffee is  $8.40 \cdot (9) = \$75.60$ .

The amount of coffee in the mixture is  $(x + 9)$  pounds.

The mixture is sold at \$7.95 per pound.

Hence, the total sale price of the mixture is  $7.95 \cdot (x + 9)$

Use the distributive property to open the parentheses.

The distributive states that for any real numbers  $a$ ,  $b$  and  $c$ ;

$$a(b \pm c) = a \cdot b \pm a \cdot c$$

So, rewrite the equation as follows;

$$75.60 + 7.28x = 7.95(x + 9)$$

$$75.60 + 7.28x = (7.95) \cdot x + (7.95) \cdot 9$$

$$75.60 + 7.28x = 7.95x + 71.55$$

Subtract 75.60 from both sides of the equation;

$$75.60 + 7.28x = 7.95x + 71.55$$

$$75.60 + 7.28x - 75.60 = 7.95x + 71.55 - 75.60$$

$$75.60 - 75.60 + 7.28x = 7.95x + 71.55 - 75.60 \quad \text{Collect the like terms}$$

$$7.28x = 7.95x - 4.05 \quad \text{Combine the like terms}$$

Subtract  $7.95x$  from both sides of the equation;

$$7.28x = 7.95x - 4.05$$

$$7.28x - 7.95x = 7.95x - 4.05 - 7.95x$$

$$7.28x - 7.95x = 7.95x - 7.95x - 4.05 \quad \text{Collect the like terms}$$

$$-0.67x = -4.05 \quad \text{Combine the like terms}$$

Divide both sides of the equation by  $-0.67$ ;

$$-0.67x = -4.05$$

$$\frac{-0.67x}{-0.67} = \frac{-4.05}{-0.67}$$

$$x \approx 6.04 \quad \text{Cancel the common factors}$$

Round up the number to the nearest whole number.

$$\text{So, } x = 6$$

Therefore, the amount of \$7.28 coffee in the mixture is about 6 pounds.

### Answer 55E.

Analyze the problem.

Two airplanes leave the same airport at the same time, both flying in opposite directions.

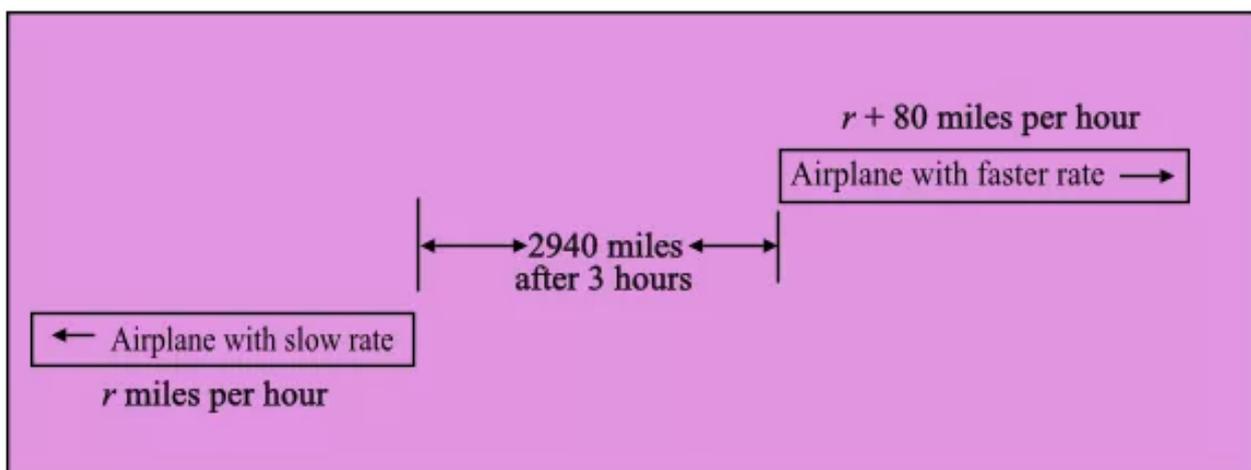
One of the planes travels faster than the other by 80 miles per hour.

Suppose the rate of the plane that is slower is  $r$  miles per hour.

Hence, the rate of the faster plane is  $r + 80$  miles per hour.

The distance between the planes after 3 hours is 2940 miles.

Represent the information with the help of the following diagram;



If  $d$  is the distance,  $r$  is the rate at which a vehicle travels and  $t$  is the time, the relationship between distance, speed (rate) and time is as follows;

$$d = rt$$

Hence, the distance  $d_s$  travelled by the slower plane at the rate of  $r$  miles per hour in time 3 hours is;

$$\begin{aligned}d_s &= r \cdot 3 \\ &= 3r\end{aligned}$$

Similarly, the distance  $d_f$  travelled by the faster plane at the rate of  $(r + 80)$  miles per hour in time 3 hours is;

$$\begin{aligned}d_f &= (r + 80) \cdot 3 \\ &= 3(r + 80)\end{aligned}$$

Write the processed information in the following table;

Airplane	$r$	( $t$ ) time in hours	$d = rt$
Slower	$r$	3	$3r$
Faster	$r + 80$	3	$3(r + 80)$

The planes flying in the opposite directions are 2940 miles apart after 3 hours.

Hence, the distance travelled by the slower plane the distance travelled by the faster plane equals the distance between the planes after 3 hours.

From the table, the distance between the planes after 3 hours is;

$$3r + 3(r + 80)$$

After 3 hours the planes are 2940 miles apart.

So, write the equation as follows;

$$3r + 3(r + 80) = 2940$$

Solve the equation for  $r$ .

Use the distributive property to open the parenthesis.

The property states that for any real numbers  $a$ ,  $b$  and  $c$ ;

$$a \cdot (b \pm c) = a \cdot b \pm a \cdot c$$

So, write the equation as follows;

$$3r + 3(r + 80) = 2940$$

$$3r + 3 \cdot r + 3 \cdot 80 = 2940$$

$$3r + 3r + 240 = 2940 \text{ Multiply}$$

Combine the like terms and rewrite the equation as follows;

$$3r + 3r + 240 = 2940$$

$$6r + 240 = 2940$$

Subtract 240 from both sides of the equation;

$$6r + 240 = 2940$$

$$6r + 240 - 240 = 2940 - 240$$

$$6r = 2700 \text{ Combine the like terms}$$

Divide both sides of the equation by 6;

$$6r = 2700$$

$$\frac{6r}{6} = \frac{2700}{6}$$

$$r = 450 \text{ Cancel the common factors}$$

Thus, the speed of the slower plane is 450 miles per hour.

The speed of the faster plane is  $450 + 80 = 530$  miles per hour

Therefore, the speeds of the planes are 450 mph and 530 mph.