RATE MEASURE

- If x is any variable, dx / dt represents the rate of change of x at time 't'.
- If y = f(x) then dy/dx is the rate of change of y w.r.t. x.
- If S is the distance travelled by a particle in time t. The relation between s and t can be expressed as s = f(t).
- v = ds/dt is the rate of change of distance s and is called velocity.

  NOTE:

  (1) V = 0 => the particle moving on a straight line comes to rest and the distances becomes maximum and it changes its direction after V=0

  (2) V > 0 => s increases

  (3) V < 0 => s decreases
- The rate of change in velocity is called the acceleration of the particle at 't' and is denoted by a

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{ds}{dt}\right]$$

$$= \frac{d^2s}{dt^2} = \frac{dv}{ds}.\frac{ds}{dt} = v.\frac{dv}{ds}$$

  It is a vector . It is measured in units /Sec^2

  NOTE:

  (1) a=0 => velocity v becomes maximum

  (2) a>0 => v increases. S Min.

  (3) a<0 => v decreases. S Max
- A particle moving on a straight line comes to rest

$$If\ \frac{ds}{dt} = 0\ \&\ \frac{d^2s}{dt^2} = 0$$

- A particle moving on a straight line is at rest momentarily.

$$If\ \frac{ds}{dt} = 0\ \&\ \frac{d^2s}{dt^2} \neq 0$$

- A particle, projected vertically upwards, attains the maximum height when ds/dt = 0.
- **RETARDATION:** If the acceleration of a particle is negative, it is called Retardation.
- **ANGULAR VELOCITY AND ANGULAR ACCELERATION:** If P is any point which moves on a curve and θ is the angle made by OP with the positive direction of the initial line $\overline{OX}$, the angular velocity of p at O = $d\theta/dt$. It is denoted by ω.

  The angular acceleration of p at

$$O = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$$

- The equations of motion of a particle p(x,y) on a plane curve are given by x = f(t),

  y = g(t) then the velocity of the particle is given by $\frac{ds}{dt} = \sqrt{\left[f'(t)\right]^2 + \left[g'(t)\right]^2}$

LEVEL-1

1. If a particle moves along a straight line according to the equation s = t^3 - $9t^2$ + 24t, then the velocity of of the particle in 2 secs is

   1) 0 2) 18 3) 24 4) 36

2. If s = $2t^3$ - $3t^2$ + 15t - 8, then the initial velocity is

   1) 8 2) 10 3) 15 4) 36

3. If s = e^t (sint - cost), then the acceleration is

   1) $2e^t$ (Cost - sint) 2) $2e^t$ (cost + sint)
   3) e^t (cost + sint) 4) e^t(cost-sint)

4. If the displacement in time t of a particle is given by s = $ae^t + be^{-t}$, then the acceleration is equal to

   1) velocity 2) displacement
   3) initial velocity 4) ae^t

5. The velocity v of a particle moving along a straight line when it is at a distance X from the point of start is given by $a+bv^2=x^2$, then the acceleration is

   1) $\frac{X}{b}$ 2) $\frac{X}{b^2}$ 3) $\frac{b}{X}$ 4) $\frac{b^2}{X}$

6. A particle moves along a straight line according to the equation s = 8 cos2t + 4 sint. The initial velocity is

   1) -5 units/sec 2) -4units/sec
   3) 4 units/sec 4) 5 units/sec

7. A particle moves along a straight line such that s = $6t^2$ - $t^3/2$. then the acceleration after 3 seconds is

   1) 5 units/sec^2 2) 27 units/sec^2
   3) 18 units/sec^2 4) 3 units/sec^2

8. The motion of a particle along a straight line is given by $v^2 = u^2 + 90s$. If the particle starts from rest, then the acceleration is

1) 15 units/sec^2 2) 30 units/sec^2
3) 45 units/sec^2 4) 75 units/sec^2

9. A particle moves along the X-axis with velocity v = dx/dt = f(x) then the acceleration of the particle is

1) $f^{11}(x)$ 2) $\frac{dv}{dx}$
3) $f(x).f^{1}(x)$ 4) $f^{1}(x)$

10. A particle moves along a line by $s = \frac{1}{3}t^3 - 3t^2 + 8t + 5$, it changes its direction when

1) t = 1, t = 2 2) t = 2, t = 4
3) t = 0, t = 4 4) t = 2, t = 3

11. A particle moves along a line by $s = t^3 - 9t^2 + 24t$, then S is decreasing when $t \in$

1) $(2, 4)$ 2) $(-\infty, 2) \cup (4, \infty)$
3) $(-\infty, 2)$ 4) $(4, \infty)$

12. The displacement of a particle in time 't' is given by $s = t^3 - t^2 - 8t - 18$. The acceleration of the particle when its velocity vanishes is

1) 15 units/sec^2 2) 10 units/sec^2
3) 5 units/sec^2 4) 20 units/sec^2

13. The displacement 's' of a particle measured from a fixed point '0' on a line is given by $s = 16 + 48t - t^3$. After 4sec, the direction of motion of the particle.

1) is towards '0' 2) is away from '0'
3) is at rest 4) can not be decided

14. A particle moves in a straight line according to the law $v^2 = 4a(x\sin x + \cos x)$ where v is the velocity of a particle at a distance X from the fixed point then the accleration is

1) 2aXsinx 2) aXsinX
3) aXcosX 4) 2aXcosX

15. The distance 's' travelled by a particle in 't' seconds is given by s = acosht + bsinht. Then its acceleration at any time is

1) s 2) 2s 3) 3s 4) 4s

16. A particle is travelling according to the law $v^2 = 36 - x^2$ where X is its displacement and 'v' is its velocity. Then its maximum velocity = ... mts/sec.

1) 16 2) 6 3) 26 4) 20

17. The angle 'θ' turned by a fly wheel at time in sec. is given by $\theta = 9000t - 3t^2$. Then fly wheel comes to rest after a time of

1) 1000 sec 2) 1500 sec
3) 9000 sec 4) 3000 sec

18. A particle moves at distance 's' in time t where $s = \sqrt{t}$. The acceleration at t =1 second is

1) 2 units/sec^2 2) 1/4 unit/sec^2
3) -2 unit/sec^2 4) -1/4 unit/sec^2

19. The velocity v of a particle is given by $v^2 = s^2 + 4s + 4$. The acceleration of the particle when it is 30 cms away from the starting point is

1) 30 cms/sec^2 2) 32 cms/sec^2
3) 34 cms/sec^2 4) 35 cms/sec^2

20. If the rate of change of $x^3 - 5x^2 + 5x + 8$ is twice the rate of change of x then the values of x are

1) 3, -3 2) 3, 1/3 3) -1/3, -3 4) 2,3

21. A particle moves on a parabola $y^2 = 8x$. At what point on the curve the abscissa and the ordinate increase at the same rate.

1) (4,2) 2) (4,-2) 3) (2,4) 4) (-2, -5)

22. A point moves on the parabola $x^2 = 6y$. If the abscissa of the point increase at 2cms/sec. Then the rate of increase of the ordinate when x = 6 cms is

1) 4 cms/sec 2) 6 cms/sec
3) 3 cms/sec 4) 5 cms/sec

23. The distance travelled by a particle in time t secs with velocity v is given by s = 1/2 vt. The acceleration of the particle is

1) 2s/t 2) v/t 3) vt 4) t/v

24. The rate of change of sine is n, then the rate of change of tan is

1) n^2 2) $1/n^2$ 3) 1/n 4) n^3

25. A particle is moving along a line so that $s = 3t^3 + 8t + 1$. The instant at which velocity and acceleration are equal are given by

1) t = 0, 2/3 2) t = 1/3, 4/3
3) t = 2/3, 4/3 4) t = 0, 5/3

26. At what value of an angle, the rates of change in sine and tangent of the same angle are equal.

1) $2n\pi$ 2) $n\pi$ 3) $n\pi/2$ 4) $3n\pi/2$

27. An angle is increasing at a constant rate. The rate of increase of tan when the angle is $\pi/3$ is

1) 4 times the increase of sine
2) 8 times the increase of cosine
3) 8 times the increase of sine
4) 4 times the increase of cosine

28. Each side of a square increases at 1cm/sec. The rate of increase in the area of the square when the side is 15 cms is
1) 20 sq.cms/sec 2) 25 sq.cms/sec
3) 30 sq.cms/sec 4) 40 sq.cms/sec

29. The rate of change of area of a square plate is equal to that of the rate of change of its perimeter. Then length of the side is
1) 2 units 2) 3 units
3) 4 units 4) 6 units

30. The radius of a circular plate in increased at 0.01 cms/sec. If the area is increased at the rate of $\pi/10$ sq.cms/sec. Then its radius is
1) 5 cms 2) 10 cms 3) 15cms 4) 20 cms

31. In a circle of radius 'r' units, the rate of change of area of a sector is
1) Directly proportional to the rate of change of the angle.
2) Inversely proportional to the rate of change of the angle of the sector
3) A constant
4) Directly proportional to the radius

32. A light is hung 6mts. Vertically above, a man is walking away from the light at the rate of 54 mt/min. If the height of the man is 2mts then the rate at which the shadow increases is
1) 24 mt/min 2) 27 mt/min
3) 30 mt/min 4) 36 mt/min

33. In the isothermal expansion of a gas is given by pv = 200. V increases at the rate of 40cucms/sec. The rate of decrease in the pressure p when v = 20 cu.cms is
1) 10 lbs/sec 2) 15 lbs/sec
3) 20 lbs/sec 4) 25 lbs/sec

34. If the radius of a sphere is equal to the height of the cylinder then the ratio of the rates of increases of the volumes of the sphere and cylinder is
1) 4:3 2) 3:4 3) $4:3\pi$ 4) $3\pi:4$

35. Water is drawn out at the rate of 27000 cu.cms/sec from a cylindrical vessel. When the radius is 30cms. The rate of falling of water level in the vessel is
1) 30π cms/sec 2) $\pi/30$ cms/sec
3) $30/\pi$ cms/sec 4) $20/\pi$ cms/sec

36. If a particle moving along a line following the law $t = as^2 + bs + c$ then the retardation of the particle is proportional to
1) Square of displacement
2) Square of velocity
3) Cube of displacement
4) Cube of velocity

37. The side of a cube is equal to the radius of a sphere. If the side and the radius increases at the same rate then the rate of change of surface areas of the cube is ---- the rate of change of surface area of sphere
1) < 2) >
3) = 4) cannot be decided

38. If the abscissa of the point increases at the rate of 2 cms /sec on thr parabola $X^2 = 2y$: then the rate at which the ordinate increases at X=3 is
1) 4 cm/sec 2) 6 cm /sec
3) 3 cm/sec 4) 5 cm/sec

39. A particle moves along a line OA which is at a distance 5 cm from O where $s = 6t^2 - \frac{t^3}{2}$, then then greatest velocity along OA is
1) 32cm/s 2) 24 cm/s
4) 18cm/s 4) None

40. If s= 8cot2t + 4sint, then the intial velocity of the particle is
1) 8 units/sec 2) 4 units/sec
3) 6 units/sec 4) None

41. A particle moves along a straight line and its velocity at a distance 'x' from the orign $k\sqrt{a^2 - x^2}$. Then accelertion of the particle is
1) k 2) $-k^2$ 3) kx 4) $-k^2x$

KEY

1) 1	2) 3	3) 2	4) 2	5) 1
6) 3	7) 4	8) 3	9) 3	10) 2
11) 1	12) 2	13) 2	14) 4	15) 1
16) 2	17) 2	18) 4	19) 2	20) 2
21) 3	22) 1	23) 2	24) 2	25) 3
26) 1	27) 3	28) 3	29) 1	30) 1
31) 1	32) 2	33) 3	34) 1	35) 3
36) 4	37) 1	38) 2	39) 2	40) 2
41) 4				

HINTS

1. $V = \frac{ds}{dt} = 3t^2 - 18t + 24$

$(V)_{t=2} = 12 - 36 + 24 = 0$

2. $V=\frac{ds}{dt}=6t^2-6t+15$

$(V)_{t=0}=15$

3. $a=\frac{d^2s}{dt^2}$

5. $a+bv^2=x^2$

$b.2v.\frac{dv}{dt}=2x.\frac{dx}{dt}=2x.v$

$\Rightarrow \frac{dv}{dt}=a=\frac{x}{b}$

8. $V^2=u^2+90s$

$2V.a=90\frac{ds}{dt}(\text{Q } u=0)$

$\Rightarrow a=45$

9. $V=f(x)$

$a=\frac{dv}{dt}=f^1(x).\frac{dx}{dt}=f^1(x).f(x)$

10. $V=t^2-6t+8$

$v=0\Rightarrow t=2, t=4$

11. $V<0$

16. $V^2=36,\ V=6$

17. $\frac{d\theta}{dt}=0\Rightarrow 6t=9000$

$t=1500 \sec$

20. $\frac{d}{dt}\left(x^3-5x^2+5x+8\right)=2\times\frac{d}{dt}(x)$

21. $y^2=8x$

$\Rightarrow 2y\left(\frac{dy}{dt}\right)=8\left(\frac{dx}{dt}\right)$

$\text{Q } \frac{dy}{dt}=\frac{dx}{dt}\Rightarrow y=4,\ x=2$

24. The rate of change of sine is cos i.e., n

The rate of change tan is $\sec^2$ i.e. $\frac{1}{n^2}$.

25. $\frac{ds}{dt}=\frac{d^2s}{dt^2}$

26. $\frac{d}{dt}(\sin\theta)=\frac{d}{dt}(\tan\theta)$

$\Rightarrow \cos\theta.\frac{d\theta}{dt}=\sec^2\theta.\frac{d\theta}{dt}$

$\Rightarrow \cos^3\theta=1$

$\Rightarrow \theta=2n\pi$

27. $\frac{d\theta}{dt}=k,$ If $\theta=\frac{\pi}{3}$,

$\frac{d}{dt}(\tan\theta)=\sec^2\theta\times\frac{d\theta}{dt}$

$=4\times\frac{d\theta}{dt}$

$=8\left[\frac{1}{2}\times\frac{d\theta}{dt}\right]$

$=8\left[\frac{d}{dt}(\sin\theta)\right]$

28. Let side = x, Area = A

$\frac{dx}{dt}=1 cm/\sec$

If x = 15 cms, $\frac{dA}{dt}=?$

$\frac{dA}{dt}=\frac{d}{dt}\left(x^2\right)=2x.\frac{dx}{dt}=30 sq.cm/\sec$

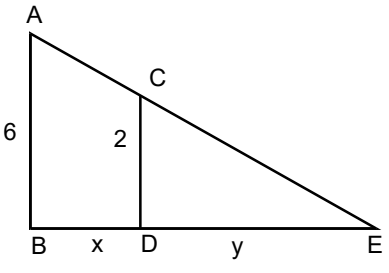
29. $\frac{d}{dt}$ (area) = $\frac{d}{dt}$ (perimeter)

32. $\frac{2}{6}=\frac{y}{x+y}$

$3y=x+y$

$2y=x$

$\Rightarrow 2\frac{dy}{dt}=\frac{dx}{dt}$



$\Rightarrow \frac{dy}{dt}=\frac{1}{2}\times 54=27 mt/\sec$

35. $\frac{dv}{dt} = 27,000 \text{ c.cms} / \sec, r = 30\text{cms}.$

$\frac{d}{dt}\left(\pi r^2 h\right) = 27,000$

$\frac{dh}{dt} = \frac{30}{\pi} cms / \sec$

37. x = r and $\frac{dx}{dt} = \frac{dr}{dt}$ (given)

$\frac{d}{dt}\left(6x^2\right) - - - \frac{d}{dt}\left(4\pi r^2\right)$

$\Rightarrow 12x.\frac{dx}{dt} - - - 8\pi r.\frac{dr}{dt}$

$\Rightarrow 12r < 8\pi r$ (Since h=r)

LEVEL-2

1. A stone projected vertically upwards rises 's' feets in 't' seconds where $s = 112t - 16t^2$. The maximum height it reached is
   1) 195 ft 2) 194 ft 3) 196 ft 4) 216 ft
2. If the motion of a particle in a straight line is given by $s = \sqrt{1+t}$, then the acceleration is proportional to
   1) s^2 2) s^3
   3) cube of velocity v 4) 4s
3. If the velocity v of a particle varies as the square of its displacement x then the acceleration varies as
   1) x^2 2) x^3 3) v^2 4) v^3
4. The side of a cube is equal to the diameter of a sphere. If the side and radius increase at the same rate. Then the ratio of the rates of change of their surface areas is
   1) π:6 2) 2π:3 3) 3:2π 4) 3:π
5. A point 'P' is moving with constant velocity V along a line AB. O is a point on the line perpendicular to AB at A and at a distance "l" from A. The Angular velocity of P about O is
   1) $\frac{lv}{op}$ 2) $\frac{lv}{op^2}$ 3) $\frac{lv^2}{op}$ 4) $\frac{op^2}{lv}$
6. The equation of motion of a particle p(x,y) on a plane are given by x = 4 + bcost, y = 5 + bsint. Its velocity at time 't' is
   1) 4 2) 5 3) b 4) tan t
7. A man on a wharf which is 20 meters above the water level, pulls in a rope to which about is tied at the rate of 4 mts/sec. The rate at which it approaches the shore. When there is still 25 meters of rope out is.
   1) 25/3 meters/sec 2) 20/3 meters/sec
   3) 5 meters/sec 4) 19/3 meters/sec
8. The volume of a ball increases at 4π c.c/sec. The rate of increase of radius when the volume is 288π c.cms is.
   1) 1/6 cm/sec 2) 1/36 cm/sec
   3) 1/9 cm/sec 4) 1/49 cm/sec
9. A particle moves along a line by $s = \frac{t^3}{3} - 3t^2 + 8t$ then the distance travelled by theparticle before if first comes to rest is.
   1) 20 2) 20/3 3) 3 4) 60
10. If the velocity of a falling body is $k\sqrt{s}$ at the instant when the body has fallen a distance 's'. Then its acceleration is
    1) $\frac{k^2}{2}$ 2) $\frac{k}{2}$ 3) k 4) $2k$
11. If the distance travelled by a particle is $x = \sqrt{at^2 + 2bt + c}$ then the acceleration is proportional to
    1) $\frac{1}{x}$ 2) $\frac{1}{x^2}$ 3) $\frac{1}{\sqrt{x}}$ 4) $\frac{1}{x^3}$
12. If $y = x^3$ and x increases at the rate of 3 units per sec; then the rate at which the slope increases when x is 3 is
    1) 9 2) 18 3) 27 4) 54
13. A point s moving along the cubical parabola $12y = x^3$. The rate of ordinate is less than the rate of abscissa when
    1) x < -2 or x > 2 2) x = $\pm$ 2
    3) - 2 < x < 2 4) 0 < x < 1
14. The area of an expanding circular region increases at a constant rate then the rate of increase of the circumference.
    1) Varies inversely as the radius
    2) Varies directly as the radius
    3) Varies directly as the square
    4) Varies inversely as the square of the radius
15. If the rate of change in the circumference of a circle is 0.5 cm/sec. Then the rate of change in the area when the circumference is 10π cm is
    1) 2.5 sq.cm/sec 2) 5 sq. cm/sec
    3) 1.5 sq.cm/sec 4) 2 sq.cm/sec

16. A spherical balloon is being inflated so that its volume increases at the rate of 40 c.c/sec. Then the rate of increase in the surface area when the radius is 8cm. is (in sq.cm/sec).

1) 10 2) 15 3) 20 4) 25

17. The side of a cube is equal to the radius of a sphere. If the rate at which volume of the cube increasing is k, then the rate at which volume of the sphere increasing, is

1) $\frac{4\pi k}{3}$ 2) $4\pi k$ 3) $\frac{k}{3}$ 4) $\frac{k}{4}$

18. If the semi vertical angle of a cone is 45^0, then the rate of change of its volume is

1) The area times the rate of change of r.

2) The base area times, the rate of change of h

3) The base area times the rate of change of r.

4) The area times of the rate of change in h.

19. In any sphere the rate of change of its volume with respect to its radius is equal to

1) It radius 2) The square of its radius

3) It surface area

4) The square of its surface area

20. A particle has velocity 6 cm/s and acceleration 4 cm/s^2 at time t = 2 sec according to $s = at^2 + bt$, then value of 'a' and 'b' are

1) 2, -2 2) 3,2

3) 4,2 4) 2,2

21. The area of an equilateral triangle of side 'a' feet is increasing at the rate of 4 sq.ft./sec. The rate at which the perimeter is increasing is

1) $\frac{3\sqrt{8}}{2}$ 2) $\frac{8\sqrt{3}}{a}$ 3) $\frac{\sqrt{3}}{a}$ 4) $\frac{2\sqrt{3}}{a}$

22. If the distance travelled by particle in 't' seconds is given by $s = \cos(\pi t/2)$ then its acceleration at time 't' is

1) $\frac{\pi^2}{2}s$ 2) $\frac{-\pi^2}{12}s$ 3) $\frac{-\pi^2}{4}s$ 4) $\frac{-\pi^2}{3}s$

23. Water is poured into an inverted conicel vessel of depth 20cm and base radius 10cm at the rate of 4cc per minute. The rate of increase of water level when the depth is 5cm is

1) $\frac{16}{25\pi}cm/\min$ 2) $\frac{16\pi}{25}cm/\min$

3) $\frac{3\pi}{8}cm/\min$ 4) $\frac{3}{8\pi}cm/\min$

24. The side of a square is equal to the diameter of a circle. If the side and radius change at the same rate, then the ratio of the change of their areas is

1) $1:\pi$ 2) $\pi:1$ 3) $2:\pi$ 4) 1:2

25. The two measurements of a cylinder are varying in such a way that the volume is kept constant. If the rates of change of the radius and height are equal and opposite in sign then which of the following is true.

1) r = 2h 2) h = 2r 3) r = h 4) $r = \frac{h}{3}$

26. A point is moving along the cubical parabola $12y = x^3$ then $|x| < 2$.

1) Rate of ordinate is more than the rate of abscissa.

2) Rate of ordinate is equal to the rate of abscissa.

3) Rate of ordinate is less than the rate of abscissa.

4) No relationship exists between rate of ordiante and rate of abscissa.

27. In a simple pendulum, if the rate of change in the time peirod is equal to the rate of change in the length then the length of the pendulum is

1) $\frac{\pi}{g}$ 2) $\frac{\pi^2}{g}$ 3) $\pi^2 g$ 4) πg^2

28. A wheel rotates so that the angle of rotation is proportional to the square of the time. The first revolution was performed by the wheel for 8 seconds the angular velocity at this timeis

1) π rad/sec 2) 2π rad/sec

3) $\pi/2$ rad/sec 4) $\pi/3$ rad/sec

29. If k is the diameter of a circle and A is the area of a sector of the circle whose vertical angle is θ then dA/dt =

1) $\frac{k^2}{8}\left(\frac{d\theta}{dt}\right)$ 2) $\left(\frac{k^2}{4}\right)\left(\frac{d\theta}{dt}\right)$

3) $\frac{d\theta}{dt}$ 4) $k\left(\frac{d\theta}{dt}\right)$

30. A particle moving on a straight line so that its distnace 's' from a fixed point at any time 't' is proportional to 't^n' if 'v' be the velocity and 'a' the acceleration at any time then $\frac{nas}{(n-1)} =$

1) v 2) v^2 3) v^3 4) 2v

31. The diametre and the altitude of a right circular cylinder are 10cm and 20cm. If the diameter increases at the rate of 2 cms/sec then the rate of decrease in the altitude when the volume remaining constant is

1) 4 cm/sec 2) 8 cm/sec

3) 12 cm/sec 4) 16 cm/sec

32. The surface area of a sphere increases at 1 sq.cm/sec. If the radius is 3cm, then the rate of increase in its volume is

1) 1/2 cu.cm/sec 2) 1 cu.cm/sec

3) 3/2 cu.cm/sec 4) 2 cu.cm/sec

33. A point p moves with an angular velocity 2 radians/sec on the circumference of a circle with centre O and radius 2 cms. PM is pendicular to the diameter of the circle such the $\angle POM = \theta$. If the velocity of the point M is zero, then values of θ are

1) 0, π 2) $\pi/2, \pi$

3) $\pi/3, \pi/6$ 4) $\pi/4, 3\pi/4$

34. A man is walking at the rate of 8 kmph towards the foot of the tower 60 mts high . The rate at which he approaches the top of the tower when he is 80 mts. from the foot of the tower is

1) 3.2 kmph 2) 4.8 kmph

3) 6.4 kmph 4) 2.3 kmph

35. The radius of the base of a cone increases at 3 cm/sec and the altitude decreases at 4 cm/sec. The rate of change of lateral surface area when radius = 7cms and altitude = 24cm is

1) 54π sq.cm/sec 2) 27π sq.cm/sec

3) 7π sq.cm/sec 4) 20π sq.cm/sec

36. A variable triangle is inscribed in a circle of radius R. If the rate of change of a side is R times the rate of change of the opposite angle, then the opposite angle is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$

3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

37. The point on the ellipse $16x^2 + 9y^2 = 400$, at which the ordinate decreases at the same rate at which the abscissa increases is

1) $\left(3, \frac{16}{3}\right)$ 2) $\left(-3, \frac{16}{3}\right)$

3) $\left(3, \frac{-16}{3}\right)$ 4) $\left(-4, \frac{-16}{3}\right)$

38. At the point (2,3) on the curve $y = x^3 - 2x + 1$, the gradient of the curve increases

1) 6 times as fast as x

2) 10 times as fast as x

3) 12 times as fast as x

4) 8 times as fast as x

39. A particle moving on a straight line so that its distance 's' from a fixed point at any time 't' is proportional to t^n. If 'V' be the velocity and 'a' is the acceleration at any time then $\frac{nas}{n-1} =$

1) v 2) v^2 3) v^3 4) 2v

40. An angle θ through which a pulley turns with time 't' is completed by $\theta = t^2+3t-5$.sqcms/min. Then the angular velocity for t = 5sec.

1) 5^c/sec 2) 13^c/sec 3) 23^c/sec 4) 35^c/sec

41. A particle 'p' moves along a straight line away from a fixed point 'O'obeying the realation $S = 16+48t-t^3$. The direction of 'P' at t= 4 is

1) $\overrightarrow{OP}$ 2) $\overrightarrow{PO}$

3) Rest at the instant

4) pependicular to $\overrightarrow{OP}$

42. A variable triangle ABC is inscribed in a circle of diameter x units. If the rate of increase in one of the sides 'a'of the triangle is 'x/2' times the increase in $\angle A$, then $\angle A =$

1) $\pi/6$ 2) $\pi/3$

3) $\pi/4$ 4) $\pi/2$

KEY

1) 3	2) 3	3) 2	4) 4	5) 2
6) 3	7) 2	8) 2	9) 2	10) 1
11) 4	12) 4	13) 3	14) 1	15) 1
16) 1	17) 1	18) 3	19) 3	20) 1
21) 2	22) 3	23) 1	24) 3	25) 1
26) 3	27) 2	28) 3	29) 1	30) 2
31) 2	32) 3	33) 1	34) 3	35) 1
36) 3	37) 1	38) 3	39) 2	40) 2
41) 1	42) 2			

HINTS

1. $S = 112t - 16t^2$

$\Rightarrow V = 112 - 32t,\ V = 0$

$\Rightarrow$ we will get the value of 't', then substitute 't' value in S, we will get maximum height.

3. $V \propto x^2$

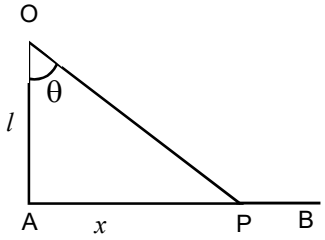
$V = kx^2 \longrightarrow (1)$

$a = k.2x.kx^2$ from (1)

$= \left(2k^2\right)x^3$

$\Rightarrow a \propto x^3$

5. $\tan\theta = \frac{x}{l}$



$\Rightarrow \sec^2\theta.\frac{d\theta}{dt} = \frac{1}{l}\times\frac{dx}{dt} = \frac{V}{l}$

$\Rightarrow \frac{d\theta}{dt} = \cos^2\theta.\frac{V}{l} = \frac{V}{l}\times\frac{l^2}{op^2} = \frac{Vl}{op^2}$

6. $\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

8. $\frac{dv}{dt} = 4\pi\ c.c/\sec$

$V = 288\pi$

$\frac{4}{3}\pi r^3 = 288\pi$

$\Rightarrow r = 6$

$\therefore \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{4}{3}\pi\mathrm{r}^3\right) = ?$

9. $V = \frac{ds}{dt} = 0$

10. $V = k\sqrt{s}$, then $a = \frac{dv}{dt}$

11. Squaring and then differentiate two times

12. x = 3, $\frac{dx}{dt} = 3\ units/\sec$,

Then $\frac{d}{dt}\left(\frac{dy}{dx}\right)$

13. $\frac{dy}{dt} < \frac{dx}{dt} \Rightarrow 3x^2 < 12$

14. $\frac{dA}{dt} = const$, use $4\pi A = C^2$, then find $\frac{dc}{dt}$.

15. $C = 10\pi$ cm, $\frac{dc}{dt} = 0.5\ cm/\sec$,

then use $4\pi A = C^2$, find $\frac{dA}{dt}$

16. $\frac{dv}{dt} = 40\ c.c/\sec$; $r = 8cm$, use

$V = \frac{1}{6\sqrt{\pi}}(S)^{3/2}$

Then find $\frac{ds}{dt}$.

17. $x = r,\ \frac{dv_1}{dt} = k$ (cube)

$\frac{dv_2}{dt} = ?$ (sphere)

18. $\alpha = 45^o$ then $\frac{dv}{dt} = ?$

$V = \frac{1}{3}\pi r^3\ (\mathrm{Q}\ h = r)$

19. $V = \frac{4}{3}\pi r^3$

then $\frac{dv}{dr} = ?$

20. $V = 6\ m/s,\ A = 4cm/\sec^2$ at $t = 2\sec$

given $s = at^2 + bt$

differentiate

21. $x = a\ ft, \frac{dA}{dt} = 4sq.ft/\sec$

$\frac{dc}{dt} = ?$

use $12\sqrt{3}A = C^2$

22. differentiate 2 times

$\frac{dv}{dt} = 4cc/\min$

$\frac{H}{R} = \frac{h}{r} = 2 \Rightarrow r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h = \frac{\pi h^3}{12}$

at h = 5 cm find $\frac{dh}{dt}$

24. $x = 2r$

$\frac{dx}{dt} = \frac{dr}{dt}$ then $\frac{dA_1}{dt} : \frac{dA_2}{dt} = 2 : \pi$

(A_1 - Area of square)

(A_2 - Area of circle)

25. $\frac{dv}{dt}$ = constant, $\frac{dh}{dt} = -\frac{dr}{dt}$

use $V = \pi r^2 h$ then differentiate w.r.t 't'.

26. See the problem No. 13

27. $T = 2\pi\sqrt{\frac{l}{g}}, \frac{dT}{dt} = \frac{dl}{dt}$

28. $\theta \propto t^2$

$\Rightarrow \theta = kt^2$ (k constant)

$k = \frac{2\pi}{64} \Rightarrow k = \frac{\pi}{32}$

$\frac{d\theta}{dt} = k.2t = \frac{\pi}{32} \times 2 \times 8 = \frac{\pi}{2} R/\sec$

29. $k = 2r$, $A = \frac{1}{2}r^2\theta = \frac{k^2\theta}{8}$

30. $S \propto t^n$

$\Rightarrow s = k\left(t^n\right)$, differentiate

31. d= 10 cm, h = 20 cm

$\frac{dd}{dt} = 2\,cm/\sec$ then $\frac{dh}{dt} = ?$, V is given constant

Use $V = \pi r^2 h$

32. Use $V = \frac{1}{6\sqrt{\pi}}(s)^{3/2}$

33. $OM = \cos\theta, \frac{d(OM)}{dt} = 0$

34. z = 60 mt

x = 80 mt

$\therefore$ y = 100 mt.

$x^2 + 60^2 = (y)^2$ differentiate and find $\frac{dy}{dt}$

35. $S = \pi rl$ where $l = \sqrt{h^2 + r^2}$

$\frac{ds}{dt} = ?$

36. Use $a = 2R\sin A$

37. $\frac{dy}{dt} = \frac{-dx}{dt}$

38. $f(x) = \frac{dy}{dx} = 3x^2 - 2$

then $\frac{df(x)}{dt} = ?$

LEVEL-3

1. A conical vessel of height of 10 mts and radius 5mts. is being filled with water at uniform rate of 3/2 cu.mts/min. The rate at which the level of water rises when the depth of water in the vessle is 4mts is

1) 3π mts/min 2) 3/4π mts/min

3) 3/8π mts/min 4) 3/2 π mts/min

2. A is a fixed point on the circumference of a circle with centre '0' and radius 'r', A particle starts at A and moves on the circumference with an angular velocity 4 radians/sec. If PM is perpendicular to OA and $\angle POM = \pi/3$, then the rate at which area of ΔPOM decreases is

1) $\frac{r^2}{2} sq.cms/\sec$ 2) r^2 $sq.cms/\sec$

3) $\frac{3r^2}{2} sq.cms/\sec$ 4) $2r^2$ $sq.cms/\sec$

3. A ladder AB of 10 mts long moves with its ends

on the axes. When the end A is 6 mts from the origin, it moves away from it at 2mts/ minute. The rate of decrease of the area of the ΔOAB is

1) $\frac{4}{3}\, sq.mts/\min$ 2) $\frac{8}{3}\, sq.mts/\min$

3) $\frac{14}{3}\, sq.mts/\min$ 4) $\frac{7}{2}\, sq.mts/\min$

4. A kite flying at a height 'h' mts has "x" meters of string paid out at a time t seconds. If the kite moves horzontally with constant velocit v mts/sec. Then the rate at which the string is paid out is

1) $\frac{\sqrt{x^2-h^2}}{v}\, mt/\sec$ 2) $\sqrt{x^2-h^2}\, mt/\sec$

3) $\frac{v\sqrt{x^2-h^2}}{x}\, mt/\sec$ 4) $\frac{\sqrt{x^2-h^2}}{h}\, mt/\sec$

5. A balloon is in the shape of a cone surmounted by a semi sphere. The radius of a sphere is equal to the height of the cone. If the height of the balloon is 2 then the rate of change in its volume is --------- times the rate of change in its height.

1) 4 π 2) 6 π 3) 12 π 4) 8 π

6. A source of light is hung h mts., directly above a straight horizontal path on which a boy 'a' mts., in height is walking. If a boy walks at a rate of b mts/sec. from the light then the rate at which his shadow increases.

1) $\frac{ab}{h-a}\, mt/\sec$ 2) $\frac{ab}{h+a}\, mt/\sec$

3) $\frac{ab}{2(h-a)}\, mt/\sec$ 4) $\frac{ab}{2(h+a)}\, mt/\sec$

7. Two parallel sides of a rectangle are being lengthened at the rate of 2cms/sec. While the other 2 parallel sides are shortened in such a way that the area of the rectangle is $50cm^2$. The rate of change of perimeter when the length of an increasing side is 5cm is (cm/sec).

1) 2 2) 6 3) -2 4) -4

8. Two cars started from a place, one moving due East and other due to North with equal speed v. Then the rate at which they were being seperated from each other is

1) $\frac{\sqrt{2}}{v}$ 2) $\frac{v}{\sqrt{2}}$ 3) $\frac{1}{\sqrt{2v}}$ 4) $\sqrt{2}\, v$

9. A is an end of diameter of a cirlce with centre O and radius 2 units. If a particle 'p'. Starting from A moves on a circle with angular velocity 4 radians/sec and M is the foot of the perpendicular of 'p' on the diameter then the rate at which M moving on the diameter when it is at a distance of 1 unit from O is

1) $4\sqrt{3}$ units/sec 2) $-4\sqrt{3}$ units/sec

3) 4 units/sec 4) -4 units/sec

10. Sand is falling on the ground from a height and taking the shape of right circular cone always such that the height is equal to the radius of the base. If the sand is falling at the rate of 6 cu.ft/sec; the rate at which the height is rising when the height is 5ft is

1) $\frac{3}{25\pi}\, ft/\sec$ 2) $\frac{6}{25\pi}\, ft/\sec$

3) $\frac{6}{20\pi}\, ft/\sec$ 4) $\frac{12}{25\pi}\, ft/\sec$

11. The slant height of a cone is fixed at 7cm. The rate of increase in the volume of the cone corresponding to the rate of increase of 0.3 cm/s in the height when h = 4cm is

1) $\frac{\pi}{10}\, cc/s$ 2) $\frac{3\pi}{10}\, cc/s$

3) $\frac{\pi}{5}\, cc/s$ 4) $\frac{7\pi}{10}\, cc/s$

12. An inverted conical vessel of semi vertical angle $\pi/4$ is being filled with water at the rate of k cc/sec. Then the rate of change in water level when it is 'h' cms is

1) $\frac{k^2}{\pi h^2} cc/s$ 2) $\frac{k^2}{r^2} cc/s$

3) $\frac{k}{r^2} cc/s$ 4) $\frac{k}{\pi h^2} cc/s$

13. A particle moves along a straight line according to the equation $s = at^2 + bt + c$. In 1 sec. It travelled a distance 11 cms, and at the end of 2 secs. If velocity is 16cm/sec and acceleration is $6cms/sec^2$. The values of a, b, c are

1) 3, 4, 4 2) 3, -4, 4 3) 4, 3, 4 4) 1, 4, 4

14. A man is approaching the foot of a pole of height 'h' and at a speed of 6 mints/sec. The rate at which the man is approaching the peak of the

pole when his distance from the foot is l units is

1) $\frac{b}{\sqrt{l^2+h^2}}$ units /sec

2) $\frac{bl}{\sqrt{l^2+h^2}}$ units/sec

3) $\frac{l}{b\sqrt{l^2+h^2}}$ units /sec

4) $\frac{lb}{\sqrt{l^2+h^2}}$ units/sec

15. Two parallel sides of a rectangle are being lengthened at the rate of 2 cm/sec. While the other two sides are shortened so that the area of the rectnagle is 50 sq.cms. The rate of change of the perimeter when the length of an increasing side 5 cms is

1) 4 cm/sec 2) -4 cm/sec

3) $4\frac{1}{3}$ cm/sec 4) $-4\frac{1}{3}$ cm/sec

KEY

1) 3	2) 2	3) 4	4) 3	5) 3
6) 1	7) 4	8) 4	9) 2	10) 2
11) 1	12) 4	13) 1	14) 4	15) 2

HINTS

1. $\frac{H}{R}=\frac{h}{r}\Rightarrow\frac{10}{5}=\frac{h}{r}\Rightarrow r=\frac{h}{2}$

use $V=\frac{1}{3}\pi r^2h=\frac{\pi h^3}{12}$

then at h = 4 mts $\frac{dv}{dt}=?$

2. $OM=r\cos\theta$

$PM=r\sin\theta$

$A=\frac{1}{2}(OM)(PM)=\frac{r^2\sin 2\theta}{4}$

$\frac{dA}{dt}=\frac{r^2}{4}(2\cos 2\theta).\frac{d\theta}{dt}=-r^2$

3. $x^2+y^2=100\Rightarrow y=8mts$

$x\frac{dx}{dt}+y\frac{dy}{dt}=0$

$\frac{dy}{dt}=\frac{-3}{2}mts/\min$

$A=\frac{1}{2}xy$

$\frac{dA}{dt}=\frac{1}{2}\left(x\frac{dy}{dt}+y\frac{dx}{dt}\right)$

$=\frac{7}{2}sq.mts/\min$

4. $x^2=y^2+h^2$

$\Rightarrow x\frac{dx}{dt}=v.y$

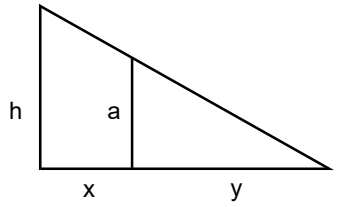
$\Rightarrow\frac{dx}{dt}=\frac{vy}{x}=\frac{v\sqrt{x^2-h^2}}{x}$

5. $r=h$

$V=\frac{1}{3}\pi r^2h+\frac{2}{3}\pi r^3$

$V=\pi r^3=\pi h^3$

$\frac{dv}{dt}=3\pi h^2\frac{dh}{dt}$



6. $\frac{a}{h}=\frac{y}{x+y}$

$\Rightarrow ax+ay=hy$

$\Rightarrow(h-a)\frac{dy}{dt}=ab$

$\Rightarrow\frac{dy}{dt}=\frac{ab}{(h-a)}$

7. Let length = x, breadth = y

x = 5, $\frac{dx}{dt}=2cm/\sec$, $xy=50$ (given)

$y=10$

$\therefore xy=50$

$$x\frac{dy}{dt}+y\frac{dx}{dt}=0\Rightarrow\frac{dy}{dt}=-4$$

$$C=2(x+y)$$

$$\frac{dc}{dt}=2\left(\frac{dx}{dt}+\frac{dy}{dt}\right)=-4$$

8. $\frac{dx}{dt}=V,\ \frac{dy}{dt}=V$ But $x=y$

$\therefore$ the distance, $D=\sqrt{x^2+x^2}=\sqrt{2}x$

$$\therefore\frac{dD}{dt}=\sqrt{2}.v$$

9. $x=r\cos\theta,\ x=1,\ r=2,\ y=\sqrt{3}$

$$\frac{dx}{dt}=-r\sin\theta.\frac{d\theta}{dt}$$

$$=-y\frac{d\theta}{dt}=-4\sqrt{3}$$

10. $h=r,\frac{dv}{dt}=cu.\,ft/\sec$

Use $V=\frac{1}{3}\pi r^2h$ when $h=5\,ft$

11. $V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(l^2-h^2\right)h$

$$=\frac{1}{3}\pi\left(l^2h-h^3\right)$$

$$\Rightarrow\frac{dv}{dt}=\frac{1}{3}\pi\left(l^2-3h^2\right)\frac{dh}{dt}$$

12. $r=h,V=\frac{1}{3}\pi r^2h=\frac{\pi}{3}h^3$

$$K=\pi h^2\frac{dh}{dt}$$

13. $a+b+c=11\rightarrow(1)$

$4a+b=16\rightarrow(2)$

$2a=6\rightarrow(3)$

Solve (1), (2), (3)

PREVIOUS EAMCET PROBLEMS

1982

1. A concial vessel of height 10 feet and semi vertical angle 30^0 is full of water. It empties in such a way that the height of water in the vessel decreases at constant rate of inch per minute. The rate of which the volume of the water in the vessel decreases when its height is 6 feet is

1) π c.ft/minute 2) 4πc.ft/minute

3) 8π c.ft/minute 4) 12π c.ft/minute

1993

2. The acceleration of a particle moving on a line is given by a(t) = 2t + 1. If it has started with an initial velocity 3cms/sec the distance it travelled in 1 sec is

1) $4\ cms$ 2) $\frac{23}{6}\ cms$

3) $5\ cms$ 4) 21/5 cms

1995

3. A particle is projected vertically upwards such that its height h and time is given by h = $60t - 16t^2$. Velocity with which it hits the ground is

1) 30 units/sec 2) 60 units/sec

3) 90units/sec 4) 180 units/sec

4) A car starts from rest and attains the speed of 1 km/hr and 2 k.ms/hr at the end of 1st and 2nd minutes. If the car moves on a straight road, the distance travelled in 2 minutes is

1) $\frac{1}{4}\ km$ 2) $\frac{1}{3}\ km$

3) $15\ km$ 4) $20\ km$

5) A particle rotates with an angular velocity'ω'

on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, Linear Velocity is

1) $\omega\sqrt{a^2+b^2}$ 2) $\frac{\omega}{\sqrt{a^2+b^2}}$

3) $\omega^3\sqrt{a^2+b^2}$ 4) $\omega^2\sqrt{a^2+b^2}$

1997

6) For a particle moving on a straight line it is observed that the distance s in time t secs

is given by $s=6t-\frac{t^2}{2}$. The maximum velocity

during the motion is

1) 3 units/sec 2) 6 units/sec

3) 9 units/sec 4) 12 units/sec

1995&98

7) The side of an equilateral triangle expands at the rate of 2 cms/sec. The rate of increase of its area when each side is 10cms. is

1) $10\sqrt{2}$ sq.cms/sec

2) $10\sqrt{3}$ sq.cms/sec

3) 10 sq.cms/sec

4) 5 sq.cms/sec

1999

8) A cylindrical vessel of radius 0.5mtrs. is filled with oil at the rate of 0.25π cu.mt/min. The rate at which the surface of the oil is increasing is

1) π mtrs/min. 2) 2 mtrs/min.

3) 5 mtrs/min. 4) 1.25 mtrs/min.

2000

9) The distance moved by the particle intime 't' is given by $S = t^3 - 12t^2 + 6t + 8$. At the instant, when its acceleration is zero. The velocity is

1) 42 2) -42

3) 48 4) -48

KEY

1) 1	2) 2	3) 2	4) 2	5) 1
6) 2	7) 2	8) 1	9) 2	