1. જો a, b, c સમાંતર શ્રેણીમાં હોય, તો નિશ્ચાયક $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ = ...

(A) 0 (B) 1 (C) x (D) $2x$

જવાબ (A) 0

➠ $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$

$= \begin{vmatrix} x+2 & x+3 & x+2a \\ 1 & 1 & 2b-2a \\ 2 & 2 & 2c-2a \end{vmatrix}$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$= \begin{vmatrix} -1 & x+3 & x+2a \\ 0 & 1 & 2b-2a \\ 0 & 2 & 2c-2a \end{vmatrix}$ $c_1 \to c_1 - c_2$

$= (-1)\ [2c - 2a - 4b + 4a]$

$= (-1)\ [2a + 2c - 4b]$

$= (-1)\ [2(a + c) - 4b]$ a, b, c સમાંતર શ્રેણીમાં છે.

$\therefore\ a + c = 2b$

$= (-1)\ [4b - 4b]$

$= 0$

2. જો x, y, z શૂન્યેતર વાસ્તવિક સંખ્યાઓ હોય, તો $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ નો વ્યસ્ત શ્રેણિક ....

(A) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(B) $xyz\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(C) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

(D) $\frac{1}{xyz}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

જવાબ (C) $\frac{1}{xyz}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

➠ $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

$\therefore\ |A| = xyz$

$A_{11} = (-1)^{1+1} yz = yz$ | $A_{21} = (-1)^{2+1} 0 = 0$
$A_{12} = (-1)^{1+2} 0 = 0$ | $A_{22} = (-1)^{2+2} xz = xz$
$A_{13} = (-1)^{1+3} 0 = 0$ | $A_{23} = (-1)^{2+3} 0 = 0$
$A_{31} = (-1)^{3+1} 0 = 0$
$A_{32} = (-1)^{3+2} 0 = 0$
$A_{33} = (-1)^{3+3} xy = xy$

$$\therefore adj\ A = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

હવે $A^{-1} = \frac{1}{|A|} adj\, A$

$$= \frac{1}{xyz}\begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

3. **જો $0 \le \theta \le 2\pi$ માટે $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ હોય, તો**

(A) Det(A) = 0 (B) Det(A) $\in$ $(2, \infty)$
(C) Det(A) $\in$ $(2, 4)$ (D) Det(A) $\in$ $[2, 4]$

જવાબ (D) Det(A) $\in$ $[2, 4]$

➠ $$\text{Det}(A) = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$$

$= 1(1 + \sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$
$= 2(1 + \sin^2\theta)$

હવે $-1 \le \sin\theta \le 1$

$\therefore\ 0 \le \sin^2\theta \le 1$

$\therefore\ 1 \le 1 + \sin^2\theta \le 2$

$\therefore\ 2 \le 2(1 + \sin^2\theta) \le 4$

$\therefore\ 2 \le \text{Det}(A) \le 4$

$\therefore\ \text{Det}(A) \in [2, 4]$

4. **સાબિત કરો કે નિશ્ચાયક $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ નું મૂલ્ય θ થી મુક્ત છે.**

➥ $$A = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$= x(-x^2 - 1) - \sin\theta(-x \sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x \cos\theta)$
$= -x^3 - x + x \sin^2\theta - \sin\theta \cos\theta - \cos\theta \sin\theta + x \cos^2\theta$
$= -x^3 - x + x (\sin^2\theta + \cos^2\theta)$

$= -x^3 - x + x\ (1)$

$= -x^3$ જે θ થી સ્વતંત્ર છે.

5. $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$ **નું મૂલ્ય શોધો.**

➥ $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix} = \begin{vmatrix} 1 & x & y \\ 0 & y & 0 \\ 0 & 0 & x \end{vmatrix}$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$= 1\ (xy - 0)$

$= xy$

6. નિશ્ચાયકનું વિસ્તરણ કર્યા સિવાય સાબિત કરો : $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

➥ $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix}$ $(R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3)$

$= \frac{abc}{abc}\begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$ (c_3 માંથી abc સામાન્ય લેતાં)

$= -\begin{vmatrix} a^2 & 1 & a^3 \\ b^2 & 1 & b^3 \\ c^2 & 1 & c^3 \end{vmatrix}$ $(c_2 \leftrightarrow c_3)$

$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$ $(c_1 \leftrightarrow c_2)$

7. $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$ **નું મૂલ્ય શોધો.**

➥ $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

$= \cos\alpha\cos\beta(\cos\alpha\cos\beta - 0) - \cos\alpha\sin\beta\ (-\cos\alpha\sin\beta) - \sin\alpha(-\sin\alpha\sin^2\beta - \sin\alpha\cos^2\beta)$

$= \cos^2\alpha\cos^2\beta + \cos^2\alpha\sin^2\beta + \sin^2\alpha\sin^2\beta + \sin^2\alpha\cos^2\beta$

$= \cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha(\sin^2\beta + \cos^2\beta)$

$= \cos^2\alpha\ (1) + \sin^2\alpha\ (1)$

$= \cos^2\alpha + \sin^2\alpha$

$= 1$

8. શૂન્યેતર a માટે સમીકરણ $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$ ઉકેલો.

➡ $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$

$\therefore \begin{vmatrix} 3x+a & x & x \\ 3x+a & x+a & x \\ 3x+a & x & x+a \end{vmatrix} = 0 \ (c_1 \to c_1 + c_2 + c_3)$

$\therefore (3x+a) \begin{vmatrix} 1 & x & x \\ 1 & x+a & x \\ 1 & x & x+a \end{vmatrix} = 0$

$\therefore (3x+a) \begin{vmatrix} 0 & -a & 0 \\ 1 & x+a & x \\ 1 & x & x+a \end{vmatrix} = 0 \ (R_1 \to R_1 - R_2)$

$\therefore (3x+a)\ [a(x+a-x)] = 0$

$\therefore (3x+a)\ (a^2) = 0$

$\therefore 3x + a = 0$ અથવા $a^2 = 0$

$\therefore x = -\frac{a}{3}$ $\qquad \therefore a = 0$

જે શક્ય નથી. $[\because a \neq 0]$

9. સાબિત કરો કે $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$.

➡ ડા.બા. $= \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

$= abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

(c_1 માંથી a, c_2 માંથી b તથા c_3 માંથી c સામાન્ય લેતાં)

$= abc \begin{vmatrix} a & c & 0 \\ a+b & b & -2b \\ b & b+c & -2b \end{vmatrix} (c_3 \to c_3 - c_1 - c_2)$

$= -2b(abc) \begin{vmatrix} a & c & 0 \\ a+b & b & 1 \\ b & b+c & 1 \end{vmatrix}$

(c_3 માંથી $-2b$ લેતાં)

$= -2b(abc)\ \{a(b-b-c) - c(a+b-b)\}$

$= -2b(abc)\ (-ac - ac)$

$= -2b(abc)\ (-2ac)$

$= 4a^2\ b^2\ c^2$

= જ.બા.

10. $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$ નું મૂલ્ય શોધો.

➡ $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = \begin{vmatrix} 2(x+y) & y & x+y \\ 2(x+y) & x+y & x \\ 2(x+y) & x & y \end{vmatrix}$ $c_1 \to c_1 + c_2 + c_3$

$= 2(x+y)\begin{vmatrix} 1 & y & x+y \\ 1 & x+y & x \\ 1 & x & y \end{vmatrix}$ ($\because$ $2(x + y)$ને પ્રથમ સ્તંભમાંથી સામાન્ય લેતાં)

$= 2(x+y)\begin{vmatrix} 1 & y & x+y \\ 0 & x & -y \\ 0 & x-y & -x \end{vmatrix}$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$= 2(x + y)\ [x(-x) - (-y)(x - y)]$

$= 2(x + y)[-x^2 + xy - y^2]$

$= -2(x + y)\ (x^2 - xy + y^2)$

$= -2(x^3 + y^3)$

11. **નિશ્ચાયકના ગુણધર્મનો ઉપયોગ કરી સાબિત કરો કે** $\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$

➡ ડા.બા. $= \begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$

$= \begin{vmatrix} \alpha & \alpha^2 & \alpha+\beta+\gamma \\ \beta & \beta^2 & \alpha+\beta+\gamma \\ \gamma & \gamma^2 & \alpha+\beta+\gamma \end{vmatrix}$ $c_3 \to c_3 + c_2$

$= (\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \alpha^2 & 1 \\ \beta & \beta^2 & 1 \\ \gamma & \gamma^2 & 1 \end{vmatrix}$ ($\because c_3$ માંથી $a+b+r$ સામાન્ય લેતાં)

$= (\alpha+\beta+\gamma)\begin{vmatrix} \alpha-\beta & \alpha^2-\beta^2 & 0 \\ \beta & \beta^2 & 1 \\ \gamma & \gamma^2 & 1 \end{vmatrix}$

$= (\alpha+\beta+\gamma)(\alpha-\beta)\begin{vmatrix} 1 & \alpha-\beta & 0 \\ \beta & \beta^2 & 1 \\ \gamma & \gamma^2 & 1 \end{vmatrix}$ ($\because R_1$ માંથી $\alpha-\beta$ સામાન્ય લેતાં)

$= (\alpha+\beta+\gamma)(\alpha-\beta)\begin{vmatrix} 1 & \alpha-\beta & 0 \\ \beta-\gamma & \beta^2-\gamma^2 & 0 \\ \gamma & \gamma^2 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_3$

$$= (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)\begin{vmatrix} 1 & \alpha+\beta & 0 \\ 1 & \beta+\gamma & 0 \\ \gamma & \gamma^2 & 1 \end{vmatrix}$$

($\because$ R_2 માંથી $\beta - \gamma$ સામાન્ય લેતાં)

$= (\alpha + \beta + \gamma)(\alpha - \beta)(\beta - \gamma)\ [(\beta + \gamma) - (\alpha + \beta)]$

$= (\alpha + \beta + \gamma)(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$

$= (\beta - \gamma)(\gamma - \alpha)(\alpha - \beta)(\alpha + \beta + \gamma) =$ જ.બા.

12. નિશ્ચાયકના ગુણધર્મનો ઉપયોગ કરી સાબિત કરો કે $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+px^3 \\ z & z^2 & 1+px^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x).$

➡ $$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+px^3 \\ z & z^2 & 1+px^3 \end{vmatrix} = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & px^3 \\ y & y^2 & px^3 \\ z & z^2 & px^3 \end{vmatrix}$$

$$= \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + pxyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$ [$\because$ c_3 માંથી P અને R_1, R_2, R_3 માંથી અનુક્રમે x, y, z સામાન્ય લેતાં]

$$= -1\begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} + pxyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \ (a \leftrightarrow c_3)$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + p(xyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \ (c_2 \leftrightarrow c_3)$$

$$= (1 + pxyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$= (1+pxyz)\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix} \quad \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$$

$$= (1+pxyz)(x-y)(y-z)\begin{vmatrix} 0 & 1 & x+y \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix}$$

$= (1 + pxyz)(x - y)(y - z)((y + z) - (x + y))$

$= (1 + pxyz)\ (x - y)(y - z)(z - x)$

13. નિશ્ચાયકના ગુણધર્મનો ઉપયોગ કરી સાબિત કરો કે $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca).$

➡ ડા.બા. $= \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} c_1 \to c_1 + c_2 + c_3$$

$$= (a+b+c) \begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$ ($\because$ c_1 માંથી $a + b + c$ સામાન્ય લેતાં)

$$= (a+b+c) \begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & a+b \\ 0 & a-c & a+2c \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$= (a + b + c)[(2b + a)(a + 2c) - (a - b)(a - c)$

$= (a + b + c)[2ab + 4bc + a^2 + 2ac - a^2 + ac + ab - bc]$

$= (a + b + c)(3ab + 3bc + 3ac)$

$= 3(a + b + c)(ab + bc + ac)$

= જ.બા.

14. નિશ્ચાયકના ગુણધર્મનો ઉપયોગ કરી સાબિત કરો કે $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1.$

➡ ડા.બા. $= \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix}$

$$= \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 3 & 7+3p \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$= 1[(7 + 3p) - 3(2 + p)]$

$= 7 + 3p - 6 - 3p$

$= 1$ = જ.બા.

15. નિશ્ચાયકના ગુણધર્મનો ઉપયોગ કરી સાબિત કરો કે $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0.$

➡ ડા.બા. $= \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix}$

$$= \begin{vmatrix} \sin\alpha & \cos\alpha\cos\delta - \sin\alpha\sin\delta & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta\cos\delta - \sin\beta\sin\delta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma\cos\delta - \sin\gamma\sin\delta & \cos(\gamma+\delta) \end{vmatrix}$$

$[c_2 \to \cos\delta\ c_2 - \sin\delta\ c_1]$

$$= \begin{vmatrix} \sin\alpha & \cos(\alpha+\delta) & \cos(\alpha+\delta) \\ \sin\beta & \cos(\beta+\delta) & \cos(\beta+\delta) \\ \sin\gamma & \cos(\gamma+\delta) & \cos(\gamma+\delta) \end{vmatrix}$$

$= 0 \quad [\because c_2 = c_3]$

= જ.બા.

16. સાબિત કરો કે, $\begin{vmatrix} a+1 & 1 & 1 \\ 1 & b+1 & 1 \\ 1 & 1 & c+1 \end{vmatrix} = abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + 1\right).$

➡ **સ્વપ્રયત્ને**

17. જો $a \neq b \neq c$ તથા $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$ હોય તો સાબિત કરો કે, $abc(ab + bc + ca) = a + b + c$.

➠ સ્વપ્રયત્ને

18. સાબિત કરો કે, $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ 2\sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix} = 5\sqrt{3}(\sqrt{6} - 5)$.

➠ સ્વપ્રયત્ને

19. સાબિત કરો કે, $\begin{vmatrix} x^2 & y^2 & z^2 \\ (x+1)^2 & (y+1)^2 & (z+1)^2 \\ (x-1)^2 & (y-1)^2 & (z-1)^2 \end{vmatrix} = -4(x-y)(y-z)(z-x)$.

➠ સ્વપ્રયત્ને

20. સાબિત કરો કે, $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a + b + c)^3$.

➠ સ્વપ્રયત્ને

21. ઉકેલો : $\begin{vmatrix} 1+x & 1-x & 1-x \\ 1-x & 1+x & 1-x \\ 1-x & 1-x & 1+x \end{vmatrix} = 0$.

➠ $x = 0, 3$

22. જો $p \neq a, q \neq b, r \neq c$ અને $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$ હોય તો સાબિત કરો કે, $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 0$.

➠ સ્વપ્રયત્ને

23. જો $a + b + c = 0$ હોય તો $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ માટે x નું મૂલ્ય મેળવો.

➠ $x = 0, \pm\sqrt{\frac{3}{2}(a^2 + b^2 + c^2}$

24. ઉકેલો : $\begin{vmatrix} 4x & 6x+2 & 8x+1 \\ 6x+2 & 9x+3 & 12x \\ 8x+1 & 12x & 16x+2 \end{vmatrix} = 0$.

➠ $x = \frac{11}{97}$

25. જો $A + B + C = \pi$ હોય તો $\begin{vmatrix} \sin^2 A & \sin A \cos A & \cos^2 A \\ \sin^2 B & \sin B \cos B & \cos^2 B \\ \sin^2 C & \sin C \cos C & \cos^2 C \end{vmatrix}$ નું મૂલ્ય મેળવો.

➠ $\sin(A - B) \sin(C - A) \sin(C - B)$

26. ત્રણ અંકની સંખ્યાઓ A28, 3B9 તથા 52C છે. જ્યાં A, B, C એ 0 અને 9 વચ્ચેનો પૂર્ણાંક છે. જો આ બધી જ

સંખ્યાઓ પૂર્ણાંક K વડે વિભાજ્ય હોય તો સાબિત કરો કે, $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ નિશ્ચાયક પણ K વડે વિભાજ્ય છે.

➡ સ્વપ્રયત્ને

27. જો a, b અને c વાસ્તવિક સંખ્યાઓ હોય, અને $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$ હોય તો સાબિત કરો કે $a + b + c = 0$ અથવા $a = b = c$.

➡ $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$

$= \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} \; c_1 \to c_1 + c_2 + c_3$

$= 2(a+b+c) \begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix}$

$= 2(a+b+c) \begin{vmatrix} 1 & c+a & a+b \\ 0 & a-c & b-a \\ 0 & b-a & c-b \end{vmatrix} \; \begin{matrix} R_2 \to R_2 - R_3 \\ R_3 \to R_3 - R_1 \end{matrix}$

$= 2(a + b + c)\ [(a - c)(c - d) - (b - a)^2]$

$= 2(a + b + c)\ (ac - ab - c^2 + bc - a^2 - b^2 + 2ab)$

$= 2(a + b + c)\ (-a^2 - b^2 - c^2 + ab + bc + ac)$

$= -(a + b + c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac)$

$= -(a + b + c)[(a + b)^2 + (b - c)^2 + (c - a)^2]$

હવે $\Delta = 0$

$\therefore -(a + b + c)\ [(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$

$\therefore a + b + c = 0$ અથવા $(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

$\therefore a + b + c = 0$ અથવા $a = b, b = c, c = a$

$\therefore a + b + c = 0$ અથવા $a = b = c$

28. જો $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ અને $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ તો $(AB)^{-1}$ શોધો.

➡ $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$\therefore |B| = 1(3 - 0) - 2(-1 - 0) - 2(2 - 0)$

$= 3 + 2 - 4$

$= 1 \neq 0$

$\therefore B^{-1}$ નું અસ્તિત્વ છે.

$B_{11} = (-1)^{1+1}\, 3 = 3$ | $B_{21} = (-1)^{2+1}(-2) = 2$

$B_{12} = (-1)^{1+2}(-1) = 1$ | $B_{22} = (-1)^{2+2}\,(1) = 1$

$B_{13} = (-1)^{1+3}\,(2) = 2$ | $B_{23} = (-1)^{2+3}(-2) = 2$

$B_{31} = (-1)^{3+1}\,(+6) = +6$

$B_{32} = (-1)^{3+2}\,(-2) = 2$

$B_{33} = (-1)^{3+3}\, 5 = 5$

$$\therefore adj\ B = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|}\, adj\ B = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

હવે $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

29. $\mathbf{A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}}$ **માટે ચકાસો કે : $[adj\ A]^{-1} = adj\ (A^{-1})$**

➡ $$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$|A| = 1(15 - 1) + 2(-10 - 1) + 1(-2 - 3)$

$= 14 - 22 - 5$

$= -13 \neq 0$

$\therefore A^{-1}$ નું અસ્તિત્વ છે.

$A_{11} = (-1)^{1+1}\, 14 = 14$ | $A_{21} = (-1)^{2+1}(-11) = 11$

$A_{12} = (-1)^{1+2}(-11) = 11$ | $A_{22} = (-1)^{2+2}\,(4) = 4$

$A_{13} = (-1)^{1+3}\,(-5) = -5$ | $A_{23} = (-1)^{2+3}(3) = -3$

$A_{31} = (-1)^{3+1}\,(-5) = -5$

$A_{32} = (-1)^{3+2}\,(3) = -3$

$A_{33} = (-1)^{3+3}\,(-1) = -1$

$$\therefore adj\ A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\, adj\ A = -\frac{1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

ધારો કે $B = adj\ A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

$|B| = 14(-4 - 9) - 11(-11 - 15) - 5(-33 + 20)$
$= -182 + 286 + 65$
$= 169 \neq 0$

$B_{11} = (-1)^{1+1}(-13) = -13$ | $B_{21} = (-1)^{2+1}(-26) = 26$
$B_{12} = (-1)^{1+2}(-26) = 26$ | $B_{22} = (-1)^{2+2}(-39) = -39$
$B_{13} = (-1)^{1+3}(-13) = -13$ | $B_{23} = (-1)^{2+3}(13) = -13$
$B_{31} = (-1)^{3+1}(-13) = -13$
$B_{32} = (-1)^{3+2}(13) = -13$
$B_{33} = (-1)^{3+3}(-65) = -65$

$\therefore adj\ B = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$

➥ $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

$|A| = 1(15 - 1) + 2(-10 - 1) + 1(-2 - 3)$
$= 14 - 22 - 5$
$= -13 \neq 0$

$\therefore A^{-1}$ નું અસ્તિત્વ છે.

$A_{11} = (-1)^{1+1}\ 14 = 14$ | $A_{21} = (-1)^{2+1}(-11) = 11$
$A_{12} = (-1)^{1+2}(-11) = 11$ | $A_{22} = (-1)^{2+2}(4) = 4$
$A_{13} = (-1)^{1+3}(-5) = -5$ | $A_{23} = (-1)^{2+3}(3) = -3$
$A_{31} = (-1)^{3+1}(-5) = -5$
$A_{32} = (-1)^{3+2}(3) = -3$
$A_{33} = (-1)^{3+3}(-1) = -1$

$\therefore adj\ A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj\ A = -\frac{1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

ધારો કે $B = adj\ A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

$|B| = 14(-4 - 9) - 11(-11 - 15) - 5(-33 + 20)$
$= -182 + 286 + 65$
$= 169 \neq 0$

$B_{11} = (-1)^{1+1}(-13) = -13$ | $B_{21} = (-1)^{2+1}(-26) = 26$
$B_{12} = (-1)^{1+2}(-26) = 26$ | $B_{22} = (-1)^{2+2}(-39) = -39$
$B_{13} = (-1)^{1+3}(-13) = -13$ | $B_{23} = (-1)^{2+3}(13) = -13$
$B_{31} = (-1)^{3+1}(-13) = -13$
$B_{32} = (-1)^{3+2}(13) = -13$
$B_{33} = (-1)^{3+3}(-65) = -65$

$\therefore adj\ B = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$

30. $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ **માટે ચકાસો કે : $(A^{-1})^{-1} = A$**

➡ ધારો કે $B = A^{-1} = -\frac{1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

$$\therefore B = \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$$

$$|B| = -\frac{14}{13}\left(-\frac{4}{169} - \frac{9}{169}\right) + \frac{11}{13}\left(-\frac{11}{169} - \frac{15}{169}\right) + \frac{5}{13}\left(-\frac{33}{169} + \frac{20}{169}\right)$$

$$= -\frac{14}{13}\left(-\frac{13}{169}\right) + \frac{11}{13}\left(-\frac{26}{169}\right) + \frac{5}{13}\left(-\frac{13}{169}\right)$$

$$= +\frac{14}{169} - \frac{22}{169} - \frac{5}{169}$$

$$= -\frac{13}{169}$$

$$= -\frac{1}{13}$$

$$adj\ B = adj\begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$$

$$= \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & 65 \end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & 5 \end{bmatrix}$$

➡ ધારો કે $B = A^{-1} = -\frac{1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$

$$\therefore B = \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$$

$$|B| = -\frac{14}{13}\left(-\frac{4}{169} - \frac{9}{169}\right) + \frac{11}{13}\left(-\frac{11}{169} - \frac{15}{169}\right) + \frac{5}{13}\left(-\frac{33}{169} + \frac{20}{169}\right)$$

$$= -\frac{14}{13}\left(-\frac{13}{169}\right) + \frac{11}{13}\left(-\frac{26}{169}\right) + \frac{5}{13}\left(-\frac{13}{169}\right)$$

$$= +\frac{14}{169} - \frac{22}{169} - \frac{5}{169}$$

$$= -\frac{13}{169}$$

$$= -\frac{1}{13}$$

$$adj\ B = adj\begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$$

$$= \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & 65 \end{bmatrix}$$

$$= \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & 5 \end{bmatrix}$$

31. નીચેની સમીકરણ સંહતિનો ઉકેલ મેળવો : $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

➥ આપેલ સમીકરણ સંહતિને શ્રેણિક સ્વરૂપે દર્શાવતાં, AX = B

જ્યાં $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$ $X = \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix}$ તથા $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$|A| = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$= 150 + 330 + 720$

$= 1200 \neq 0$

$\therefore A^{-1}$ નું અસ્તિત્વ છે.

$A_{11} = (-1)^{1+1}\ 75 = 75$ | $A_{21} = (-1)^{2+1}(-150) = 150$

$A_{12} = (-1)^{1+2}(-110) = 110$ | $A_{22} = (-1)^{2+2}(-100) = -100$

$A_{13} = (-1)^{1+3}\ 72 = 72$ | $A_{23} = (-1)^{2+3}\ 0 = 0$

$A_{31} = (-1)^{3+1}\ 75 = 75$

$A_{32} = (-1)^{3+2}\ (-30) = 30$

$A_{33} = (-1)^{3+3}\ (-24) = -24$

$$\therefore adj\ A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj\ A$$

$$= \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

હવે AX = B

$\therefore A^{-1}AX = A^{-1}B$

$\therefore X = A^{-1}B$ $\quad (\because A^{-1}A = I,\ IX = X)$

➥ આપેલ સમીકરણ સંહતિને શ્રેણિક સ્વરૂપે દર્શાવતાં, AX = B

જ્યાં $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$ $X = \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix}$ તથા $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$|A| = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$= 150 + 330 + 720$

$= 1200 \neq 0$

$\therefore A^{-1}$ નું અસ્તિત્વ છે.

$A_{11} = (-1)^{1+1} 75 = 75$ | $A_{21} = (-1)^{2+1}(-150) = 150$

$A_{12} = (-1)^{1+2}(-110) = 110$ | $A_{22} = (-1)^{2+2}(-100) = -100$

$A_{13} = (-1)^{1+3} 72 = 72$ | $A_{23} = (-1)^{2+3} 0 = 0$

$A_{31} = (-1)^{3+1}\ 75 = 75$

$A_{32} = (-1)^{3+2}\ (-30) = 30$

$A_{33} = (-1)^{3+3}\ (-24) = -24$

$$\therefore adj\ A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\, adj\, A$$

$$= \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

હવે $AX = B$

$\therefore A^{-1}AX = A^{-1}B$

$\therefore X = A^{-1}B \qquad (\because A^{-1}A = I,\ IX = X)$

32. **દર્શાવો કે** $$\begin{vmatrix} \sum_{r=1}^{16} 2^r & a & 2^{16}-1 \\ 3\sum_{r=1}^{16} 4^r & b & 2(4^{16}-1) \\ 7\sum_{r=1}^{16} 8^r & c & 4(8^{16}-1) \end{vmatrix} = 0.$$

➠ **સ્વપ્રયત્ને**

33. **જો** $a_1, a_2, a_3, \ldots.\ a_r$ **સમગુણોત્તર શ્રેણીમાં હોય તો દર્શાવો કે,** $\begin{vmatrix} a_{r+1} & a_{r+5} & a_{r+9} \\ a_{r+7} & a_{r+11} & a_{r+15} \\ a_{r+11} & a_{r+7} & a_{r+21} \end{vmatrix}$ **એ** r **થી સ્વતંત્ર છે.**

➠ **સ્વપ્રયત્ને**

34. $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ **તથા** $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ **હોય તો AB મેળવો. તેનો ઉપયોગ કરી સમીકરણ સંહતિ**

$$\left.\begin{aligned} x + y + 2z &= 1 \\ 3x + 2y + z &= 7 \\ 2x + y + 3z &= 2 \end{aligned}\right\}$$ **નો ઉકેલ મેળવો.**

➠ $x = 2,\ y = 1,\ z = -1$

35. **સાબિત કરો કે,** $\begin{vmatrix} yz - x^2 & zx - y^2 & xz - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix} = \begin{vmatrix} r^2 & U^2 & U^2 \\ U^2 & r^2 & U^2 \\ U^2 & U^2 & r^2 \end{vmatrix}$ **જ્યાં** $r^2 = x^2 + y^2 + z^2$ **તથા**

$U^2 = xy + yz + zx$. **(Hint : Use** $|adj\ A| = |A|^2$**)**

➠ **સ્વપ્રયત્ને**