CHAPTER – 3

SQUARES AND SQUARE ROOTS

Exercise 3.1

1. Which of the following natural numbers are perfect squares? Give reasons in support of your answer.

(i) 729

(ii) 5488

(iii) 1024

(iv) 243

Solution:

(i) 729

We know that

3	729
3	243
3	81
3	27
3	9
3	3
	1

It can be written as

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

Here

729 is the product of pairs of equal prime factors

Therefore, 729 is a perfect square.

(ii) 5488

We know that

2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

It can be written as

$5488 = 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7$

Here

After pairing the same prime factors, one factor 7 is left unpaired.

Therefore, 5488 is not a perfect square.

(iii) 1024

We know that

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

It can be written as

$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Here

After pairing the same prime factors, there is no factor left.

Therefore, 1024 is a perfect square.

(iv) 243

We know that

3	243
3	81
3	27
3	9
3	3
	1

It can be written as

$243 = 3 \times 3 \times 3 \times 3 \times 3$

Here

After pairing the same prime factors, factor 3 is left unpaired.

Therefore, 243 is not a perfect square.

2. Show that each of the following numbers is a perfect square. Also, find the number whose square is the given number.

(i) 1296

(ii) 1764

(iii) 3025

(iv) 3969

Solution:

(i) 1296

We know that

2	1294
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

It can be written as

$1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Here

After pairing the same prime factors, no factor is left.

Therefore, 1296 is a perfect square of $2 \times 2 \times 3 \times 3 = 36$.

(ii) 1764

We know that

2	1764
2	882
7	441
7	63
3	9
3	3
	1

It can be written as

1764 = 2 × 2 × 3 × 3 × 7 × 7

Here

After pairing the same factors, no factor is left.

Therefore, 1764 is a perfect square of 2 × 3 × 7 = 42.

(iii) 3025

We know that

2	3025
2	605
11	121
11	11
	1

It can be written as

3025 = 5 × 5 × 11 × 11

Here

After pairing the same prime factors, no factor is left.

Therefore, 3025 is a perfect square of 5 × 11 = 55.

(iv) 3969

We know that

3	3969
3	1323
3	441
3	147
7	49
7	7
	1

It can be written as

$3969 = 3 \times 3 \times 3 \times 3 \times 7 \times 7$

Here

After pairing the same prime factors, no factor is left.

Therefore, 3969 is a perfect square of $3 \times 3 \times 7 = 63$.

3. Find the smallest natural number by which 1008 should be multiplied to make it a perfect square.

Solution:

We know that

2	1008
2	504
2	252
2	126
3	63
3	21
7	7
	1

It can be written as

$1008 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$

Here

After pairing the same kind of prime factors, one factor 7 is left.

Now multiplying 1008 by 7

We get a perfect square

Therefore, the required smallest number is 7.

4. Find the smallest natural number by which 5808 should be divided to make it a perfect square. Also, find the number whose square is the resulting number.

Solution:

We know that

2	5808
2	904
2	1452
2	726
3	363
11	121
11	11
	1

It can be written as

$5808 = 2 \times 2 \times 2 \times 2 \times 3 \times 11 \times 11$

Here

After pairing the same kind of prime factors, factor 3 is left.

Now dividing the number by 3, we get a perfect square.

Therefore, the square root of the resulting number is $2 \times 2 \times 11 = 44$.

Exercise 3.2

1. Write five numbers which you can decide by looking at their one's digit that they are not square numbers. Solution:

We know that

A number which ends with the digits 2, 3, 7 or 8 at its unit places is not a perfect square.

Example – 111, 372, 563, 978, 1282 are not square numbers.

2. What will be the unit digit of the squares of the following numbers?

(i) 951

(ii) 502

(iii) 329

(iv) 643

(v) 5124

(vi) 7625

(vii) 68327

(viii) 95628

(ix) 99880

(x) 12796

Solution:

(i) 951

The unit digit of the square is 1.

(ii) 502

The unit digit of the square is 4.

(iii) 329

The unit digit of the square is 1.

(iv) 643

The unit digit of the square is 9.

(v) 5124

The unit digit of the square is 6.

(vi) 7625

The unit digit of the square is 5.

(vii) 68327

The unit digit of the square is 9.

(viii) 95628

The unit digit of the square is 4.

(ix) 99880

The unit digit of the square is 0.

(x) 12796

The unit digit of the square is 6.

3. The following numbers are obviously not perfect. Give reason.

(i) 567

(ii) 2453

(iii) 5298

(iv) 46292

(v) 74000

Solution:

In the given numbers

If the square of a number does not have 2, 3, 7, 8 or 0 as its unit digit, the squares 567, 2453, 5208, 46292 and 74000 cannot be the perfect squares as they have 7, 2, 8, 2 digits at the unit place.

4. The square of which of the following numbers would be an odd number or an even number? Why?

(i) 573

(ii) 4096

(iii) 8267

(iv) 37916

Solution:

We know that

The square of an odd number is odd and a square of an even number is even.

So 573 and 8262 are odd numbers and their squares will be an odd number.

4096 and 37916 are even numbers and their square will also be even number.

5. How many natural numbers lie between square of the following numbers?

(i) 12 and 13

(ii) 90 and 91

Solution:

(i) We know that

No. of natural numbers between the squares of 12 and 13 = $(13^2 – 12^2) – 1$

By further calculation

= (13 + 12 – 1)

So we get

= 25 – 1

= 24

(ii) We know that

No. of natural numbers between the squares of 90 and 91 = $(91^2 – 90^2) – 1$

By further calculation

= (91 + 90 – 1)

So we get

= 181 – 1

= 180

6. Without adding, find the sum.

(i) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15

(ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29

Solution:

(i) We know that

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = n^2$

Here n = 8

So the sum = $8^2 = 64$

(ii) We know that

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 = n^2$

Here n = 15

So the sum = $15^2 = 225$

7. (i) Express 64 as the sum of 8 odd numbers.

(ii) 121 as the sum of 11 odd numbers.

Solution:

(i) We know that

64 as the sum of 8 odd numbers = $8^2 = n^2$

Here n = 8

It can be written as

$= 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$

(ii) We know that

121 as the sum of 11 odd numbers = $11^2 = n^2$

Here n = 11

It can be written as

$= 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$

8. Express the following as the sum of two consecutive integers.

(i) 19^2

(ii) 33^2

(iii) 47^2

Solution:

We know that

$n^2 = \frac{(n^2-1)}{2} + \frac{(n^2+1)}{2}$ is the sum of two consecutive integers when n is odd

(i) $19^2 = \frac{(19^2-1)}{2} + \frac{(19^2+1)}{2}$

We know that

$19^2 = 361$

By further calculation

$= \frac{(361-1)}{2} + \frac{(361+1)}{2}$

So we get

= 180 + 181

(ii) $33^2 = \frac{(33^2-1)}{2} + \frac{(33^2+1)}{2}$

We know that

$33^2 = 1089$

By further calculation

$= \frac{(1089-1)}{2} + \frac{(1089+1)}{1}$

So we get

$= \frac{1088}{2} + \frac{1090}{2}$

= 544 + 545

(iii) $47^2 = \frac{(47^2 - 1)}{2} + \frac{(47^2 + 1)}{2}$

We know that

$47^2 = 2209$

By further calculation

$= \frac{(2209 - 1)}{2} + \frac{(2209 + 1)}{1}$

So we get

$= \frac{2208}{2} + \frac{2210}{2}$

= 1104 + 1105

9. Find the squares of the following numbers without actual multiplication:

(i) 31

(ii) 42

(iii) 86

(iv) 94

Solution:

We know that

$(a + b)^2 = a^2 + 2ab + b^2$

(i) $31^2 = (30 + 1)^2$

We can write it as

$= 30^2 + 2 \times 30 \times 1 + 1^2$

By further calculation

$= 900 + 60 + 1$

$= 961$

(ii) $42^2 = (40 + 2)^2$

We can write it as

$= 40^2 + 2 \times 40 \times 2 + 2^2$

By further calculation

$= 1600 + 160 + 4$

$= 1764$

(iii) $86^2 = (80 + 6)^2$

We can write it as

$= 80^2 + 2 \times 80 \times 6 + 6^2$

By further calculation

$= 6400 + 960 + 36$

$= 7396$

(iv) $94^2 = (90 + 4)^2$

We can write it as

$= 90^2 + 2 \times 90 \times 4 + 4^2$

By further calculation

$= 8100 + 720 + 16 = 8836$

10. Find the squares of the following numbers containing 5 in unit's place:

(i) 45

(ii) 305

(iii) 525

Solution:

(i) $45^2 = n5^2$

It can be written as

$= n\ (n + 1)$ hundred $+ 5^2$

Substituting the values

$= 4 \times 5$ hundred $+ 25$

By further calculation

$= 2000 + 25$

$= 2025$

(ii) $305^2 = (30 \times 31)$ hundred $+ 25$

By further calculation

$= 93000 + 25$

$= 93025$

(iii) $525^2 = (52 \times 53)$ hundred $+ 25$

By further calculation

$= 275600 + 25$

$= 275625$

11. Write a Pythagorean triplet whose one number is

(i) 8

(ii) 15

(iii) 63

(iv) 80

Solution:

(i) 8

Take n = 8

So the triplet will be 2n, $n^2 - 1$, $n^2 + 1$

Here

If 2n = 8, then $n = \frac{8}{2} = 4$

Substituting the values

$n^2 - 1 = 4^2 - 1 = 16 - 1 = 15$

$n^2 + 1 = 4^2 + 1 = 16 + 1 = 17$

Therefore, the triplets are 8, 15 and 17.

(ii) 15

Take 2n = 15

So $n = \frac{n}{2}$ is not possible

$n^2 - 1 = 15$

By further calculation

$n^2 = 15 + 1 = 16 = 4^2$

n = 4

So we get

$2n = 2 \times 4 = 8$

$n^2 - 1 = 15$

$n^2 + 1 = 4^2 + 1 = 16 + 1 = 17$

Therefore, the triplets are 8, 15 and 17.

(iii) 63

Take $n^2 – 1 = 63$

By further calculation

$n^2 = 63 + 1 = 64 = 8^2$

So $n = 8$

Here

$2n = 2 \times 8 = 16$

$n^2 – 1 = 63$

$n^2 + 1 = 8^2 + 1 = 64 + 1 = 65$

Therefore, the triplets are 16, 63 and 65.

(iv) 80

Take $2n = 80$

$n = \frac{80}{2} = 40$

Here

$n^2 – 1 = 40^2 – 1 = 1600 – 1 = 1599$

$n^2 + 1 = 40^2 + 1 = 1600 + 1 = 1601$

Therefore, the triplets are 80, 1599 and 1601.

12. Observe the following pattern and find the missing digits:

$21^2 = 441$

$201^2 = 40401$

$2001^2 = 4004001$

$20001^2 = 4 – 4 – 1$

$200001^2 =$ ——

Solution:

$21^2 = 441$

$201^2 = 40401$

$2001^2 = 4004001$

$20001^2 = 400040001$

$200001^2 = 40000400001$

13. Observe the following pattern and find the missing digits:

$9^2 = 81$

$99^2 = 9801$

$999^2 = 998001$

$9999^2 = 99980001$

$99999^2 =$ 9—8—01

$999999^2 =$ 9—0—1

Solution:

$9^2 = 81$

$99^2 = 9801$

$999^2 = 998001$

$9999^2 = 99980001$

$99999^2 = 9999800001$

$999999^2 = 999998000001$

14. Observe the following pattern and find the missing digits:

$7^2 = 49$

$67^2 = 4489$

$667^2 = 444889$

$6667^2 = 44448889$

66667^2 **= 4—-8—–9**

666667^2 **= 4—-8—-8-**

Solution:

$7^2 = 49$

$67^2 = 4489$

$667^2 = 444889$

$6667^2 = 44448889$

$66667^2 = 4444488889$

$666667^2 = 444444888889$

Exercise 3.3

1. By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect squares or not? If the number is a perfect square then find its square root:

(i) 121

(ii) 55

(iii) 36

(iv) 90

Solution:

(i) We know that

Square root of 121

121 – 1 = 120

120 – 3 = 117

117 – 5 = 112

112 – 7 = 105

105 – 9 = 96

96 – 11 = 85

85 – 13 = 72

72 – 15 = 57

57 – 17 = 40

40 – 19 = 21

21 – 21 = 0

So the square root of 121 is 11

Hence, 121 is a perfect square.

(ii) We know that

Square root of 55

55 – 1 = 54

54 – 3 = 51

51 – 5 = 46

46 – 7 = 39

39 – 9 = 30

30 – 11 = 19

19 – 13 = 6

6 – 15 = – 9 is not possible

Hence, 55 is not a perfect square.

(iii) We know that

Square root of 36

36 – 1 = 35

35 – 3 = 32

32 – 5 = 27

27 – 7 = 20

20 – 9 = 11

11 – 11 = 0

Hence, 36 is a perfect square and its square root is 6.

(iv) We know that

Square root of 90

90 – 1 = 89

89 – 3 = 86

86 – 5 = 81

81 – 7 = 74

74 – 9 = 65

65 – 11 = 54

54 – 13 = 41

41 – 15 = 26

26 – 17 = 9

9 – 19 = – 10 which is not possible

Hence, 90 is not a perfect square.

2. Find the square roots of the following numbers by prime factorization method:

(i) 784

(ii) 441

(iii) 1849

(iv) 4356

(v) 6241

(vi) 8836

(vii) 8281

(viii) 9025

Solution:

(i) We know that

Square root of 784

2	784
2	392
2	196
2	98
7	49
7	7
	1

It can be written as

$\sqrt{784} = \sqrt{2 \times 2 \times 2 \times 2 \times 7 \times 7}$

So we get

$= 2 \times 2 \times 7$

= 28

(ii) We know that

Square root of 441

3	441
7	343
7	49
7	7
	1

It can be written as

$\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7}$

So we get

= 3 × 7

= 21

(iii) We know that

Square root of 1849

43	1849
43	43
	1

It can be written as

$\sqrt{1849} = \sqrt{43 \times 43}$ = 43

(iv) We know that

Square root of 4356

2	4356
2	2178
3	1089
3	363
11	121
11	11
	1

It can be written as

$\sqrt{4356} = \sqrt{2 \times 2 \times 3 \times 3 \times 11 \times 11}$

So we get

= 2 × 3 × 11

= 66

(v) We know that

Square root of 6241

79	6241
79	79
	1

It can be written as

$\sqrt{6241} = \sqrt{79 \times 79}$ = 79

(vi) We know that

Square root of 8836

2	8836
2	4418
47	2209
47	47
	1

It can be written as

$\sqrt{8836} = \sqrt{2 \times 2 \times 47 \times 47}$

So we get

= 2 × 47

= 94

(vii) We know that

Square root of 8281

7	8281
7	1183
13	169
13	13
	1

It can be written as

$\sqrt{8281} = \sqrt{7 \times 7 \times 13 \times 13}$

So we get

= 7 × 13

= 91

(viii) We know that

Square root of 9025

5	9025
5	1805
19	361
19	19
	1

It can be written as

$\sqrt{9025} = \sqrt{5 \times 5 \times 19 \times 19}$

So we get

= 5 × 19

= 95

3. Find the square roots of the following numbers by prime factorization method:

(i) $\mathbf{9\frac{67}{121}}$

(ii) $\mathbf{17\frac{13}{36}}$

(iii) 1.96

(iv) $\mathbf{0.0064}$

Solution:

(i) $9\frac{67}{121} = \frac{(9 \times 121 + 67)}{121}$

By further calculation

$= \frac{(1089 + 67)}{121}$

$= \frac{1156}{121}$

By squaring we get

$\sqrt{\frac{1156}{121}} = \frac{\sqrt{1156}}{\sqrt{121}}$

We know that

2	1156
2	578
17	289
17	17
	1

It can be written as

$= \frac{\sqrt{2\times2\times17\times17}}{\sqrt{11\times11}}$

So we get

$= \frac{(2 \times 17)}{11}$

$= \frac{34}{11}$

$= 3\,\frac{1}{11}$

(ii) $17\,\frac{13}{36} = \frac{(17 \times 36 + 13)}{36}$

By further calculation

$= \frac{(612 + 13)}{36}$

$= \frac{625}{36}$

By squaring we get

$\sqrt{\frac{625}{36}} = \frac{\sqrt{625}}{\sqrt{36}}$

We know that

5	625	2	36
5	125	2	18
5	25	3	9
5	5	3	3
	1		1

It can be written as

$= \frac{\sqrt{5\times5\times5\times5}}{\sqrt{2\times2\times3\times3}}$

So we get

$= \frac{(5\times5)}{(2\times3)}$

$= \frac{25}{6}$

$= 4\frac{1}{6}$

(iii) $1.96 = \frac{196}{100}$

By squaring we get

$\sqrt{\frac{196}{100}} = \frac{\sqrt{196}}{\sqrt{100}}$

We know that

2	196
2	98
7	49
7	7
	1

2	100
2	50
5	25
5	5
	1

It can be written as

$= \frac{\sqrt{2\times2\times7\times7}}{\sqrt{2\times2\times5\times5}}$

So we get

$= \frac{(2\times7)}{(2\times5)}$

$= \frac{14}{10}$

= 1.4

(iv) $0.0064 = \frac{64}{10000}$

By squaring we get

$\sqrt{\frac{64}{10000}} = \frac{\sqrt{64}}{\sqrt{10000}}$

We know that

2	10000
2	5000
2	2500
2	12500
5	625
5	125
5	25
5	5
	1

It can be written as

$$= \frac{\sqrt{2\times2\times2\times2\times2\times2}}{\sqrt{2\times2\times2\times2\times5\times5\times5\times5}}$$

So we get

$$= \frac{(2 \times 2 \times 2)}{(2 \times 2 \times 5 \times 5)}$$

$$= \frac{8}{100}$$

= 0.08

4. For each of the following numbers, find the smallest natural number by which it should be multiplied so as to get a perfect square. Also, find the square root of the square number so obtained:

(i) 588

(ii) 720

(iii) 2178

(iv) 3042

(v) 6300

Solution:

(i) 588 = 2 × 2 × 3 × 7 × 7

We know that

2	588
2	294
3	147
7	49
7	7
	1

By pairing the same kind of factors, one factor 3 is left unpaired.

So to make it a pair we must multiply it by 3

Required least number = 3

Square root of 588 × 3 = 1764

Here

2 × 3 × 7 = 42

(ii) 720 = 2 × 2 × 2 × 2 × 3 × 3 × 5

We know that

2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

By pairing the same kind of factors, one factor 5 is left unpaired.

So to make it a pair we must multiply it by 5

Required least number = 5

Square root of 720 × 5 = 3600

Here

2 × 2 × 3 × 5 = 60

(iii) 2178 = 2 × 3 × 3 × 11 × 11

We know that

2	2178
3	1089
3	363
11	121
11	11
	1

By pairing the same kind of factors, one factor 2 is left unpaired.

So to make it a pair we must multiply it by 2

Required least number = 2

Square root of 2178 × 2 = 4356

Here

2 × 3 × 11 = 66

(iv) 3042 = 2 × 3 × 3 × 13 × 13

We know that

2	3042
3	1521
3	507
13	169
13	13
	1

By pairing the same kind of factors, one factor 2 is left unpaired

So to make it a pair we must multiply it by 2

Required least number = 2

Square root of 3042 × 2 = 6084

Here

2 × 3 × 13 = 78

(v) 6300 = 2 × 2 × 3 × 3 × 5 × 5 × 7

We know that

2	6300
2	3150
3	1575
3	525
3	175
5	35
7	7
	1

By pairing the same kind of factors, one factor 7 is left unpaired

So to make it a pair we must multiply it by 7

Required least number = 7

Square root of 6300 × 7 = 44100

Here

2 × 3 × 5 × 7 = 210

5. For each of the following numbers, find the smallest natural number by which it should be divided so that this quotient is a perfect square. Also, find the square root of the square number so obtained:

(i) 1872

(ii) 2592

(iii) 3380

(iv) 16224

(v) 61347

Solution:

(i) $1872 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 13$

It can be written as

2	1872
2	936
2	468
2	234
3	117
3	39
13	13
	1

By pairing the same kind of factors, one factor 13 is left unpaired

Required least number = 13

The number 1872 should be divided by 13 so that the resultant number will be a perfect square

Resultant number = $1872 \div 13 = 144$

Square root = $2 \times 2 \times 3 = 12$

(ii) $2592 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

It can be written as

2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

By pairing the same kind of factors, one factor 2 is left unpaired

Required least number = 2

The number 2592 should be divided by 2 so that the resultant number will be a perfect square

Resultant number = 2592 ÷ 2 = 1296

Square root = 2 × 2 × 3 × 3 = 36

(iii) 3380 = 2 × 2 × 5 × 13 × 13

It can be written as

2	3380
2	1690
5	845
13	169
13	13
	1

By pairing the same kind of factors, one factor 5 is left unpaired

Required least number = 5

The number 3380 should be divided by 5 so that the resultant number will be a perfect square

Resultant number = 3380 ÷ 5 = 676

Square root = 2 × 13 = 26

(iv) 16224 = 2 × 2 × 2 × 2 × 2 × 3 × 13 × 13

It can be written as

2	16224
2	8112
2	4056
2	2028
2	1014
3	507
13	169
13	13
	1

By pairing the same kind of factors, two factors 2 and 3 is left unpaired

Required least number = 2 × 3 = 6

The number 16224 should be divided by 6 so that the resultant number will be a perfect square

Resultant number = 16224 ÷ 6 = 2704

Square root = 2 × 2 × 13 = 52

(v) 61347 = 3 × 11 × 11 × 13 × 13

It can be written as

3	61347
11	20449
11	1859
13	169
13	13
	1

By pairing the same kind of factors, one factor 3 is left unpaired

Required least number = 3

The number 61347 should be divided by 3 so that the resultant number will be a perfect square

Resultant number = 61347 ÷ 3 = 20449

Square root = 11 × 13 = 143

6. Find the smallest square number that is divisible by each of the following numbers:

(i) 3, 6, 10, 15

(ii) 6, 9, 27, 36

(iii) 4, 7, 8, 16

Solution:

(i) 3, 6, 10, 15

Number which is divisible by

3, 6, 10, 15 = LCM of 3, 6, 10, 15

It can be written as

2	3, 6, 10, 15
3	3, 3, 5, 15
5	1, 1, 5, 5
	1, 1, 1, 1,

So we get

$= 2 \times 3 \times 5$

$= 30$

(ii) 6, 9, 27, 36

Number which is divisible by

6, 9, 27, 36 = LCM of 6, 9, 27, 36

It can be written as

3	6, 9, 27, 36
3	2, 3, 9, 12
2	2, 1, 3, 4
	1, 1, 3, 2,

So we get

= 3 × 3 × 2 × 2 × 3

= 108

Here the smallest square

= 108 × 3

= 324

(iii) 4, 7, 8, 16

Number which is divisible by

4, 7, 8, 16 = LCM of 4, 7, 8, 16

It can be written as

2	4, 7, 8, 16
2	2, 7, 4, 8
2	1, 7, 2, 4
	1, 7, 1, 2,

So we get

= 2 × 2 × 2 × 2 × 7

= 112

Here the smallest square

= 112 × 7

= 784

7. 4225 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Solution:

It is given that

Total number of plants = 4225

Here

Number of rows = Number of plant in each row

So the number of rows = square root of 4225

It can be written as

5	4225
5	845
13	169
13	13
	1

So we get

$= \sqrt{5 \times 5 \times 13 \times 13}$

$= 5 \times 13$

$= 65$

Hence, the number of rows is 65 and the number of plants in each row is 65.

8. The area of rectangle is 1936 sq. m. If the length of the rectangle is 4 times its breadth, find the dimensions of the rectangle.

Solution:

It is given that

Area of rectangle = 1936 sq. m

Take breadth = x m

Length = 4x m

So we get

$4x^2 = 1936$

By further calculation

$x^2 = \frac{1936}{4} = 484$

We know that

2	484
2	242
11	121
11	11
	1

It can be written as

$$x = \sqrt{484} = \sqrt{2 \times 2 \times 11 \times 11}$$

By further calculation

= 2 × 11

= 22

Here

Length = 4x = 4 × 22 = 88 m

Breadth = x = 22 m

9. In a school a P.T. teacher wants to arrange 2000 students in the form of rows and columns for P.T. display. If the number of rows is equal to number of columns and 64 students could not be accommodated in this arrangement. Find the number of rows.

Solution:

It is given that

Total number of students in a school = 2000

The P.T. teacher arranges in such a way that

No. of rows = no. of students in each row

So 64 students are left

Required number of students = 2000 – 64 = 1936

No. of rows = $\sqrt{1936}$

We know that

2	1936
2	968
2	484
2	242
11	121
11	11
	1

It can be written as

$= \sqrt{2 \times 2 \times 2 \times 2 \times 11 \times 11}$

By further calculation

$= 2 \times 2 \times 11$

$= 44$

10. In a school, the students of class VIII collected ₹2304 for a picnic. Each student contributed as many rupees as the number of students in the class. Find the number of students in the class.

Solution:

It is given that

Amount collected for picnic = ₹2304

We know that

No. of students = no. of rupees contributed by each student = $\sqrt{2304}$

Here

2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

It can be written as

$= \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3}$

By further calculation

= 2 × 2 × 2 × 2 × 34= 48

Therefore, the number of students in class VIII is 4811.

11. The product of two numbers is 7260. If one number is 15 times the other number, find the numbers.

Solution:

It is given that

Product of two numbers = 7260

Consider one number = x

Second number = 15x

It can be written as

$15x \times x = 7260$

$15x^2 = 7260$

By further calculation

$x^2 = \frac{7260}{15} = 484$

$x = \sqrt{484} = \sqrt{2 \times 2 \times 11 \times 11}$

So we get

$= 2 \times 11$

$= 22$

Here

One number = 22

Second number = $22 \times 15 = 330$

12. Find three positive numbers in the ratio 2: 3: 5, the sum of whose squares is 950.

Solution:

It is given that

Ratio of three positive numbers = 2: 3: 5

Sum of their squares = 950

Consider

First number = 2x

Second number = 3x

Third number = 5x

It can be written as

$(2x)^2 + (3x)^2 + (5x)^2 = 950$

By further calculation

$4x^2 + 9x^2 + 25x^2 = 950$

$38x^2 = 950$

So we get

$x^2 = \frac{950}{38} = 25$

$x = \sqrt{25} = 5$

Here

First number = $2 \times 5 = 10$

Second number = $3 \times 5 = 15$

Third number = $5 \times 5 = 25$

13. The perimeter of two squares is 60 metres and 144 metres respectively. Find the perimeter of another square equal in area to the sum of the first two squares.

Solution:

It is given that

Perimeter of first square = 60 m

Side = $\frac{60}{4} = 15$ m

Perimeter of second square = 144 m

Side = $\frac{144}{4} = 36$ m

So the sum of perimeters of two squares = 60 + 144 = 204 m

Sum of areas of these two squares = $15^2 + 36^2$

= 225 + 1296

= 1521 m^2

Here

Area of third square = 1521 m^2

We know that

3	1521
3	507
13	169
13	13
	1

So we get

Side = $\sqrt{\text{Area}} = \sqrt{1521}$

It can be written as

= $\sqrt{3 \times 3 \times 13 \times 13}$

= 3 × 13

= 39 m

Here

Perimeter = 4 × side

Substituting the values

= 4 × 39

= 156 m

Exercise 3.4

1. Find the square root of each of the following by division method:

(i) 2401

(ii) 4489

(iii) 106929

(iv) 167281

(v) 53824

(vi) 213444

Solution:

(i) $\sqrt{2401} = 49$

By division method

	49
4	$\overline{24}\ \overline{01}$
	16
89	801
	801
	0

(ii) $\sqrt{4489} = 67$

By division method

	67
6	$\overline{44}\ \overline{89}$
	36
127	889
	889
	0

(iii) $\sqrt{106929} = 327$

By division method

	327
3	$\overline{10}\ \overline{69}\ \overline{29}$
	9
62	169
	124
647	4529
	4529
	0

(iv) $\sqrt{167281} = 409$

By division method

	409
4	$\overline{16}\ \overline{72}\ \overline{81}$
	16
809	7281
	7281
	0

(v) $\sqrt{53824} = 232$

By division method

	232
2	$\overline{5}\ \overline{38}\ \overline{2}$
	4
43	138
	129
462	92
	92
	0

(vi) $\sqrt{213444}$ = 462

By division method

	462
4	$\overline{21}\ \overline{34}\ \overline{44}$
	16
86	534
	516
922	1844
	1844
	0

2. Find the number of digits in the square root of each of the following (without any calculation):

(i) 81

(ii) 169

(iii) 4761

(iv) 27889

(v) 525625

Solution:

(i) 81

We know that

In 81, a group of two's is 1

Therefore, its square root has one digit.

(ii) 169

We know that

In 169, group of two's are 2

Therefore, its square root has two digits.

(iii) 4761

We know that

In 4761, group of two's are 2

Therefore, its square root has two digits.

(iv) 27889

We know that

In 27889, groups of two's are 3

Therefore, its square root has three digits.

(v) 525625

We know that

In 525625, groups of two's are 3

Therefore, its square root has three digits.

3. Find the square root of the following decimal numbers by division method:

(i) 51.84

(ii) 42.25

(iii) 18.4041

(iv) 5.774409

Solution:

(i) $\sqrt{51.84} = 7.2$

By division method

	7.2
7	$\overline{42}.\overline{84}$
	49
142	284
	284
	0

(ii) $\sqrt{42.25} = 6.5$

By division method

	6.5
6	$\overline{42}.\overline{25}$
	36
125	625
	625
	0

(iii) $\sqrt{18.4041} = 4.29$

By division method

	4.29
4	$\overline{18}.\overline{40}\,\overline{41}$
	16
81	240
	164
849	7641
	7641
	0

(iv) $\sqrt{5.774409} = 2.403$

By division method

	2.403
2	$\bar{5}.\overline{77}\,\overline{44}\,\overline{36}$
	4
81	177
	176
849	14409
	14409
	0

4. Find the square root of the following numbers correct to two decimal places:

(i) 645.8

(ii) 107.45

(iii) 5.462

(iv) 2

(v) 3

Solution:

(i) $\sqrt{645.8}$ = 25.41

It can be written as

	25.41
2	$\overline{6}\,.\,\overline{45}\;\overline{80}\;\overline{00}$
	4
45	245
	225
504	2080
	2016
5081	6400
	5081
	1319

(ii) $\sqrt{107.45}$ = 10.36

It can be written as

	10.36
1	$\overline{1}\,.\,\overline{07}\;\overline{40}\;\overline{00}$
	1
203	740
	609
2066	13100
	12396
	704

(iii) $\sqrt{5.462}$ = 2.337 = 2.34

It can be written as

	2.337
2	$\bar{5}.\overline{46}\,\overline{20}\,\overline{00}$
	4
43	146
	129
463	1720
	1389
4667	33100
	32669
	431

(iv) $\sqrt{2}$ = 1.41

It can be written as

	1.41
1	$\bar{2}.\overline{00}\,\overline{00}$
	1
24	100
	96
281	400
	281
	119

(v) $\sqrt{3}$ = 1.73

It can be written as

	1.73
1	$\overline{3}\,.\,\overline{00}\;\overline{00}$
	1
27	200
	189
343	1100
	1029
	71

5. Find the square root of the following fractions by division method:

(i) $\frac{841}{1521}$

(ii) $8\frac{257}{529}$

(iii) $16\frac{169}{441}$

Solution:

(i) $\frac{841}{1521}$

By squaring

$$\sqrt{\frac{841}{1521}} = \frac{\sqrt{841}}{\sqrt{1521}} = \frac{29}{39}$$

It can be written as

	29
2	$\overline{8}\;\overline{41}$
	4
49	441
	441
	0

	39
3	$\overline{15}\;\overline{21}$
	9
69	621
	621
	0

(ii) $8\frac{257}{529}$

By squaring

$$\sqrt{8\frac{257}{529}} = \sqrt{\frac{4232+257}{529}} = \sqrt{\frac{4489}{529}}$$

So we get

$= \frac{67}{23} = 2\frac{21}{23}$

It can be written as

	23
2	$\overline{5}\ \overline{29}$
	4
43	129
	129
	0

	67
6	$\overline{44}\ \overline{89}$
	36
127	889
	889
	0

(iii) $16\frac{169}{441}$

By squaring

$$\sqrt{16\frac{169}{441}} = \sqrt{\frac{7056+169}{441}} = \sqrt{\frac{7225}{441}}$$

So we get

$= \frac{\sqrt{7225}}{\sqrt{441}}$

$= \frac{85}{21} = 4\frac{1}{21}$

It can be written as

	21
2	$\overline{4}\,\overline{41}$
	4
41	41
	41
	0

	85
8	$\overline{72}\,\overline{25}$
	64
165	825
	825
	0

6. Find the least number which must be subtracted from each of the following numbers to make them a perfect square. Also find the square root of the perfect square number so obtained:

(i) 2000

(ii) 984

(iii) 8934

(iv) 11021

Solution:

(i) 2000

We know that

	44
4	$\overline{20}\,\overline{00}$
	16
84	400
	336
	64

By taking square root, 64 is left as remainder

Subtracting 64 from 2000

We get 1936 which is a perfect square and its square root is 44.

(ii) 984

We know that

	31
3	$\overline{9}\ \overline{84}$
	9
61	84
	61
	23

By taking square root, 23 is left as remainder

Subtracting 23 from 984

We get 961 which is a perfect square and its square root is 31.

(iii) 8934

We know that

	94
9	$\overline{89}\ \overline{34}$
	81
184	834
	736
	98

By taking square root, 98 is left as remainder

Subtracting 98 from 894

We get 8934 – 98 = 8836 which is a perfect square and its square root is 94.

(iv) 11021

We know that

	104
1	$\overline{1}\ \overline{10}\ \overline{21}$
	1
204	1021
	816
	205

By taking square root, 205 is left as remainder

Subtracting 205 from 11021

We get 11021 – 205 = 10816 which is a perfect square and its square root is 104.

7. Find the least number which must be added to each of the following numbers to make them a perfect square. Also, find the square root of the perfect square number so obtained:

(i) 1750

(ii) 6412

(iii) 6598

(iv) 8000

Solution:

(i) 1750

We know that

	42
4	$\overline{17}\ \overline{50}$
	16
82	150
	164
	14

By taking square root

41^2 is less than 1750

So by taking 42^2

164 – 150 = 14 less

Adding 14 we get a square of 42 which is 1764.

(ii) 6412

We know that

	81
8	$\overline{64}\ \overline{12}$
	64
161	12
	161
	149

By taking square root

80^2 is less than 6412

So by taking 81^2

161 – 12 = 14 less

Adding 149 we get a square of 81 which is 6561.

(iii) 6598

We know that

$$\begin{array}{r|l} & 82 \\ \hline 8 & \overline{65}\ \overline{98} \\ & 64 \\ \hline 162 & 198 \\ & 324 \\ \hline & 126 \\ \hline \end{array}$$

By taking square root

81^2 is less than 6598

So by taking 82^2

324 – 198 = 126 less

Adding 126 we get a square of 82 which is 6724.

(iv) 8000

We know that

$$\begin{array}{r|l} & 89 \\ \hline 8 & \overline{80}\ \overline{00} \\ & 64 \\ \hline 169 & 1600 \\ & 1521 \\ \hline & 79 \\ \hline \end{array}$$

By taking square root

89^2 is less than 8000

So by taking 90^2

8100 – 8000 = 100 less

Adding 100 we get a square of 90 which is 8100.

```
90 | 8000 | 90
    -8100
      100
```

8. Find the smallest four-digit number which is a perfect square.

Solution:

It is given that

Smallest four – digit number = 1000

We know that

```
        31
   3 | 10 00
     |  9
  61 | 100
     |  61
     |  39
```

By taking square root, we find that 39 is left.

If we subtract any number from 1000 we get 3 digit number

Take $32^2 = 1024$

Here 1024 – 1000 = 24 is to be added to get a perfect square of least 4 digit number

Therefore, the required 4 digit smallest number is 1024.

9. Find the greatest number of six digits which is a perfect square.

Solution:

It is given that

Greatest six digit number = 999999

We know that

	999
9	$\overline{99}\ \overline{99}\ \overline{99}$
	81
189	1899
	1701
1989	19899
	17901
	1998

By taking square root, we find that 1998 is left

If we subtract 1998 from 999999 we get 998001 which is a perfect square.

Therefore, required six digit greatest number is 998001.

10. In a right triangle ABC, $\angle B = 90^0$.

(i) If AB = 14 cm, BC = 48 cm, find AC.

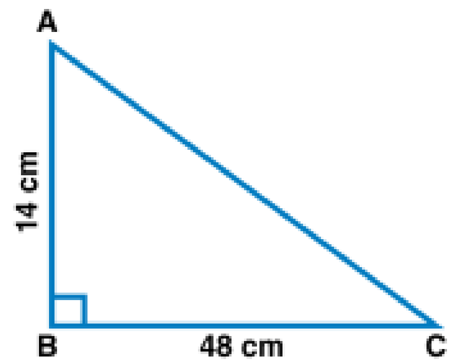
(ii) If AC = 37 cm, BC = 35 cm, find AB.

Solution:

(i) In a right angled triangle ABC

It is given that

AB = 14 cm and BC = 48 cm



Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$= 14^2 + 48^2$

By further calculation

$= 196 + 2304$

$= 2500$

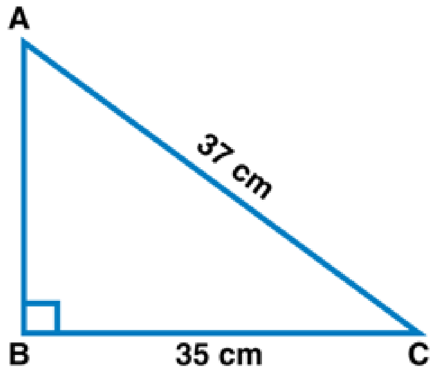
So we get

$AC = \sqrt{2500} = 50$ cm

	50
5	$\overline{25}\ \overline{00}$
	25
	0

(ii) In right triangle ABC

$B = 90^0$, AC = 37 cm, BC = 35 cm



Using Pythagoras Theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$37^2 = AB^2 + 352$

By further calculation

$1369 = AB^2 + 1225$

$AB^2 = 1369 – 1225 = 144$

So we get

$AB = \sqrt{144} = 12$ cm

	12
1	$\overline{1}\ \overline{44}$
	1
22	44
	44
	0

11. A gardener has 1400 plants. He wants to plant these in such a way that the number of rows and number of columns remains same. Find the minimum number of plants he needs more for this.

Solution:

It is given that

Total number of plants = 1400

We know that

	37
3	$\overline{14}\ \overline{00}$
	9
67	500
	469
	31

Here

Number of columns = Number of rows

By taking the square root of 1400

$37^2 < 1400$

So take $38^2 = 1444$

We need 1444 – 1400 = 44 plants more

Therefore, the minimum number of plants he needs more for this is 44.

12. There are 1000 children in a school. For a P.T. drill they have to stand in such a way that the number of rows is equal to number of columns. How many children would be left out in this arrangement?

Solution:

It is given that

No. of total children in a school = 1000

For a P.T. drill, children have to stand in such a way that

No. of rows = No. of columns

Take the square root of 1000

39 is left as remainder

Left out children = 39

	31
3	$\overline{10}\ \overline{00}$
	9
61	100
	61
	39

Hence, 39 children would be left out in this arrangement.

13. Amit walks 16 m south from his house and turns east to walk 63 m to reach his friend's house. While returning, he walks diagonally from his friend's house to reach back to his house. What distance did he walk while returning?

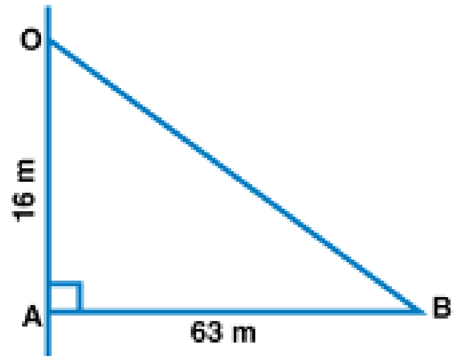
Solution:

It is given that

Amit walks 16 m south from his house and turns east to walk 63 m to reach his friend's house

Consider O as the house and A and B as the places

OA = 16 m, AO = 63 m



Using Pythagoras theorem

$OB^2 = OA^2 + AB^2$

Substituting the values

$= 16^2 + 63^2$

By further calculation

$= 256 + 3969$

$= 4225$

So we get

$OB = \sqrt{4225} = 65$

	65
6	$\overline{42}\,\overline{25}$
	36
125	625
	625
	0

Therefore, Amit has to walk 65 m to reach his house.

14. A ladder 6 m long leaned against a wall. The ladder reaches the wall to a height of 4.8 m. Find the distance between the wall and the foot of the ladder.

Solution:

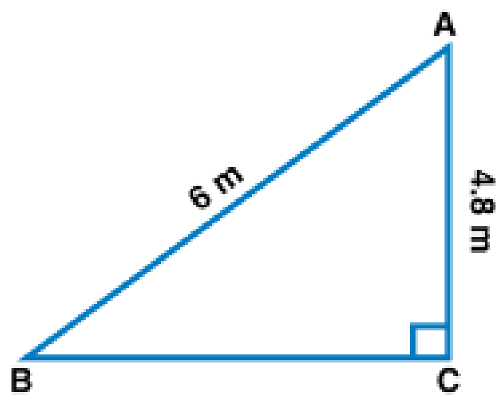
It is given that

Length of ladder = 6 m

Ladder reaches the wall to a height of 4.8 m

Consider AB as the ladder and AC as the height of the wall

AB = 6 m and AC = 4.8 m



Distance between the foot of ladder and wall is BC

Using Pythagoras theorem,

$AB^2 = AC^2 + BC^2$

Substituting the values

$6^2 = 4.8^2 + BC^2$

By further calculation

$BC^2 = 6^2 - 4.8^2$

$BC^2 = 36 - 23.04 = 12.96$

So we get

$BC = \sqrt{12.96} = 3.6$ m

$$\begin{array}{r|l} & 3.6 \\ \hline 3 & \overline{12}.\overline{96} \\ & 9 \\ \hline 66 & 396 \\ & 396 \\ \hline & 0 \\ \hline \end{array}$$

Hence, the distance between the wall and the foot of the ladder is 3.6 m.

Mental Maths

Question 1: Fill in the blanks:

(i) A number ending in ……… is never a perfect square.

(ii) On combining two consecutive triangular number, we get a ………

(iii) If a number has digits ……… in the unit's place, then its square ends in 1.

(iv) Sum of first 10 odd natural numbers is ………

(v) Number of non-square numbers between 11^2 and 12^2 is ………

(vi) Number of zeros in the end of the square of 400 is ………

(vii) Square of any ……… number can be expressed as the sum of two consecutive natural numbers.

(viii)For a natural number m > 1, $(2m, m^2 – 1, m^2 + 1)$ is called ………

Solution:

(i) A number ending in 2,3,7 and 8 is never a perfect square.

(ii) On combining two consecutive triangular number, we get a square number.

(iii) If a number has digits 1 or 9 in the unit's place, then its square ends in 1.

(iv) Sum of first 10 odd natural numbers is $(10)^2 = 100$.

(v) Number of non-square numbers between 11^2 and 12^2 is $(12^2 – 11^2) – 1 = 12 + 11 – 1 = 22$.

(vi) Number of zeros in the end of the square of 400 is 0000 (four zeros).

(vii) Square of any odd number can be expressed as the sum of two consecutive natural numbers.

(viii) For a natural number m > 1, (2m, $m^2 - 1$, $m^2 + 1$) is called Pythagorean triplet.

Question 2: State whether the following statements are true (T) or false (F):
(i) All natural numbers are not perfect squares.
(ii) A perfect square can never be expressed as the product of pairs of equal prime factors.
(iii) A number having 2,3,7 or 8 at its unit place is never a square number.
(iv) A number having 0, 1, 4, 5, 6 or 9 at its unit place is always a square number.
(v) A number ending in an even number of zeros is always a perfect square.
(vi) Square of an odd number is always an odd number.
(vii) 1, 3, 6, 10, 15, are called triangular numbers.
(viii)There are 2n non-square numbers between the squares of consecutive numbers n and (n + 1).
(ix) (4, 6, 8) is a Pythagorean triplet.
Solution:
(i) All natural numbers are not perfect squares. (True)
(ii) A perfect square can never be expressed as
the product of pairs of equal prime factors. (True)
(iii) A number having 2, 3, 7 or 8 at its unit place is never a square number. (Time)
(iv) A number having 0, 1, 4, 5, 6 or 9 at its unit place is always a square number. (False)
Correct:
As 24, 14, 34, 26, 19, 50, 61, 35, etc are not a perfect square
(v) A number ending in an even number of zeros is always a perfect square. (False)

Correct:
As 200, 500, 8000, etc. are not perfect squares.
(vi) Square of an odd number is always an odd number. (True)
(vii) 1, 3, 6, 10, 15, are called triangular numbers. (True)
(viii) There are 2n non-square numbers between
the squares of consecutive numbers n and (n + 1). (True)
(ix) (4, 6, 8) is a Pythagorean triplet. (False)
Correct:
As $4^2 + 6^2 \neq 8^2 \Rightarrow 16 + 36 \neq 64$
$\Rightarrow 52 \neq 64$

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 15):

Question 3: How many natural numbers lie between 25^2 and 26^2?
(a) 49
(b) 50
(c) 51
(d) 52
Solution:
Natural numbers between 25^2 and 26^2
25 + 26 – 1 = 50 (b)

Question 4: Square of an even number is always
(a) even
(b) odd
(c) even or odd
(d) none of these
Solution:
Square of an even number is always even. (a)

Question 5: 1+ 3 + 5 + 7 + ……….. up to n terms is equal to
(a) $n^2 – 1$
(b) $(n + 1)^2$
(c) $n^2 + 1$
(d) n^2
Solution:
1 + 3 + 5 + 7 + …….. Up to n terms is equal to $n^2 + 1$ (c)

Question 6. $\sqrt{\{\mathbf{208}\} + \sqrt{\{\mathbf{2304}\}}}$ **is equal to**
(a) 18
(b) 16
(c) 14
(d) 22
Solution:

$\sqrt{208 + \sqrt{2304}}$

$= \sqrt{208 + 48}$

$= \sqrt{256} = 16$ (b)

	16
1	$\overline{2}\ \overline{56}$
	1
26	156
	156
	0

$$
\begin{array}{r|l}
 & 48 \\ \hline
4 & \overline{23}\;\overline{04} \\
 & 16 \\ \hline
66 & \;\;704 \\
 & \;\;704 \\ \hline
 & \;\;0 \\ \hline
\end{array}
$$

Question 7. $\sqrt{0.0016}$ is equal to
(a) 0.04
(b) 0.004
(c) 0.4
(d) none of these
Solution:
$\sqrt{0.0016} = 0.04$

$$
\begin{array}{r|r}
 & 0.04 \\ \hline
04 & \overline{0}\,.\,\overline{00}\;\overline{16} \\
 & 16 \\ \hline
 & 0 \\ \hline
\end{array}
$$

Question 8: The smallest number by which 75 should be divided to make it a perfect square is
(a) 1
(b) 2
(c) 3
(d) 4
Solution:
$75 = 3 \times 5 \times 5$
Factor 3 is unpaired
$\therefore$ By dividing 75 by 3, we get a perfect square of 5.

Question 9. $\sqrt{3\frac{6}{25}}$ **is equal to**

(a) $\frac{5}{9}$ **(b)** $\frac{4}{5}$

(c) $\frac{9}{5}$ **(d)** $\frac{5}{4}$

Solution:

$$\sqrt{3\frac{6}{25}} = \sqrt{\frac{81}{25}} = \sqrt{\frac{81}{25}}$$

$$= \frac{9}{5} = 1\frac{4}{5} \qquad \text{(c)}$$

Question 10: The smallest number by which 162 should be multiplied to make it a perfect square is
(a) 4
(b) 3
(c) 2
(d) 1
Solution:
162 = 2 × 3 × 3 × 3 × 3
For 2 is left unpairs. So, by multiplying 162 by 2,
we get a perfect square.
∴ Required least number to be multiplied = 2 (c)

Question 11: If the area of a square field is 961 unit^2, then the length of its side is
(a) 29 units
(b) 41 units
(c) 31 units
(d) 39 units

Solution:

Area of a square = 961 unit^2

∴ It's side = $\sqrt{961}$ unit = 31 unit (c)

	31
3	$\overline{9}\ \overline{61}$
	9
61	61
	61
	0

Question 12: The smallest number that should be subtracted from 300 to make it a perfect square is

(a) 11

(b) 12

(c) 13

(d) 14

Solution:

300

Taking the square root of 300,

we see that 11 is left unpaired.

∴ 11 be subtracted. (a)

	17
1	$\overline{3}\ \overline{00}$
	1
66	200
	189
	11

Question 13: If one number of Pythagoream triplet is 6, then the triplet is

(a) (4, 5, 6)

(b) (5, 6, 7)
(c) (6, 7, 8)
(d) (6, 8, 10)
Solution:
One number of a Pythagorean triplet is 6
Let 2n = 6 ⇒ n = 3
$n^2 - 1 = 3^2 - 1 = 8$ and $n^2 + 1 = 3^2 + 1 = 10$
∴ Triplet is (6, 8, 10) (d)

Question 14: n^{th} triangular number is

(a) $\frac{n(n+1)}{2}$ **(b)** $\frac{n(n-1)}{2}$

(c) $\frac{(n-1)(n+1)}{2}$ **(d) none of these**

Solution:
nth triangular number = $\frac{n(n+1)}{2}$ (a)

Value Based Questions

Question 1: In a school, students of class VIII collected ₹9216 to give a donation to an NGO working for the education of poor children. If each student donated as many rupees as the number of students in class VIII. Find the number of students in class VIII.

Why should we donate money for the education of poor children? What values are being promoted?

Solution:

Amount collected by students = ₹ 9216 and each donated amount equal to the number of students

Number of students = $\sqrt{9261}$ = ₹ 96

$$
\begin{array}{r|l}
 & 96 \\
\hline
9 & \overline{92}\ \overline{16} \\
 & 81 \\
\hline
186 & 1116 \\
 & 1116 \\
\hline
 & \quad 0 \\
\hline
\end{array}
$$

Donation to an NGO, who is working for the education of poor children is a noble cause.

Question 2: A person wants to plant 2704 medicinal plants with a board depicting the diseases in which that can be used. He planted these in the form of rows. If each row contains as many plants as the number of rows, then find the number of rows. Why should we plant medicinal plants? What values are being promoted?

Solution:

Total number of plants = 2704

These are planted in such a way that

Number of rows = number of plants in each row

∴ Number of rows = $\sqrt{2704}$

$$
\begin{array}{r|l}
 & 52 \\
\hline
5 & \overline{27}\ \overline{04} \\
 & 25 \\
\hline
102 & \ \ 204 \\
 & \ \ 204 \\
\hline
 & \qquad 0 \\
\hline
\end{array}
$$

Medicinal plants = 52

These plants give us many kinds of medicines

Which are useful for different diseases.

So, it is a noble cause for society.

Higher Order Thinking Skills (Hots)

Question 1: A square field is to be ploughed. Ramu get it ploughed in ₹34560 at the rate of ₹15 per sq. m. Find the length of side of square field.

Solution:

Total expenditure for ploughing the field = ₹34560

The rate of ploughing = ₹15 per sq. m

∴ Total area of the square field = $\frac{34560}{15}$

= 2304 m^2

Side of the square field = $\sqrt{2304}$ m

= 48 m

	48
4	$\overline{23}\ \overline{04}$
	16
88	704
	704
	0

Question 2: Lalit has some chocolates. He distributed these chocolates among 13 children in such a way that he gave one chocolate to first child, 3 chocolates to second child, 5 chocolates to third and so on. Find the number of chocolates Lalit had.

Solution:

Number of children = 13

Lalit gave chocolates to the children in such a way that he gives one chocolate to first child, 3 chocolates to the second child 5 chocolates to a third child and so on

∴ 1 + 3 + 5 + up to 13 terms

(∵ 1 + 3 + 5 + 7 + ……… n terms = n^2)

Check Your Progress

Question 1: Show that 1089 is a perfect square. Also find the number whose square is 1089.

Solution:

$1089 = 3 \times 3 \times 11 \times 11$

3	1089
3	363
11	121
11	11
	1

∵ Prime factors are in pairs and no factor is left.

∴ It is a perfect square and its square root $= 3 \times 11 = 33$

Question 2: Find the smallest number which should be multiplied by 3675 to make it a perfect square. Also find the square root of this perfect square.

Solution:

3675

Factorising, we get

3	3675
5	1225
5	245
7	49
7	7
	1

$3675 = 3 \times 5 \times 5 \times 7 \times 7$

Pairing the same kinds of factors, one factor 3 is left unpaired.

3675 should be multiplied by 3
We get 11025 is a perfect square of $3 \times 5 \times 7 = 105$

Question 3: Express 121 as the sum of 11 odd numbers.
Solution:
$121 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$

Question 4: How many numbers lie between 99^2 and 100^2?
Solution:
Total numbers which lie between 99^2 and 100^2
$= (99 + 100 - 1) = 198$

Question 5: Write a Pythagorean triplet whose one number is 17.
Solution:
One number of Pythagorean triplet = 17
Let $n^2 + 1 = 17 \Rightarrow n^2 = 17 - 1 = 16 = (4)^2$
$\therefore n = 4$
$\therefore$ Numbers will be $2n = 2 \times 4 = 8$
$n^2 - 1 = 4^2 - 1 = 15$ and $n^2 + 1 = 17$
$\therefore$ Triplet is (8, 15, 17)

Question 6: Find the smallest square number which is divisible by each of the numbers 6, 8, 9.
Solution:
smallest Square which is divisible by 6, 8, 9

2	6, 8, 9
3	3, 4, 9
	1, 4, 3

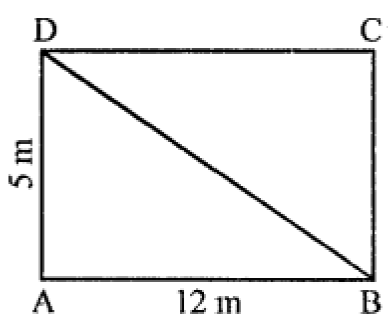
LCM of 6, 8, 9 = 2 × 3 × 3 × 4
= 2 × 2 × 2 × 3 × 3 = 72
Smallest square = 2 × 72 = 144

Question 7: In an auditorium the number of rows is equal to number of chairs in each row. If the capacity of the auditorium is 1764. Find the number of chairs in each row.

Solution:
In an auditorium, there are
Number of rows = Number of chairs in each row
But, capacity is = 1764 persons
∴ Number of chairs = $\sqrt{1764} = 42$

$$\begin{array}{r|l} & \ \ 42 \\ \hline 4 & \overline{17}\ \overline{64} \\ & 16 \\ \hline 82 & \ \ 164 \\ & \ \ 164 \\ \hline & \ \ \ \ 0 \end{array}$$

Question 8: Find the length of diagonal of a rectangle whose length and breadth are 12 m and 5 m respectively.
Solution:
Length of a rectangle = 12 m
and breadth = 5 m



∴ Length of diagonal = $\sqrt{AB^2 + AD^2}$

$= \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$

$= \sqrt{169} = 13$ m

	13
1	$\overline{1}\ \overline{69}$
	1
82	69
	69
	0

Question 9: Find the square root of 144 by successive subtraction.
Solution:

Square root of 144 = $\sqrt{144}$
144 = 144 – 1 = 143
143 = 143 – 3 = 140
140 = 140 – 5 = 135
135 = 135 – 7 = 128
128 = 128 – 9= 119
119 = 119 – 11 = 108
108 = 108 – 13 = 95
95 = 95 – 15 = 80
80 = 80 – 17 = 63
63 = 63 – 19 = 44
44 – 21 = 23
23 – 23 = 0
∴ Square root = 12

Question 10: Find the square root of following numbers by prime factorisation:

(i) 5625
(ii) 1521
Solution:
(i) 5625 = 3 × 3 × 5 × 5 × 5 × 5
Square root of 5625 = $\sqrt{5625}$
= 3 × 5 × 5 = 75

3	5625
3	4875
5	625
5	125
5	25
5	5
	1

(ii) 1521 = 3 × 3 × 13 × 13
Square root of 1521 = $\sqrt{1521}$
= 3 × 13 = 39

3	1521
3	507
13	169
13	13
	1

Question 11: Find the square root of following numbers by long division method:
(i) 21904
(ii) 108241
Solution:
(i) $\sqrt{21904}$ = 148

	148
1	$\overline{3}\ \overline{19}\ \overline{04}$
	1
24	119
	96
288	2304
	2304
	0

(ii) $\sqrt{108241}$ = 329

	329
3	$\overline{10}\ \overline{82}\ \overline{41}$
	9
62	182
	124
649	5841
	5841
	0

Question 12: Find the square root of following decimal numbers:
(i) 17.64
(ii) 13.3225
Solution:
(i) $\sqrt{17.64}$ = 4.2

	4.2
4	$\overline{17}.\overline{64}$
	16
82	164
	164
	0

(ii) $\sqrt{13.3225} = 3.65$

$$\begin{array}{r|l} & 3.65 \\ \hline 3 & \overline{13}\,.\,\overline{32}\,\overline{25} \\ & 9 \\ \hline 66 & 432 \\ & 396 \\ \hline 725 & 3625 \\ & 3625 \\ \hline & 0 \\ \hline \end{array}$$

Question 13: Find the square root of following fractions:

(i) $\mathbf{1\frac{25}{44}}$ **(ii)** $\mathbf{11\frac{225}{576}}$

Solution:

(i) $1\frac{25}{144} = \frac{144+25}{144} = \frac{169}{144}$

$\therefore \sqrt{\frac{169}{144}} = \frac{\sqrt{169}}{\sqrt{144}} = \frac{\sqrt{13\times13}}{\sqrt{12\times12}}$

$= \frac{13}{12} = 1\frac{1}{12}$

(ii) $11\frac{225}{576} = \frac{11\times576+225}{576}$

$= \frac{6336+225}{576} = \frac{6561}{576}$

$$\begin{array}{r|l} & 81 \\ \hline 8 & \overline{65}\,.\,\overline{61} \\ & 64 \\ \hline 161 & 161 \\ & 161 \\ \hline & 0 \\ \hline \end{array}$$

$$
\begin{array}{r|l}
 & 24 \\ \hline
2 & \overline{5}\,.\,\overline{76} \\
 & 4 \\ \hline
44 & 176 \\
 & 176 \\ \hline
 & \quad 0
\end{array}
$$

$= \sqrt{\frac{6561}{576}} = \frac{\sqrt{656}}{\sqrt{576}} = \frac{81}{24}$

$= 3\,\frac{9}{24} = 3\,\frac{3}{8}$

Question 14: Find the least number which must be subtracted from 2311 to make it a perfect square.

Solution:

2311

Taking square root, we see that 7 is left as remainder.

So, 7 is to be subtracted from 2311.

$$
\begin{array}{r|l}
 & 48 \\ \hline
4 & \overline{23}\,.\,\overline{11} \\
 & 16 \\ \hline
88 & \;\;711 \\
 & \;\;704 \\ \hline
 & \quad 7
\end{array}
$$

Question 15: Find the least number which must be added to 520 to make it a perfect square.

Solution:

520

Taking square root of 520, we see that

$(22)^2 < 520$

Then we should take $(23)^2$ which is 529
So, 529 – 520 = 9 is to be added
So, that we resultant number will be a perfect square.
∴ Required number = 9

	22
2	$\overline{5}.\overline{20}$
	4
42	120
	84
	36

Question 16: Find the greatest number of 5 digits which is a perfect square.

Solution:
Greatest 5 digits number = 99999
Taking square root we see that 143 is left as remainder.
So, by subtracting 143 from 99999,
we get the greatest 5 digits which is a perfect square.
Required number = 99999 – 143 = 99856

	316
3	$\overline{9}.\overline{99}\,\overline{99}$
	9
61	99
	61
626	3899
	3756
	143