Exercise 14.1

Answer 1E.

Consider the function $W = f(T, v)$.

Here, W is the wind chill index, T is the actual temperature, and v is the wind speed.

Observe the table given in the text book on page 903, which shows the wind chill values at different values of temperature T and the wind speed v.

(a)

Find the value of $f(-15, 40)$ using the table.

Wind speed (km/h)

Actual temperature (°C)

T \ v	5	10	15	20	25	30	40	50	60	70	80
5	4	3	2	1	1	0	−1	−1	−2	−2	−3
0	−2	−3	−4	−5	−6	−6	−7	−8	−9	−9	−10
−5	−7	−9	−11	−12	−12	−13	−14	−15	−16	−16	−17
−10	−13	−15	−17	−18	−19	−20	−21	−22	−23	−23	−24
−15	−19	−21	−23	−24	−25	−26	−27	−29	−30	−30	−31
−20	−24	−27	−29	−30	−32	−33	−34	−35	−36	−37	−38
−25	−30	−33	−35	−37	−38	−39	−41	−42	−43	−44	−45
−30	−36	−39	−41	−43	−44	−46	−48	−49	−50	−51	−52
−35	−41	−45	−48	−49	−51	−52	−54	−56	−57	−58	−60
−40	−47	−51	−54	−56	−57	−59	−61	−63	−64	−65	−67

From the table, notice that $f(-15, 40) = \boxed{-27}$, which is shown in red color.

The meaning of the function $f(-15, 40) = -27$ is, if the temperature T is -15°C and the wind speed v is 40 km/h, then the apparent severity of the cold W is about -27°C with no wind.

(b)

Find the value of v, that satisfies the equation $f(-20, v) = -30$ using the table.

Wind speed (km/h)

Actual temperature (°C)

T \ v	5	10	15	20	25	30	40	50	60	70	80
5	4	3	2	1	1	0	−1	−1	−2	−2	−3
0	−2	−3	−4	−5	−6	−6	−7	−8	−9	−9	−10
−5	−7	−9	−11	−12	−12	−13	−14	−15	−16	−16	−17
−10	−13	−15	−17	−18	−19	−20	−21	−22	−23	−23	−24
−15	−19	−21	−23	−24	−25	−26	−27	−29	−30	−30	−31
−20	−24	−27	−29	−30	−32	−33	−34	−35	−36	−37	−38
−25	−30	−33	−35	−37	−38	−39	−41	−42	−43	−44	−45
−30	−36	−39	−41	−43	−44	−46	−48	−49	−50	−51	−52
−35	−41	−45	−48	−49	−51	−52	−54	−56	−57	−58	−60
−40	−47	−51	−54	−56	−57	−59	−61	−63	−64	−65	−67

First write the meaning of the equation $f(-20, v) = -30.$

If the temperature T is -20°C, find the value of the wind speed which gives the apparent severity of the cold W about -30°C with no wind.

From the table, notice that $f(-20, 20) = -30$ which is shown in red color.

The meaning of the function $f(-20, 20) = -30$ is, if the temperature T is -20°C and the wind speed v is 20 km/h, then the apparent severity of the cold W is about -30°C with no wind.

(c)

Find the value of T, that satisfies the equation $f(T,20)=-49$ using the table 1.

Wind speed (km/h)

Actual temperature (°C)

T \ v	5	10	15	20	25	30	40	50	60	70	80
5	4	3	2	1	1	0	−1	−1	−2	−2	−3
0	−2	−3	−4	−5	−6	−6	−7	−8	−9	−9	−10
−5	−7	−9	−11	−12	−12	−13	−14	−15	−16	−16	−17
−10	−13	−15	−17	−18	−19	−20	−21	−22	−23	−23	−24
−15	−19	−21	−23	−24	−25	−26	−27	−29	−30	−30	−31
−20	−24	−27	−29	−30	−32	−33	−34	−35	−36	−37	−38
−25	−30	−33	−35	−37	−38	−39	−41	−42	−43	−44	−45
−30	−36	−39	−41	−43	−44	−46	−48	−49	−50	−51	−52
−35	−41	−45	−48	−49	−51	−52	−54	−56	−57	−58	−60
−40	−47	−51	−54	−56	−57	−59	−61	−63	−64	−65	−67

First write the meaning of the equation $f(T,20)=-49$.

If the wind speed v is 20 km/h, find the value of the temperature which gives the apparent severity of the cold W about -49°C with no wind.

From the table, notice that $f(-35,20)=-49$ which is shown in red color in the table.

The meaning of the function $f(-35,20)=-49$ is, if the temperature T is -35°C and the wind speed v is 20 km/h, then the apparent severity of the cold W is about -49°C with no wind.

(d)

Describe the meaning of the function $W=f(-5,v)$.

Here, temperate T is a constant -5°C.

And wind speed v is a variable.

So W is the function of wind speed v with constant temperature -5°C and that gives the wind chill values.

(e)

Describe the meaning of the function $W=f(T,50)$.

Here, temperate T is a variable.

And wind speed v is a constant 50 km/h.

So W is the function of temperature T with wind speed 50 km/h and that gives the wind chill values.

Answer 2E.

The temperature humidity index I is defined as

$I = f(T,h)$

Where T is the actual temperature and h is the relative humidity.

The following table shows the values of I:

Relative humidity(%)

Actual Temperature(°F)

T \ h	20	30	40	50	60	70
80	77	78	79	81	82	83
85	82	84	86	88	90	93
90	87	90	93	96	100	106
95	93	96	101	107	114	124
100	99	104	110	120	132	144

(a)

The value of $f(95,70)$ can be found by looking at the table as, the row where the actual temperature is 95 degrees Fahrenheit and the column where the relative humidity is 70%.

From the table, observe that the apparent temperature is 124 degrees Fahrenheit

This means is that, though the actual temperature is 95 degrees Fahrenheit, a humidity level of 70 percent will make it 125 degrees Fahrenheit.

(b)

Look at the row where the actual temperature is 90 degrees, and where the apparent temperature is 100 degrees Fahrenheit. This happens at 60 percent humidity.

Therefore, the value of h such that $f(90,h)=100$ is $\boxed{60\%}$

(c)

Look at the column where the humidity is 50 percent and where the apparent temperature is 88 degrees Fahrenheit. This happens at 85 degrees Fahrenheit.

Therefore, the value of T such that $f(T,50)=88$ is $\boxed{85\%}$

(d)

The function of $I = f(80, h)$ is a function of apparent temperature, with respect to the humidity, specifically when the actual temperature is 80 degrees Fahrenheit.

The function $I = f(100, h)$, likewise, is the same function, except when the actual temperature is 100 degrees Fahrenheit.

Comparing the above two functions, see that $I = f(80, h)$ has a smaller range of apparent temperatures and it escalates slower, as well. $I = f(100, h)$, on the other hand, has a much wider range and the temperatures escalate at a faster rate.

Answer 3E.

We have $P(L, K) = 1.47L^{0.65}K^{0.35}$.

Replace L with 120 and K with 20 to find $P(120, 20)$.

$$\begin{aligned} P(120, 20) &= 1.47(120)^{0.65}(20)^{0.35} \\ &= 1.47(22.463)(2.853) \\ &\approx 94.2 \end{aligned}$$

Thus, we get $\boxed{P(120, 20) = 94.2}$. This means that the manufacturer's yearly production is 94.2 million dollars when 120,000 labor hours are spent and 20 million dollars is invested.

Answer 4E.

Consider the Cobb-Douglas Production function:

$$P(L, K) = 1.01L^{0.75}K^{0.25}.$$

Where P is the total production, L is the amount of labor, and K is the amount of capital invested.

If the amounts of labor and capital are doubled, replace L and K with $2L$ and $2K$ in the function $P(L, K) = 1.01L^{0.75}K^{0.25}$.

$$\begin{aligned} P(2L, 2K) &= 1.01(2L)^{0.75}(2K)^{0.25} \\ &= 1.01 \times 2^{0.75} L^{0.75} \times 2^{0.25} K^{0.25} \end{aligned}$$

$= 1.01 \times 2^{0.75+0.25} \times L^{0.75} \times K^{0.25}$ Since $a^m \cdot a^n = a^{m+n}$.

$= 2 \times (1.01L^{0.75}K^{0.25})$

Replace $1.01L^{0.75}K^{0.25}$ with $P(L, K)$.

$$P(2L, 2K) = 2P(L.K)$$

Therefore, the production is $\boxed{\text{doubled}}$ when the amounts of labor and capital are doubled.

The general production formula is:

$P(L,K)=1.01L^{\alpha}K^{1-\alpha}$.

Where P is the total production, L is the amount of labor, and K is the amount of capital invested.

If the amounts of labor and capital are doubled, replace L and K with $2L$ and $2K$ in the function $P(L,K)=1.01L^{\alpha}K^{1-\alpha}$.

$$P(2L,2K)=1.01(2L)^{\alpha}(2K)^{1-\alpha}$$
$$=1.01\times 2^{\alpha}L^{\alpha}\times 2^{1-\alpha}K^{1-\alpha}$$

$=1.01\times 2^{\alpha+1-\alpha}\times L^{\alpha}\times K^{1-\alpha}$ Since $a^{m}\cdot a^{n}=a^{m+n}$.

$$=2\times\left(1.01L^{\alpha}K^{1-\alpha}\right)$$

Replace $1.01L^{\alpha}K^{1-\alpha}$ with $P(L,K)$.

$$P(2L,2K)=2P(L.K)$$

Therefore, the production is doubled for general production function.

Answer 5E.

(a) We have $S=f(w,h)=0.1091w^{0.425}h^{0.725}$.

Replace w with 160 and h with 70 to find $f(w,h)$.

$$f(160,70) = 0.1091(160)^{0.425}(70)^{0.725}$$
$$= 0.1091(8.6447)(21.762)$$
$$\approx 20.5$$

Thus, we get $f(w,h)=20.5$. This means that the surface area of a 70 inches tall person who weighs 160 pounds is about 20.5 square feet.

(b) Let the weight be 121 pounds and the height be 67 inches.

Substitute 121 for w and 67 for h to find $f(w,h)$.

$$f(121,67) = 0.1091(121)^{0.425}(67)^{0.725}$$
$$= 0.1091(7.6769)(21.0816)$$
$$\approx 17.66$$

Thus, the surface area is obtained as $\boxed{17.66\text{ square feet}}$.

Answer 6E.

The wind-chill index W has been modeled as:

$$W(T,v)=13.12+0.6215T-11.37v^{0.16}+0.3965Tv^{0.16}.$$

Where W a subjective temperature that depends on the actual temperature is T and the wind speed $v.$

Determine the values for the wind-chill index for few values of T and $v.$

If the temperature is $-5^{\circ}C$ and the wind speed is 50km/h, then determine the wind-chill index value at $W(-5,50).$

Plug in $T=-5$ and $v=50$ to the function of $W(T,v).$

$$W(-5,50)=13.12+0.6215\times(-5)-11.37(50)^{0.16}+0.3965(-5)(50)^{0.16}$$

Use Maple to determine the value.

$$> 13.12+0.6215\cdot(-5)-11.37\cdot 50^{0.16}+0.3965\cdot(-5)\cdot 50^{0.16}$$

$$-14.95630671$$

Similarly determine the values for different values of T and $v.$

The table represents the values of wind-chill index as a function of air temperature and wind speed:

$T \setminus v$	5	10	15	20	25	30	40	50	60	70	80
5	4.1	2.7	1.8	1.1	0.52	0.05	-0.71	-1.33	-1.85	-2.29	-2.70
0	-1.6	-3.3	-4.4	-5.2	-5.9	-6.5	-7.4	-8.1	-8.8	-9.3	-9.8
-5	-7.3	-9.3	-10.6	-11.6	-12.3	-12.9	-14.1	-14.9	-15.7	-16.4	-16.9
-10	-12.9	-15.3	-16.8	-17.9	-18.8	-19.5	-20.8	-21.8	-22.6	-23.3	-24.0
-15	-18.6	-21.2	-22.9	-24.2	-25.2	-26.0	-27.4	-28.6	-29.5	-30.4	-31.1
-20	-24.3	-27.2	-29.1	-30.5	-31.6	-32.6	-34.1	-35.4	-36.5	-37.4	-38.2
-25	-29.9	-33.2	-35.2	-36.8	-38.0	-39.1	-40.8	-42.2	-43.4	-44.4	-45.3
-30	-35.6	-39.2	-41.4	-43.1	-44.5	-45.6	-47.5	-49.0	-50.3	-51.4	-52.4
-35	-41.3	-45.1	-47.6	-49.4	-50.9	-52.1	-54.2	-55.8	-57.2	-58.5	-59.5
-40	-46.9	-51.1	-53.7	-55.7	-57.3	-58.7	-60.9	-62.7	-64.2	-65.5	-66.6

The values in the table are fairly close (within 0.5) to the values of the original table.

Answer 7E.

(a) f(40,15)=25.It means that wave height is 25 at 40 knots wind speed and 15 hours duration.

(b) h= f(30,t) is describing change in wave height according to the variation in time from 5 to 50 hours at windspeed 30 knots. In this range wave height is varying from 9 to 19, therefore this function s increasing.

(c) h=f(v,30) is describing change in wave height according to variation in wind speed at 30th hour. In this range wave height is increasing from 2 to 62, therefore this function is increasing.

Answer 8E.

(a) It is given that the cost for making a small box is $2.50. Then, the total cost to make x small boxes is $2.50x$. Similarly, the total cost to make y medium box is $4y$ and z large boxes is $4.50z$.

Now, the total cost for making x small boxes, y medium boxes, and z large boxes is given by

$$\begin{aligned} C &= f(x, y, z) \\ &= 8000 + (2.50x + 4y + 4.50z) \end{aligned}$$

(b) Plug in x with 3000, y with 5000, and z with 4000.

$$\begin{aligned} f(3000, 5000, 4000) &= 8000 + [2.50(3000) + 4(5000) + 4.50(4000)] \\ &= 8000 + [7500 + 20000 + 18000] \\ &= 53,500 \end{aligned}$$

Thus, we get $f(3000, 5000, 4000) = 53,500$.

(c) Assume D to be a set of ordered pairs of real numbers. If to each ordered pair (x, y) in D there corresponds a unique real number $f(x,y)$, then f is called a function of x and y.

The set D is the domain of the function and the corresponding set of values for $f(x,y)$ is the range of f.

We note that the function is defined for all points (x, y, z) such that $x \ge 0, y \ge 0, z \ge 0$.

Therefore, the domain of the function is $\boxed{\{(x, y, z): x \ge 0, y \ge 0, z \ge 0\}}$.

Answer 9E.

(a) Replace x with 2 and y with -1 to find $g(2, -1)$.

$$g(2, -1) = \cos(2 - 2)$$
$$= \cos 0$$
$$= 1$$

Thus, we get $\boxed{g(2, -1) = 1}$.

(b) We know that the cosine function is defined for all real numbers. Also, we note that $x + 2y$ is defined for all x and y.

Therefore, the domain of the functions is defined as $\mathbb{R}^2$.

(c) Since the value of a cosine function always lies in the interval $[-1, 1]$, we can say that the range of the given function is $[-1, 1]$.

Answer 10E.

(a) Evaluate $F(3,1)$ for the function $F(x, y) = 1 + \sqrt{4 - y^2}$.

To evaluate $F(3,1)$ substitute 3 for x and 1 for y in the given function,

$$F(x, y) = 1 + \sqrt{4 - y^2}$$
$$\Rightarrow F(3,1) = 1 + \sqrt{4 - 1^2}$$
$$= 1 + \sqrt{3}$$

Therefore $F(3,1) = \boxed{1 + \sqrt{3}} \approx 2.732.$

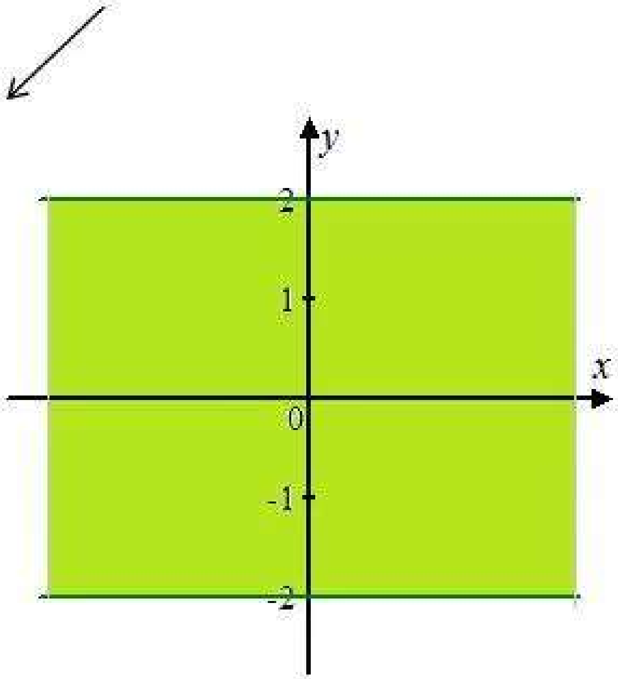
(b) The expression for F makes sense only when $4-y^2 \geq 0$

It implies that $y^2 \leq 4 \Rightarrow -2 \leq y \leq 2.$

However there is no restriction on *x* because there is no *x* term in the expression $1+\sqrt{4-y^2}$

and hence the domain is $\boxed{D=\{(x,y)\in\mathbb{R}\times\mathbb{R}:-2\leq y\leq 2, x\in\mathbb{R}\}}$

This is nothing but half plane and it is shown below (shaded part is the domain for the given function)



Note: The notation $\mathbb{R}$ denotes set of Real numbers.

(c) Range is the set of all possible values that F takes on the domain.

Observe the expression for the function $F(x,y)=1+\sqrt{4-y^2}$

The term $\sqrt{4-y^2}$ is always positive for $-2\leq y\leq 2$, and its value always less than or equal to 2.

So it can be written as

$$0\leq\sqrt{4-y^2}\leq 2$$

$$\Rightarrow 1+0\leq 1+\sqrt{4-y^2}\leq 1+2 \quad \text{Add 1}$$

$$\Rightarrow 1\leq 1+\sqrt{4-y^2}\leq 3 \quad \text{Simplify}$$

$$\Rightarrow 1\leq F(x,y)\leq 3 \quad \text{Since } F(x,y)=1+\sqrt{4-y^2}$$

The last inequality shows that the values that F lies between 1 and 3.

Therefore the range of the given function $F(x,y)=1+\sqrt{4-y^2}$ is $\boxed{R=[1,3]}$.

Answer 11E.

Consider the following function:

$$f(x,y,z)=\sqrt{x}+\sqrt{y}+\sqrt{z}+\ln\left(4-x^2-y^2-z^2\right)$$

(a) Calculate $f(1,1,1)$.

Replace *x* with 1, *y* with 1, and *z* with 1 to find $f(1,1,1)$.

$$f(x,y,z)=\sqrt{x}+\sqrt{y}+\sqrt{z}+\ln\left(4-x^2-y^2-z^2\right)$$

$$\begin{aligned} f(1,1,1) &= \sqrt{1}+\sqrt{1}+\sqrt{1}+\ln\left(4-1^2-1^2-1^2\right) \\ &= 3+\ln(1) \\ &= 3+0 \\ &= 3 \end{aligned}$$

$$f(1,1,1)=\boxed{3}$$

(b) Find and describe the domain of f.

Assume *D* to be a set of ordered triads of real numbers. If to each ordered triad (x,y,z) in *D* there corresponds a unique real number $f(x,y,z)$, then f is called a function of x,y, and z.

The set *D* is the domain of the function and the corresponding set of values of $f(x,y,z)$ is the range of f.

The function f is defined for all points (x,y,z) such that $x \ge 0,\ y \ge 0,\ z \ge 0$ and $4-x^2-y^2-z^2>0$.

Therefore, the domain of the function f is $\left\{(x,y,z):x^2+y^2+z^2<4,\ x\ge 0, y\ge 0, z\ge 0\right\}$.

Answer 12E.

Given $g(x,y,z) = x^3y^2z\sqrt{10-x-y-z}$

(a) Replace x with 1, y with 2, and z with 3 in $g(x,y,z) = x^3y^2z\sqrt{10-x-y-z}$.

$$\begin{aligned} g(1,2,3) &= \left(1^3\right)\left(2^2\right)(3)\sqrt{10-1-2-3} \\ &= (12)\sqrt{4} \\ &= 24 \end{aligned}$$

Thus, we get $\boxed{g(1,2,3)=24}$.

(b) Assume D to be a set of ordered pairs of real numbers. If to each ordered pair (x, y) in D there corresponds a unique real number $f(x, y)$, then f is called a function of x and y. The set D is the domain of the function and the corresponding set of values for $f(x, y)$ is the range of f.

We note that the function is defined for all (x, y, z) such that $10 - x - y - z \geq 0$.

Therefore, the domain of the function is $\boxed{\{(x, y, z) : 10 - x - y - z \geq 0\}}$.

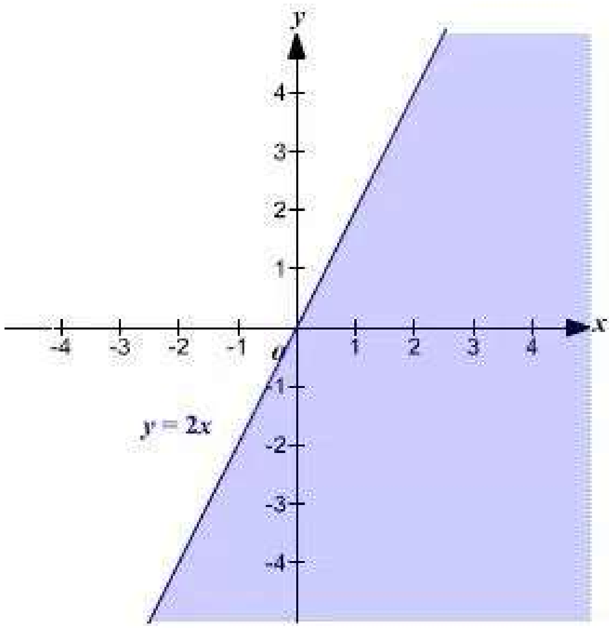
Answer 13E.

Assume D to be a set of ordered pairs of real numbers. If to each ordered pair (x, y) in D there corresponds a unique real number $f(x, y)$, then f is called a function of x and y. The set D is the domain of the function and the corresponding set of values for $f(x, y)$ is the range of f.

We note that the function is defined for all points (x, y) such that $2x - y \geq 0$ or $y \leq 2x$.

Therefore, the domain of the function is $\boxed{\{(x, y) : y \leq 2x\}}$.

Let us sketch the domain.



Answer 14E.

Consider the following function:

$$f(x,y)=\sqrt{xy}$$

The object is to find and sketch the domain of the function.

Take $f(x,y)=\sqrt{xy}>0.$

The inequality depends up on the following region:

$$\left\{\begin{array}{c} y<0, x\le 0 \\ x\le 0, x\ge 0, y=0 \\ x\ge 0, y>0 \end{array}\right\}.$$

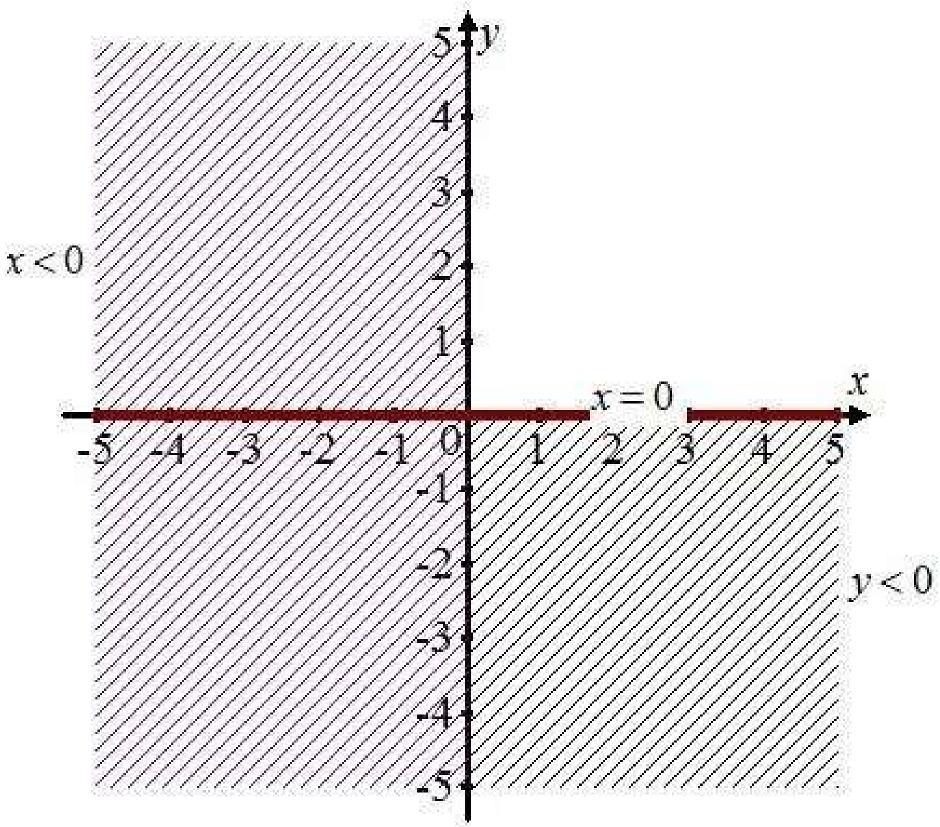
Sketch the graph of the given function in three cases as follows:

Case1:

If $y<0$ then $x\le 0.$

So, the inequality $xy\ge 0.$

The graph of the region is as shown below:

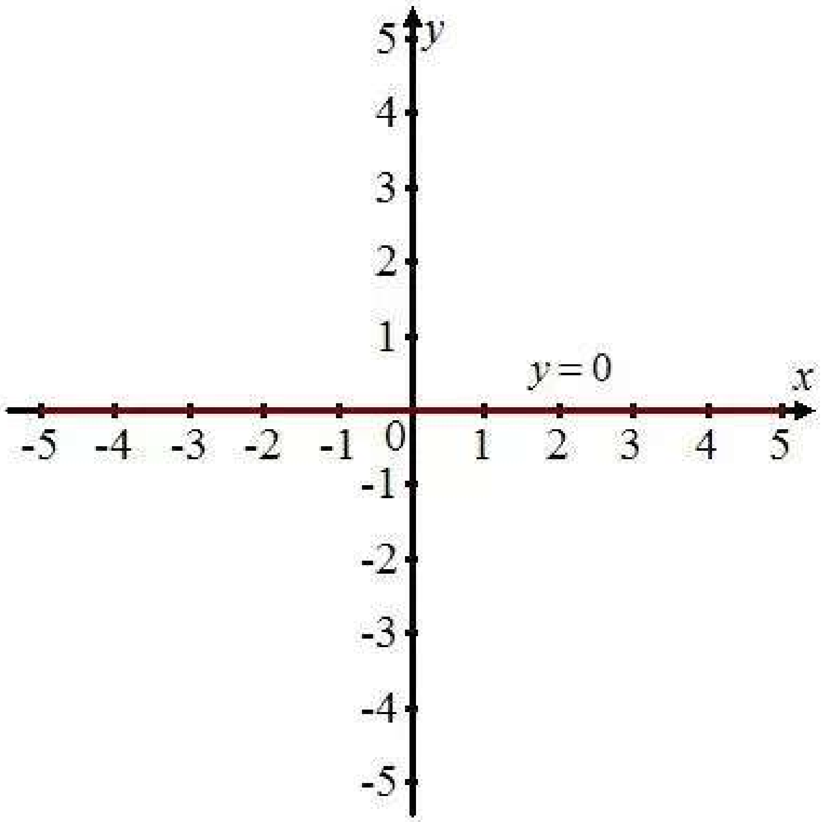


Therefore, the domain of the graph is $D=\{(x,y)\mid x\le 0, y<0\}.$

Case2:

If $y = 0$ and $x \le 0, x \ge 0$ then the inequality $xy \ge 0.$

The graph of the region is as shown below:

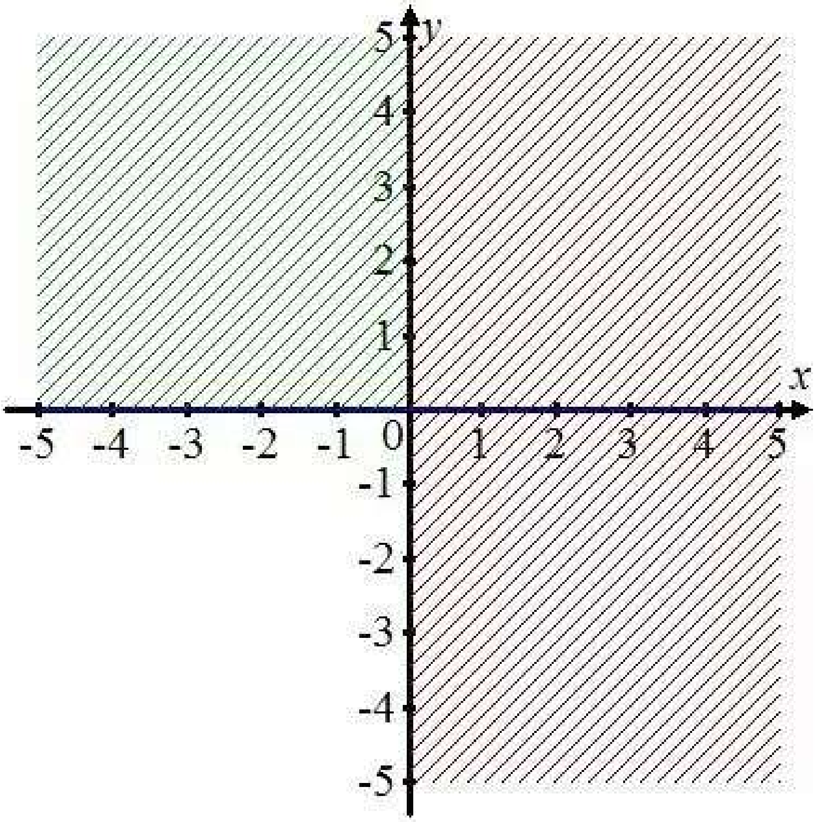


In this case the domain of the graph is $y = 0.$

Case3:

If $y > 0$ and $x \geq 0$ then the inequality $xy \geq 0$.

The graph of the region is as shown below:



In this case the domain of the given function is $D = \{(x, y) \mid x \geq 0, y > 0\}$.

Answer 15E.

Consider the following function;

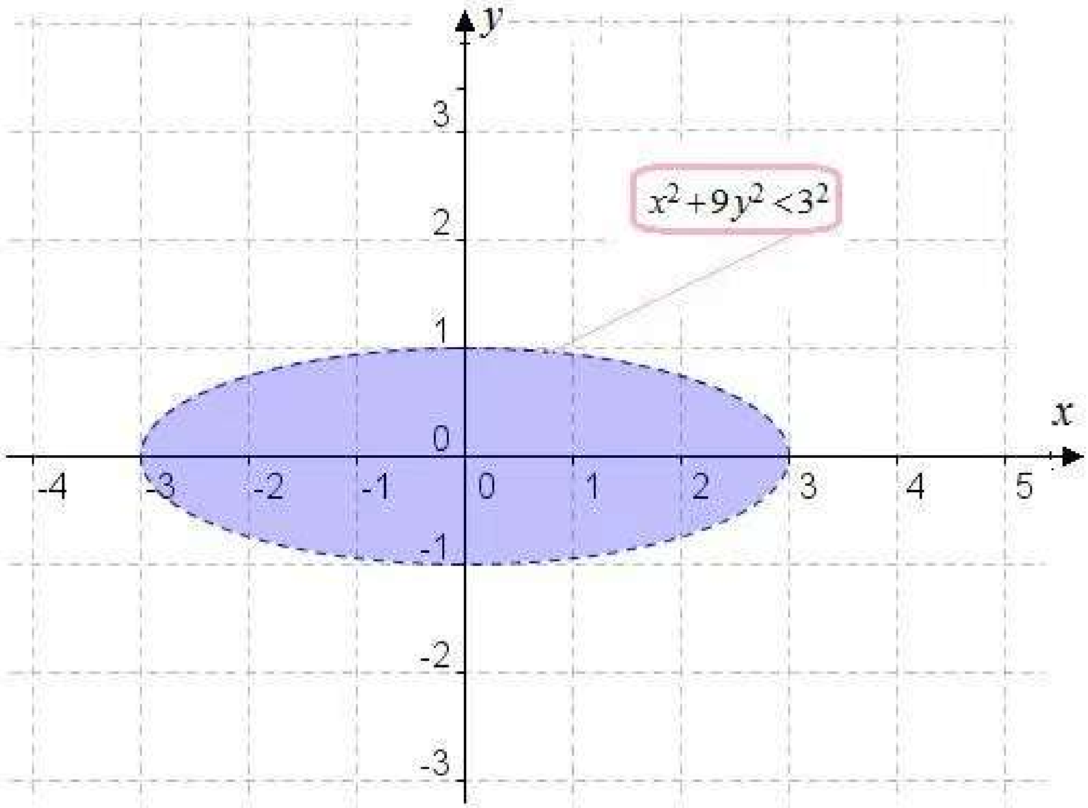
$$f(x,y)=\ln\left(9-x^2-9y^2\right)$$

Note that, the expression inside the logarithmic function must be positive. Use this knowledge, the domain is the region determined by the following inequality.

$$\begin{aligned}9-x^2-9y^2&>0\\9&>x^2+9y^2\\3^2&>x^2+9y^2\end{aligned}$$

Hence, the domain of the set is $\boxed{\left\{(x,y)\mid x^2+9y^2<3^2\right\}}$.

The graph of this region is as follows.



From the above graph, the shaded region is the domain of the function, $f(x,y)$, which is the set of all points in the inside of an ellipse, $x^2+9y^2<3^2$.

Answer 16E.

Consider the function,

$$f(x,y)=\sqrt{x^2-y^2}$$

The objective is to sketch the domain of the function.

The domain of a function is the set of all points for which the function is defined.

Here the function has Square root function is defined for non-negative values.

Hence the domain of $f(x,y)=\sqrt{x^2-y^2}$ is $\boxed{x^2-y^2\geq 0}$.

Rewrite it as $(x+y)(x-y)\geq 0$.

Product of two factors is non-negative when both are positive or both negative.

So, the domains are given as

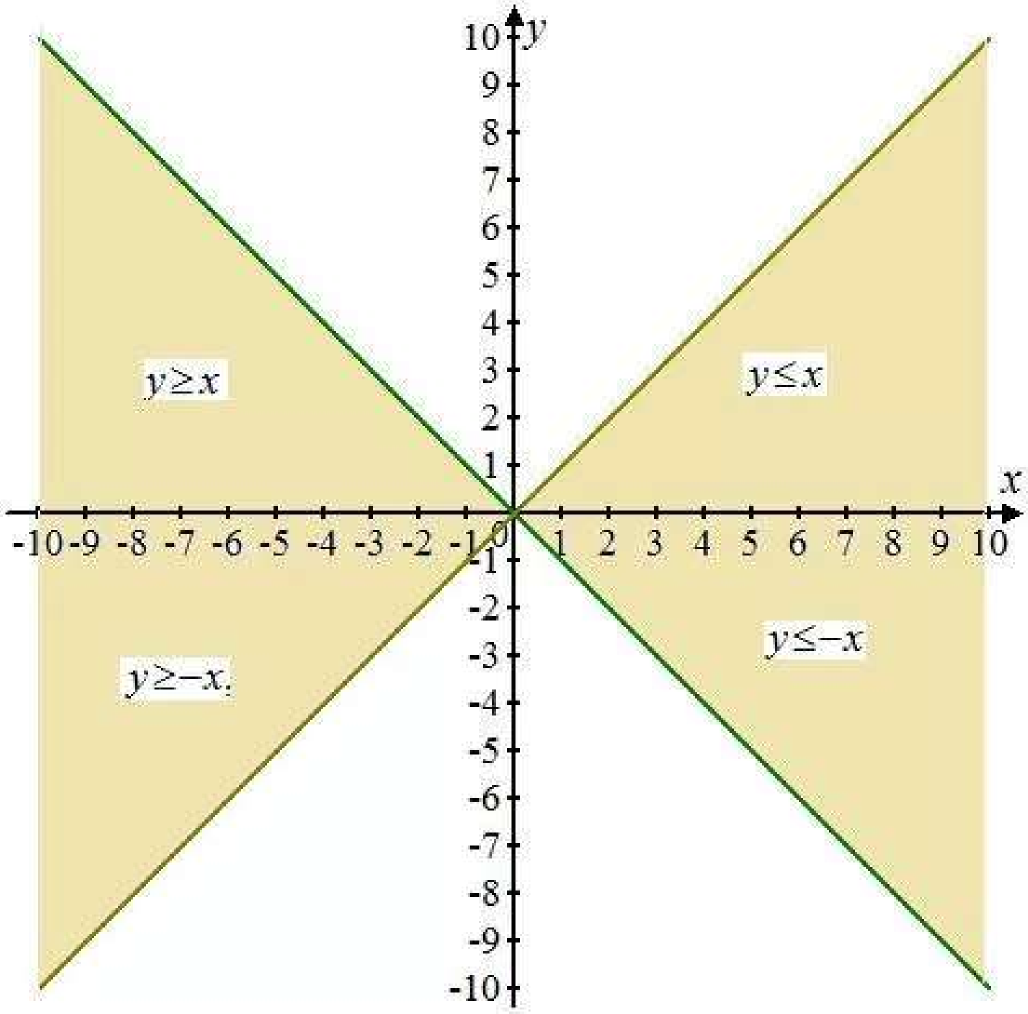
$$x+y\geq 0, x-y\geq 0 \text{ or } x+y\leq 0, x-y\leq 0 \quad x\geq -y, x\geq y \text{ or } x\leq -y, x\leq y$$

$$x\geq -y, x\geq y \text{ or } x<-y, x<y$$

$$y\geq -x, y\leq x \text{ or } y\leq -x, y\geq x$$

Therefore, the domain of the function is $\{(x, y): y\leq \pm x \text{ or } y\geq \pm x\}$.

The graph of the domain $x^2-y^2\geq 0$ is shown below.



Answer 17E.

Consider the following function:

$$f(x,y)=\sqrt{1-x^2}-\sqrt{1-y^2}$$

Find the domain of the given function.

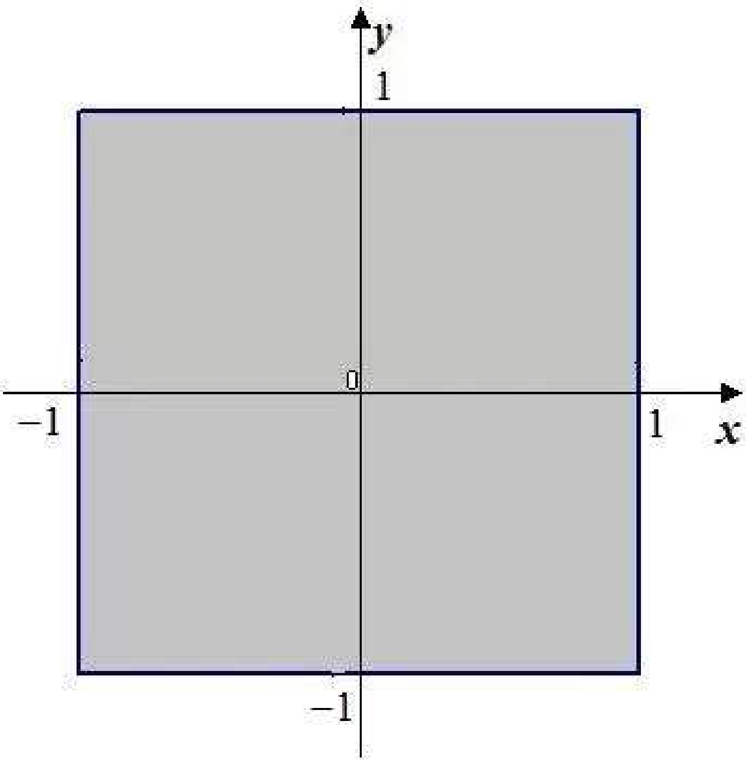
Recall that the expression inside any square root must be non-negative.

$$1-x^2\ge 0 \text{ and } 1-y^2\ge 0$$
$$x^2-1\le 0 \text{ and } y^2-1\le 0$$
$$(x-1)(x+1)\le 0 \text{ and } (y-1)(y+1)\le 0$$
$$-1\le x\le 1 \text{ and } -1\le y\le 1$$

So, the domain of the function $f(x,y)=\sqrt{1-x^2}-\sqrt{1-y^2}$ is,

$$\boxed{\{(x,y)\in\mathbb{R}^2 \mid -1\le x\le 1 \text{ and } -1\le y\le 1\}}.$$

Sketch the domain $-1\le x\le 1$ and $-1\le y\le 1$ of the given function as shown below:



Answer 18E.

Consider the function $f(x,y)=\sqrt{y}+\sqrt{25-x^2-y^2}$.

Find the domain of the function $f(x,y)$.

Let $f(x,y)=g(x,y)+h(x,y)$.

Here, $g(x,y)=\sqrt{y}$ and $h(x,y)=\sqrt{25-x^2-y^2}$.

Clearly, $g(x,y)=\sqrt{y}$ is defined only when $y\ge 0$.

So the domain of the function $g(x,y)$ is $\{(x,y)\mid y\ge 0\}$.

Similarly, $h(x,y)=\sqrt{25-x^2-y^2}$ is defined only when $25-x^2-y^2\ge 0$.

That is,

$$25-x^2-y^2\ge 0$$
$$25\ge x^2+y^2$$
$$x^2+y^2\le 25$$

So the domain of the function $h(x,y)$ is $\{(x,y)\mid x^2+y^2\le 25\}$.

Since the function $f(x,y)$ is the summation of both the functions $g(x,y)$ and $h(x,y)$, therefore, the domain of the function $f(x,y)$ is $D=\{(x,y)\mid y\ge 0 \text{ and } x^2+y^2\le 25\}$.

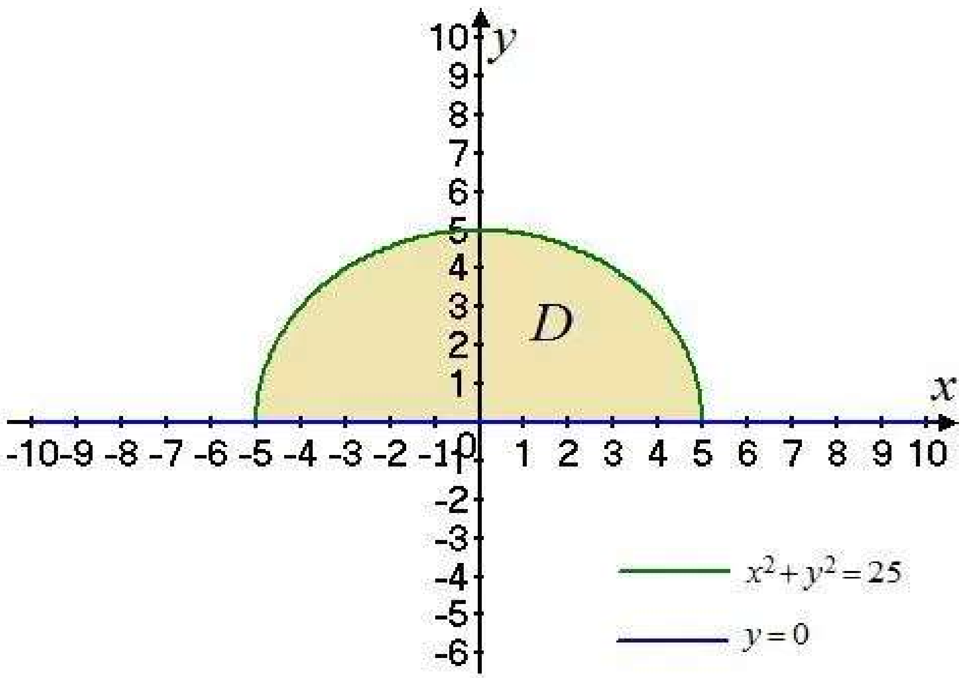
Sketch the domain of the function $f(x,y)=\sqrt{y}+\sqrt{25-x^2-y^2}$.

The equation $x^2+y^2\le 5^2$ represents a disk of radius 5.

And the inequality $y\ge 0$ represents the region above the *x*-axis.

Shade the region inside the circle of radius 5 above the *x*-axis to sketch the domain of the function $f(x,y)$.

The graph of the domain of the function $f(x,y)=\sqrt{y}+\sqrt{25-x^2-y^2}$ is shown below:



Answer 19E.

Consider the following function;

$$f(x,y)=\frac{\sqrt{y-x^2}}{1-x^2}.$$

Note that, the expression inside any square root must be non-negative.

Use this knowledge, the domain is the region determined by the following inequality.

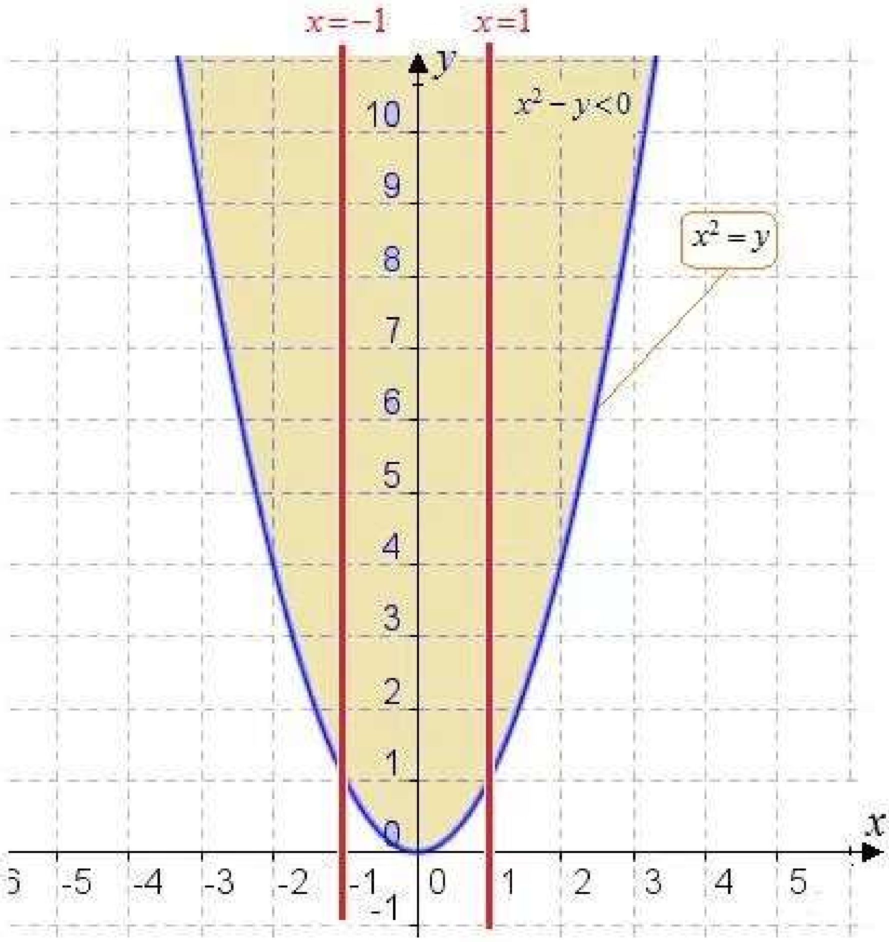
$$y-x^2\geq 0$$
$$y\geq x^2$$

Also, division by zero is undefined, so the denominator cannot be zero.

$$1-x^2\neq 0$$
$$1\neq x^2$$
$$x\neq \pm 1$$

Hence, the domain is the set $\boxed{\{(x,y)\mid y\geq x^2; x\neq \pm 1\}}$.

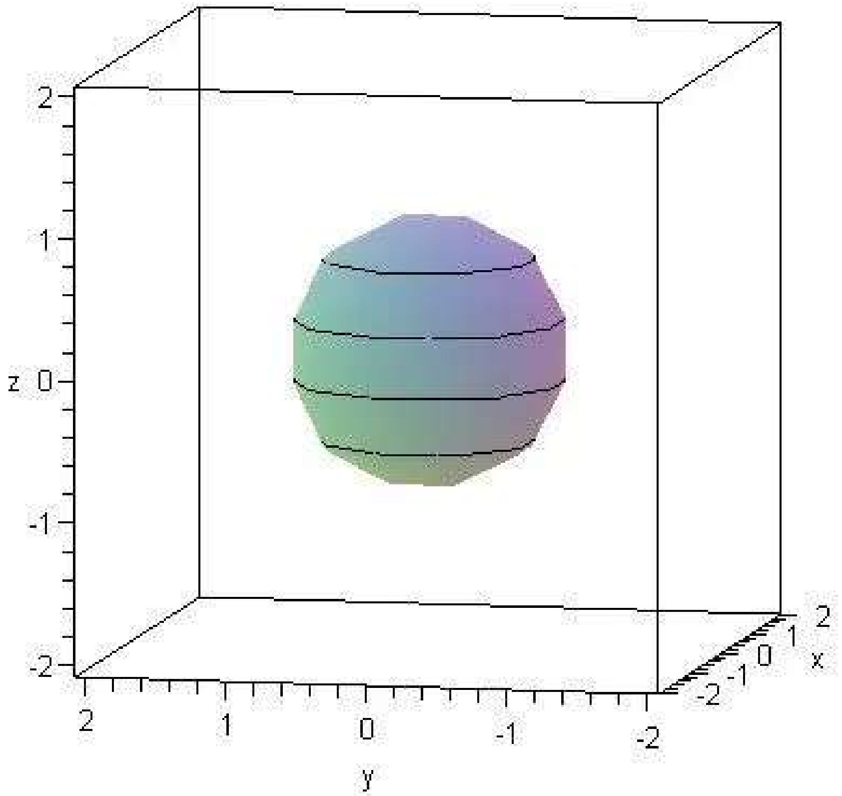
The graph of this region is as follows.



From the above graph, the red colored vertical lines are intentionally omitted points, since, $x\neq \pm 1$.

Answer 21E.

The graph of the domain of the given function $f(x,y,z)=\sqrt{1-x^2-y^2-z^2}$ is:



Answer 22E.

Consider the following function:

$$f(x,y,z)=\ln\left(16-4x^2-4y^2-z^2\right)$$

Find the domain of the given function.

Recall that the expression inside the radical must be non-negative.

$$16-4x^2-4y^2-z^2>0$$

$$4x^2+4y^2+z^2<16$$

$$\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{16}<1 \quad \text{divide by 16.}$$

$$\frac{x^2}{2^2}+\frac{y^2}{2^2}+\frac{z^2}{4^2}<1$$

This is an ellipsoid.

So, the domain of the function $f(x,y,z)=\ln\left(16-4x^2-4y^2-z^2\right)$ is,

$$\boxed{\left\{(x,y,z)\in\mathbb{R}^3 \mid \frac{x^2}{2^2}+\frac{y^2}{2^2}+\frac{z^2}{4^2}<1\right\}}.$$

Answer 23E.

To sketch the graph of $f(x,y)=1+y$:

Suppose the above surface as $z=1+y$, in this equation the variable *x* is missed, so this plane is parallel to *x*-axis.

To find *y*-intercept of the plane, put $x=0, z=0$ in the equation of the surface,

$$z=1+y$$
$$0=1+y$$
$$y=-1$$

So, the *y*-intercept is $(0,-1,0)$

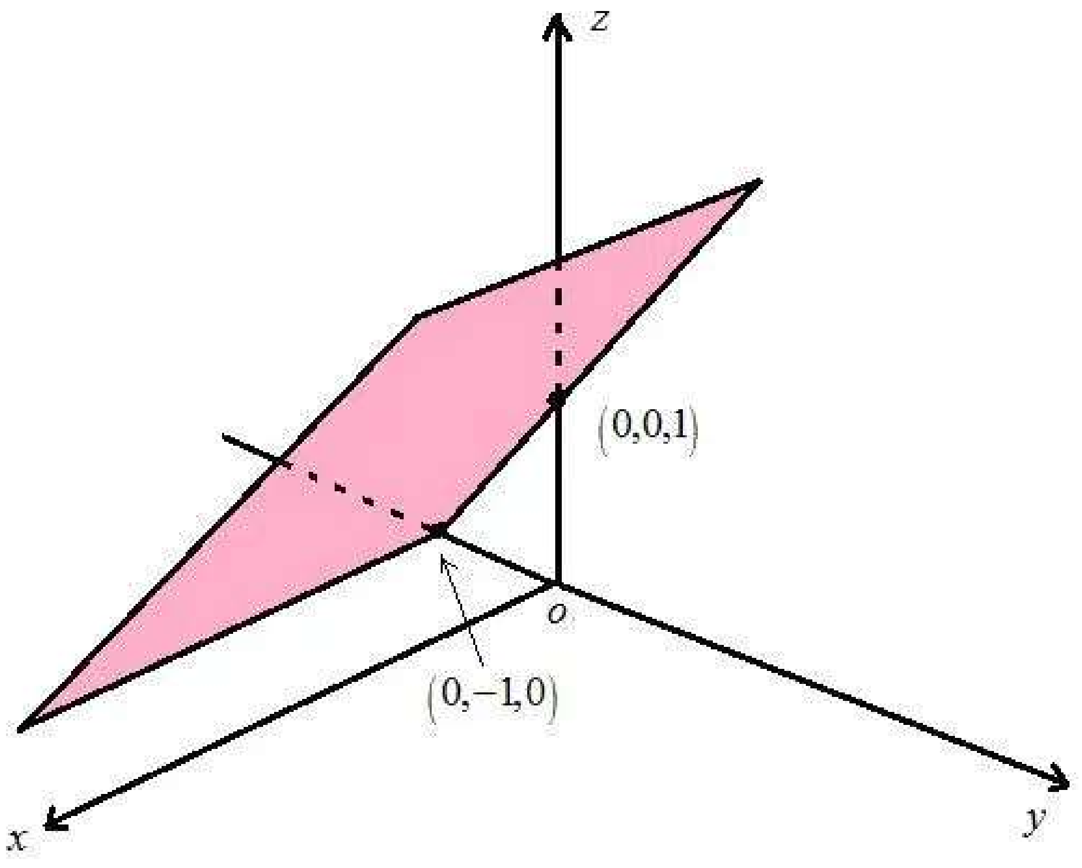
To find *z*-intercept of the plane, put $x=0, y=0$ in the equation of the surface,

$$z=1+y$$
$$z=1+0$$
$$z=1$$

So, the *z*-intercept is $(0,0,1)$

So, the plane $z=1+y$ is passing through the points $(0,-1,0)$, $(0,0,1)$ and is parallel to *x*-axis.

The sketch of the graph of $f(x,y)=1+y$ is shown below:



Answer 24E.

Consider the function:

$f(x,y)=2-x$.

The objective is to sketch the graph of the function.

It can be written as $z=f(x,y)$

So, the graph of *f* has the equation $z=2-x$ or $x+z=2$, which represents a plane.

To graph the plane we first find the intercepts.

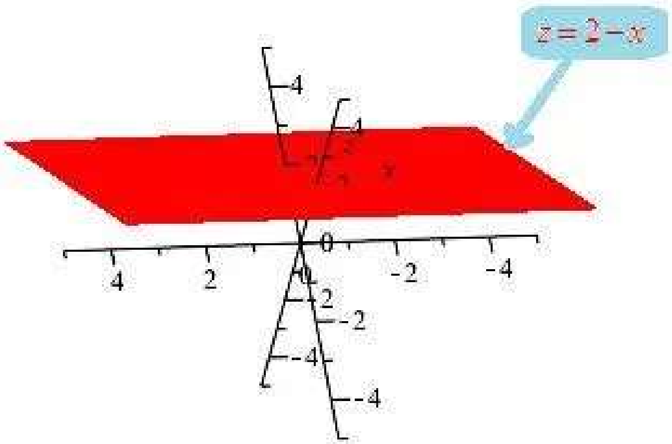
Take $x=0 \Rightarrow z=2$

Take $z=0 \Rightarrow x=2$

Hence $x-$intercept is $2, z-$intercept is 2

This information helps to sketch the graph of equation $f(x,y)=2-x$.

The graph of the function $f(x,y)=2-x$ is shown below:



Answer 25E.

Consider the function

$f(x,y)=10-4x-5y$

The objective is to graph the function.

The graph of *f* has the equation $z=10-4x-5y$ or $4x+5y+z=10$, which represents a plane.

To graph the plane first finds the intercepts.

To find the *x* intercept, Put $y=0, z=0$ in the equation $z=10-4x-5y$.

$$0=10-4x$$
$$4x=10$$
$$\Rightarrow x=\frac{5}{2}$$

Therefore, the *x*-intercept is $x=\frac{5}{2}$.

Similarly, to find the *y* intercept, Put $x=0, z=0$ in the equation $z=10-4x-5y$.

$$0=10-0-5y$$
$$5y=10$$
$$\Rightarrow y=2$$

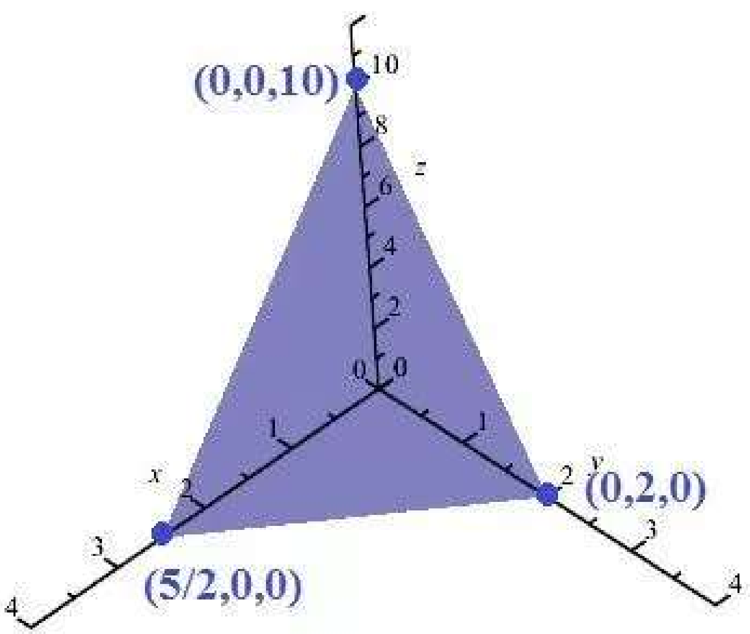
Therefore, the *y*-intercept is 2.

Similarly, to find the *z* intercept, Put $y=0, x=0$ in the equation $z=10-4x-5y$.

$$z=10-0-0$$
$$z=10$$

This information helps to sketch the graph of equation $f(x,y)=10-4x-5y$.

The graph of the function $f(x,y)=10-4x-5y$ looks like this:

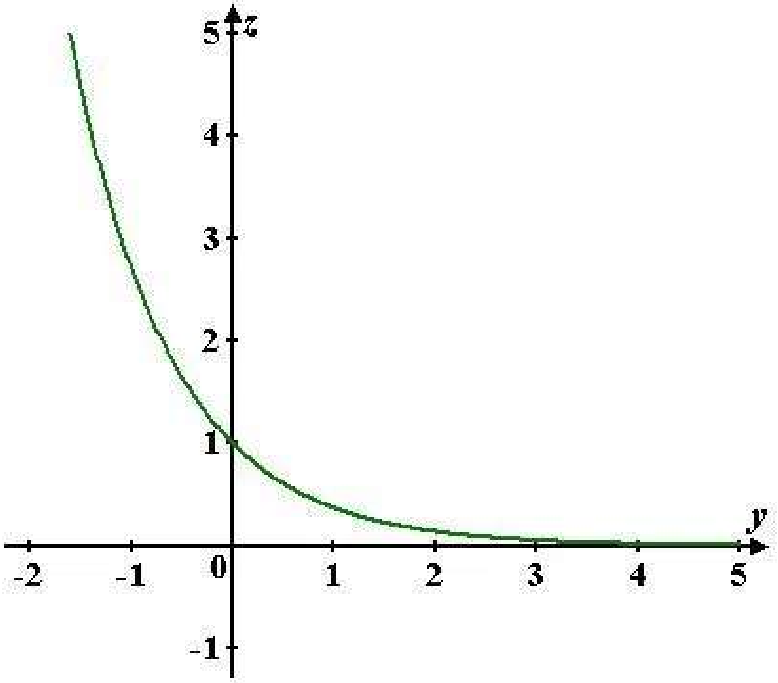


Answer 26E.

Consider the function $f(x,y) = e^{-y}$

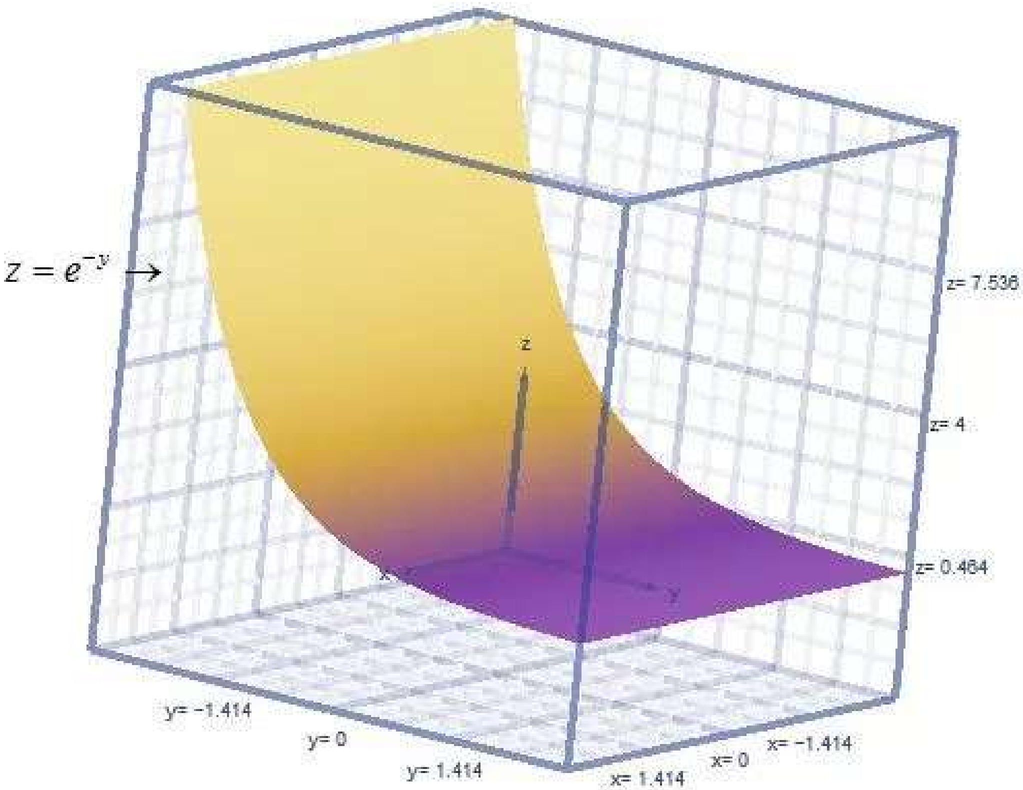
To sketch the graph of $z = f(x,y) = e^{-y}$

Observe that $z = e^{-y}$ is a curve upon *yz* – plane looks like



So, the surface parallel to *x* – axis including this curve is the required surface.

Therefore, the surface looks like



Answer 27E.

Consider the following function;

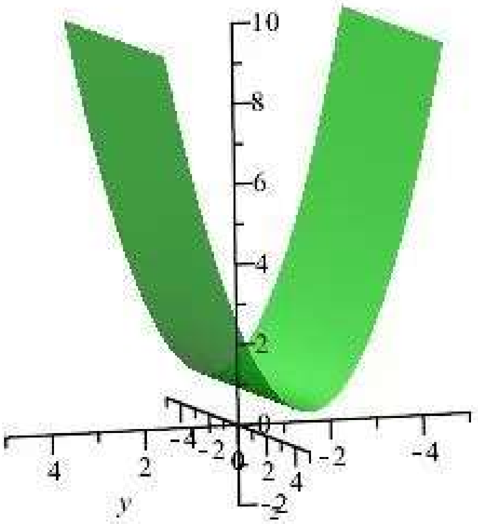
$f(x,y)=y^2+1$

The graph has an equation, $z=y^2+1$, which is a parabola in the $yz-$plane.

The vertex at $(1,0)$, and also, the graph of the function $f(x,y)$ has an equation

$f(x,y)=y^2+1$ or $z=y^2+1$, which represents a paraboloid in three dimension.

The graph of the function $f(x,y)=y^2+1$ is as shown below.



The graph of the function $f(x,y)=y^2+1$

Answer 28E.

Consider the following function:

$f(x,y)=1+2x^2+2y^2$

Let the surface be $z=f(x,y)=1+2x^2+2y^2$.

As $1+2x^2+2y^2>0$, $z>0$

So, the graph of the surface is along the positive *z*-axis.

Put $x=0$ in the equation of the surface $z=1+2x^2+2y^2$,

$z=1+2y^2$

So, yz-plane intersects the surface in a parabola.

Put $x=k(\text{constant})$ in $z=1+2x^2+2y^2$,

$z=1+2k^2+2y^2$

This means that, if slice the graph with any plane parallel to yz-plane, a parabola obtained that opens upward.

Put $y = k(\text{constant})$ in $z = 1 + 2x^2 + 2y^2$.

$z = 1 + 2x^2 + 2k^2$

So, again a parabola is obtained that opens upward.

Put $z = k(\text{constant})$ in $z = 1 + 2x^2 + 2y^2$.

$k = 1 + 2x^2 + 2y^2$

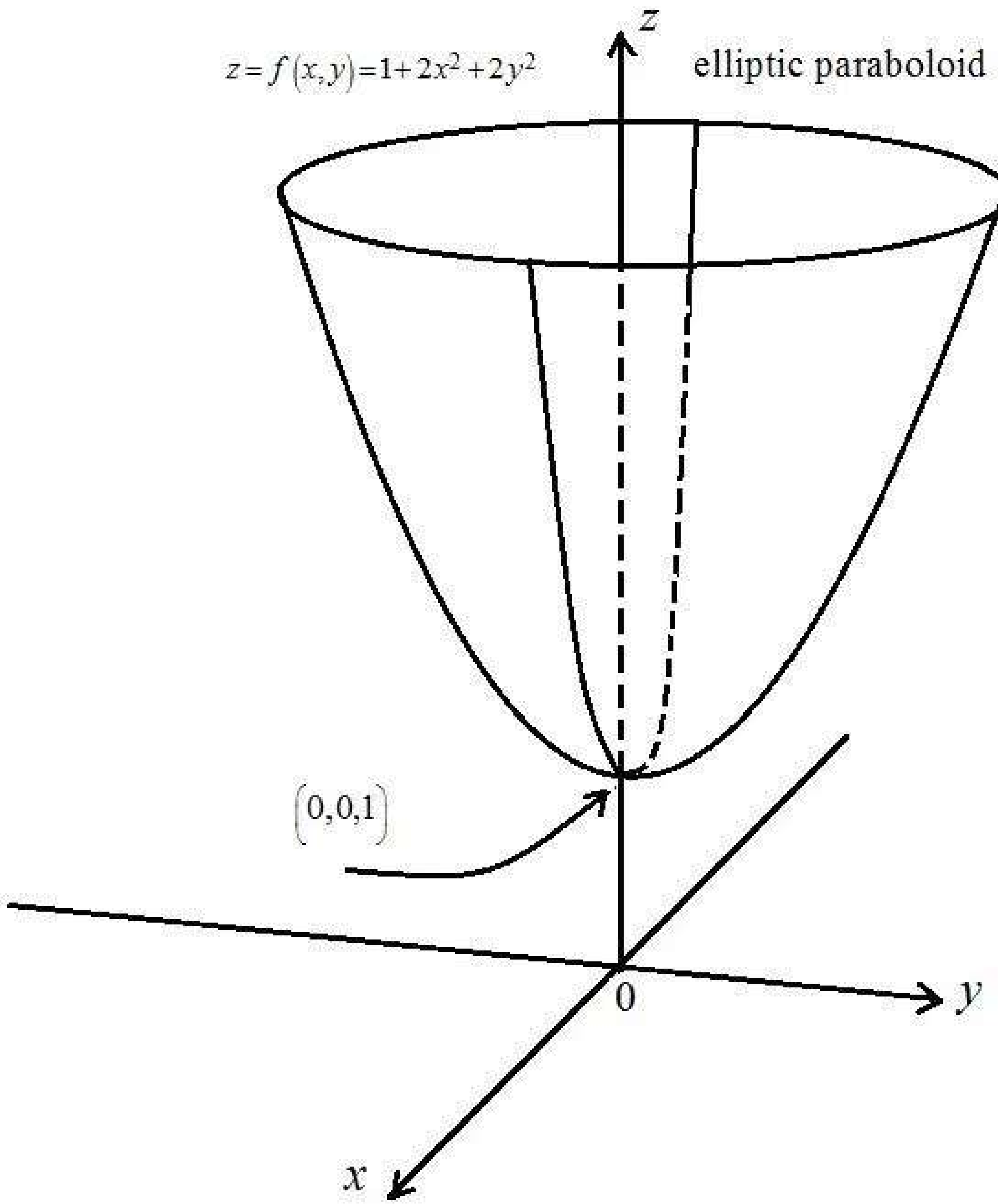
$k - 1 = 2x^2 + 2y^2$

Or $1 = \dfrac{x^2}{\frac{k-1}{2}} + \dfrac{y^2}{\frac{k-1}{2}}$

This equation represents the family of ellipses for different values of k.

So, by this description, the surface is an infinite elliptic paraboloid.

The graph of the surface $z = 1 + 2x^2 + 2y^2$ is shown below:



Answer 29E.

Consider the surface $z = 9 - x^2 - 9y^2$

This equation can be written as,

$$x^2 + 9y^2 = 9 - z$$

As $x^2 + 9y^2 \geq 0$, $9 - z \geq 0$

So, $z \leq 9$

So the surface is along negative *z*-axis below the plane $z = 9$

Put $x = 0, y = 0$ in $z = 9 - x^2 - 9y^2$,

$$z = 9$$

So the vertex of the surface is $(0,0,9)$

Put $z = k$ in $z = 9 - x^2 - 9y^2$,

$$x^2 + 9y^2 = 9 - k$$

So, for the different values of *k*, this equation represents the family of ellipses.

Put $x = k$ in $z = 9 - x^2 - 9y^2$,

$$z = -9y^2 + (9 - k^2)$$

So, for the different values of *k*, this equation represents the family of parabolas opens downward.

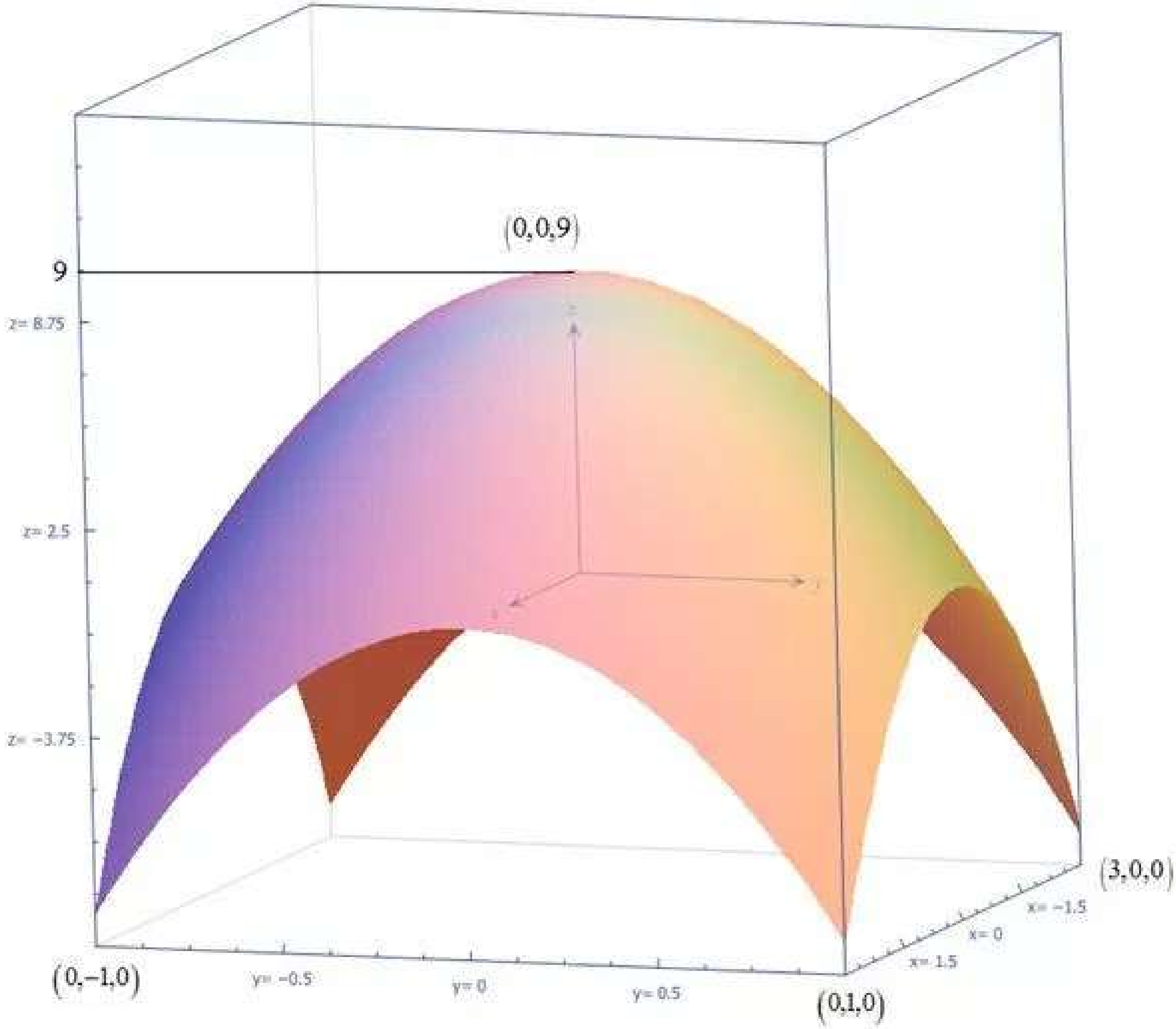
Put $y = k$ in $z = 9 - x^2 - 9y^2$,

$$z = -x^2 + (9 - 9k^2)$$

So, for the different values of *k*, this equation also represents the family of parabolas opens downward.

In particular, for $x = 0, z = 0$,
$$\begin{aligned} z &= 9 - x^2 - 9y^2 \\ 0 &= 9 - 0 - 9y^2 \\ 9y^2 &= 9 \\ y &= \pm 1 \end{aligned}$$
And, for $y = 0, z = 0$,
$$\begin{aligned} z &= 9 - x^2 - 9y^2 \\ 0 &= 9 - x^2 - 0 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$
So, the surface is passing through the points $(0,1,0),(0,-1,0)$, and $(3,0,0),(-3,0,0)$ So, from this description, the surface represents an elliptic paraboloid opens downward.

The sketch of the graph of the surface $z = 9 - x^2 - 9y^2$ is shown below:



Answer 30E.

To sketch the graph of the function $z=\sqrt{4x^2+y^2}$, first simplify this equation as follows:

$$z=\sqrt{4x^2+y^2} \quad \ldots\ldots (1)$$

$$z^2=4x^2+y^2 \quad \text{(Squaring on both sides)}$$

This is can be written as,

$$\frac{z^2}{1^2}=\frac{x^2}{\left(\frac{1}{2}\right)^2}+\frac{y^2}{1^2} \quad \ldots\ldots (2)$$

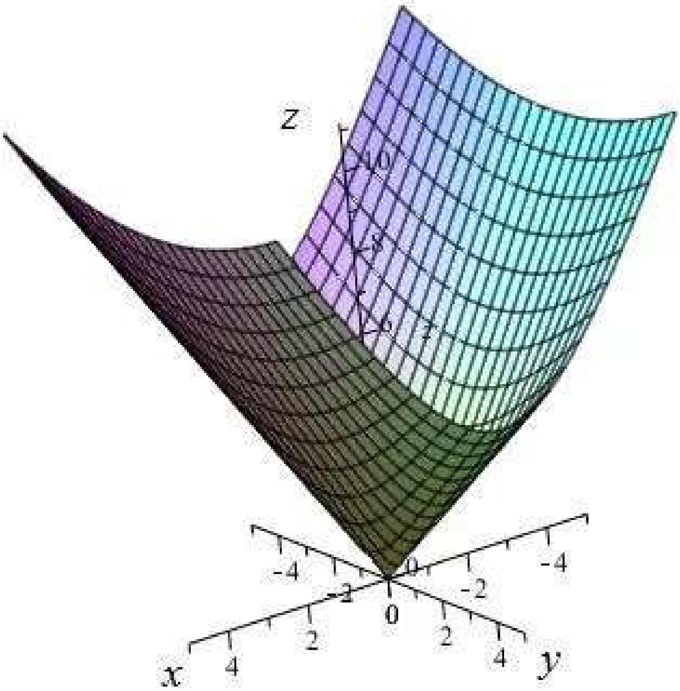
The equation of a cone is,

$$\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2} \quad \ldots\ldots (3)$$

So by comparing the equations (2), and (3), the equation (1) represents the equation of a cone.

But by (1), since $z \geq 0$, the graph of z is just the top half of this cone.

The graph of the cone $z=\sqrt{4x^2+y^2}$ is shown as follows:



Answer 31E.

To sketch the graph of the function $z=\sqrt{4-4x^2-y^2}$, first simplify this equation as follows:

$$z=\sqrt{4-4x^2-y^2}$$

$$z^2=4-4x^2-y^2 \quad \text{(Squaring on both sides)}$$

$$4x^2+y^2+z^2=4$$

$$\frac{x^2}{1^2}+\frac{y^2}{2^2}+\frac{z^2}{2^2}=1 \quad \text{(Dividing both sides by 4)} \ \ldots\ldots (1)$$

The equation of the ellipsoid is,

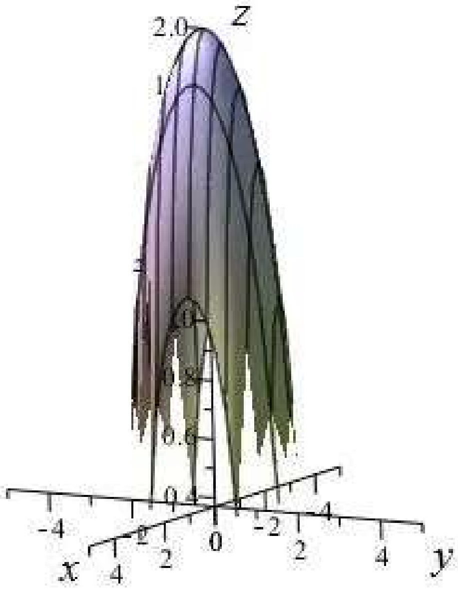
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \ \ldots\ldots (2)$$

By comparing the equations (1), (2), the equation (1) represents an ellipsoid.

But, $z=\sqrt{4-4x^2-y^2}$

Since $z\geq 0$, this equation represents top half of the ellipsoid.

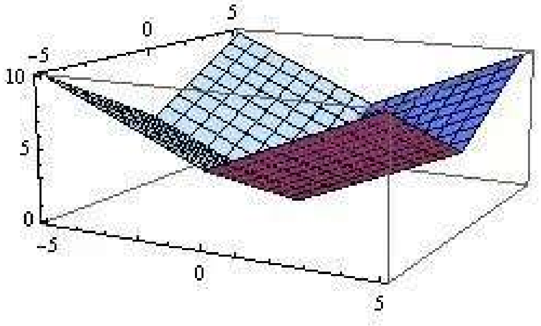
The sketch of the function $z=\sqrt{4-4x^2-y^2}$ is shown as follows:



Answer 32E.

a) $f(x, y) = |x| + |y|$ the graph of this function is VI

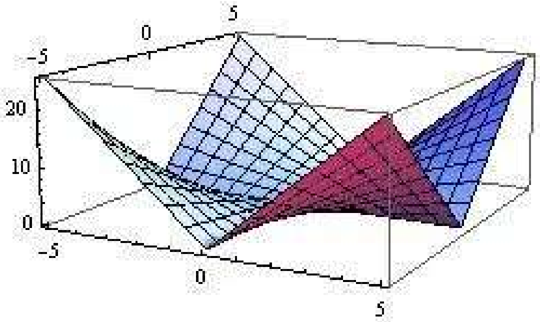
When x=0 z=| y| and when y=0 then z= |x|



b) $f(x, y) = |xy|$ match with V

When x=1 then z=|y| and when y=1 then z=|x|

When x=0 then z=0 and when y=0 then z=0

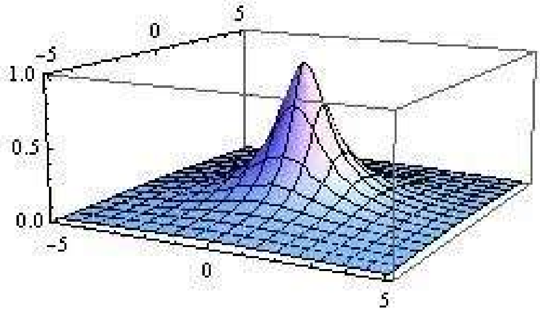


c) $f\left(x, y\right) = \frac{1}{1 + x^2 + y^2}$ its graph is l

When x=0 $z = \frac{1}{1 + y^2}$ and when y=0 $z = \frac{1}{1 + x^2}$.

When x=y=0 then z=1

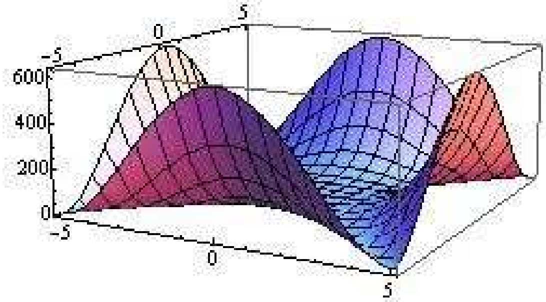
When $x \longrightarrow \infty$ or $y \longrightarrow \infty$ then $z \longrightarrow 0$



d) $f\left(x, y\right) = \left(x^2 - y^2\right)^2$ match with IV

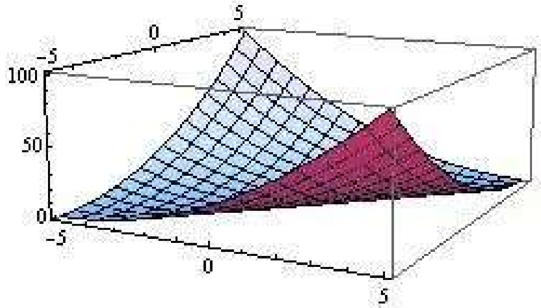
When x=±y then z=0

When x=0 then z=y4 and when y=0 then z=x4



e) $f\left(x, y\right) = (x-y)^2$ match with II

When x=0 z=x2 and if y=0 then z=x2
If x=y then z=0

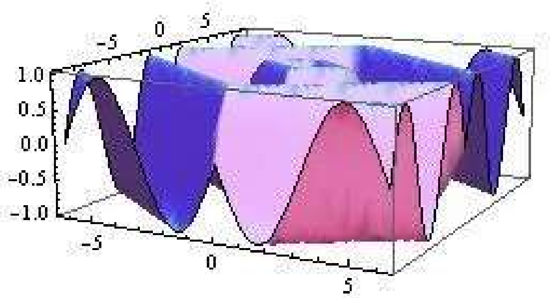


f) $f(x, y) = \sin(|x| + |y|)$ match with III

We have a sine function.

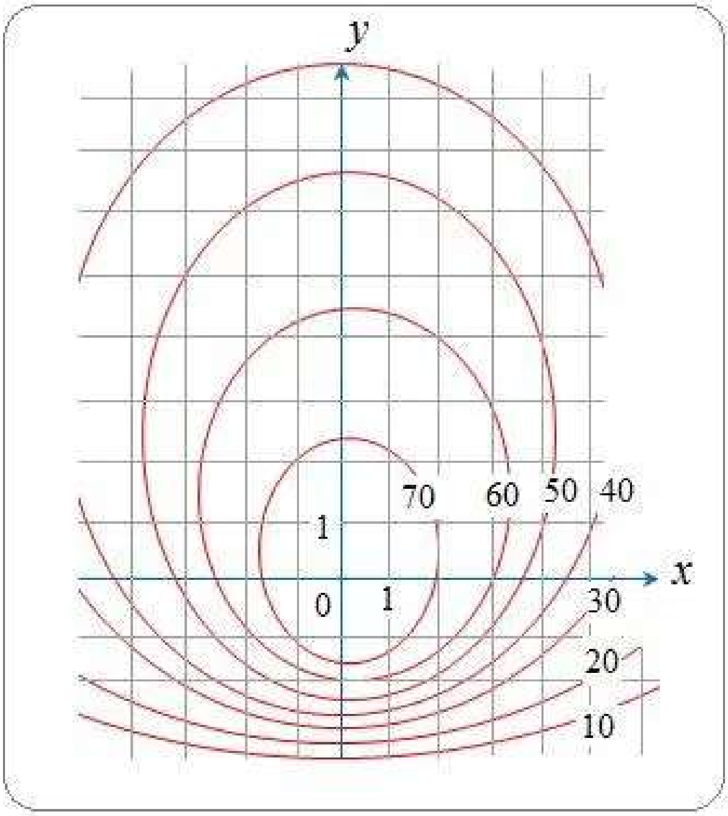
When x=0 z=sin|y| and when y=0 z=|x|

When x=y=0 then z=0



Answer 33E.

Sketch a contour map of the following function f as follows:



The point $(-3,3)$ lies between the level curves $f(x,y)=50$ and $f(x,y)=60$. The point is little closer to $f(x,y)=60$, so, the estimated value is as follows:

$f(-3,3)\approx 56.$

Similarly the point $(3,-2)$ appears to be just about halfway between the level curves $f(x,y)=30$ and $f(x,y)=40$. So, the estimated value is as follows:

$f(3,-2)\approx 35.$

Therefore, the values of $f(-3,3)\approx \boxed{56}$ and $f(3,-2)\approx \boxed{35}$.

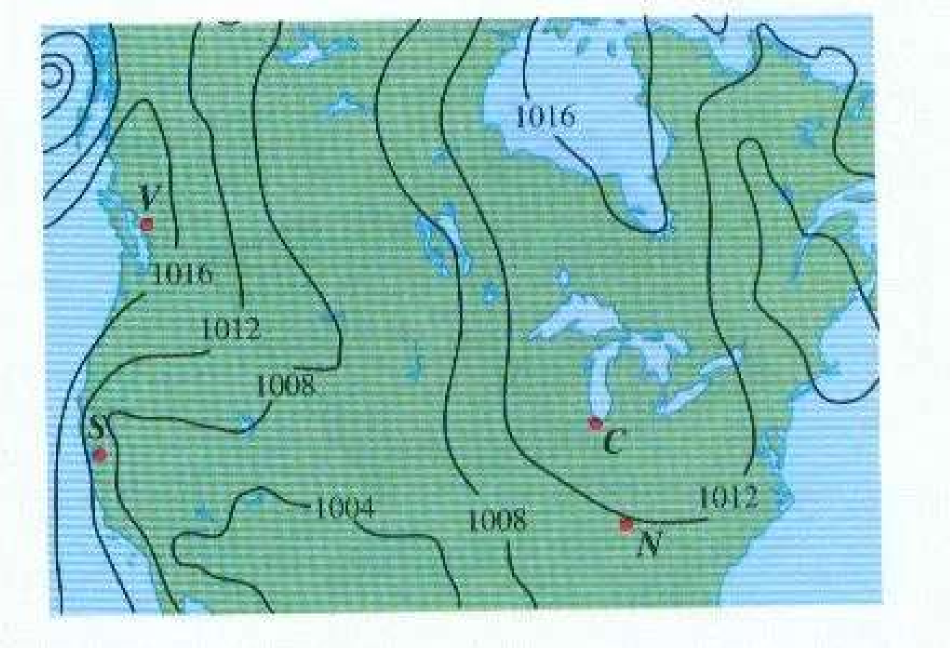
From the graph, observe that the level curves rises when it approaches towards zero, gradually from above and steeply from below.

Answer 34E.

2460-14.1-34E

SA Code: 767
SR Code: 733
RID: 587

Given below is the diagram.



(a) Let z denote the level curves.

We note that the point C lies between the level curves $z = 1016$ and $z = 1012$. Since the point is little closer to the level curve $z = 1012$, we estimate that the atmospheric pressure at Chicago is about 1013 mb.

Similarly the point N lies near the curve $z = 1012$. Thus, we can say that the atmospheric pressure at Nashville is about 1011.9 mb.

The point S lies between $z = 1008$ and $z = 1012$. Since the point is nearer to $z = 1012$, atmospheric pressure at San Francisco is about 1011 mb.

Now, the point V lies close to the level curve $z = 1016$. Therefore, the atmospheric pressure at V is about 1017 mb.

(b) The regions where the isobars are close together are the areas where the winds are strongest. From the given map, we can say that stronger winds blow at San Francisco.

Answer 35E.

Let $z = f(x, y)$ be the required function, where x is the depth of the lake, y is the time of the year, and z is the water temperature.

We have to find $f(160, 10)$.

We note that the point (160, 10) lies between the level curves $z = 8$ and $z = 12$. Since the point is little closer to the level curve $z = 12$, we estimate that $f(160, 10) \approx 11°\text{C}$.

Similarly, the point (180, 5) lies between the curves $z = 16$ and $z = 20$. Also, the point is closer to the curve $z = 20$. Thus, we can say that $f(180, 5) \approx 19.5°C$.

Answer 36E.

Consider the following two contour maps:

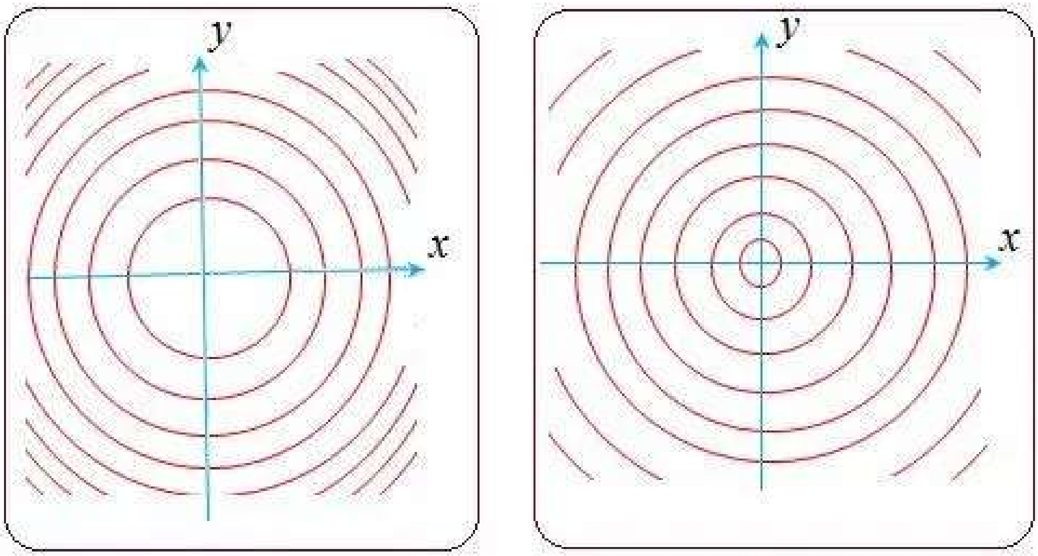


Fig I Fig II

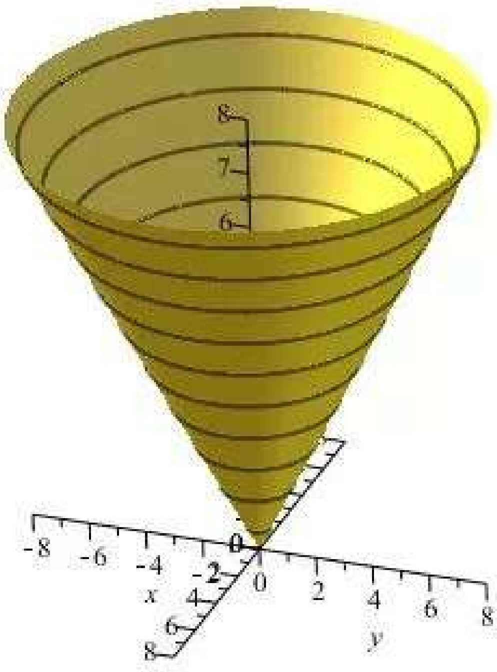
Suppose the surface has the following equation:

$z = f(x, y).$

A contour is obtained by slicing the surface with a horizontal plane with equation, $z = k.$

The z values of the cone centered at the origin increase at a constant rate, so the level curves of the cone are equally spaced.

Sketch the graph of the cone is as shown below



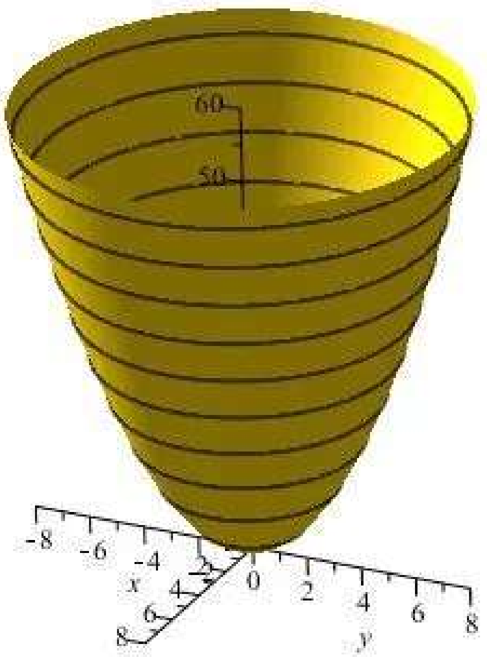
Suppose the surface has the following equation

$z = g(x, y).$

A contour is obtained by slicing the surface with a horizontal plane with equation $z = k.$

A paraboloid has vertex at the origin, on the other hand the values of z changes slowly near the origin and move quickly as move farther away. Thus, the level curves of the paraboloid near the origin to be spaced more widely apart than those from the origin.

Sketch the graph of the paraboloid is as shown below



Therefore, the contour map in Fig I corresponds to the paraboloid and contour map in Fig II is corresponds to the cone.

Answer 37E.

On the map of the Lonesome Mountain, the surface of a graph is steep where the level curves are close together and more gradual where they are farther away. Notice that the level curves are relatively close together near *A* and relatively far apart near *B*. Therefore, the terrain is steep near *A* and gradual near *B*.

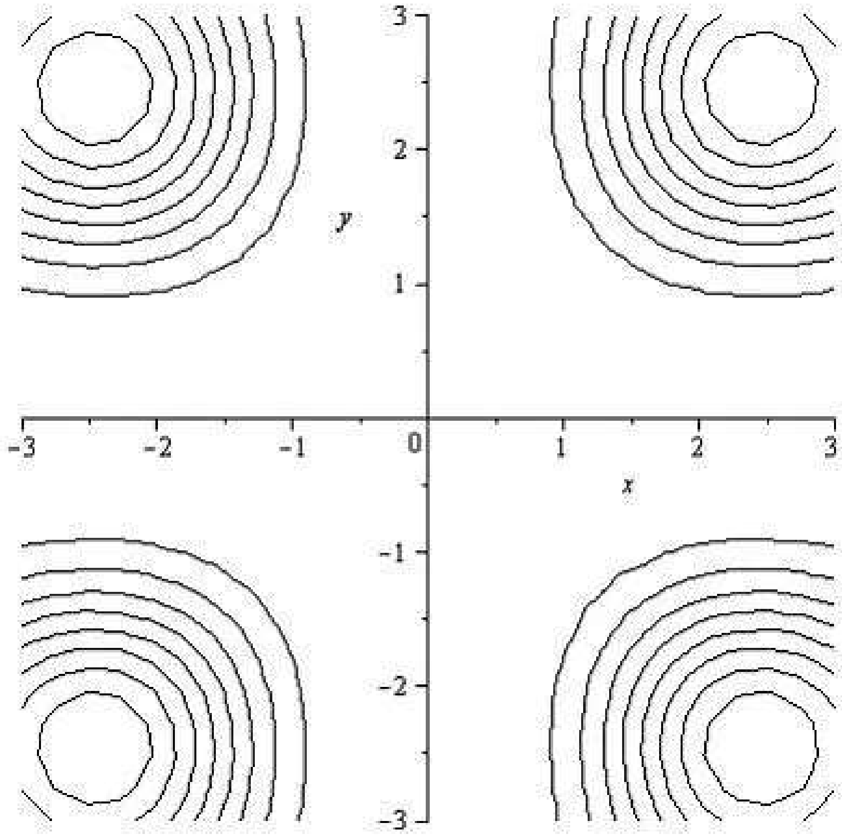
Therefore the level curves at point *A* are very close together, indicating that the terrain is $\boxed{\text{quite steep}}$ and at point *B* the level curves are much farther apart, and then expect that the terrain is less steep than at the point *A*. So the terrain near *B* is $\boxed{\text{almost flat}}$.

Answer 38E.

From the give figure, we can say that the contour map has four peaks.
Thus, the function representing the given graph will be of the

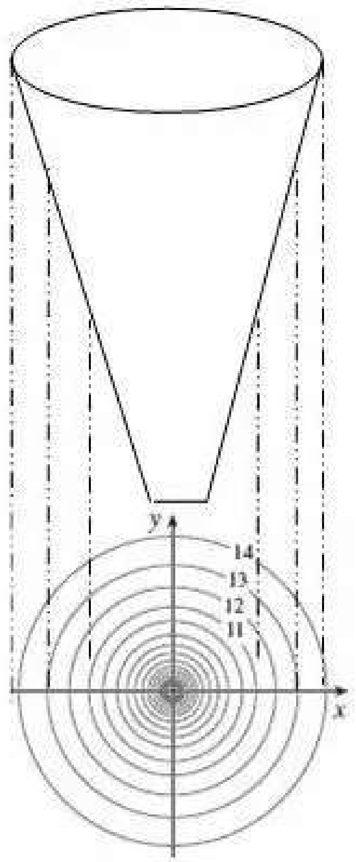
$$f(x,y) = x^4y^4e^{-(x^2+y^2/3)}$$

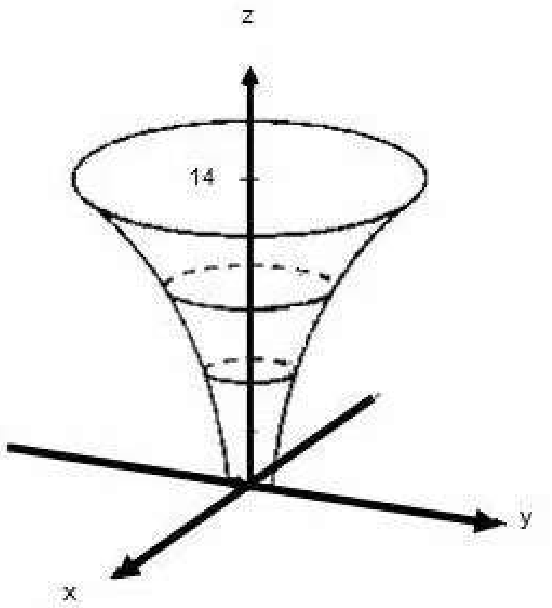
Now, sketch the contour map of the function.



Answer 39E.

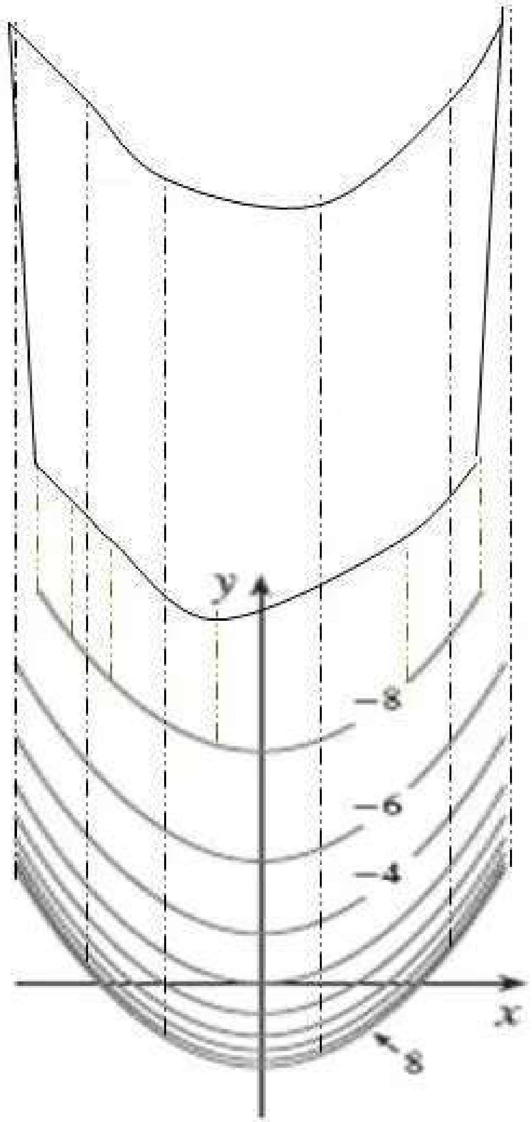
The level curves f(x,y) = k are just the traces of the graph of f in the horizontal plane z = k projected down to the x-y plane.The surface is steep where the level curves are close together and somewhat flatter where they are farthere apart.

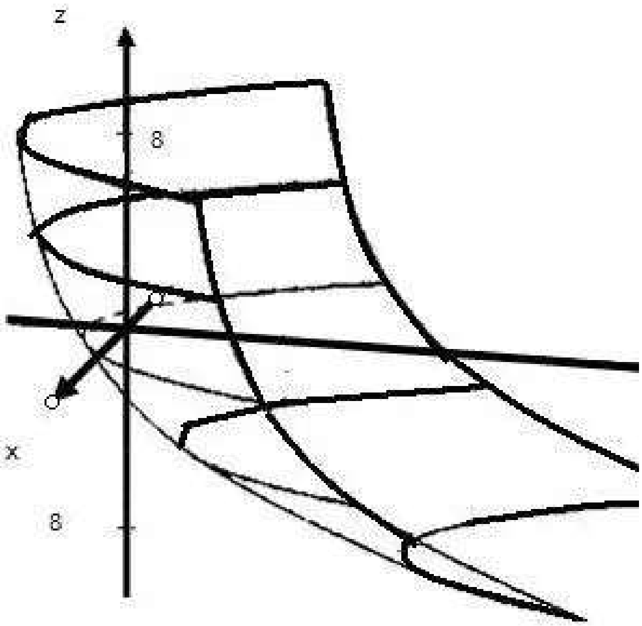




Answer 40E.

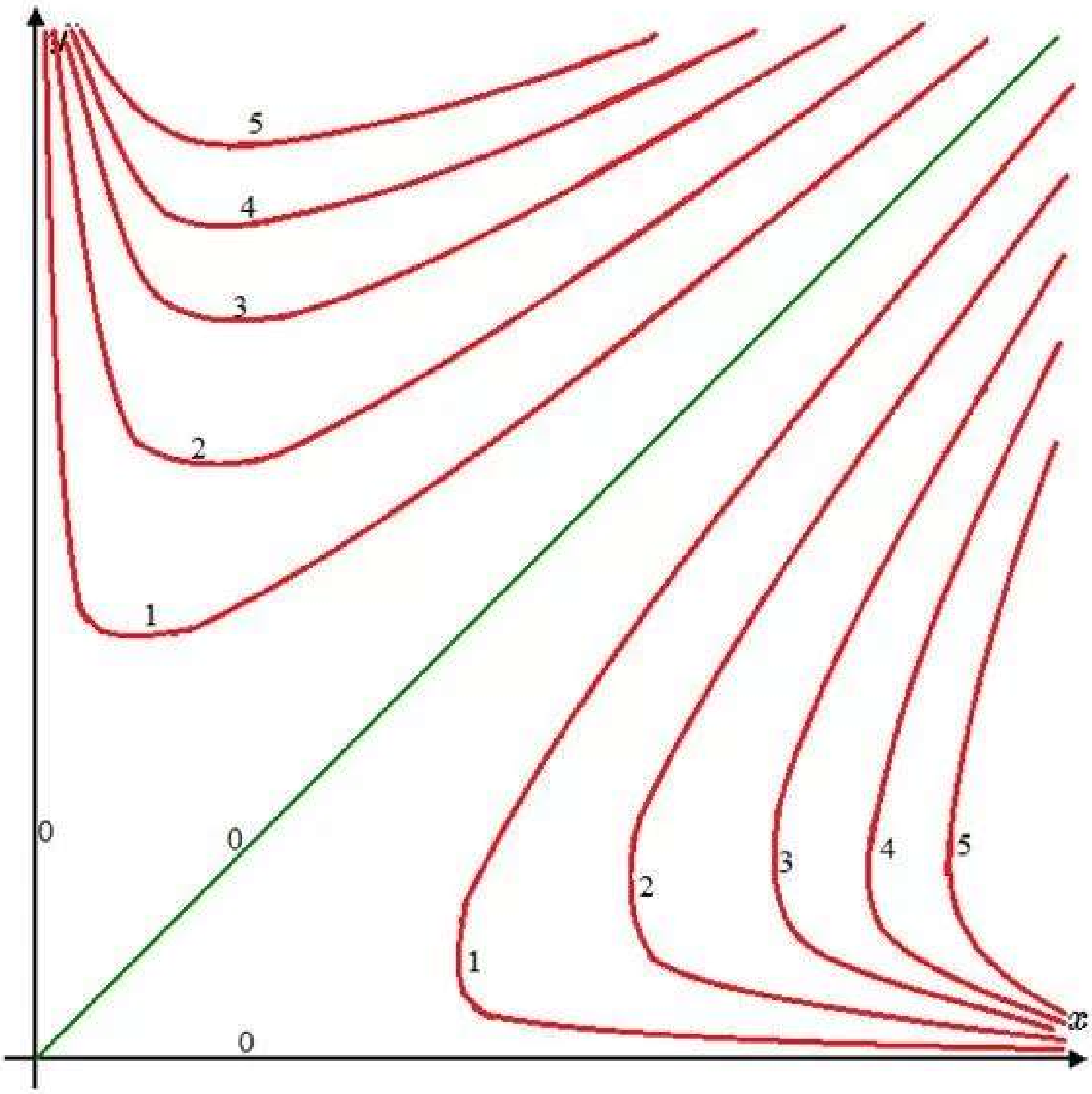
The level curves f(x,y) = k are just the traces of the graph of f in the horizontal plane z = k projected down to the x-y plane. The surface is steep where the level curves are close together and somewhat flatter where they are farthere apart.





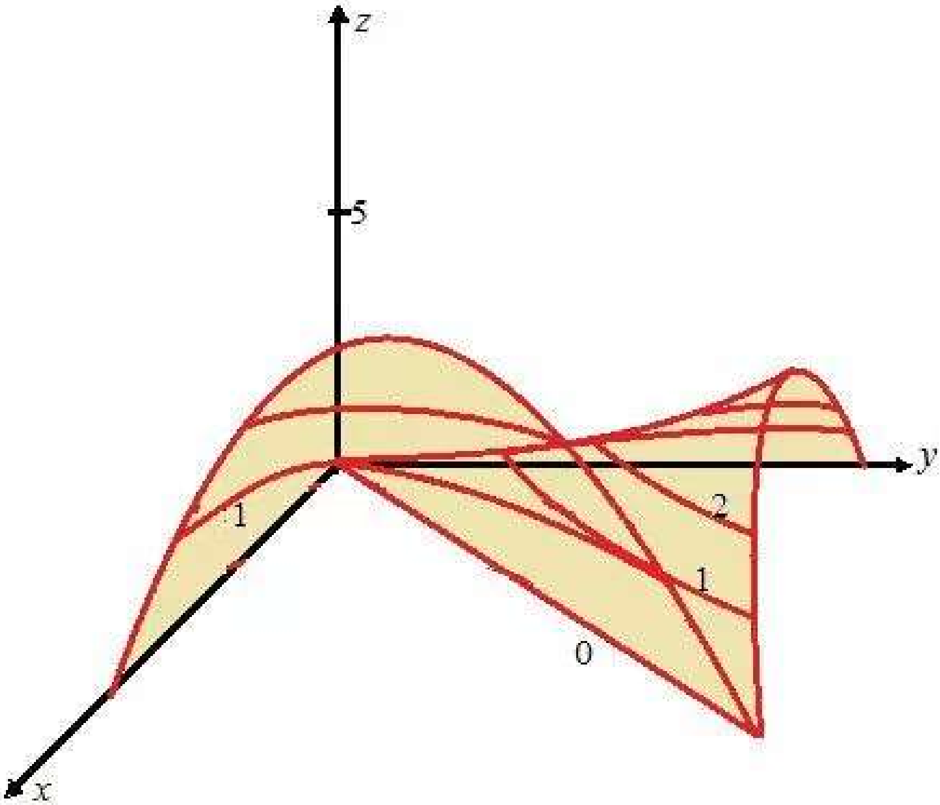
Answer 41E.

Consider the following contour map:



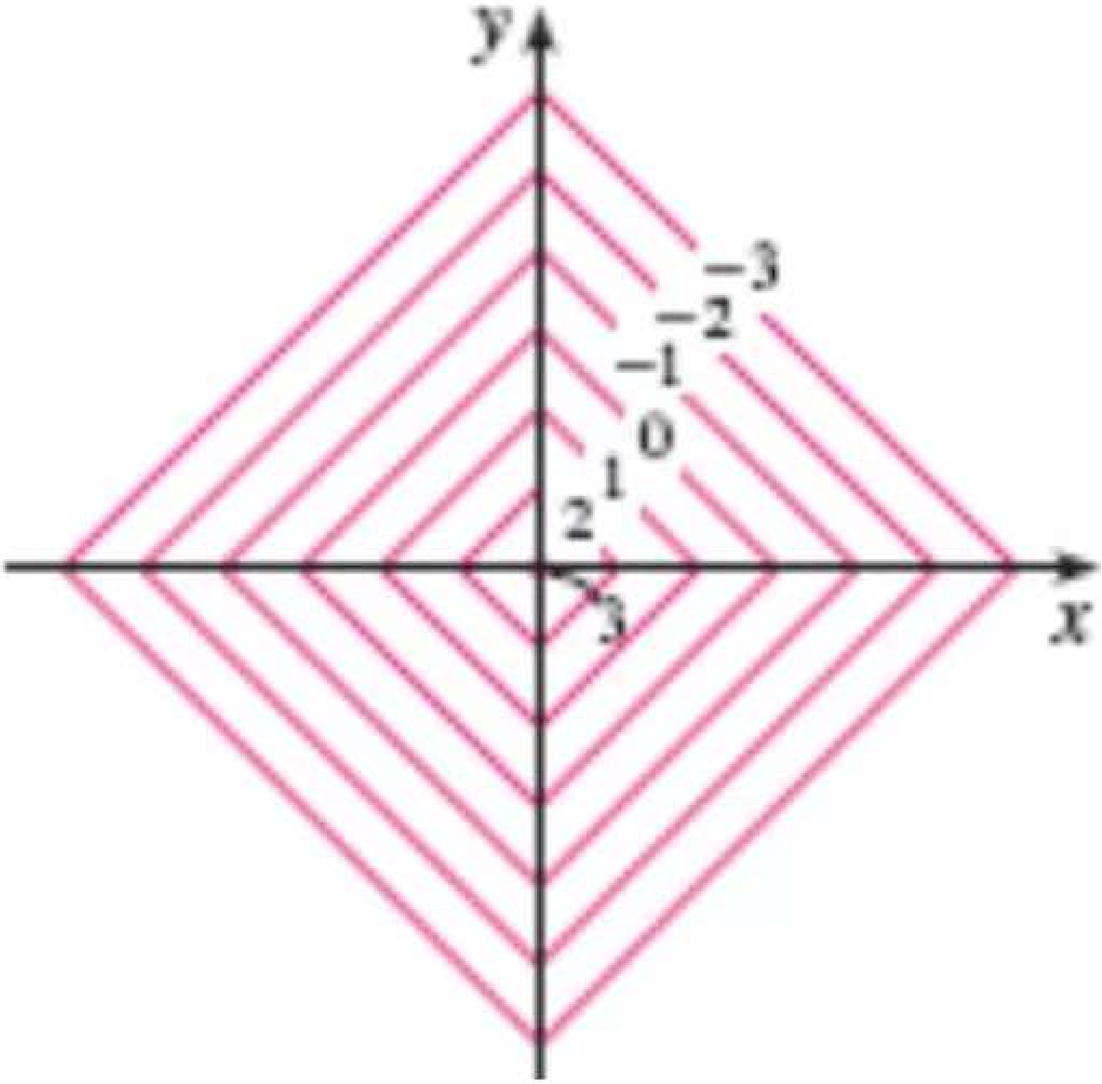
Observe that each curve is symmetric about the line $y = x$. So, each curve is possesses an inverse in the 1st quadrant itself. Assuming the label at each curve is the height, the surface looks like two furls symmetric about the line $y = x$. The label 0 along the line,

$y = x$ shows the symmetric furl that touches the xy - plane along $y = x$ and bulges from there. Each contour is labelled on the surface as well. This is shown as follows:



Answer 42E.

Consider the following contour map:

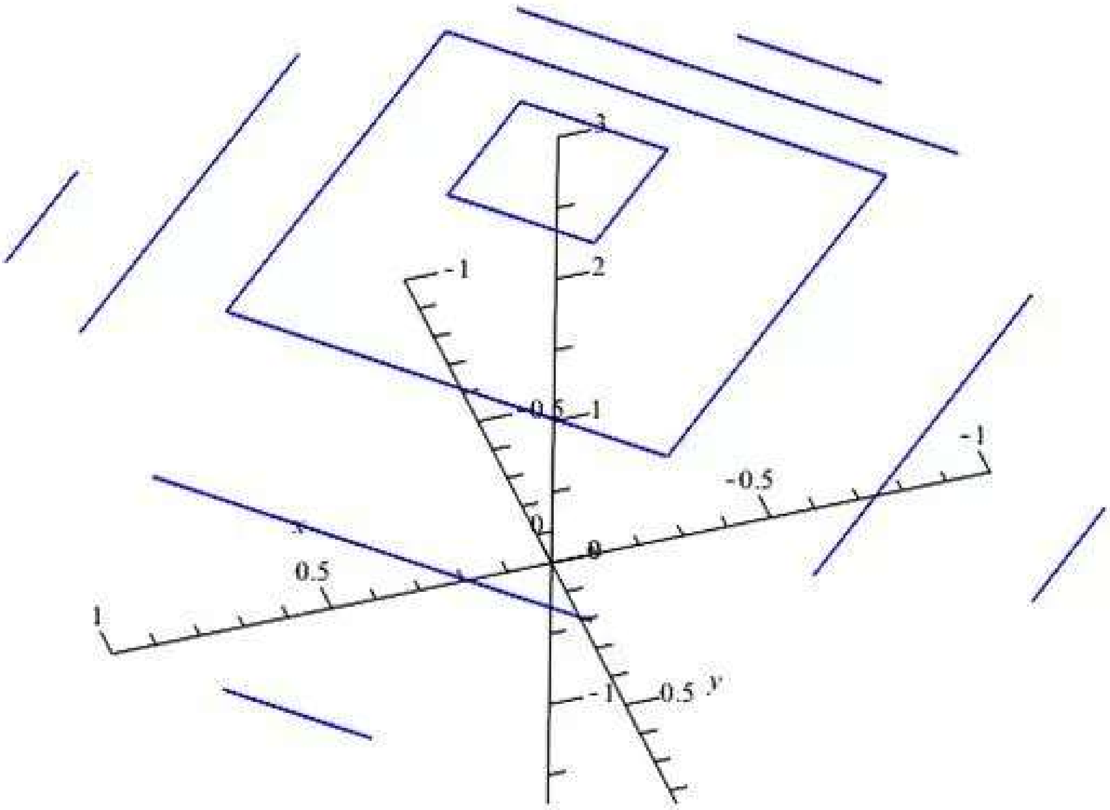


The objective is to make rough sketch of the graph of *f*.

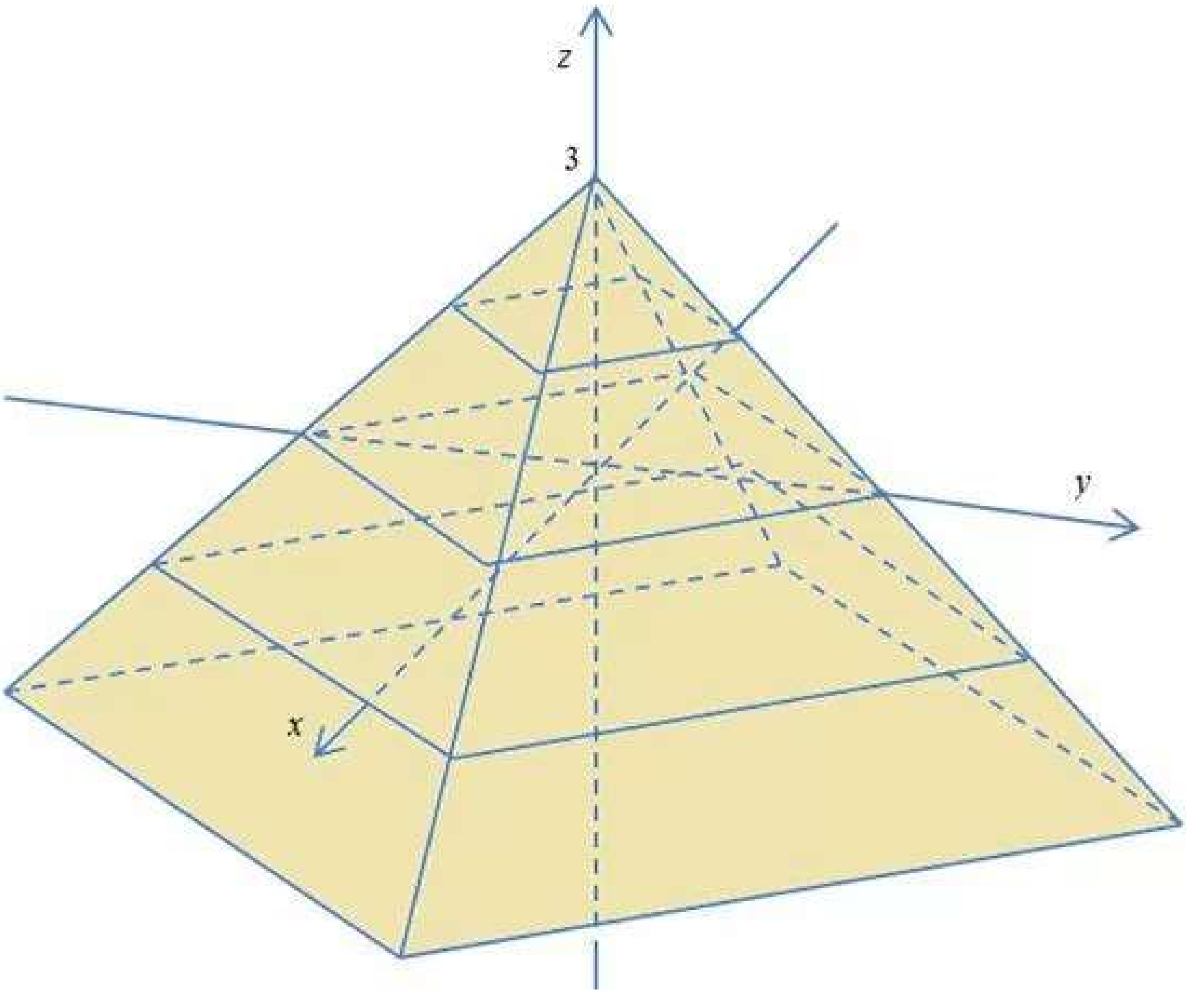
Observe that, the surface of a contour map of a function is a pyramid with four walls and it is like the top half of an octahedron.

In fact, it is the graph of the function such as $f(x,y)=3-|x|-|y|$

Use the contour map of a function, the rough sketch of the graph of *f* is shown below:



The horizontal traces are squares and the level curves are equally spaces, which means that the steepness along the graph remains constant. The surface is given by a pyramid with a square base.



Answer 43E.

Consider the following function:

$$f(x,y)=(y-2x)^2.$$

The objective is to draw the contour map of the function and show that the several level curves.

Use the result.

The level curves of a function f of two variables are the curves with equations $f(x,y)=k$ where k is a constant.

The given function is,

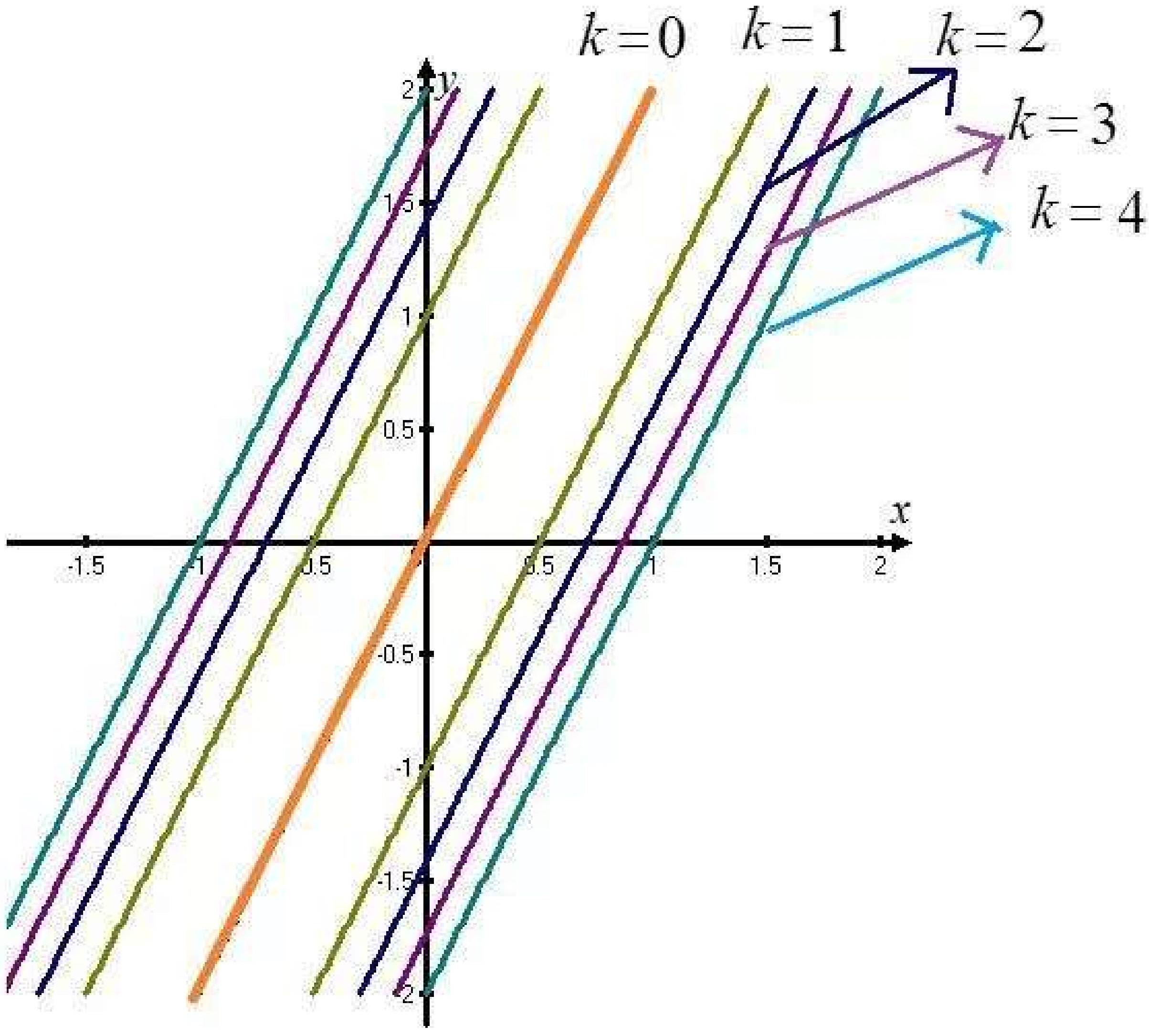
$$f(x,y)=(y-2x)^2$$

The level curves of $f(x,y)$ are

$$(y-2x)^2=k,\quad k=\ldots,-3,-2,-1,0,1,2,3\ldots$$

Sketch the level curves with $k=0,1,2,3,4,\ldots$

Draw the following level curves with $k=0,1,2,3,4,\ldots$:

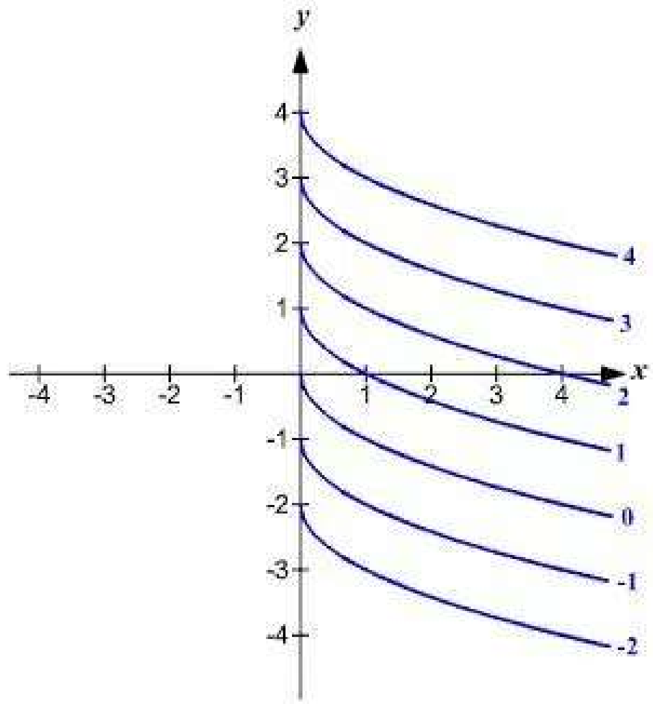


Therefore, the contour lines in the graph form a band.

Answer 45E.

We have $f(x, y) = \sqrt{x} + y$. The level curves are given by $\sqrt{x} + y = k$ or $y = -\sqrt{x} + k$.

Let us sketch the level curves by replacing k with −2, −1, 0, 1, 2, 3, and 4.

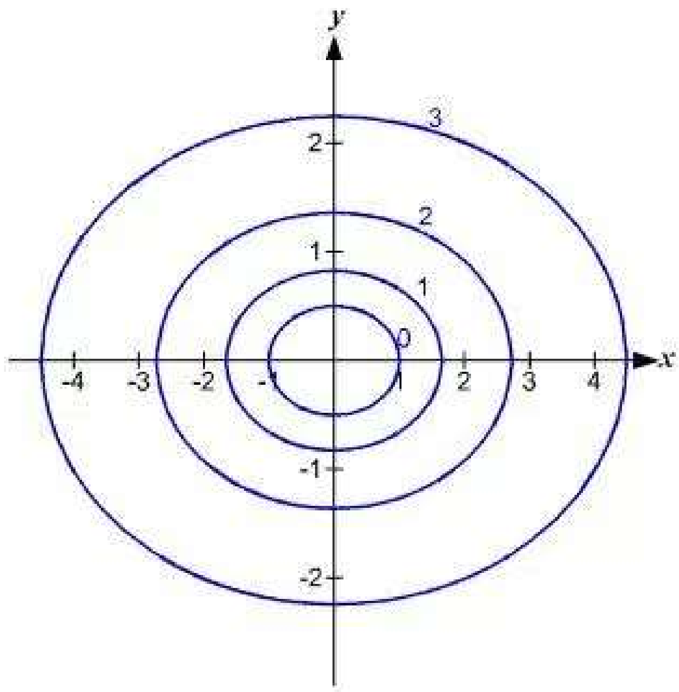


Answer 46E.

We have $f(x, y) = \ln(x^2 + 4y^2)$. The level curves are given by $\ln(x^2 + 4y^2) = k$ or

$$y = \pm\frac{\sqrt{e^k - x^2}}{2}.$$

Let us sketch the level curves by replacing k with 0, 1, 2, and 3.



Answer 47E.

Consider the following function;

$$f(x,y)=ye^x.$$

To draw a contour map of the function, show several level curves.

To complete the drawing, graph equations with varying values of c as a value of the function.

$$ye^x=c$$

To graph this equation, first solve for y.

$$ye^x=c$$

$$y=\frac{c}{e^x}$$

$$=ce^{-x}$$

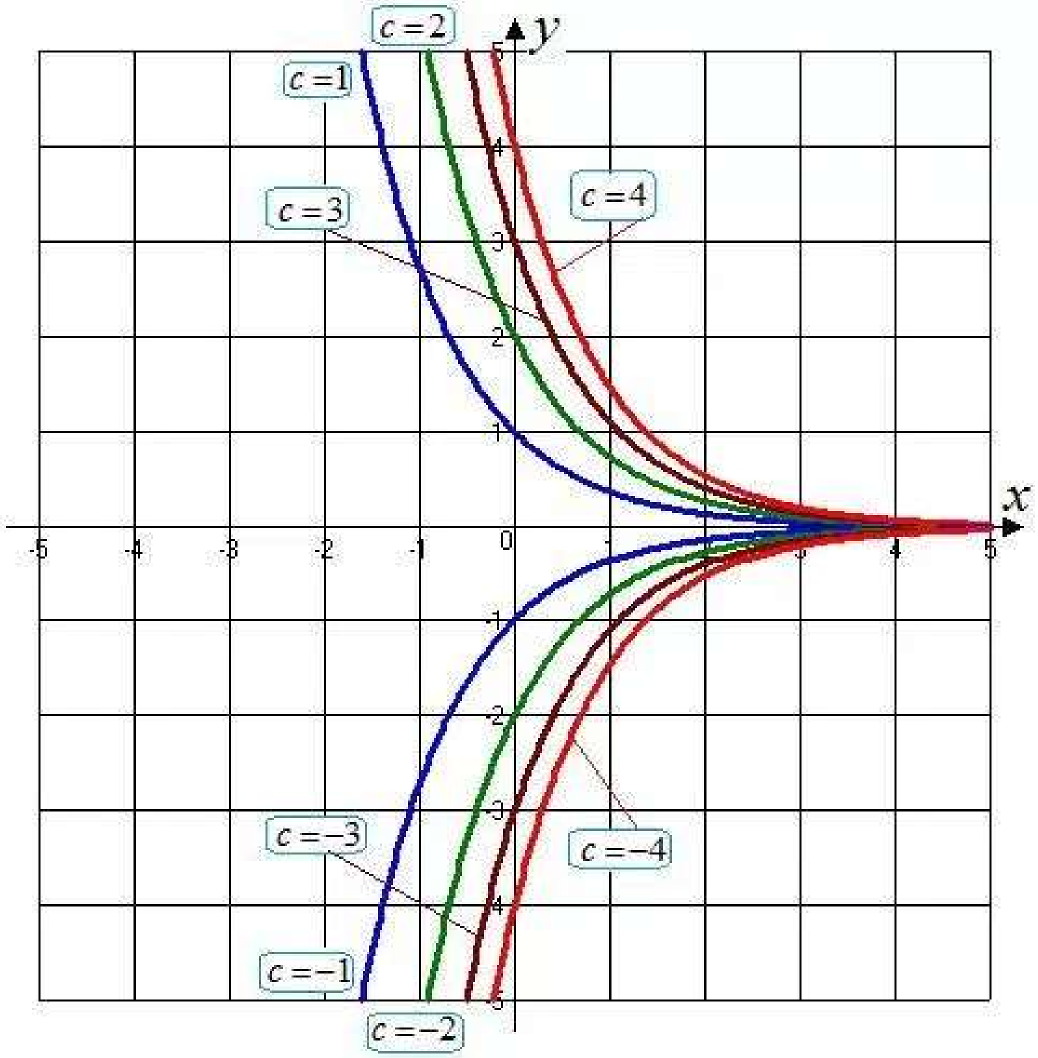
So, $y=ce^{-x}$.

For example, the equation where $c=1$ is $y=(1)e^{-x}$.

Similarly, put $c=-1,-2,-3,-4,2,3,4$ in the function, $y=ce^{-x}$.

Then graph the curves at those values of c.

The graph of the level curves with varying values of c is as shown below.



Answer 48E.

Consider the following function;

$$f(x,y) = y\sec x.$$

To draw a contour map of the function, write several level curves of the function, f.

To complete the drawing, graph equations with varying values of c as a value of the function.

Let $y\sec x = c$

To graph this equation, first solve for y.

$$y\sec x = c$$

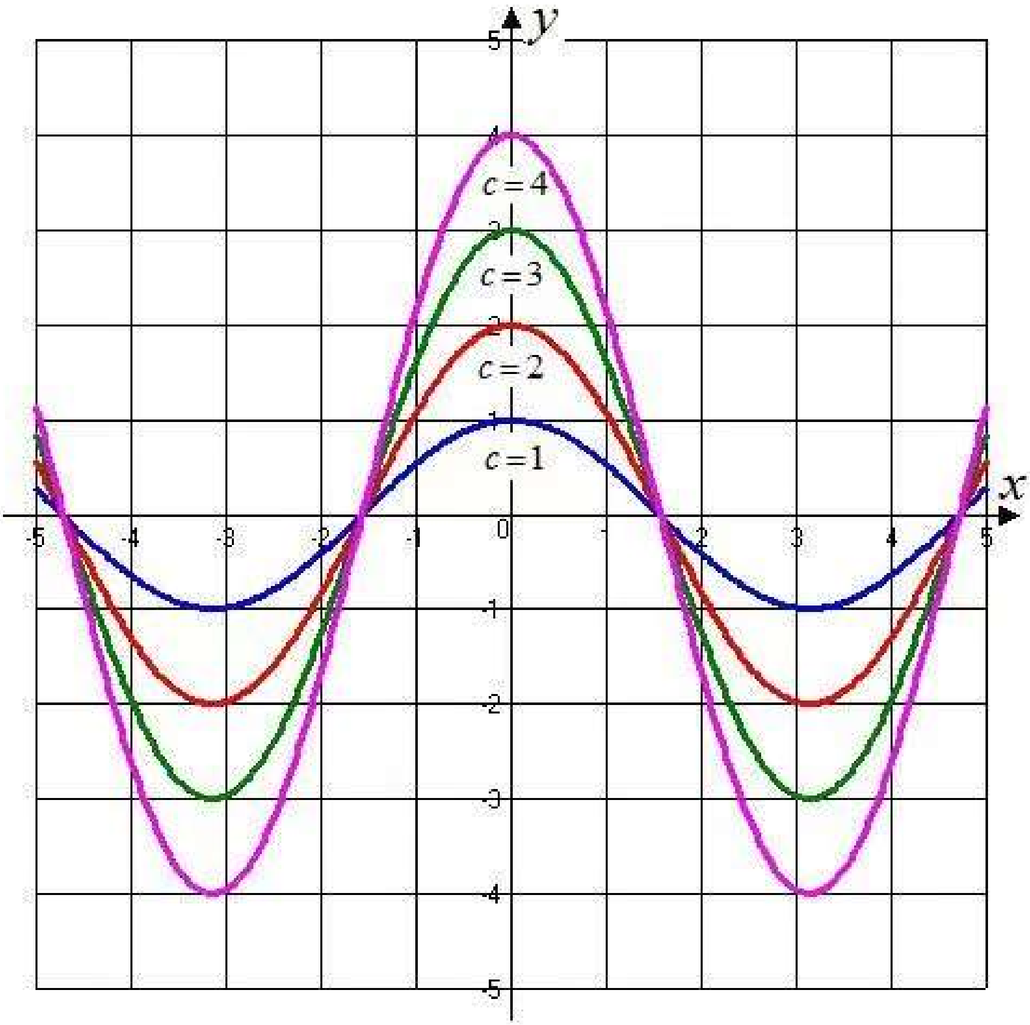
$$y = \frac{c}{\sec x}$$

$$y = c\cos x$$

For example, the equation where $c = 4$ is

$$y = 4\cos x$$

The graph of the level curves, with varying values of c is shown below.



Answer 49E.

Consider the following function.

$$f(x,y)=\sqrt{y^2-x^2}$$

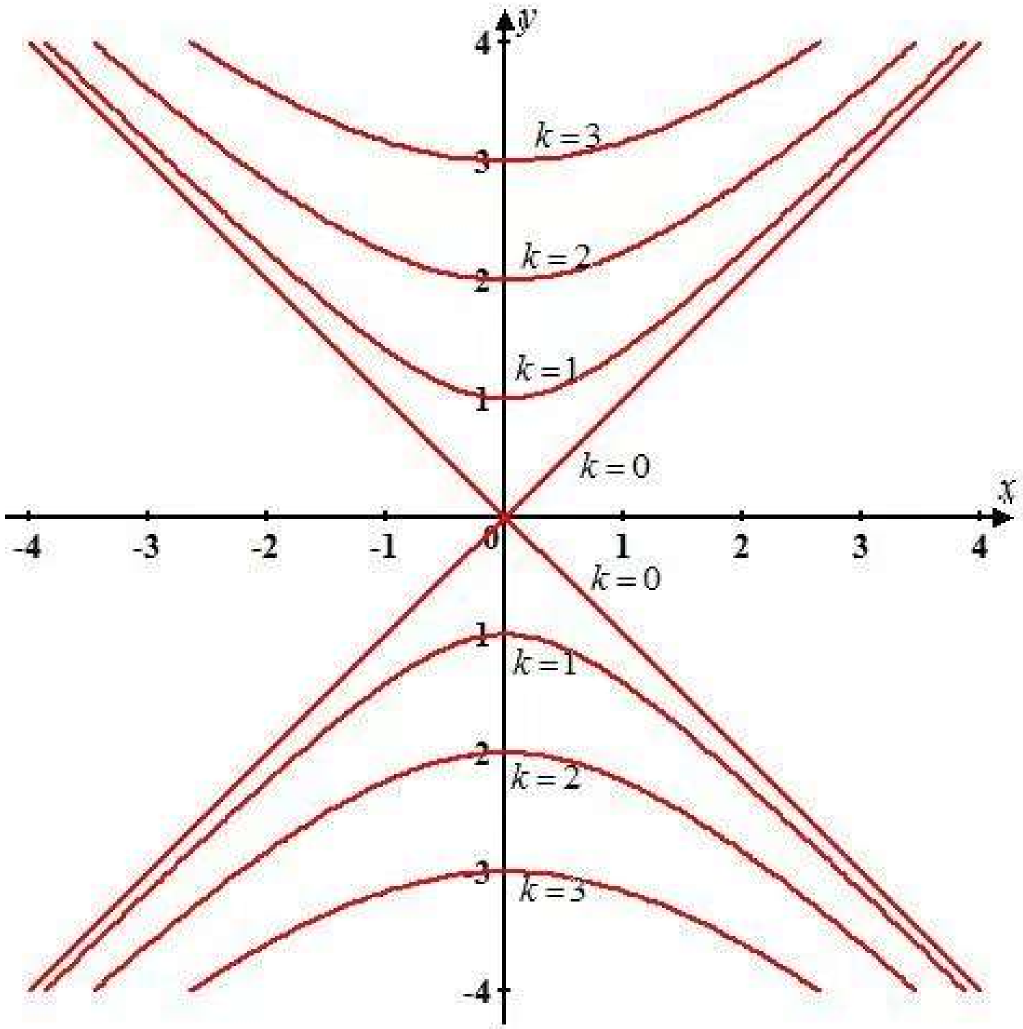
The objective is to draw the contour map of the function.

The contour map can obtain by setting up the function $f(x,y)=k$, where k is a constant.

Draw the level curves for various values of k.

The level curves of $f(x,y)$ are

$$\sqrt{y^2 - x^2} = k, k = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$



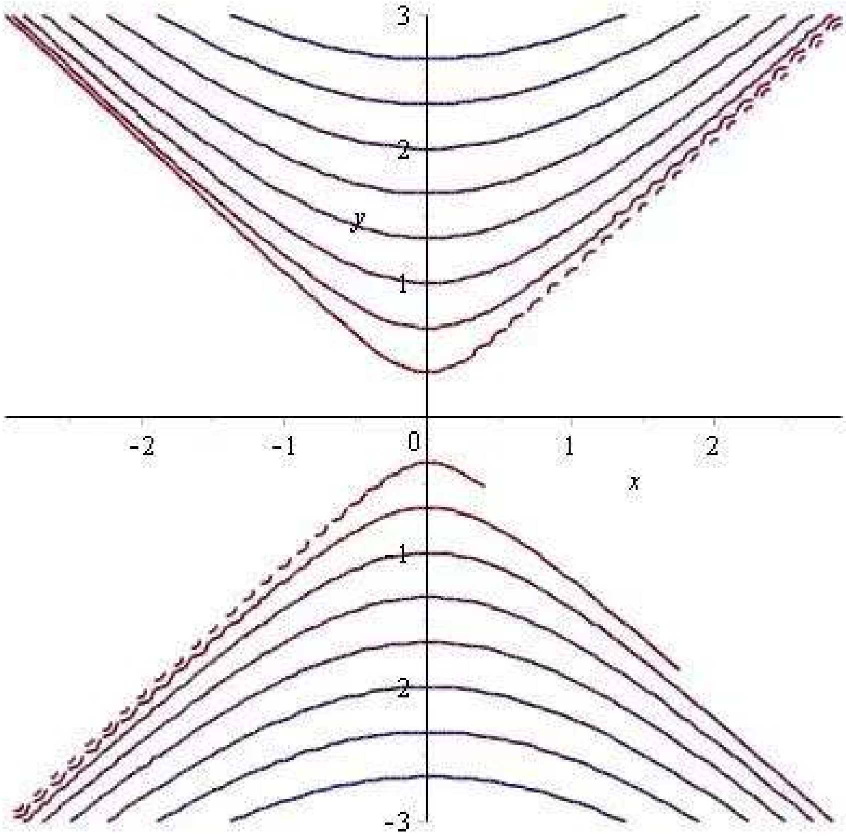
The graph of can be confirmed by using by a CAS.

Use the following Maple software commands.

with(plots);

contourplot(sqrt(-x^2+y^2), x = -3 .. 3, y = -3 .. 3);

The Maple output is shown below.



Answer 50E.

The equation of a curve is $f(x,y)=\frac{y}{(x^2+y^2)}$.

The objective is to draw a contour map of the function showing several level curves.

Recollect that the level curves of a function f of two variables are the curves with equations $f(x,y)=k$, where k is a constant.

So, the level curves of $f(x,y)$ are,

$$\frac{y}{(x^2+y^2)}=k, \quad k=\ldots,-3,-2,-1,0,1,2,3,\ldots$$

These level curves are shown below:

$$\frac{y}{(x^2+y^2)}=-3$$

$$\frac{y}{(x^2+y^2)}=-2$$

$$\frac{y}{(x^2+y^2)}=-1$$

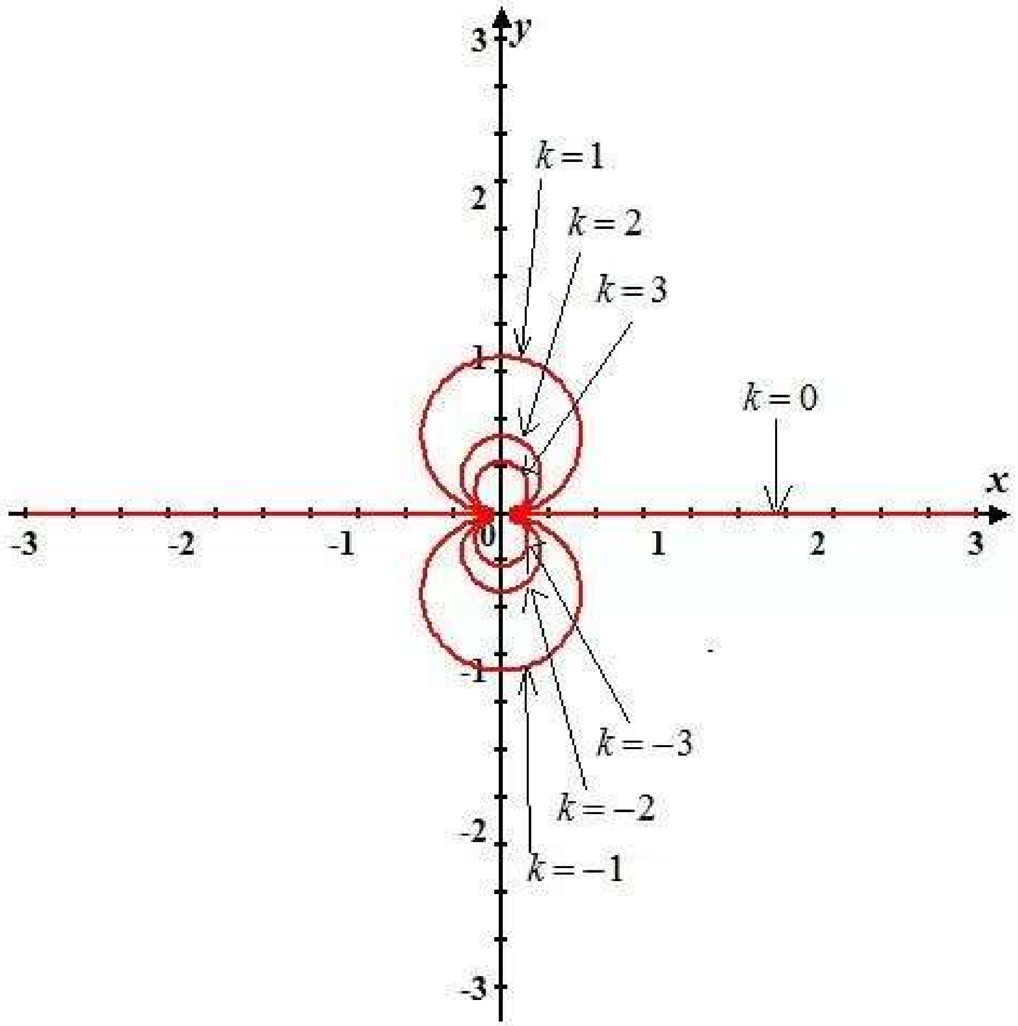
$$\frac{y}{(x^2+y^2)}=0$$

$$\frac{y}{(x^2+y^2)}=1$$

$$\frac{y}{(x^2+y^2)}=2$$

$$\frac{y}{(x^2+y^2)}=3$$

The contour map showing different level curves are shown below:



Answer 51E.

Consider the function $f(x,y)=x^2+9y^2$.

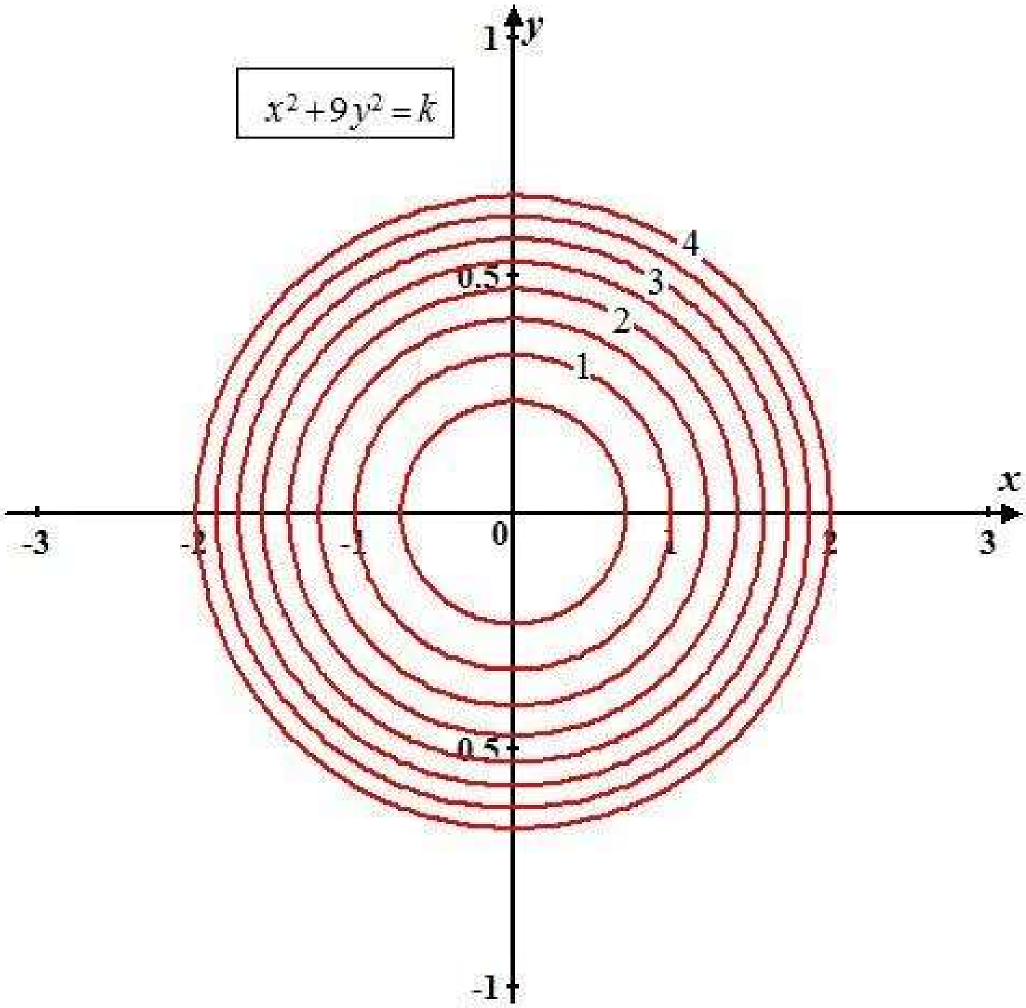
The objective is to sketch both contour map and a graph of the function and compare them.

Recall that the definition of level curves, the level curves of a function f of two variables are the curves with equations $f(x,y)=k$, where k is a constant.

The level curves of $f(x,y)$ are $x^2+9y^2=k,\ k=0,1,2,3,\ldots$.

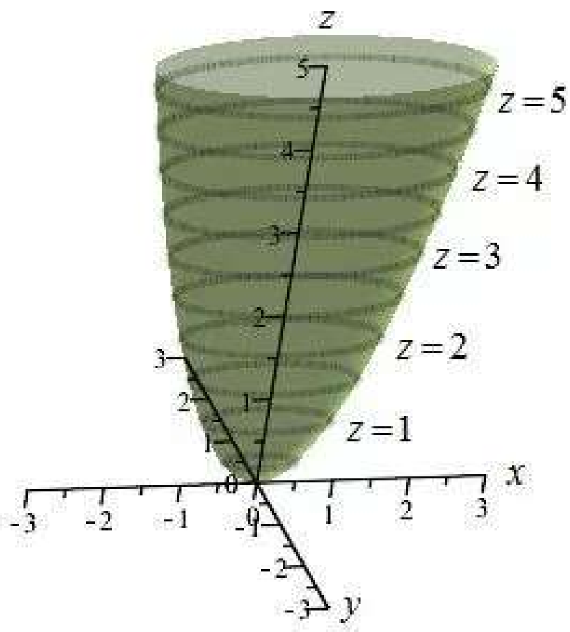
This is family of concentric circles with center $(0,0)$ and radius $\sqrt{k}$.

The cases $k=0,1,2,3,\ldots$ are shown in the below figure.



The figure shows a contour map along with level curves for the function

$f(x,y) = x^2 + 9y^2$.



A level curves $f(x,y) = k$ is the set of all points in the domain of f at which f takes on a given value k. In other words, it shows where the graph of f has height k.

Observe the two graphs, the relation between the level curves and horizontal traces.

The level curves $f(x,y) = k$ are just the traces of the graph of f in the horizontal plane $z = k$ projected down to the xy-plane.

Draw the level curves of a function and visualize them lifted up to the surface at the indicated height.

Answer 52E.

Consider the function $f(x, y) = \sqrt{36 - 9x^2 - 4y^2}$

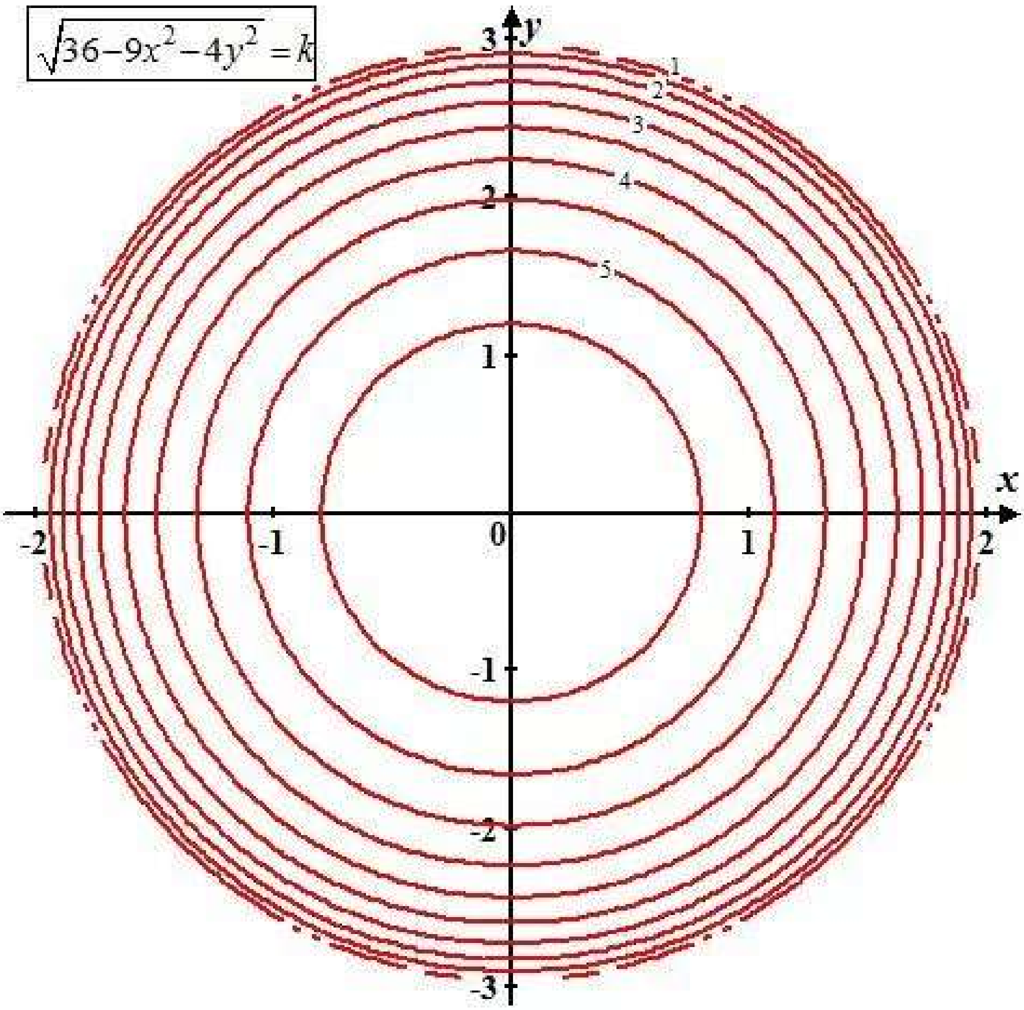
Recall that the definition of level curves:

The level curves of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant.

The level curves of $f(x, y)$ are

$$\sqrt{36 - 9x^2 - 4y^2} = k, \quad k = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

Sketch the level curves with $k = 1, 2, 3, 4, 5$:



The figure shows a contour map along with level curves for the function

$f(x,y)=\sqrt{36-9x^2-4y^2}$.

Use Maple to check the contour map showing several level curves.

Maple Input:

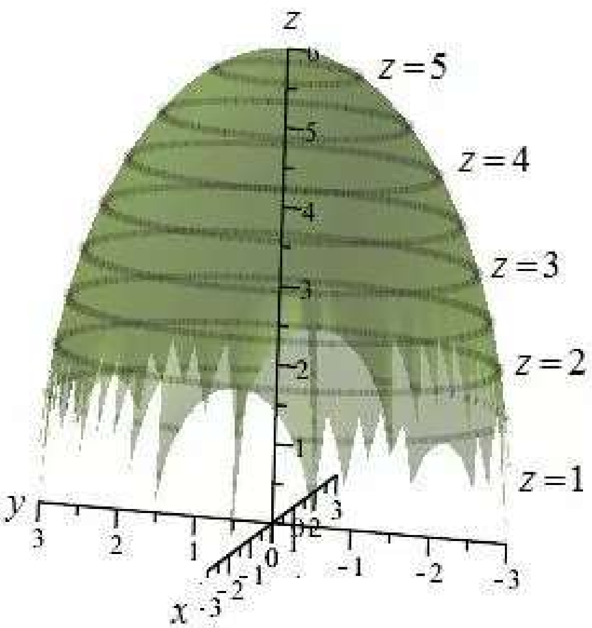
> with(plots):

> contourplot(sqrt(36-9*(x^2)-4*(y^2)), x=-3..3,y=-3..3);

Maple Output:

> *with*(*plots*) :

> $plot3d\left(\sqrt{36-9x^2-4y^2}, x=-3..3, y=-3..3\right);$



A level curves $f(x,y)=k$ is the set of all points in the domain of f at which f takes on a given value k. In other words, it shows where the graph of f has height k.

Observe the two graphs, the relation between the level curves and horizontal traces. The level curves $f(x,y)=k$ are just the traces of the graph of f in the horizontal plane $z=k$ projected down to the xy-plane. So draw the level curves of a function and visualize them being lifted down to the surface at the indicated height. The surface is steep where the level curves are apart from each other. It is somewhat flatter where they are close together.

Answer 53E.

Consider the data,

A thin metal plate, located in the xy-plane, has temperature $T(x,y)$ at the point (x,y). The level curves of T are called isothermals because at all points on an isothermal the temperature is the same.

To sketch some isothermals if the temperature function is given by

$$T(x,y)=\frac{100}{(1+x^2+2y^2)}$$

Recall that the definition of level curves:

The level curves of a function f of two variables are the curves with equations $f(x,y)=k$, where k is a constant.

From the definition the level curves are

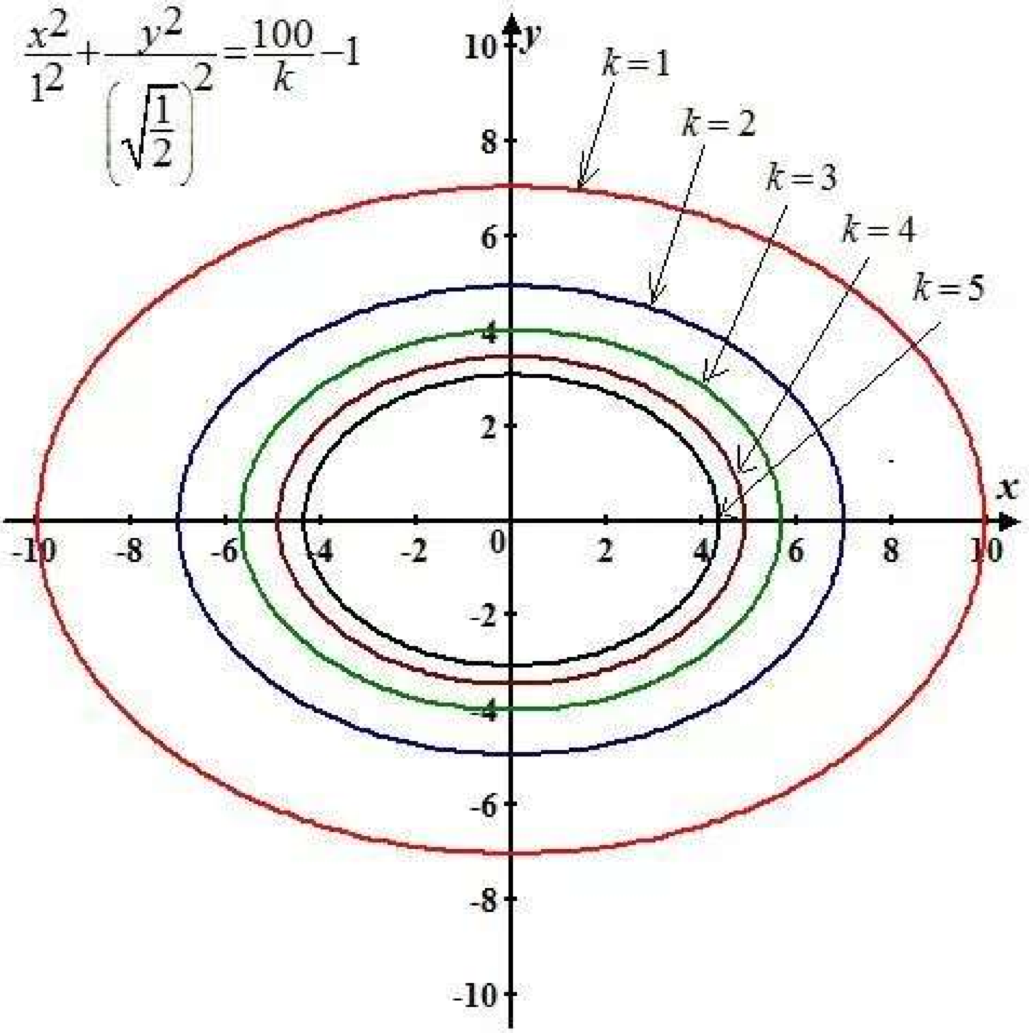
$$k=\frac{100}{(1+x^2+2y^2)}$$

$$(1+x^2+2y^2)=\frac{100}{k}$$

$$x^2+2y^2=\frac{100}{k}-1$$

$$\frac{x^2}{1^2}+\frac{y^2}{\left(\sqrt{\frac{1}{2}}\right)^2}=\frac{100}{k}-1$$

This is a family of ellipse with center $(0,0)$. The cases $k=1,2,3,4,5$ are shown in the below figure.



Answer 54E.

Consider the data,

If $V(x,y)$ is the electrical potential at a point (x,y) in the xy-plane, then the level curves of V are called equipotential curves because at all points on such a curve the electric potential is the same.

To sketch some equipotential curves if $V(x,y)=\frac{c}{\sqrt{r^2-x^2-y^2}}$, where c is a positive constant.

Recall that the definition of level curves:

The level curves of a function f of two variables are the curves with equations $f(x,y)=k$, where k is a constant.

The level curves of V have the following form, where c, k, r are positive constants.

$$V(x, y) = k$$

$$\frac{c}{\sqrt{r^2 - x^2 - y^2}} = k$$

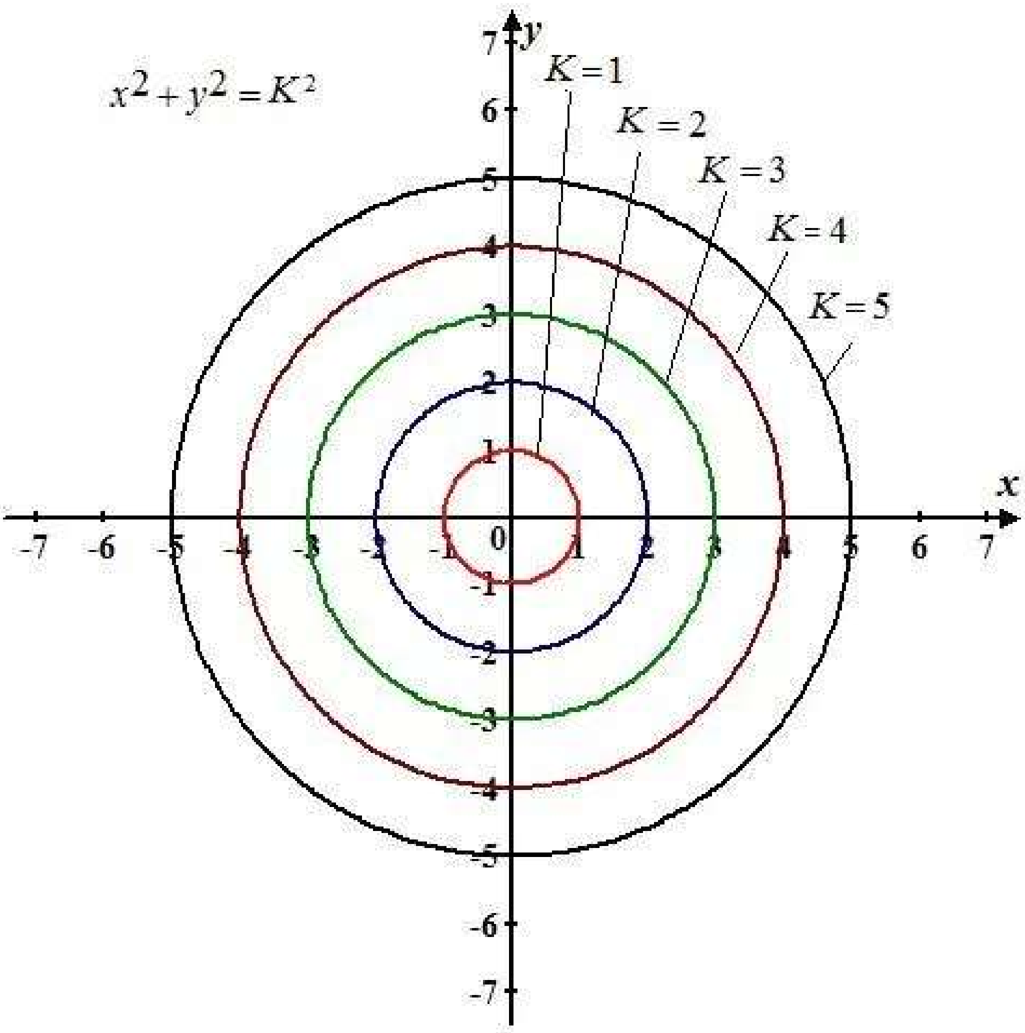
$$\sqrt{r^2 - x^2 - y^2} = \frac{c}{k}$$

$$r^2 - x^2 - y^2 = \left(\frac{c}{k}\right)^2$$

$$x^2 + y^2 = r^2 - \left(\frac{c}{k}\right)^2$$

$$x^2 + y^2 = K^2 \qquad \text{Since } r^2 - \left(\frac{c}{k}\right)^2 = K^2$$

This is a family of circles with center $(0,0)$ and the radius R. The cases $R = 1, 2, 3, 4, 5$ are shown in the following figure.



Answer 55E.

Consider the function $f(x,y)=xy^2-x^3$

Use a computer to graph the function $f(x,y)=xy^2-x^3$. Notice that we get an especially good picture of a function when rotation is used to give views from different vantage points.

Use Maple to graph the function using various domains and viewpoints and get a printout one that, gives a good view.

Maple Input:

```
> with(plots):

> plot3d(x*(y^2)-(x^3),x=-2..2,y=-2..2);
```

Maple Output:

> $with(plots):$

> $plot3d\left(x\cdot y^2-x^3, x=-2..2, y=-2..2\right);$

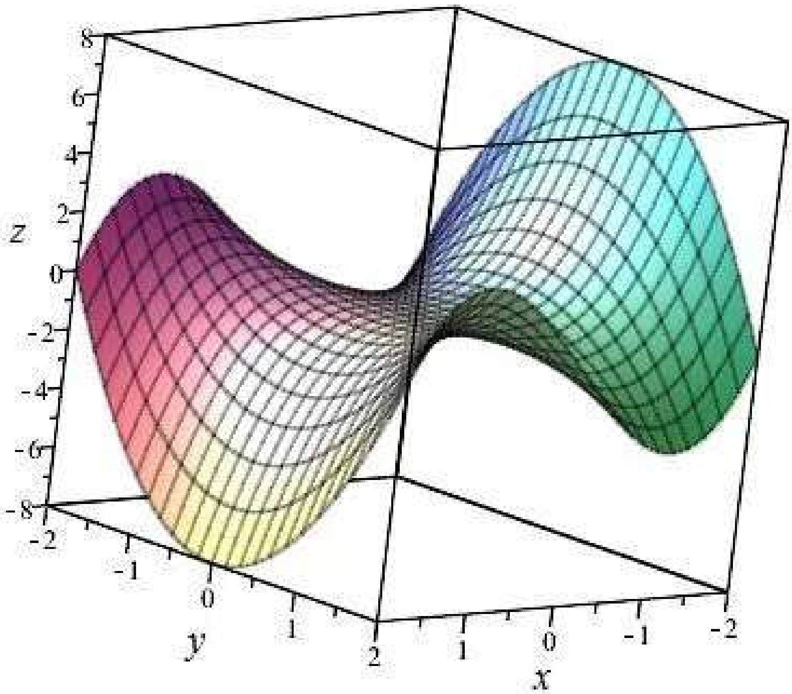


Figure-A

Maple Input:

with(plots);

contourplot(x*(y^2)-(x^3),x=-3..3,y=-3..3);

Maple Output:

> $with(plots)$:

> $contourplot\left(x \cdot y^2 - x^3, x = -3..3, y = -3..3\right)$;

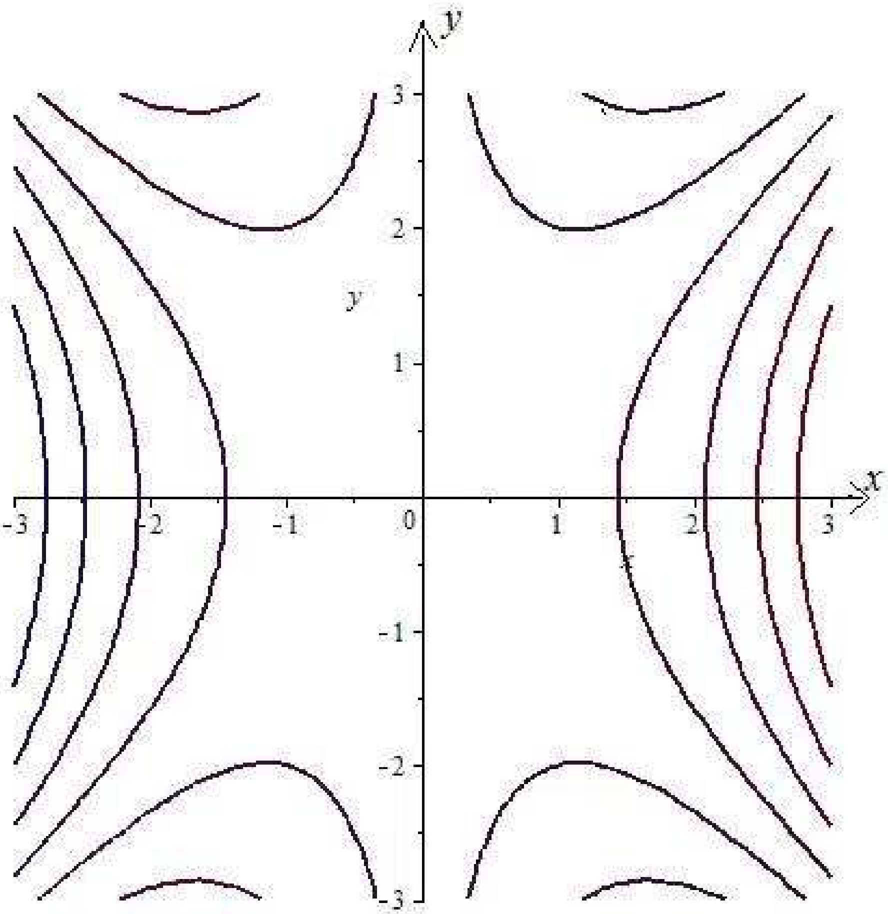


Figure-B

Observe from the figures (A) and (B) the graph of f is very flat near the origin and the contour lines deviate from the origin.

Answer 56E.

Consider the function $f(x,y)=xy^3-yx^3$

Use a computer to graph the function $f(x,y)=xy^3-yx^3$. Notice that we get an especially good picture of a function when rotation is used to give views from different vantage points.

Use Maple to graph the function using various domains and viewpoints and get a printout one that, gives a good view.

Maple Input:

```
> with(plots):
> plot3d(x*(y^3)-y*(x^3),x=-2..2,y=-2..2);
```

Maple Output:

> $with(plots)$:

> $plot3d\left(x\cdot y^2-x^3, x=-2..2, y=-2..2\right)$;

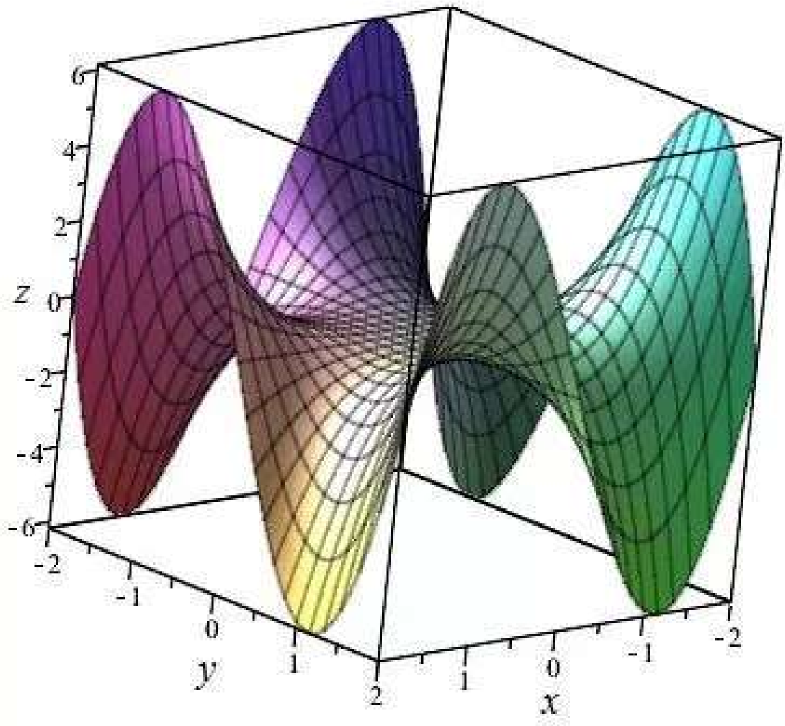


Figure-A

Maple Input:

with(plots);

contourplot(x*(y^3)-y*(x^3),x=-3..3,y=-3..3);

Maple Output:

> $with(plots):$

> $contourplot\left(x\cdot y^3 - y\cdot x^3, x=-3..3, y=-3..3\right);$

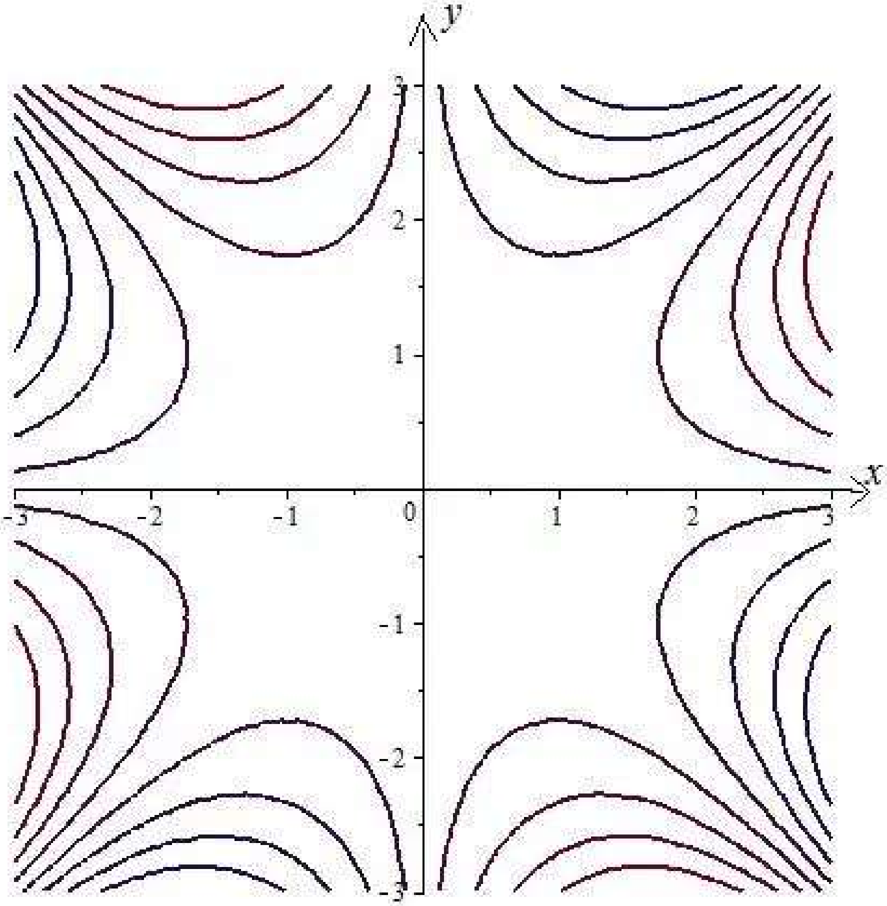


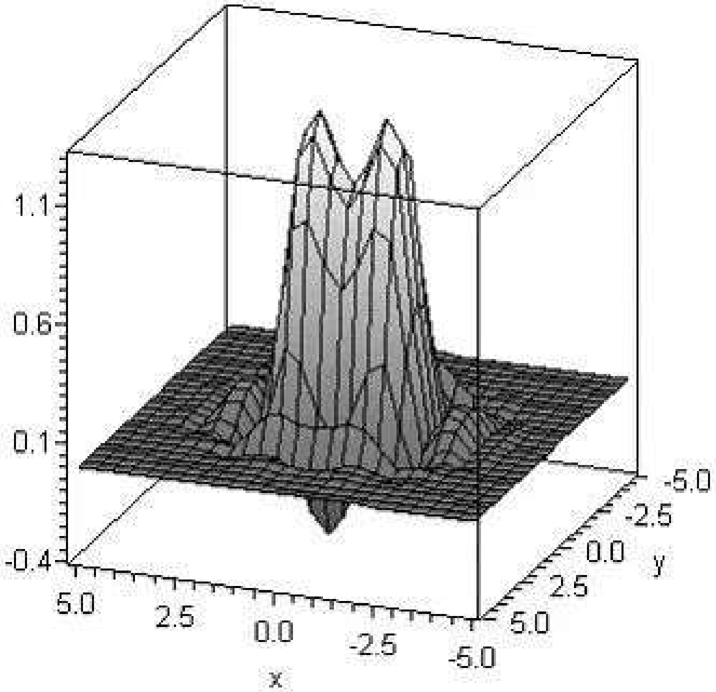
Figure-B

Observe from the figures (A) and (B) the graph of f is very flat near the origin and the contour lines deviate from the origin.

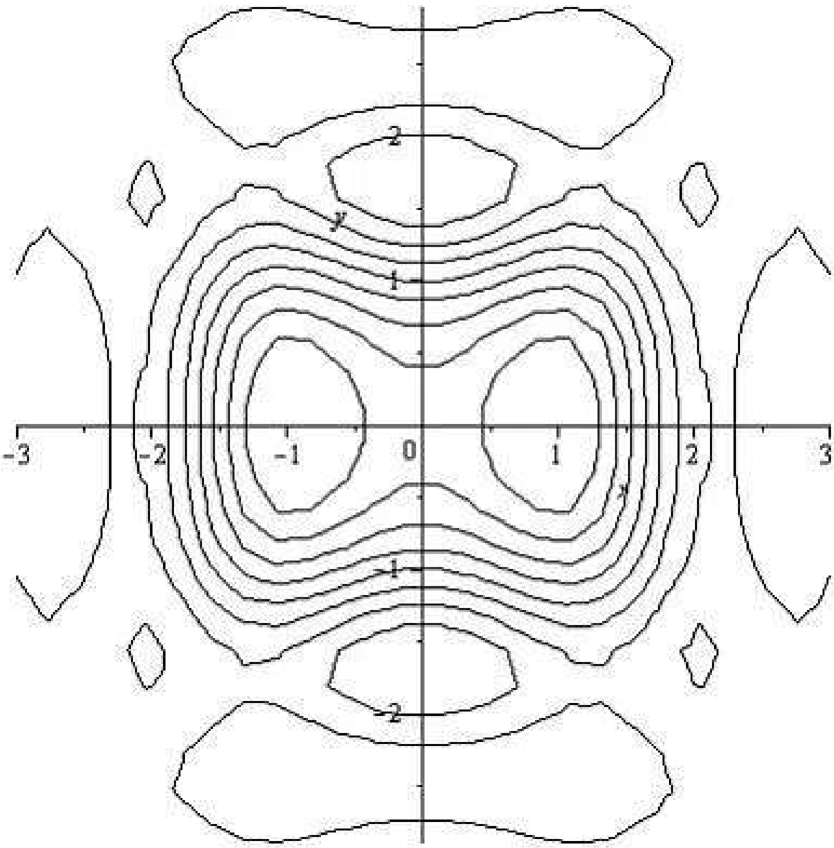
Q57E.

Given $f(x,y) = e^{-(x^2+y^2)/3}\left(\sin\left(x^2\right)+\cos\left(y^2\right)\right)$

Let us start by sketching the graph of the given function.



Now, sketch the contour lines of the function.

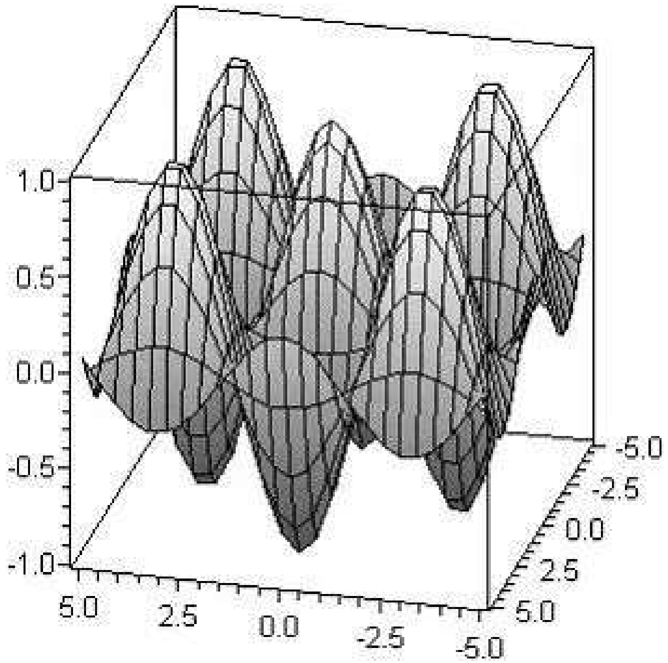


We note that the curves in the contour map are crowded towards the origin, which means the graph is steep at the origin.

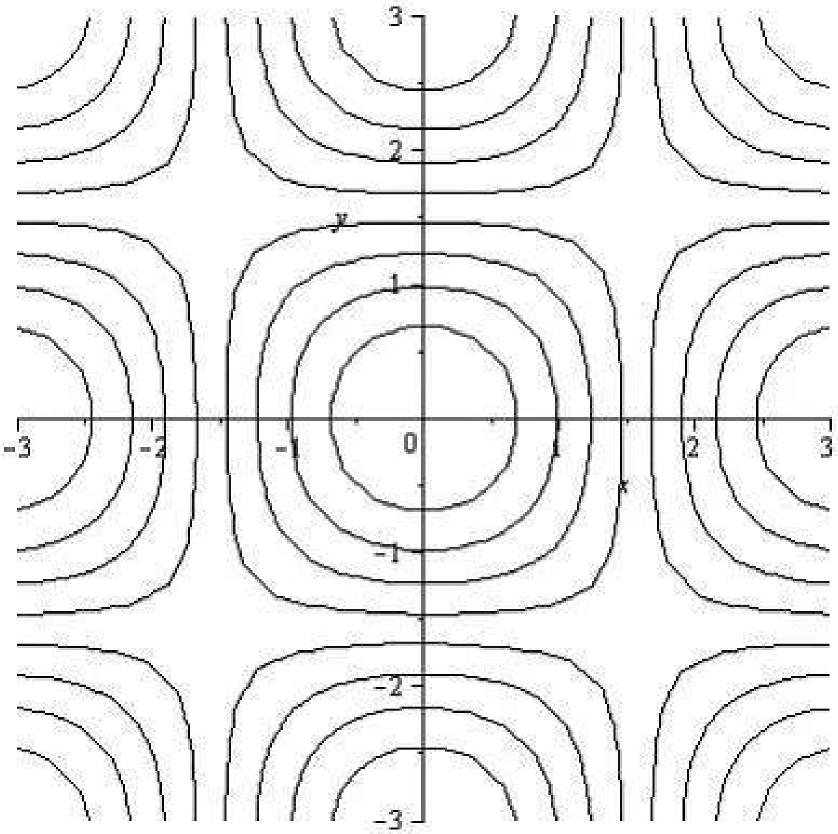
Answer 58E.

Given $f(x,y) = \cos x \cos y$

Let us start by sketching the graph of the given function.



Now, sketch the contour lines of the function.



We note that the curves in the contour map are crowded towards the origin, which means the graph is steep at the origin. Similarly, we note that the steep points in the graph are represented by closely packed curves in the contour map.

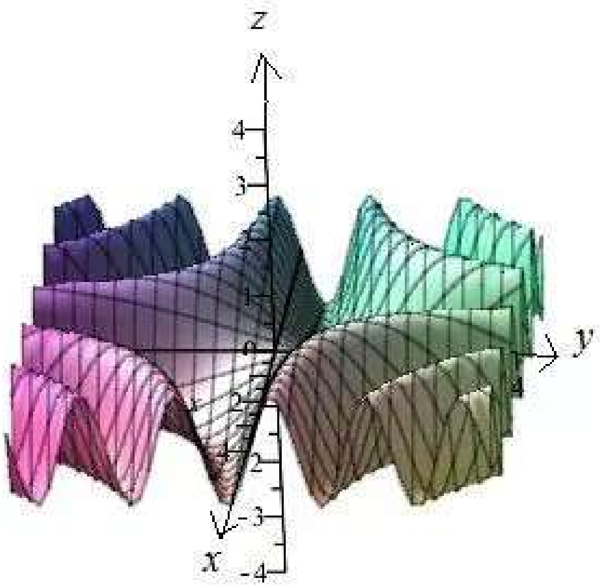
Answer 59E.

Consider the function $z = \sin(xy)$

(a)

Since the function $\sin(xy)$ is a periodic function and whose value range from -1 to 1, this follows that the function $z = \sin(xy)$ represents a surface which looks like tidal waves in a sea, that lies in between the planes $z = 1$ and $z = -1$. Generally the sine function has the smaller angles this follows that, for smaller values of x and y the surface will be almost flatter to xy-plane it means the surface will be somewhat flatter near the origin. Hence the function matches with graph **C**.

The graph of the function $z = \sin(xy)$ as shown in the below figure



(b)

Recall that the definition of level curves:

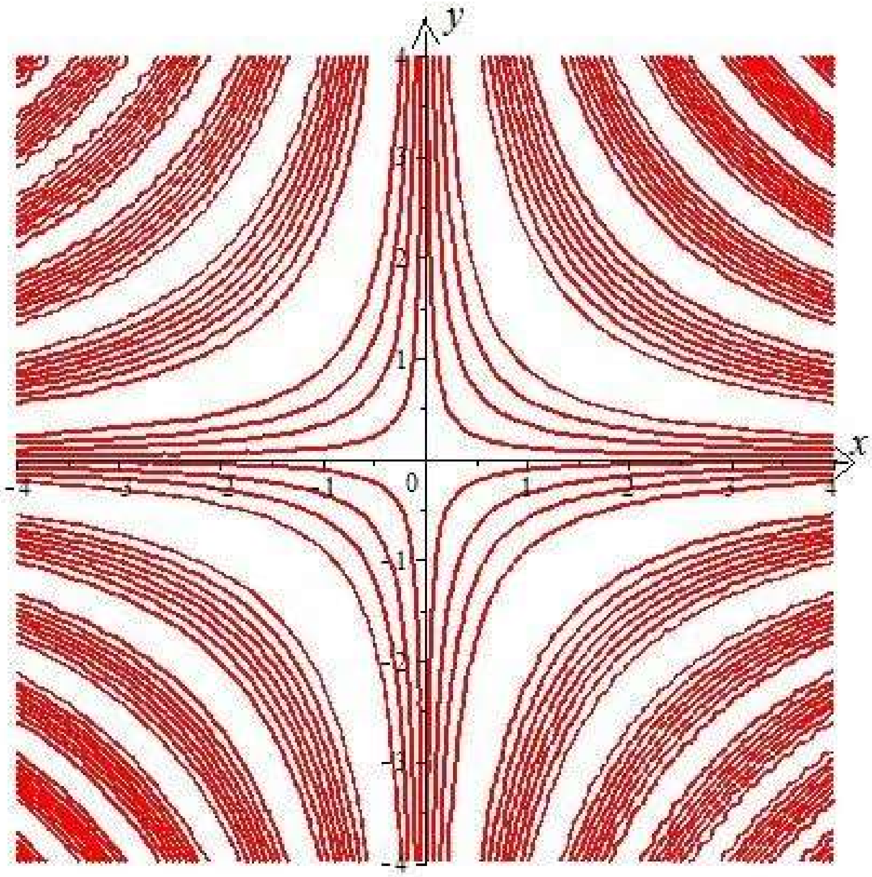
The level curves of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant.

The level set $\sin(xy) = k$ consists of infinitely many curves. To see why each level set is a bunch of curves, look at the example $\sin(xy) = \frac{1}{2}$. This is satisfied when

$xy = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \cdots$. This describes a bunch of curves.

Observe the surface of the function, which is almost flatter to xy-plane it means the surface will be somewhat flatter near the origin and the contour lines deviate from the origin.

So the contour map for the function is as follows



Hence the function matches with contour map **II.**

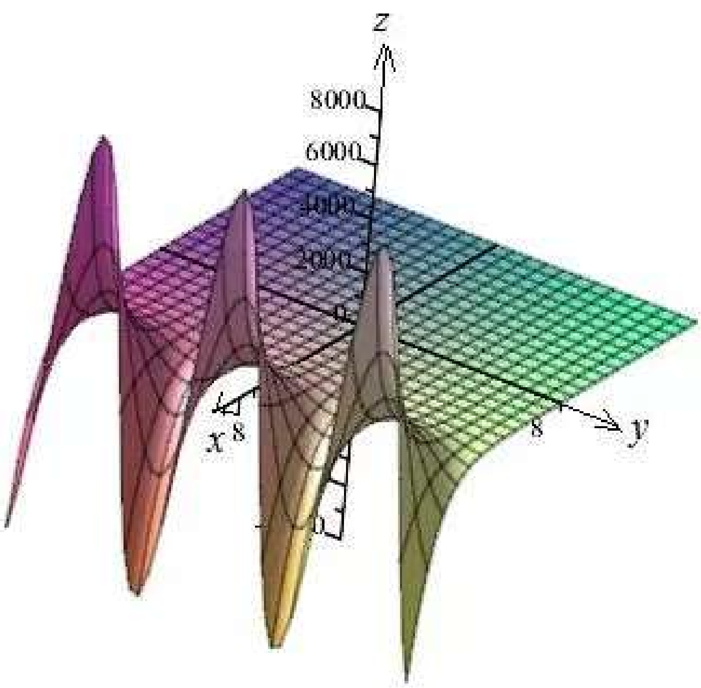
Answer 60E.

Consider the function $z = e^x \cos y$

(a)

Let the function $z = e^x \cos y$ is an increasing function. It increases exponentially as x does. Since the cosine function has a periodic and even. Hence the function matches with graph **A.**

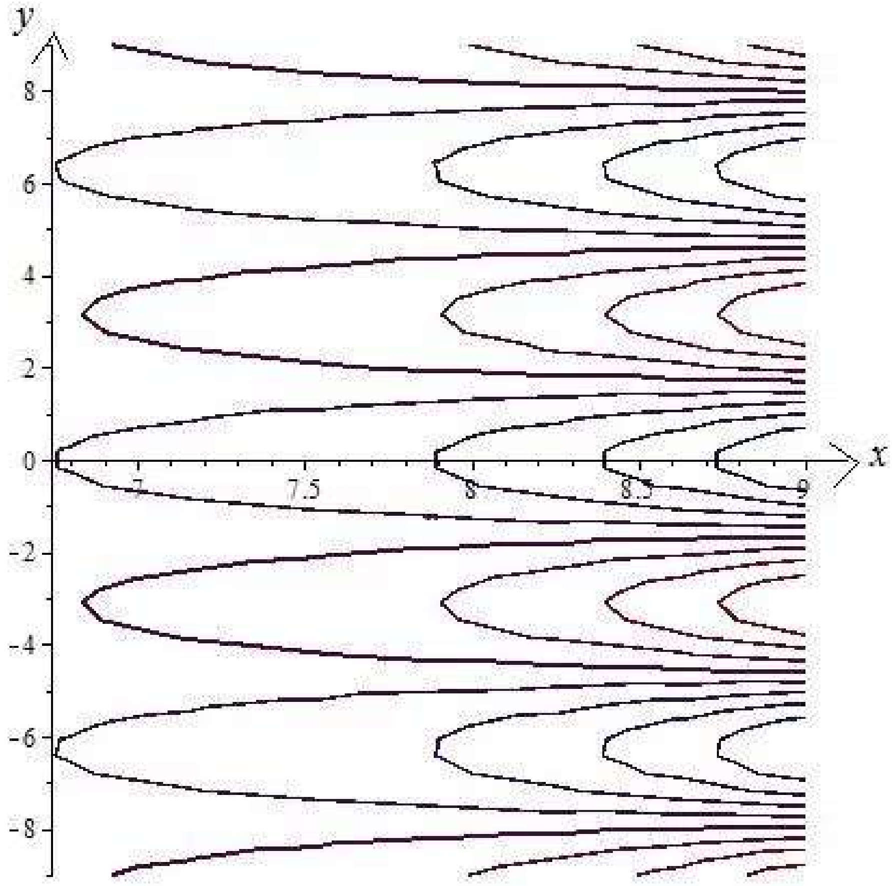
The graph of the function $z = e^x \cos y$ as shown in the below figure



(b)

Since the function $z = e^x \cos y$ is an increasing function. It increases exponentially as x does and the cosine function has a periodic and even. From this is can be seen that **A** is the correct graph of the function. Now projecting down the level curves of the graph **A** gives cross section **IV**.

So the contour map for the function is as follows



Hence the function matches with contour map **IV**.

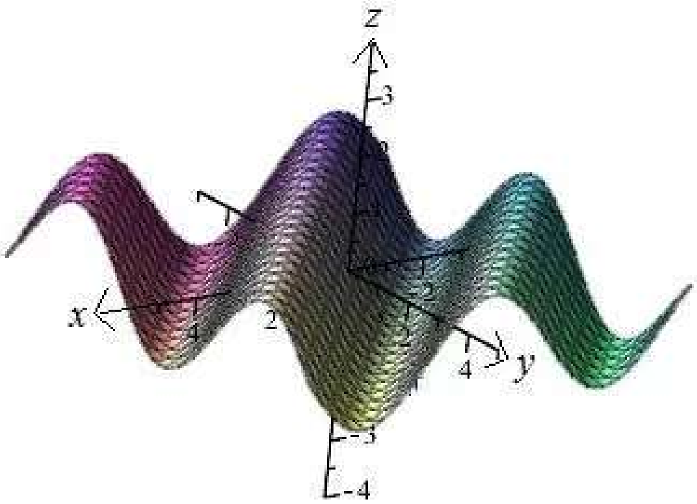
Answer 61E.

Consider the function $z = \sin(x - y)$

(a)

Since the function $\sin(x-y)$ is a periodic function and whose value range from -1 to 1, this follows that the function $z = \sin(x-y)$ represents a surface which looks like tidal waves in a sea, that lies in between the planes $z = 1$ and $z = -1$. Generally the sine function has the smaller angles this follows that, for smaller values of x and y the surface will be almost flatter to xy-plane it means the surface will be somewhat flatter near the origin. Hence the function matches with graph **F**.

The graph of the function $z = \sin(x-y)$ as shown in the below figure



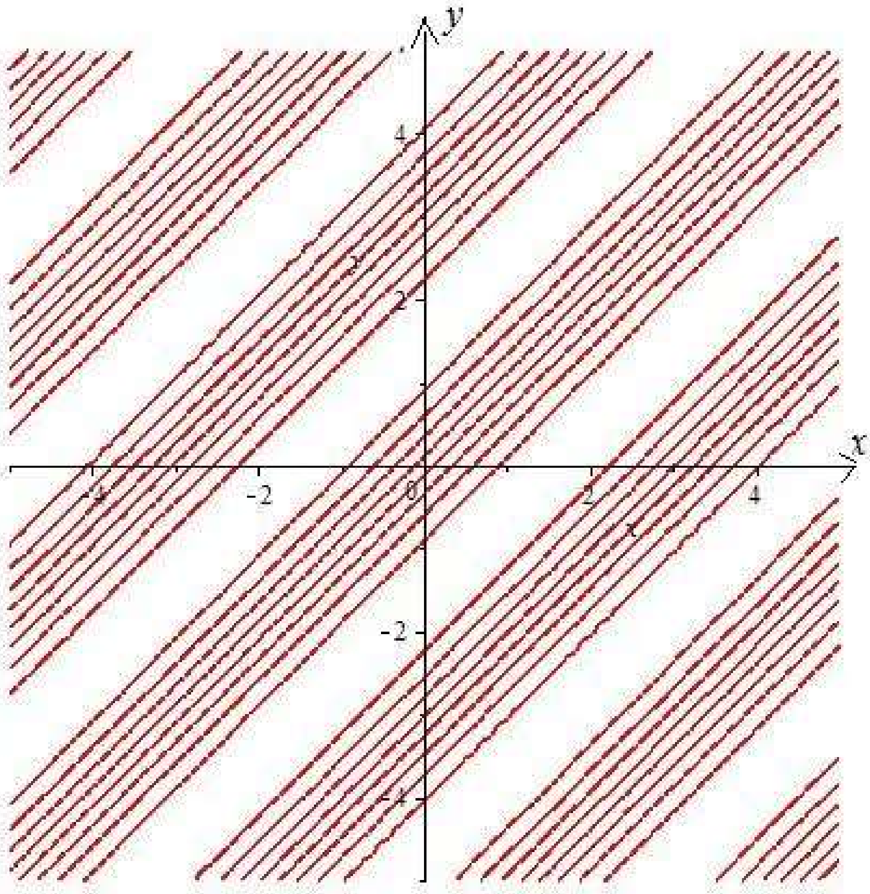
Recall that the definition of level curves:

The level curves of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant.

The level set $\sin(x-y) = k$ consists of infinitely many parallel lines. To see why each level set is a bunch of parallel lines, look at the example $\sin(x-y) = \frac{1}{2}$. This is satisfied when

Observe the surface of the function, which is almost flatter to xy-plane it means the surface will be somewhat flatter near the origin and the contour lines deviate from the origin.

So the contour map for the function is as follows



Hence the function matches with contour map **I**.

Answer 62E.

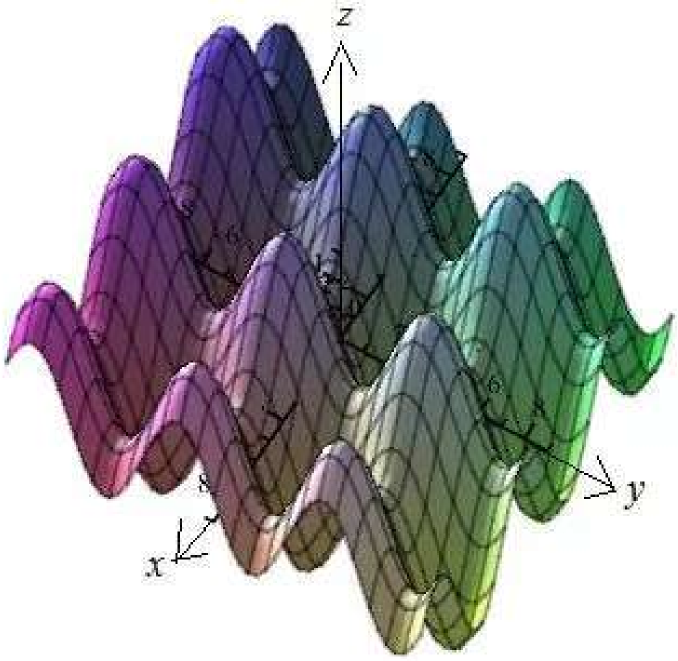
Consider the function $z = \sin x - \sin y$

(a)

Since the sine function is a periodic function and whose value range from -1 to 1, this follows that the sine function represents a surface which looks like tidal waves in a sea.

But there are two sine functions, so there should be a sine wave in either the *X* or the *Y* directions. Hence the function matches with graph $\mathbf{E}$.

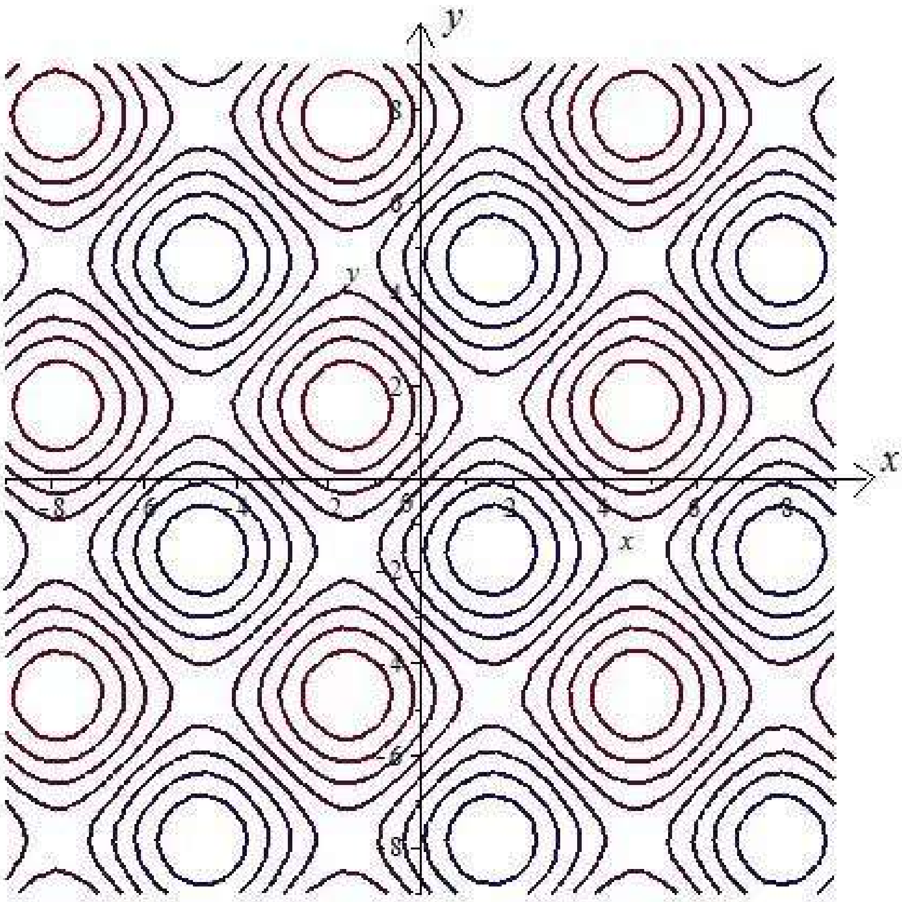
The graph of the function $z = \sin x - \sin y$ as shown in the below figure



(b)

In part (a) the figure just like in a map of a mountain range. As there are multiple sine waves all across, the contour should resemble a bunch of humps.

So the contour map for the function is as follows



Hence the function matches with contour map **III.**

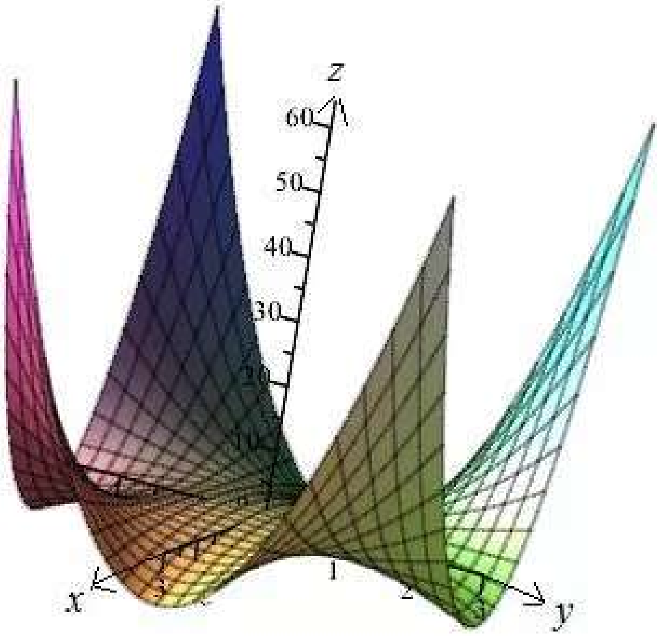
Answer 63E.

Consider the function $z=(1-x^2)(1-y^2)$

(a)

In this function, the parabola $z=(1-x^2)$ is the trace in the xz- plane and the parabola $z=(1-y^2)$ is the trace in the yz-plane. Hence the function matches with graph **B.**

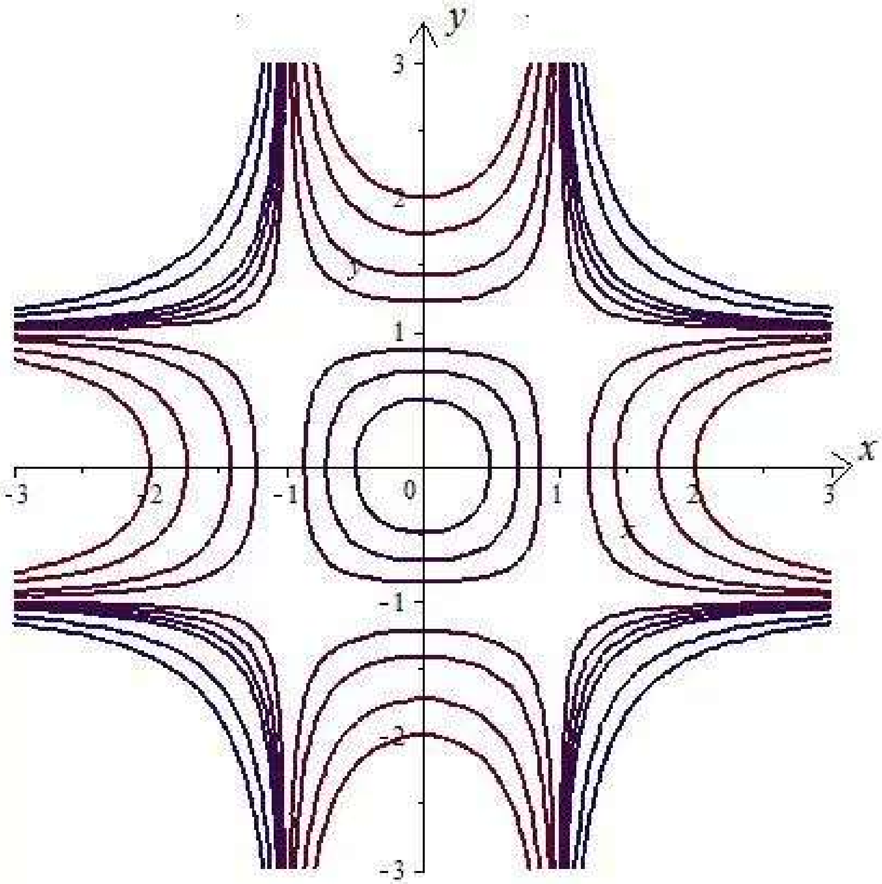
The graph of the function $z=(1-x^2)(1-y^2)$ as shown in the below figure



(b)

In part (a) Observe the surface of the function, which is almost flatter to xy-plane it means the surface will be somewhat flatter near the origin and the contour lines deviate from the origin. This function is zero along the lines $x = \pm 1$ and $y = \pm 1$.

So the contour map for the function is as follows



Hence the function matches with contour map **VI.**

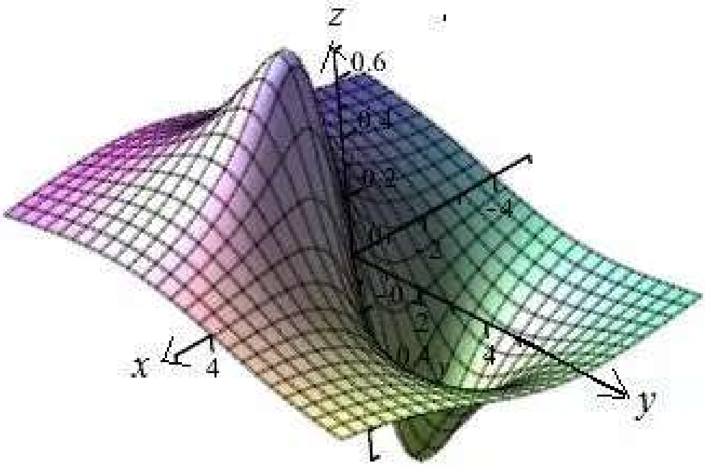
Answer 64E.

Consider the function $z = \dfrac{x-y}{1+x^2+y^2}$

(a)

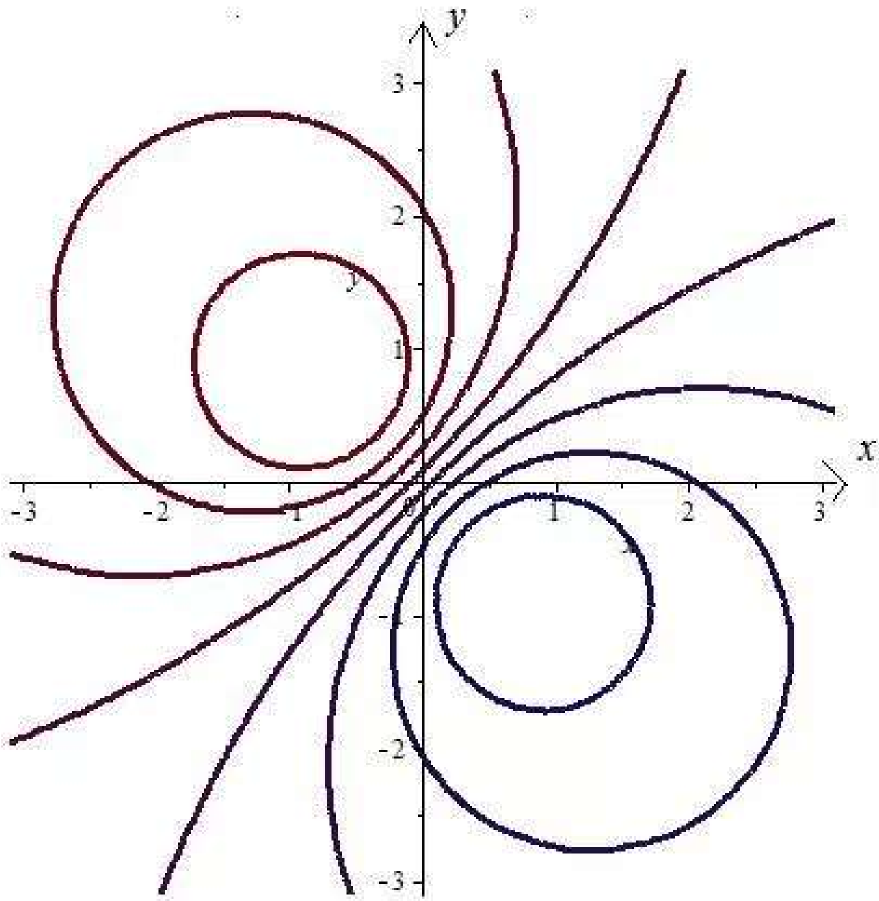
As the values of x and y increases, the function should flatten out to resemble something like a flat plane. The surface will be a xy-plane for the equal values of x and y. Hence the function matches with graph **D.**

The graph of the function $z = \dfrac{x-y}{1+x^2+y^2}$ as shown in the below figure



(b)

The contour map should show two large mountains that gets flatter and flatter more and more slowly as the x's and y's increase.



Hence the function matches with contour map **V**.

Answer 65E.

Describe the level surfaces of the function, $f(x,y,z)=x+3y+5z$

Level curves:

The level curves of a function f of two variables are the curves with equations $f(x,y)=k$, where k is a constant.

The level surfaces of $f(x,y,z)=x+3y+5z$ are $k=x+3y+5z$

The four particular level surfaces with $k=-5,0,5,10$ are,

$x+3y+5z+5=0$,

$x+3y+5z=0$,

$x+3y+5z-5=0$,

$x+3y+5z-10=0$.

An equation of this form describes a plane, so the contour surfaces are the family of parallel planes.

Use Maple to check the contour map showing several level curves.

Maple Input:

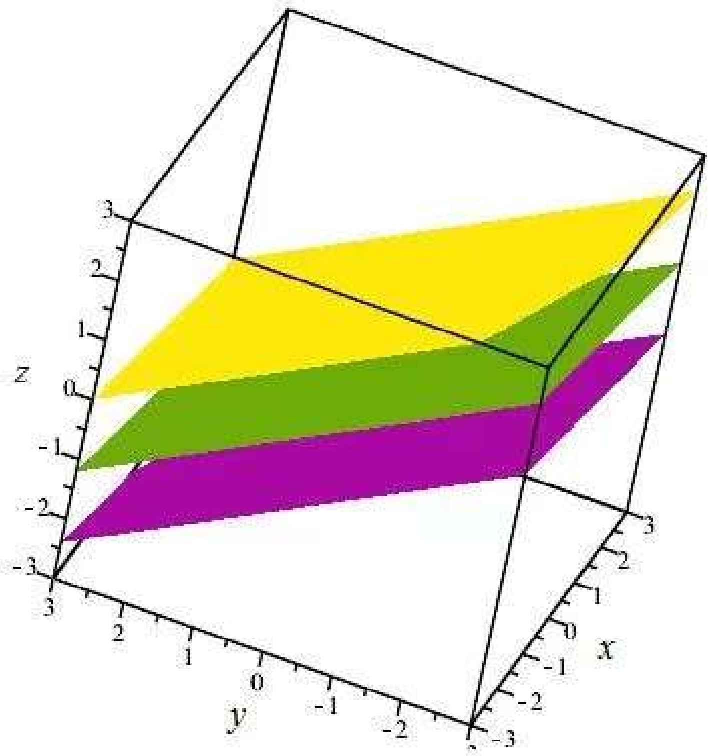
> with (plots):

> implicit plot(x+3y+5z, x=-3..3,y=-3..3,z=-3..3);

Maple Output:

> $with(plots):$

> $implicitplot3d(x+3\cdot y+5\cdot z, x=-3..3, y=-3..3, z=-3..3);$



Answer 66E.

Consider the following function:

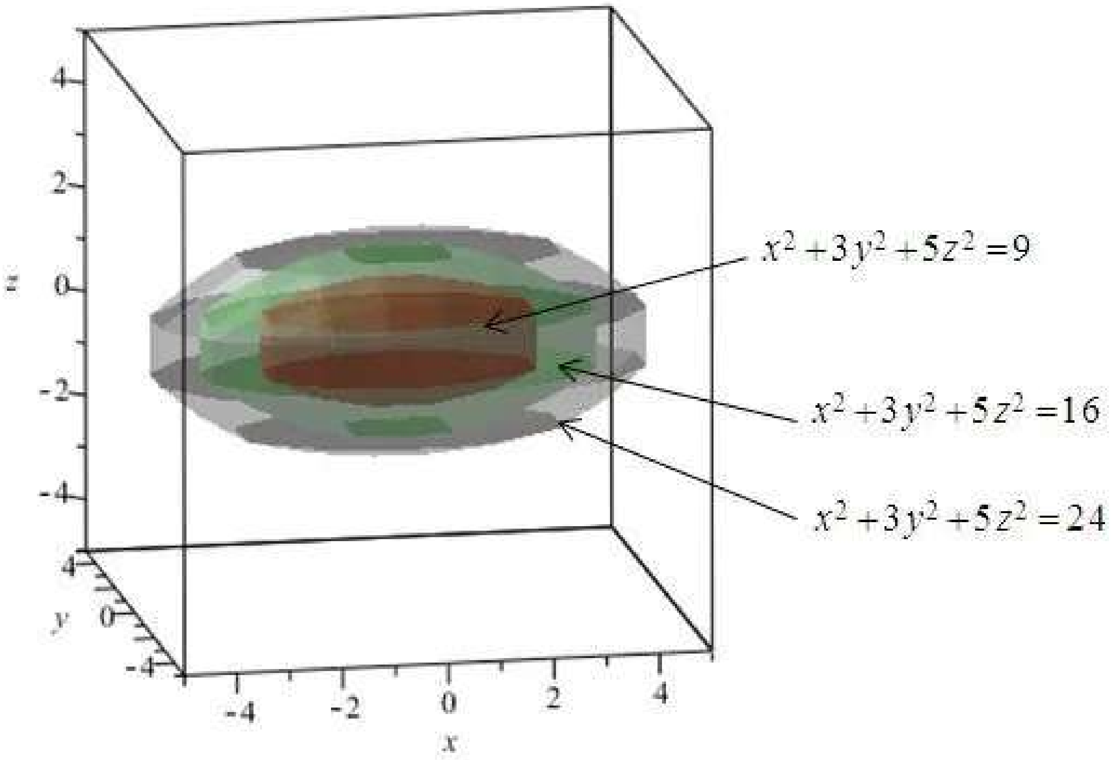
$$f(x,y,z)=x^2+3y^2+5z^2$$

Recall the following definition of level curves:

The level curves of a function f of two variables are the curves with equations $f(x,y)=k$. Here, k is a constant.

The level surfaces are $x^2+3y^2+5z^2=k$. Here, $k\geq 0$.

These are family of ellipsis for $k>0$ and this is the origin if $k=0$.



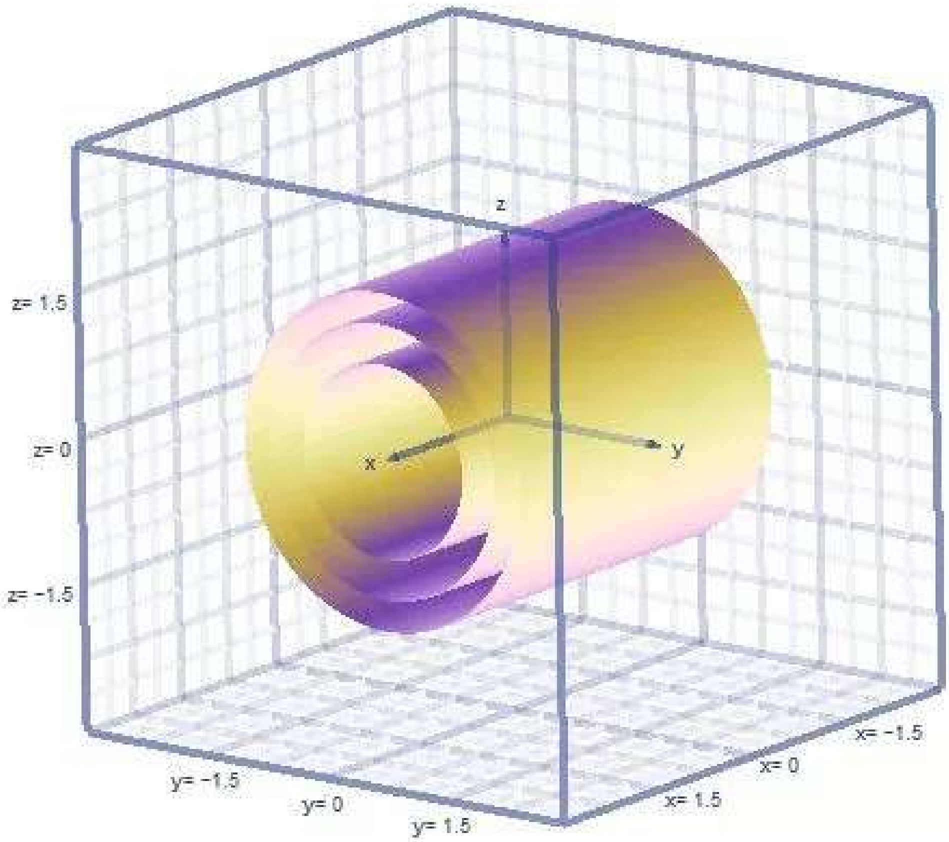
Answer 67E.

To describe the level surfaces of the function $f(x,y,z)=y^2+z^2$, need the following steps.

The level surfaces are $y^2+z^2=k$, where $k\geq 0$.

These surfaces form a family of circular cylinders with *x*-axis as the axis.

For example, the level surfaces corresponding to the values $k=1, k=2, k=3$ are shown as below:



Answer 58E.

From the given equation, we can say that each level surface has an equation of the form $x^2-y^2-z^2=k$. On substituting 0 for k, we get $x^2-y^2-z^2=0$, which represents an elliptical cone.

Similarly, the level surfaces corresponding to the values $k=1$, $k=2$, $k=3$ are family of elliptical cones.

Thus, the level surfaces for the given equation are a family of elliptical cones, for $k \geq 0$.

Answer 69E.

To describe the graph of g is obtained from the graph of f.

Recall that the Vertical and Horizontal shifts:

Suppose $c>0$. To obtain the graph of

$y=f(x)+c$, shift the graph of $y=f(x)$ a distance c units upward

$y=f(x)-c$, shift the graph of $y=f(x)$ a distance c units downward

$y=f(x-c)$, shift the graph of $y=f(x)$ a distance c units to the right

$y=f(x+c)$, shift the graph of $y=f(x)$ a distance c units to the left

Recall that the Vertical and Horizontal Stretching and Reflecting:

Suppose $c>0$. To obtain the graph of

$y=cf(x)$, stretch the graph of $y=f(x)$ vertically by a factor of c

$y=\left(\frac{1}{c}\right)f(x)$, compress the graph of $y=f(x)$ vertically by a factor of c

$y=f(cx)$, compress the graph of $y=f(x)$ horizontally by a factor of c

$y=f\left(\frac{x}{c}\right)$, stretch the graph of $y=f(x)$ horizontally by a factor of c

$y=-f(x)$, reflect the graph of $y=f(x)$ about the x-axis

$y=f(-x)$, reflect the graph of $y=f(x)$ about the y-axis

(a)

Let $g(x,y)=f(x,y)+2$

From the Vertical and Horizontal shifts,

$y=f(x)+c$, shift the graph of $y=f(x)$ a distance c units upward

Hence the graph of g is obtained from the graph of f by shift the graph of $g(x,y)=f(x,y)$ a distance 2 units upward.

(b)

Let $g(x,y)=2f(x,y)$

From the Vertical and Horizontal Stretching and Reflecting:

$y=cf(x)$, stretch the graph of $y=f(x)$ vertically by a factor of c

Hence the graph of g is obtained from the graph of f by stretch the graph of $g(x,y)=f(x,y)$ vertically by a factor of 2.

(c)

Let $g(x,y) = -f(x,y)$

From the Vertical and Horizontal Stretching and Reflecting:

$y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis

Hence the graph of g is obtained from the graph of f by reflect the graph of $g(x,y) = f(x,y)$ about the xy-plane.

(d)

Let $g(x,y) = 2 - f(x,y)$

$= -f(x,y) + 2$

From the Vertical and Horizontal shifts,

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

From the Vertical and Horizontal Stretching and Reflecting:

$y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis

Hence the graph of g is obtained from the graph of f by reflect the graph of $g(x,y) = f(x,y)$ about the xy-plane and then shift it upward 2 units.

Answer 70E.

To describe the graph of g is obtained from the graph of f.

Recall that the Vertical and Horizontal shifts:

Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

(a)

Let $g(x,y)=f(x-2,y)$

From the Vertical and Horizontal shifts,

$y=f(x-c)$, shift the graph of $y=f(x)$ a distance c units to the right

Hence the graph of g is obtained from the graph of f by shift it 2 units in the positive x-direction.

(b)

Let $g(x,y)=f(x,y+2)$

From the Vertical and Horizontal shifts,

$y=f(x+c)$, shift the graph of $y=f(x)$ a distance c units to the left

Hence the graph of g is obtained from the graph of f by shift it 2 units in the negative y-direction.

(c)

Let $g(x,y)=f(x+3,y-4)$

From the Vertical and Horizontal shifts,

$y=f(x+c)$, shift the graph of $y=f(x)$ a distance c units to the left

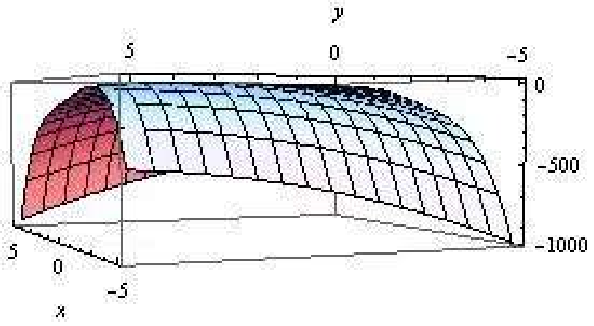
$y=f(x-c)$, shift the graph of $y=f(x)$ a distance c units to the right

Hence the graph of g is obtained from the graph of f by shift it 3 units in the negative x-direction then 4 units in the positive y-direction.

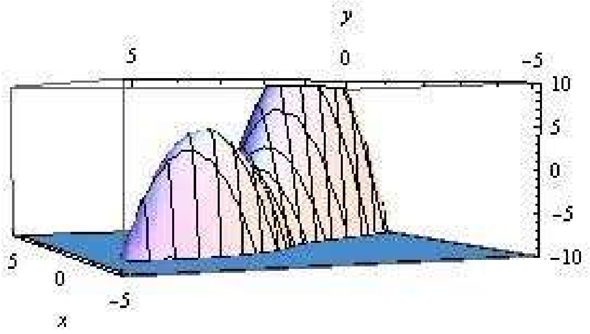
Answer 71E.

We have $f(x, y) = 3x - x^4 - 4y^2 - 10xy$

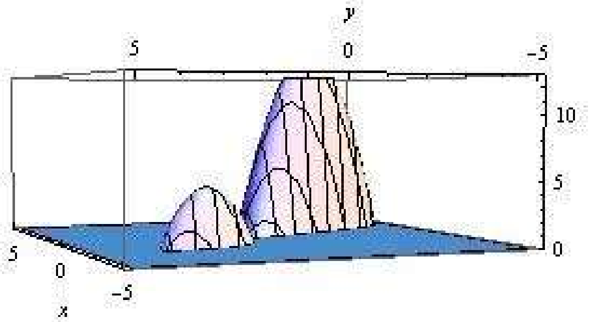
We have a polynomial function. We start with $-5 \le x \le 5$ and $-5 \le y \le 5$ and then we use variuos ranges.



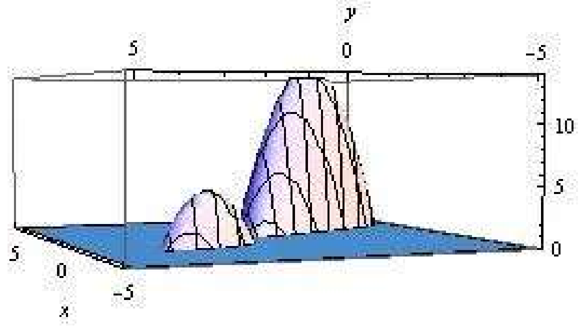
Range → Automatic



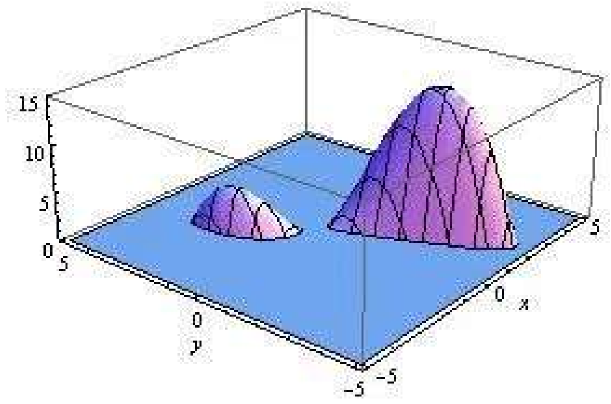
Range → {-10, 10}



Range → {0, 13}

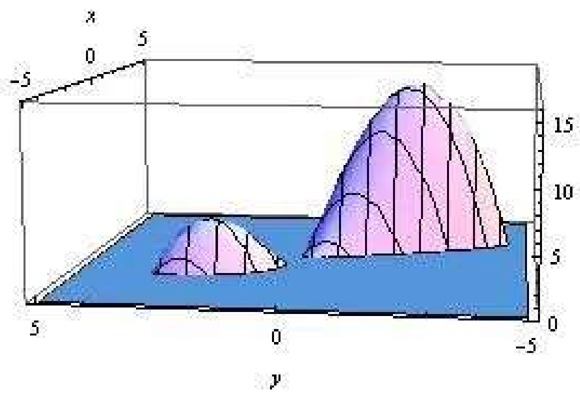


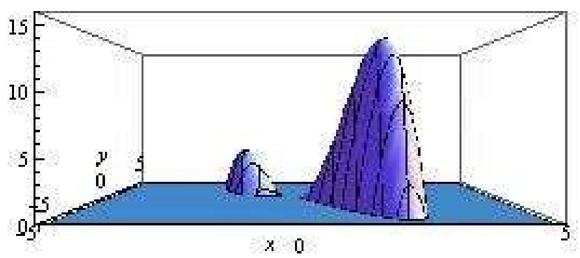
Range → {0, 14}



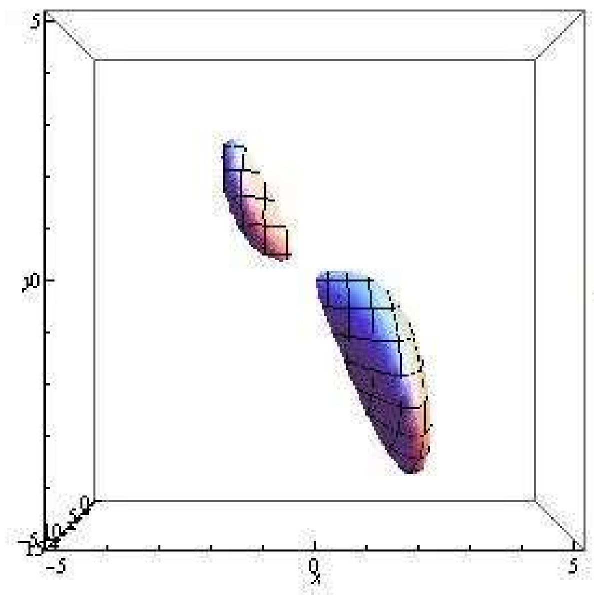
Range → {0, 15}

Now we can use differents viewpoints for the range {0,16}

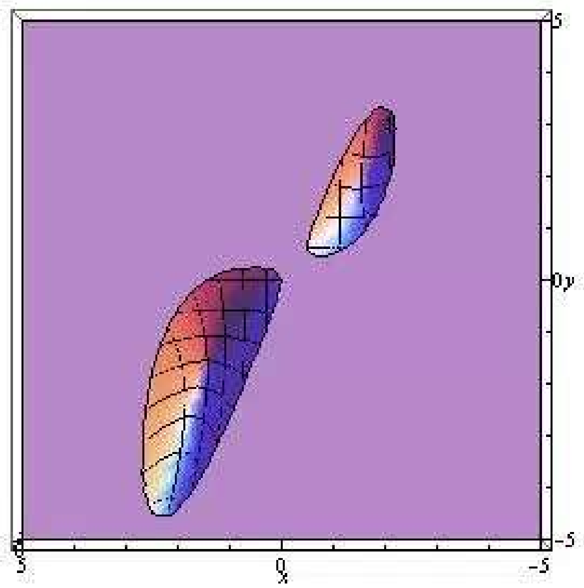




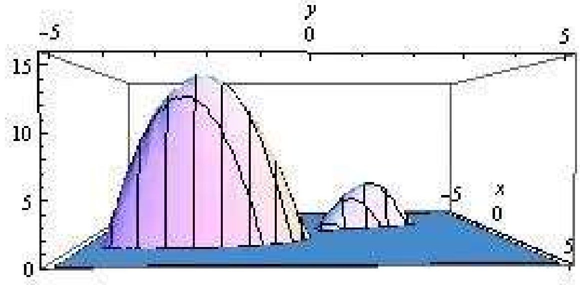
ViewPoint → Front



ViewPoint → Top



ViewPoint → Below



From the front view graph, we can say that the maximum value appears to be 15.

We can say that we can see two local maximum points. (Both hilltops)

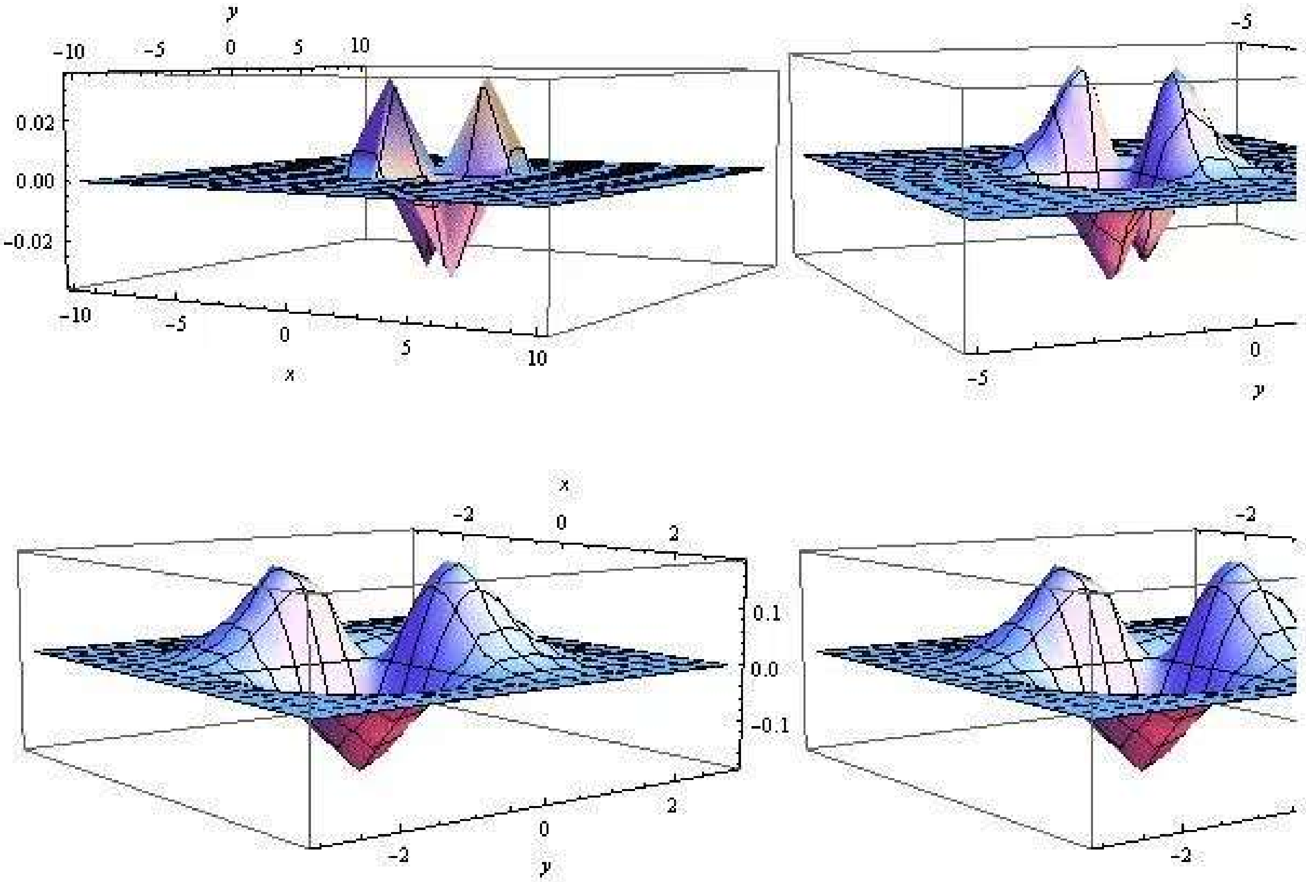
There does not appear to be any local minimum point.

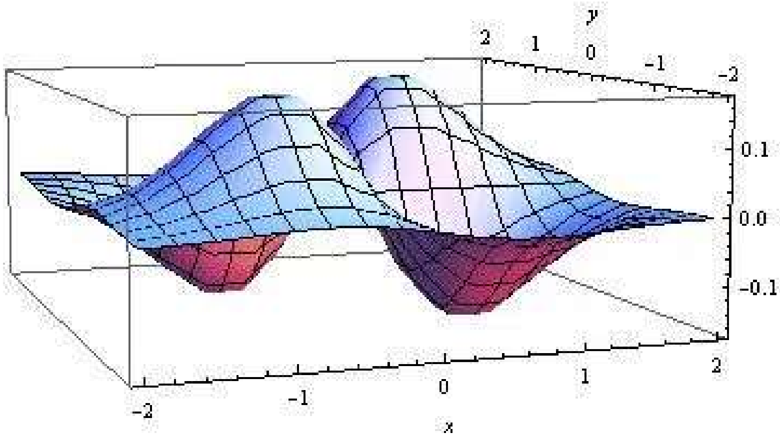
Answer 72E.

Given

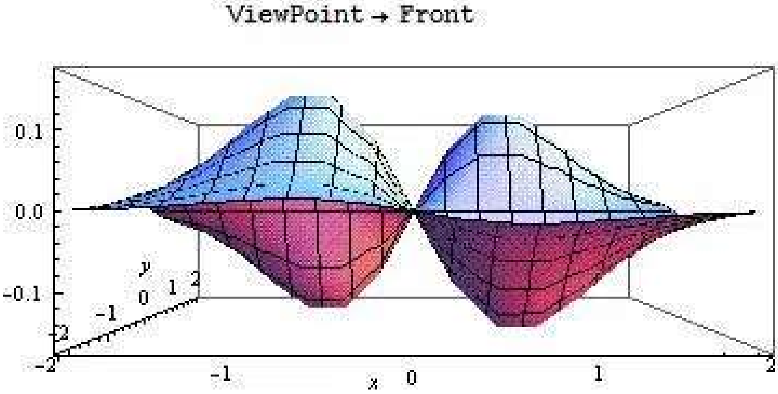
$$f(x, y) = xye^{-x^2-y^2}$$

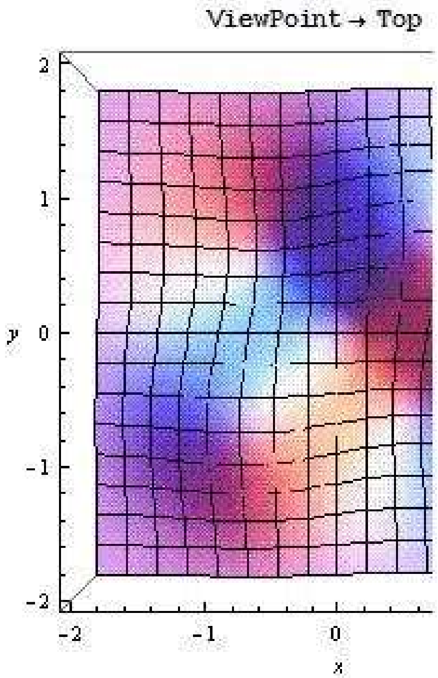
Using a computer to graph the function we get

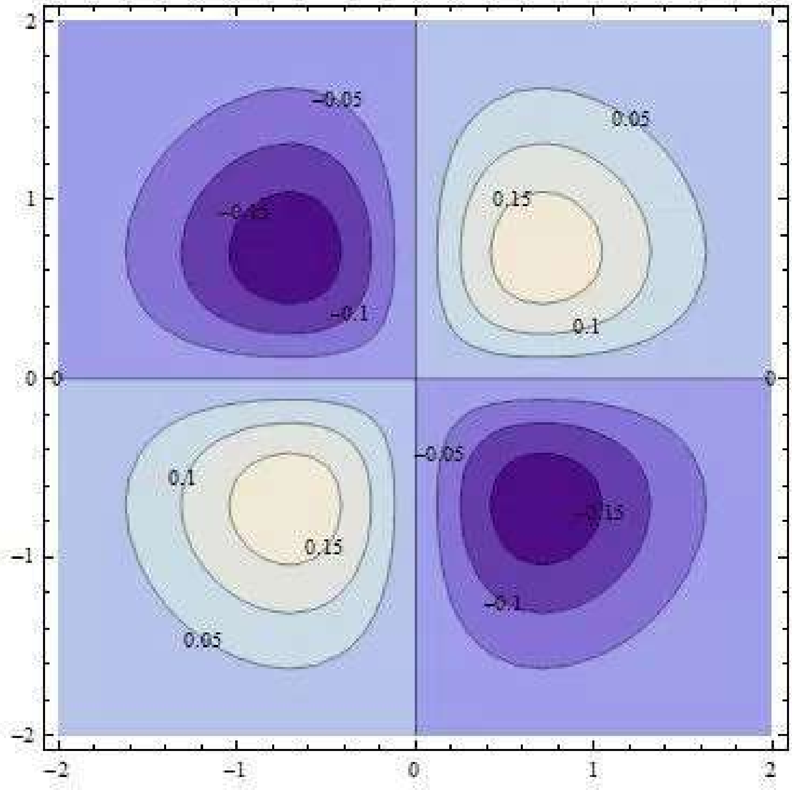



y
2
1
0
-1
-2
0.1
0.0
-0.1
-2
-1
0
1
2
x

For -2


ViewPoint → Front
0.1
0.0
-0.1
y
-2
-1
0
1
2
-1
x
0
1
2


ViewPoint → Top
2
1
y
0
-1
-2
-2
-1
0
x



From the graphs we can see that the function has two hilltops and two vally bottoms.

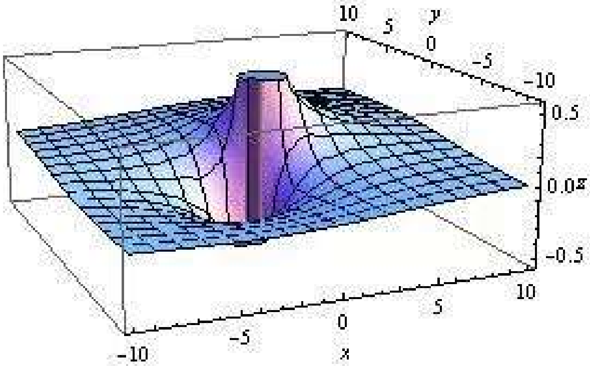
The maximum values are local maximum points z ˜ 0.18,the neighboring points give lowest values of f.

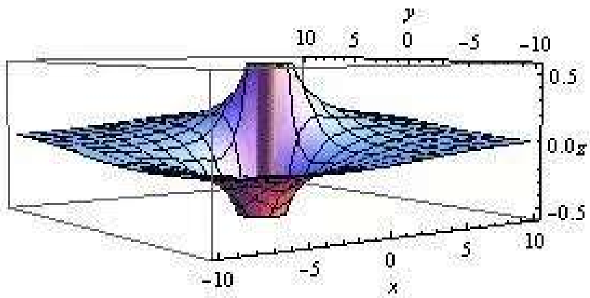
The minimum values are local minimum points (z ˜ -0.18), the neighboring points give greater values of f.

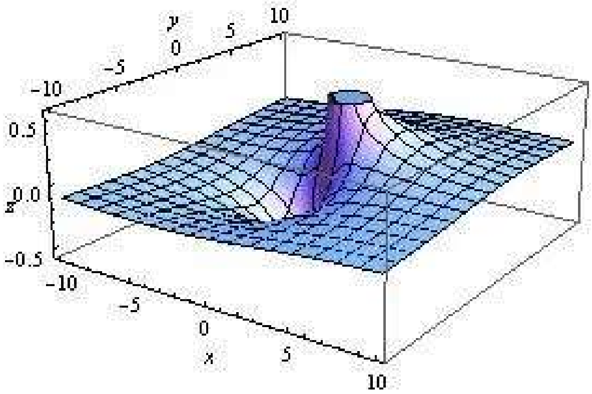
Answer 73E.

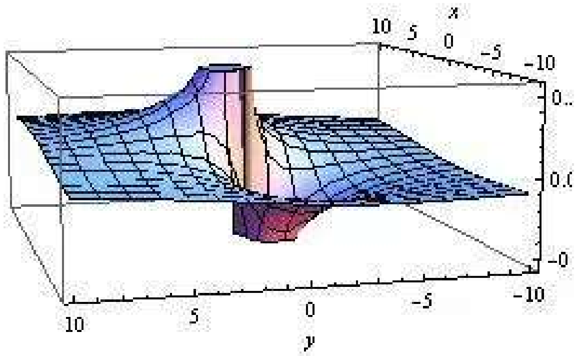
We have $f\left(x, y\right) = \frac{x+y}{x^2+y^2}$

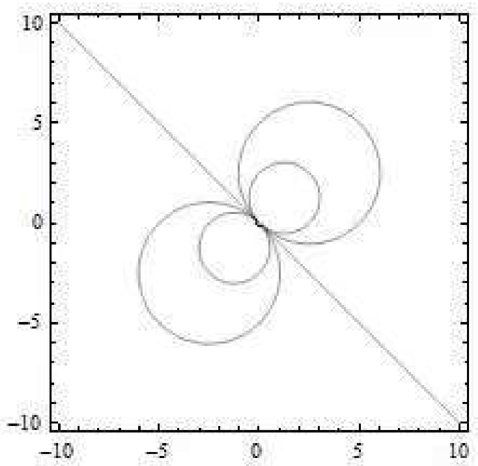
Using CAS we get:











1) As x and y increase, the function values appear to approach 0.

2) As (x,y) approaches the origin, the graph exhibits asymptotic behavior.

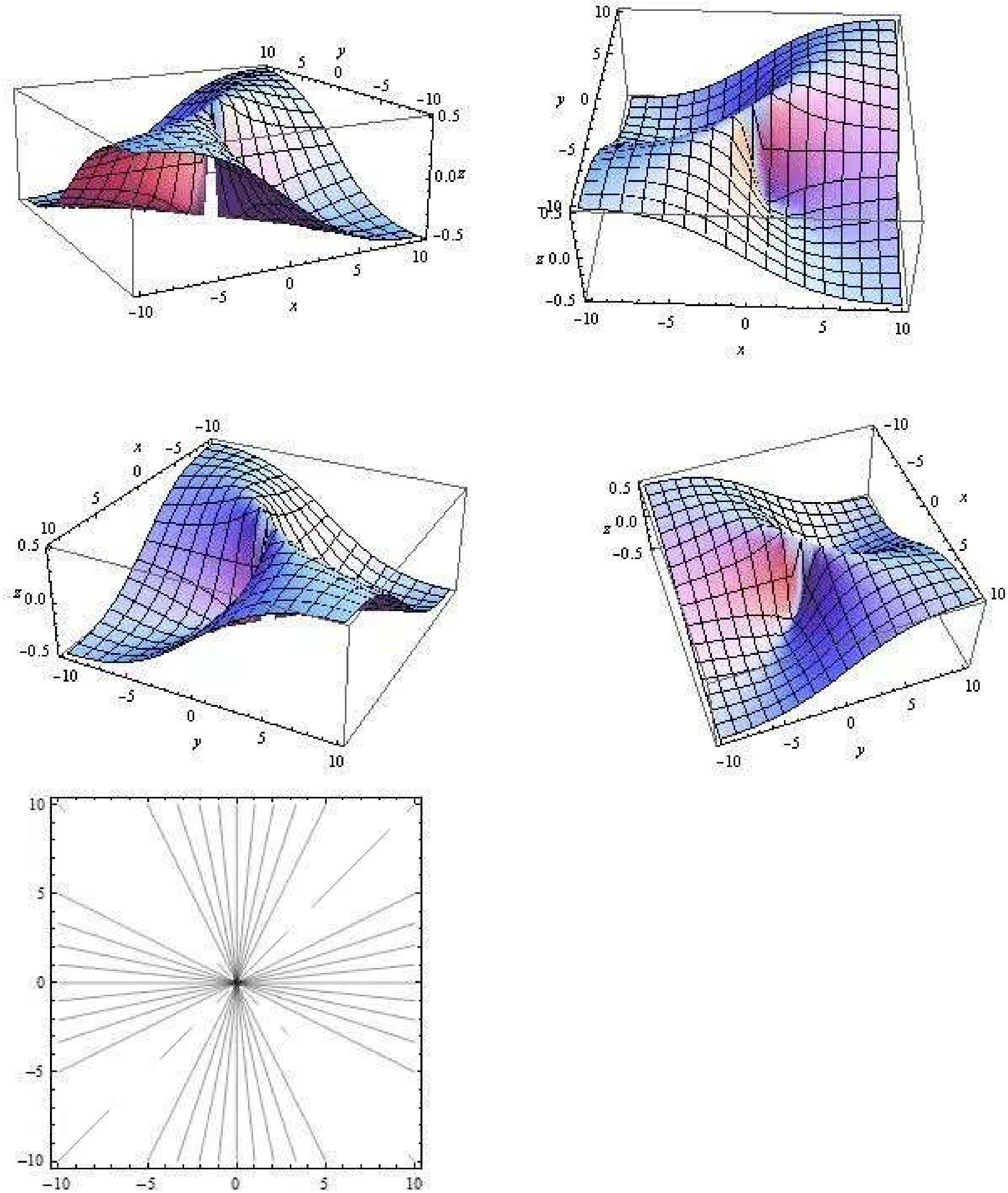
3) From some directions, $f(x,y) \rightarrow -\infty$.

4) From the contour map we can see that f(x,y) approaches 0 along the line y=-x.

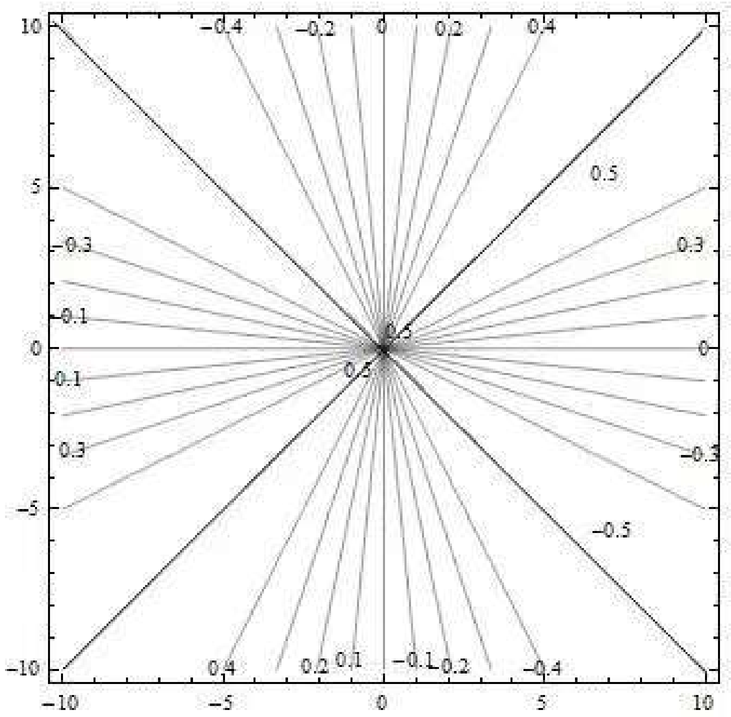
Answer 74E.

We have $f\left(x, y\right) = \frac{xy}{x^2 + y^2}$

Using CAS we get:



1) The function is undefined at the origin since the graph has different limiting values as (x,y) approaches the origin.



2) From the graph the function values is 0.5 along the line y=x

3) Along the line y=-x, the value of f is -0.5.

4) Along the axes, f(x,y)=0 for all values of (x,y) except the origin

Answer 75E.

Consider the function $f(x,y)=e^{cx^2+y^2}$

To investigate the function $f(x,y)=e^{cx^2+y^2}$ and comment on it depends on *c*.

Use Maple software,

Enter the function $f(x,y)=e^{cx^2+y^2}$, then go to plot option. We get the different plots for different values of *c*.

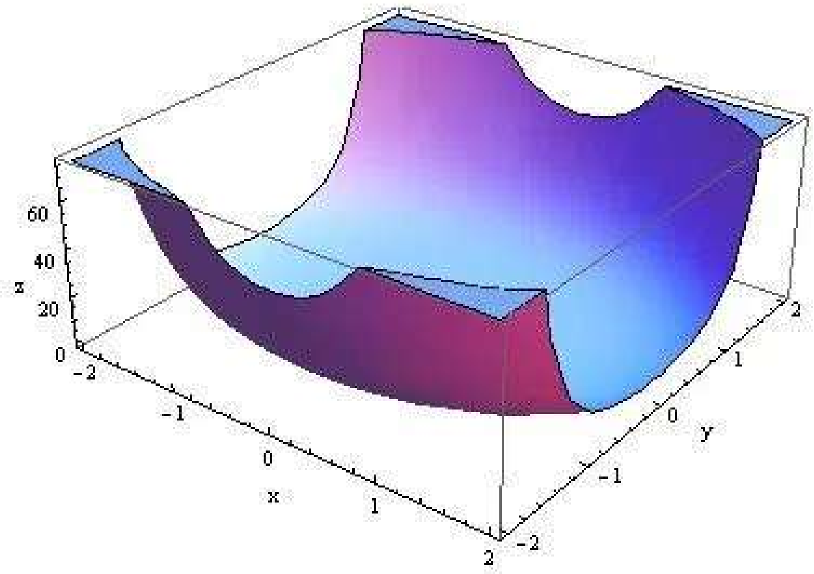
check different values for *c*, how the graph changes.

First, consider several graphs of the function.

For the function $f(x,y)=e^{cx^2+y^2}$

Take $c=0.5$

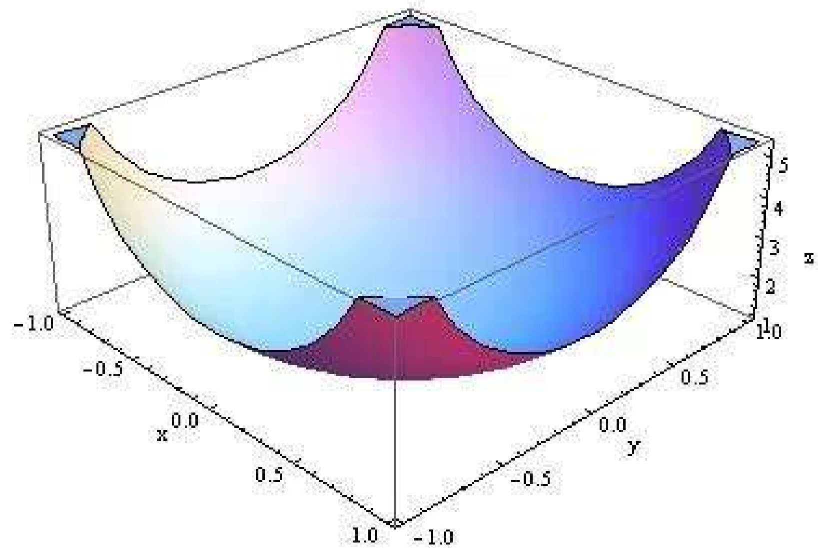
The graph of $f(x,y)=e^{\frac{1}{2}x^2+y^2}$ is



For the function $f(x,y)=e^{cx^2+y^2}$

Take $c=1$

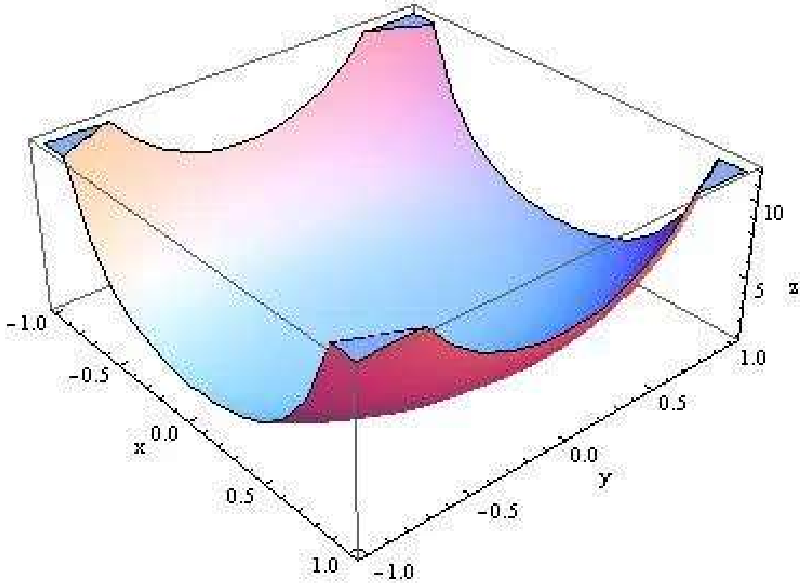
The graph of $f(x,y)=e^{x^2+y^2}$ is



For the function $f(x,y)=e^{cx^2+y^2}$

Take $c=2$.

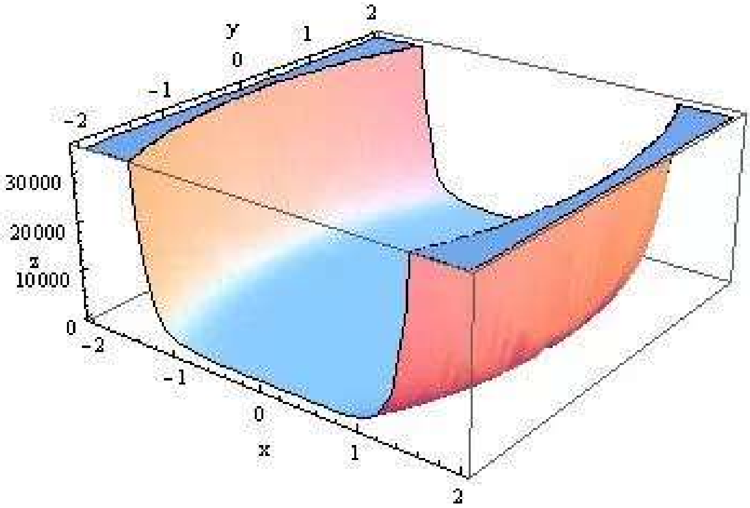
The graph of $f(x,y)=e^{2x^2+y^2}$ is



For the function $f(x,y)=e^{cx^2+y^2}$

Take $c=3$.

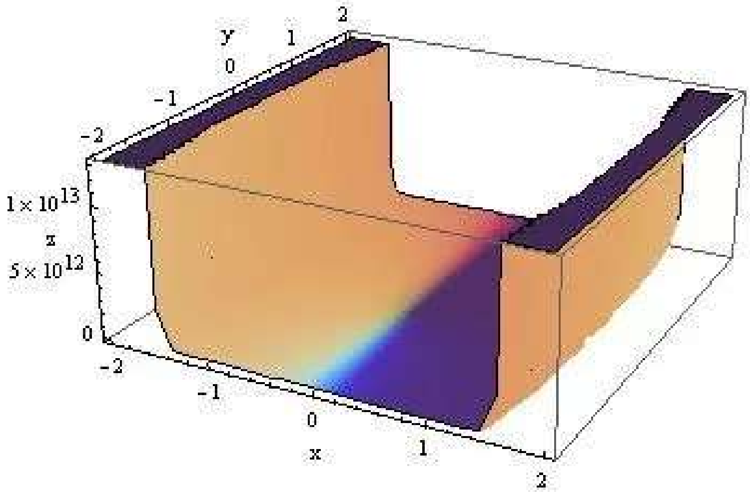
The graph of $f(x,y)=e^{x^2+y^2}$ is



For the function $f(x,y)=e^{cx^2+y^2}$

Take $c=10$.

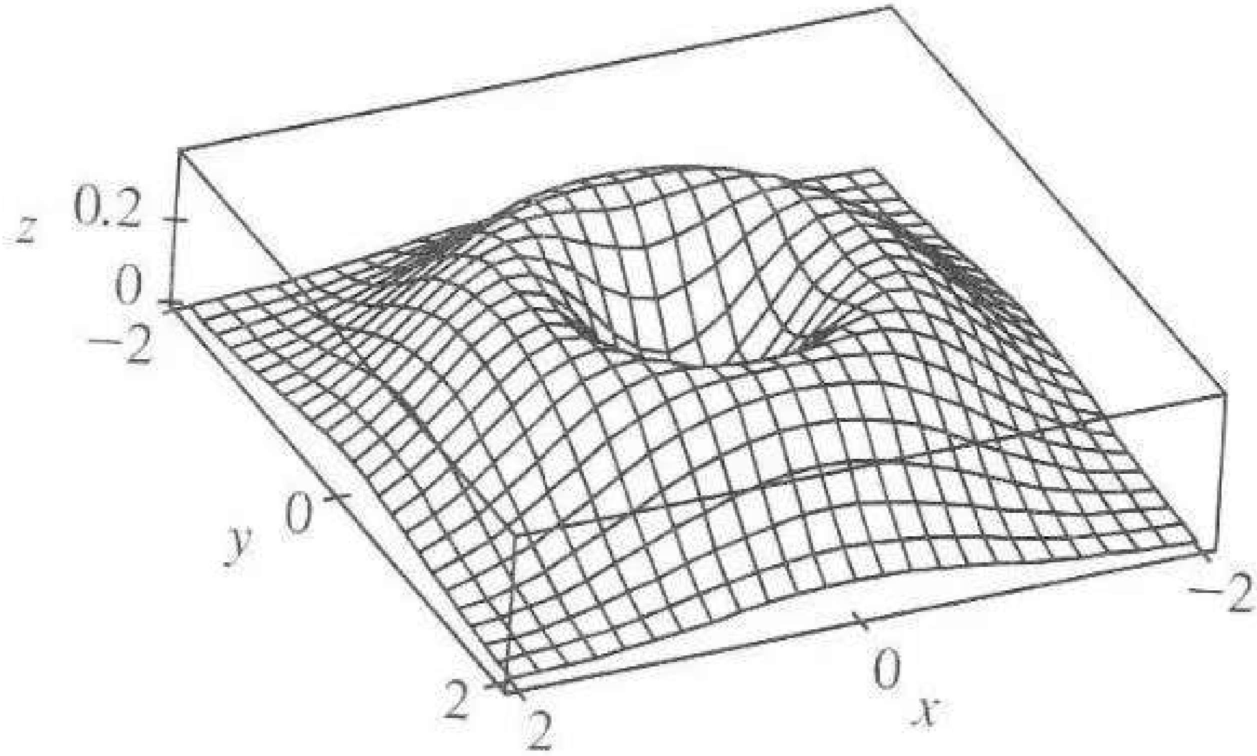
The graph of $f(x,y)=e^{10x^2+y^2}$ is



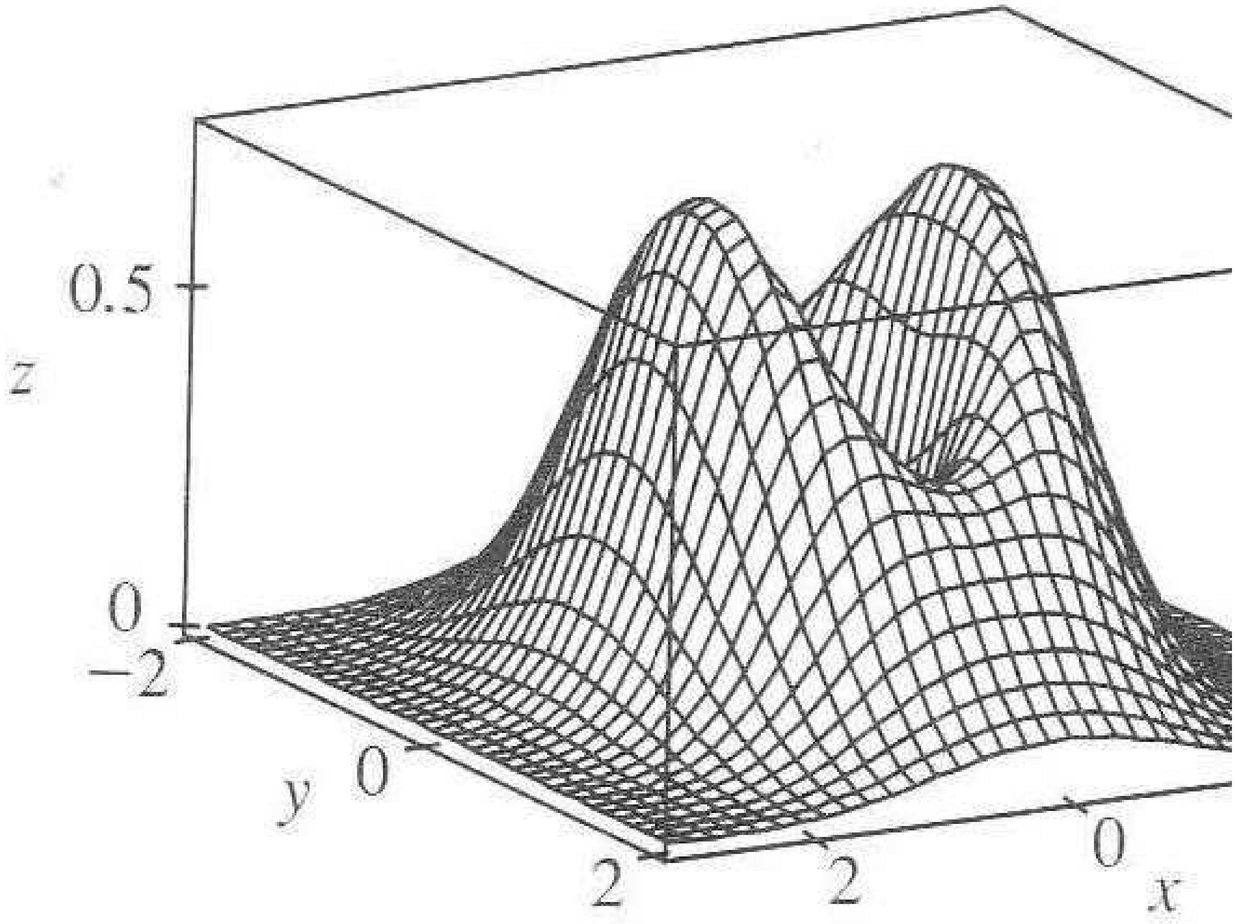
Therefore, as *c* increases, the graph is stretched towards the *x*-axis.

Answer 76E.

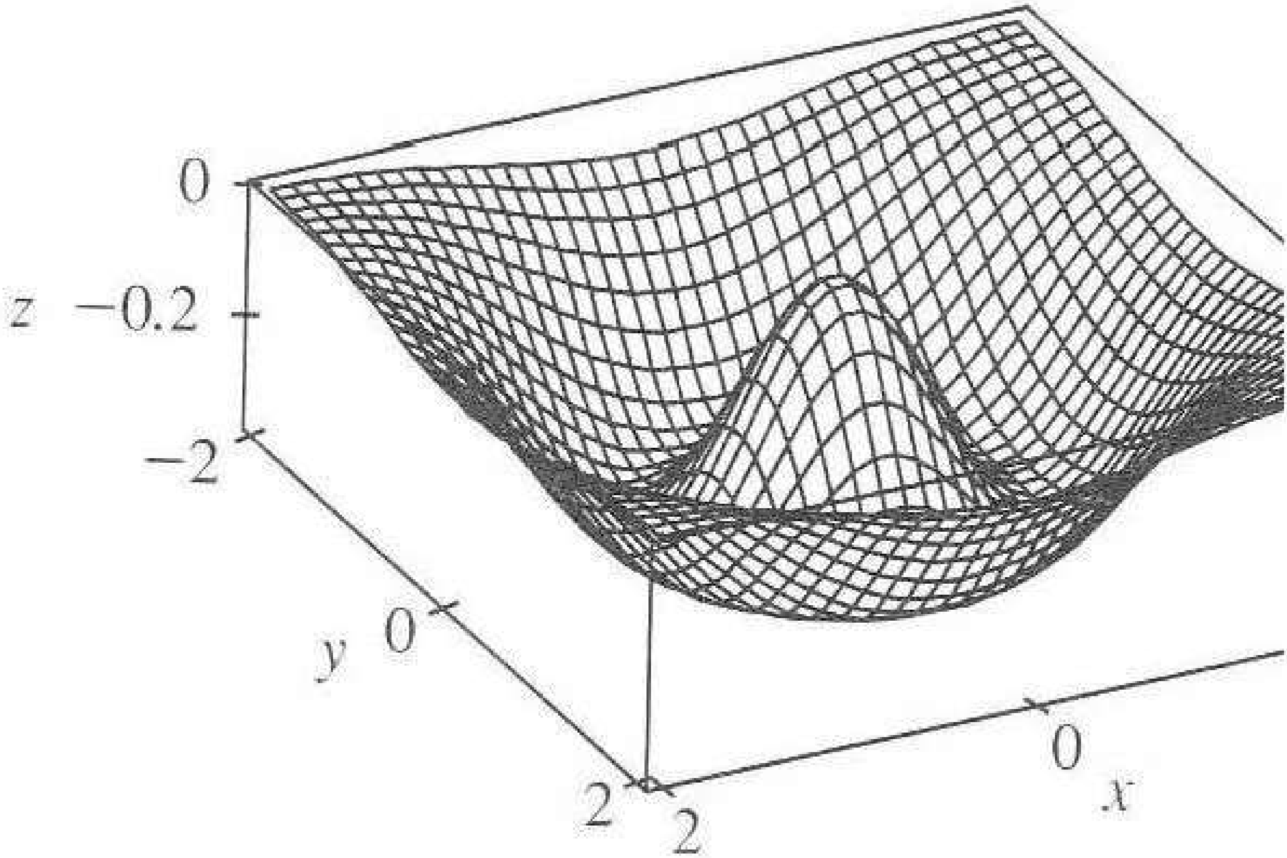
We are given the function $z = \left(ax^2 + by^2\right)e^{-x^2 - y^2}$. There are only three basic shapes which can be obtained. Interchanging a and b rotates the graph by 90o about the z-axis.



$a = 1, b = 1$



$a = 2, b = 1$



$a = -1, b = -1$

If a and b are both positive, then we see that the graph has two maximum points whose height increases as a and b increase.

If a and b have opposite signs, then the graph has two maximum points and two minimum points, and if a and b are both negative, then the graph has one maximum point and two minimum points.

Answer 77E.

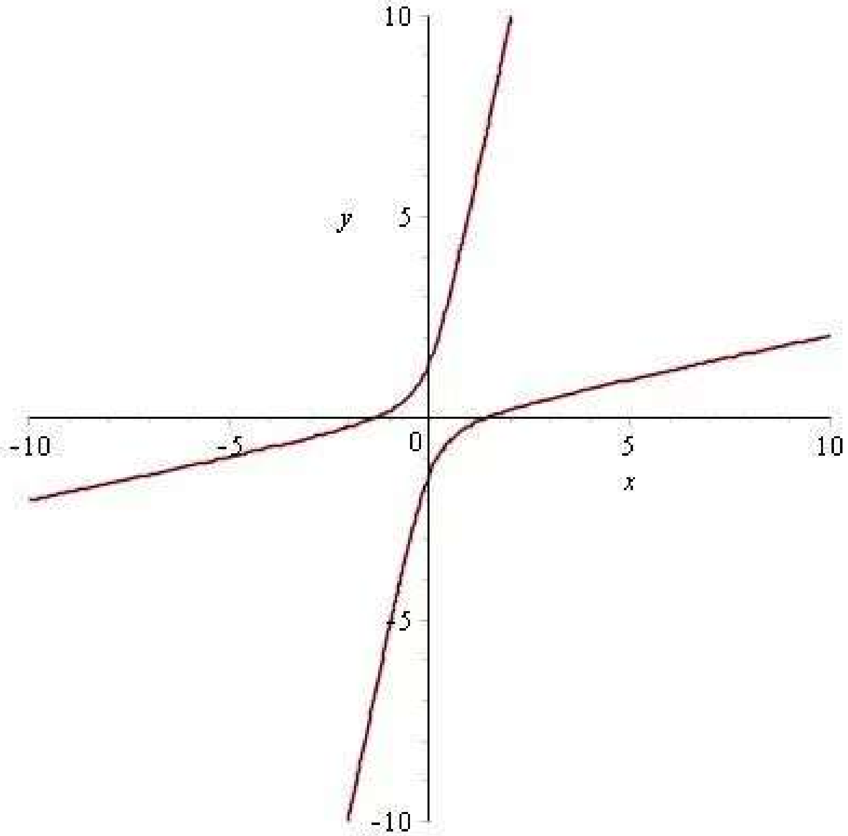
Consider the equation of family of surfaces.

$$z = x^2 + y^2 + cxy$$

Investigate the family of surfaces using a Computer.

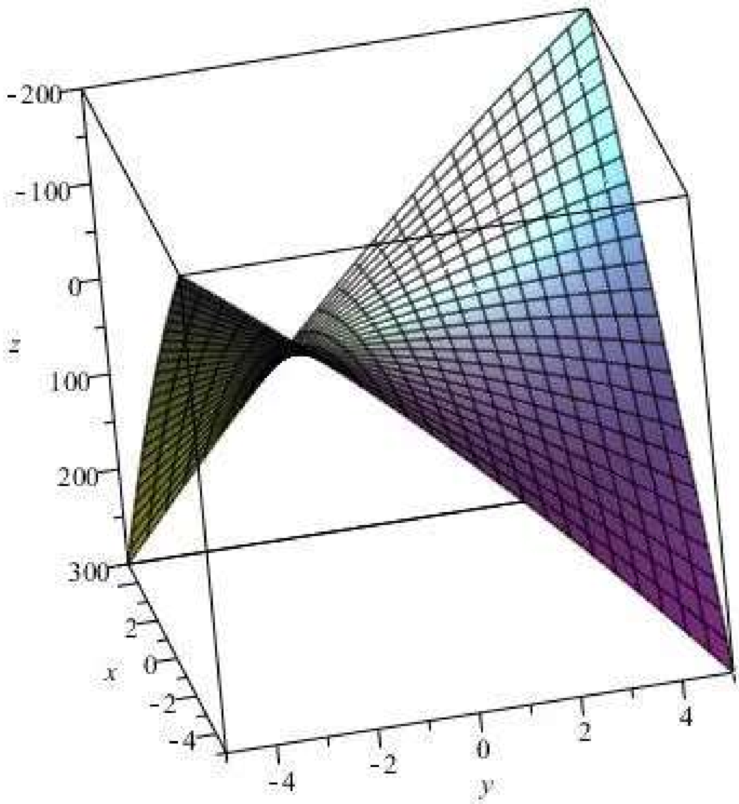
For $c < -2$:

The surface $z = x^2 + y^2 + cxy$ intersects the plane $z = k \neq 0$ in a hyperbola.

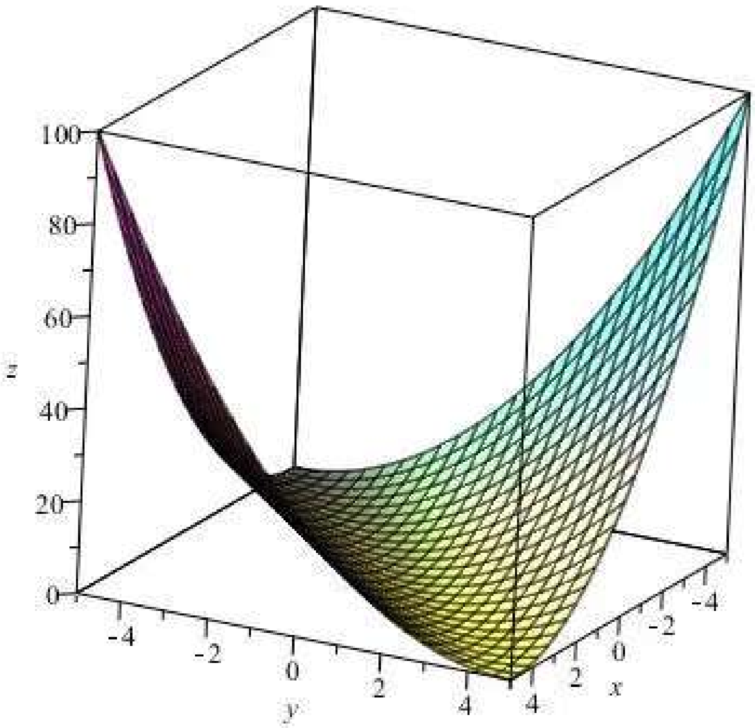


The surface $z = x^2 + y^2 + cxy$ intersects the plane $x = y$ in the parabola $z = (2+c)x^2$ and the surface $z = x^2 + y^2 + cxy$ intersects the plane $x = -y$ in the parabola $z = (2-c)x^2$. these parabolas open in opposite directions, thus the surface is hyperbolic paraboloid.

The sketch of the of the surface with $c = -10$:



The sketch of the of the surface with $c=-2$:



For $c=-2$:

So, the surface is,

$$z = x^2 + y^2 + cxy$$
$$z = x^2 + y^2 - 2xy$$
$$z = (x-2)^2$$

So, the surface is constant along each line $x-y=0$, $z=0$. The shape of cylinder is determined by its intersection with the plane $x+y=0$, where $z=4x^2$.

Thus, the cylinder is the parabolic with minima of 0 on the line $y=x$.

For $-2<c<0$ and $z>0$ for all x and y :

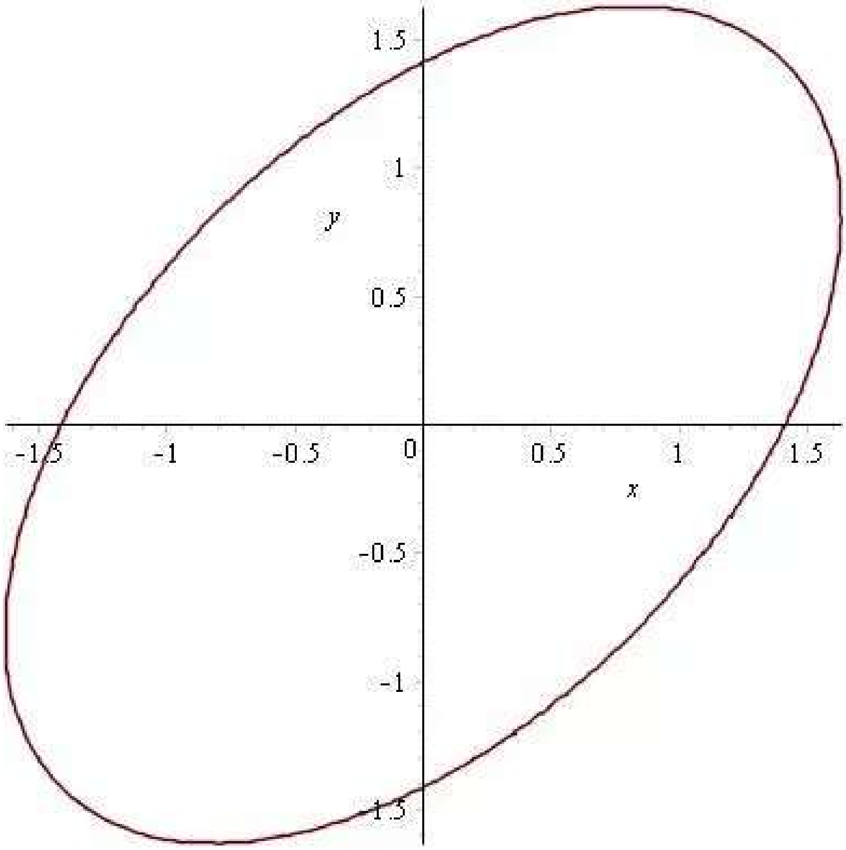
If x and y have the same sign, then

$$x^2 + y^2 + cxy \geq x^2 + y^2 - 2xy = (x-y)^2 \geq 0$$

If they x and y have the opposite signs, then

$$cxy \geq 0$$

The intersection with the surface $z = x^2 + y^2 + cxy$ and the plane $z = k > 0$ is an ellipse .



The intersection with the surface $z = x^2 + y^2 + cxy$ and the planes $x = 0$ and $y = 0$ are parabolas $z = y^2$ and $z = x^2$ respectively.

Thus, the surface is an elliptic paraboloid.

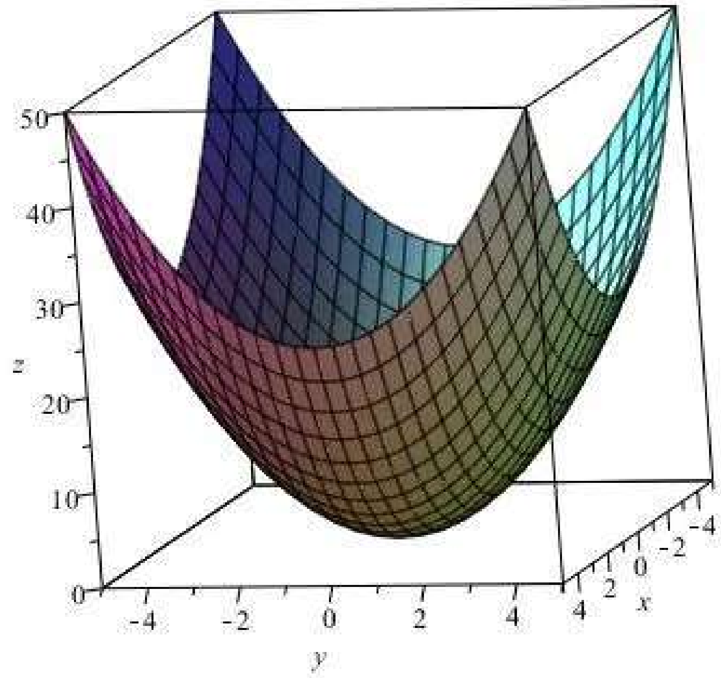
For $c > 0$:

The graphs have same slope, but they are reflected in the plane $x = 0$

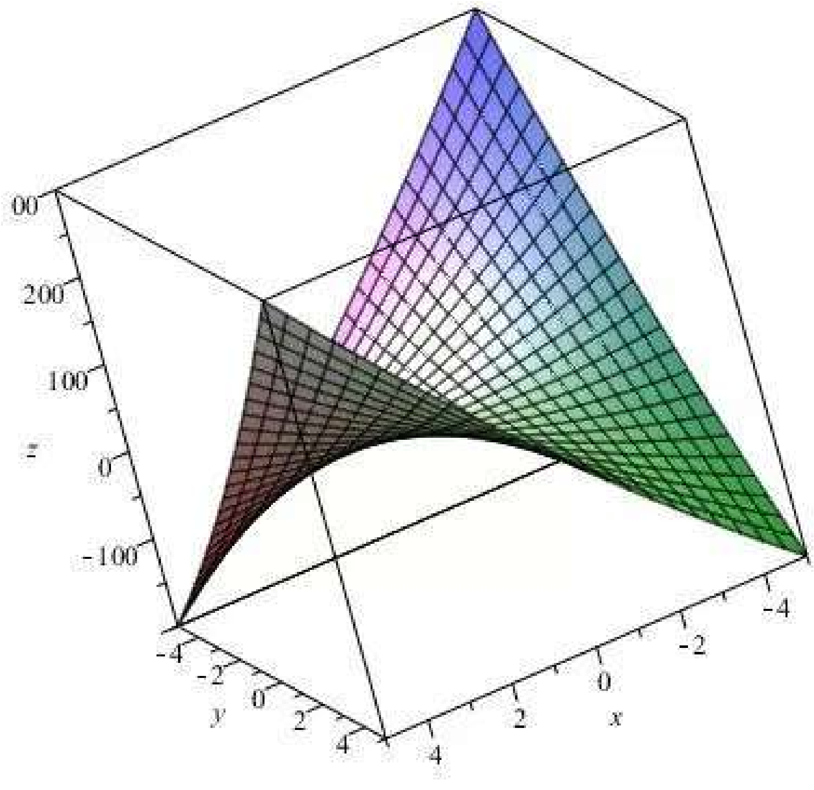
As $x^2 + y^2 + cxy = (-x)^2 + y^2 + (-c)(-x)y$.

That is the value of z is same for the for c at (x, y) as it is for $-c$ at $(-x, y)$.

The sketch of the surface $z = x^2 + y^2 + cxy$ for $c = 0$:



The graph of the surface $z = x^2 + y^2 + cxy$ for $c = 10$:



Therefore, the surface is an elliptic paraboloid for $0 < c < 2$, a parabolic cylinder for $c = 2$ and hyperbolic paraboloid for $c > 2.$

Answer 78E.

The objective is to sketch the graph of the given functions.

Consider the function $f(x,y)=\sqrt{x^2+y^2}$.

Use Maple to graph the function $f(x,y)$.

Input command:

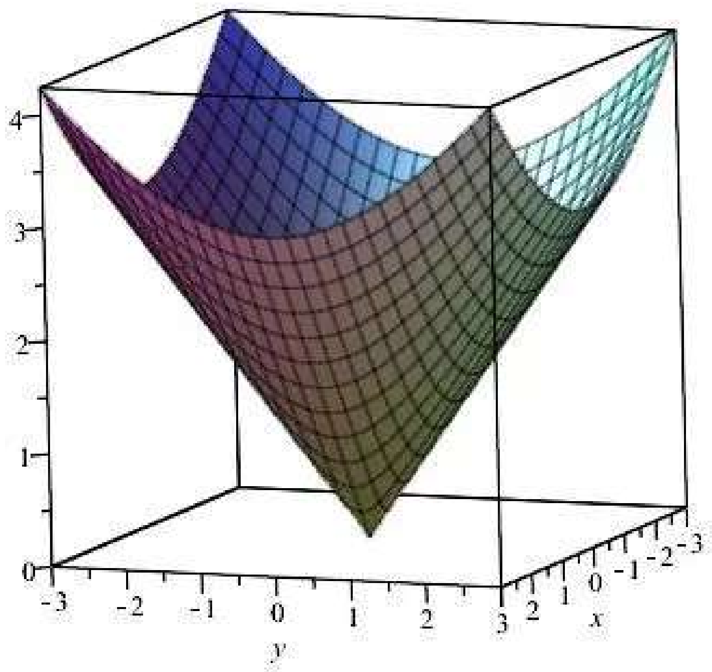
with(plots);

plot3d(sqrt(x^2+y^2), x = -3 .. 3, y = -3 .. 3);

Maple result:

> *with(plots)* :

> $plot3d\left(\sqrt{x^2+y^2}, x=-3..3, y=-3..3\right);$



Consider the function $f(x,y) = e^{\sqrt{x^2+y^2}}$.

Use Maple to graph the function $f(x,y)$.

Input command:

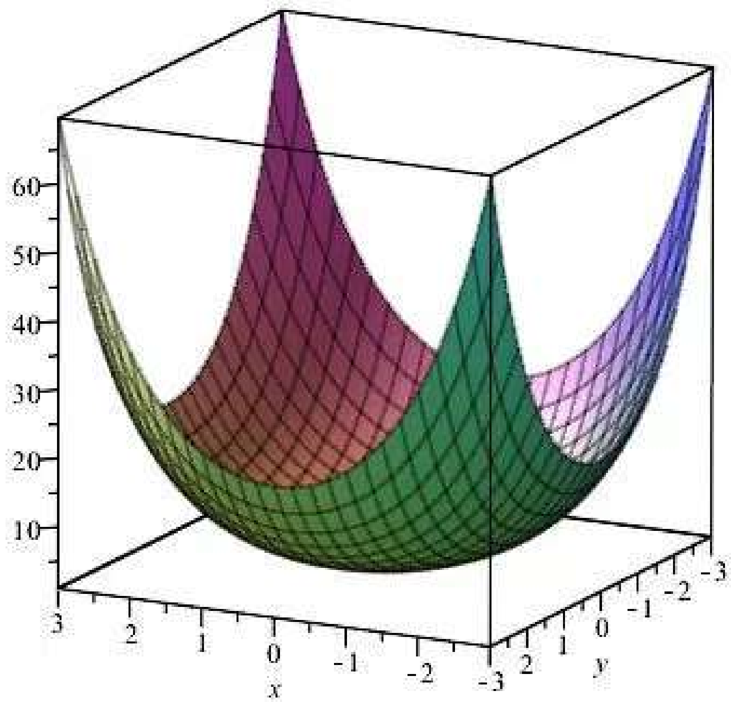
with(plots);

plot3d(exp(sqrt(x^2+y^2)), x = -3 .. 3, y = -3 .. 3);

Maple result:

> $with(plots)$:

> $plot3d\left(e^{\sqrt{x^2+y^2}}, x=-3..3, y=-3..3\right)$;



Consider the function $f(x,y)=\ln\left(\sqrt{x^2+y^2}\right)$.

Use Maple to graph the function $f(x,y)$.

Input command:

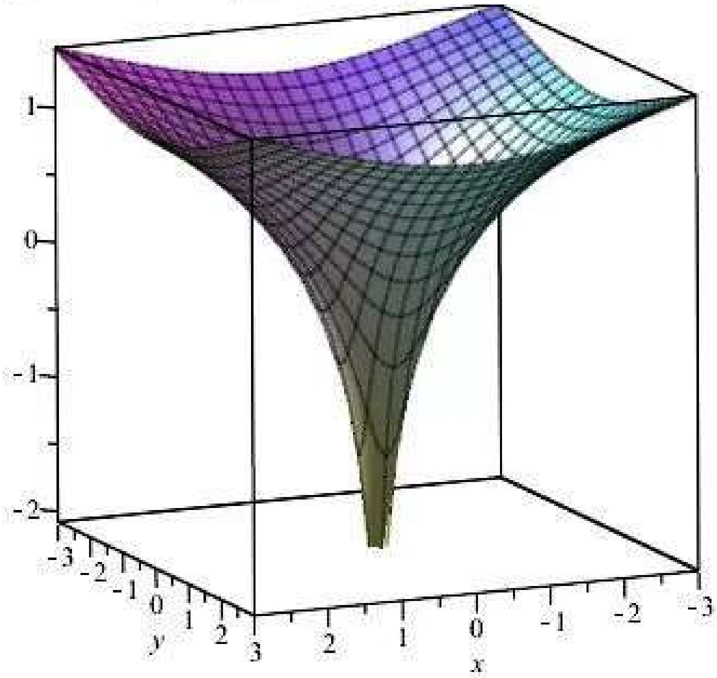
with(plots);

plot3d(ln(sqrt(x^2+y^2)), x = -3 .. 3, y = -3 .. 3);

Maple result:

> $with(plots):$

> $plot3d\left(\ln\left(\sqrt{x^2+y^2}\right), x=-3..3, y=-3..3\right);$



Consider the function $f(x,y)=\sin\left(\sqrt{x^2+y^2}\right)$.

Use Maple to graph the function $f(x,y)$.

Input command:

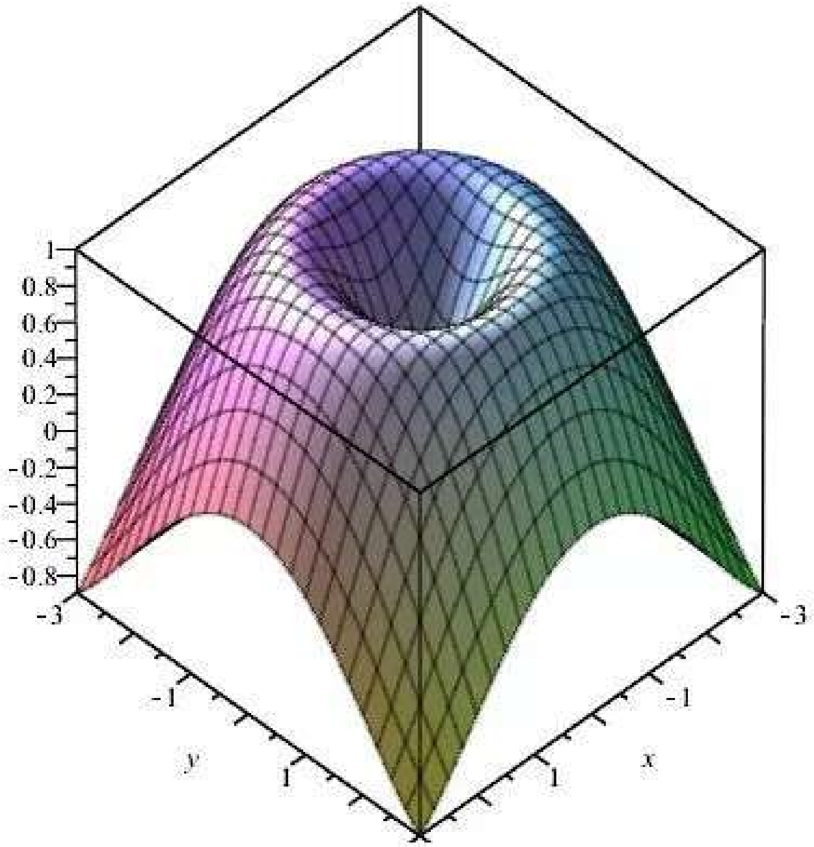
with(plots);

plot3d(sin(sqrt(x^2+y^2)), x = -3 .. 3, y = -3 .. 3);

Maple result:

> $with(plots)$:

> $plot3d\left(\sin\left(\sqrt{x^2+y^2}\right), x=-3..3, y=-3..3\right);$



Consider the function $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$.

Use Maple to graph the function $f(x,y)$.

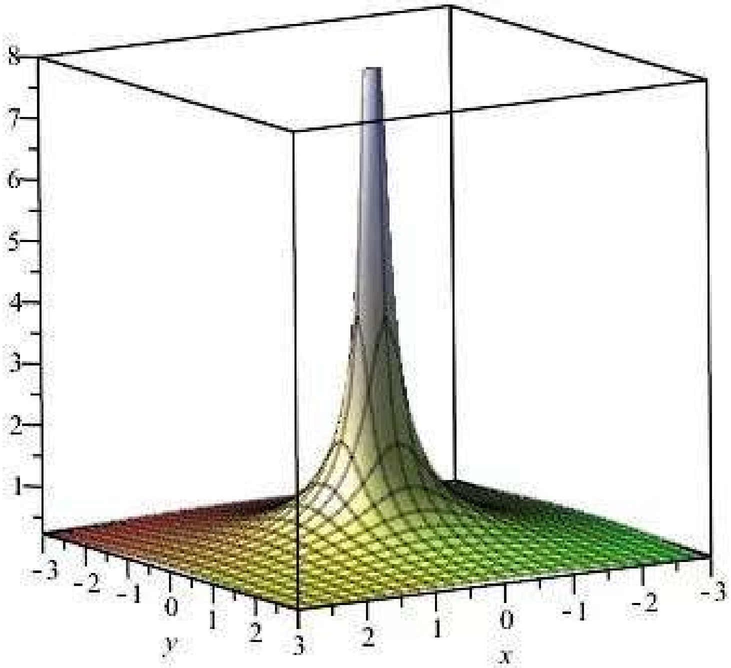
Input command:

```
with(plots);
plot3d(1/sqrt(x^2+y^2), x = -3 .. 3, y = -3 .. 3);
```

Maple result:

> $with(plots):$

> $plot3d\left(\frac{1}{\sqrt{x^2+y^2}}, x=-3..3, y=-3..3\right);$



Consider the function $f(x,y) = g\left(\sqrt{x^2+y^2}\right)$.

Let g is a function of one variable.

To graph the function $z = f(x,y) = g\left(\sqrt{x^2+y^2}\right)$, rotate the graph of $z = g(x)$ about the z-axis.

Answer 79E.

We have $P = bL^{\alpha}K^{1-\alpha}$

a)Taking logarithms

$$\ln P = \ln bL^{\alpha}K^{1-\alpha}$$

$$\ln P = \ln b + \ln L^{\alpha} + \ln K^{\alpha-1}$$

$$\ln P = \ln b + \alpha \ln L + (1-\alpha)\ln K$$

$$\ln P = \ln b + \alpha \ln L + \ln K - \alpha \ln K$$

$$\ln P - \ln K = \ln b + \alpha[\ln L - \ln K]$$

$$\ln\left[\frac{P}{K}\right] = \ln b + \alpha \ln\left[\frac{L}{K}\right]$$

b)We compute x and y from the table and the result was rounded to 2 decimal places.

Year	P	L	K	x=ln(L/K)	y=ln(P/K)
1899	100	100	100	0.00	0.00
1900	101	105	107	-0.02	-0.06
1901	112	110	114	-0.04	-0.02
1902	122	117	122	-0.04	0.00
1903	124	122	131	-0.07	-0.05
1904	122	121	138	-0.13	-0.12
1905	143	125	149	-0.18	-0.04
1906	152	134	163	-0.20	-0.07
1907	151	140	176	-0.23	-0.15
1908	126	123	185	-0.41	-0.38
1909	155	143	198	-0.33	-0.24
1910	159	147	208	-0.35	-0.27
1911	153	148	216	-0.38	-0.34
1912	177	155	226	-0.38	-0.24
1913	184	156	236	-0.41	-0.25
1914	169	152	244	-0.47	-0.37
1915	189	156	266	-0.53	-0.34
1916	225	183	298	-0.49	-0.28
1917	227	198	335	-0.53	-0.39
1918	223	201	366	-0.60	-0.50
1919	218	196	387	-0.68	-0.57
1920	231	194	407	-0.74	-0.57
1921	179	146	417	-1.05	-0.85
1922	240	161	431	-0.98	-0.59

Using CAS we get that the least squares regresion line through the points (x,y) is given by

$y = 0.75x + 0.01$ (We round the formula to 2 decimal places) c)We have

$$y = 0.75x + 0.01$$

$$y = \alpha x + \ln b$$

Comparing we get that:

ln b= 0.01 then b= $e^{0.01}$ ~ 1.01

α= 0.75

So the Cobb-Douglas production function is

$$P = bL^{\alpha} K^{1-\alpha}$$

$$P = 1.01\, L^{0.75} K^{1-0.75}$$

$$P = 1.01\, L^{0.75} K^{0.25}$$