CBSE Test Paper 05

CH-14 Mathematical Reasoning

1. Which of the following is equivalent to $(p \wedge q)$?

   a. $\sim (p \to q)$

   b. $(p \wedge \sim q)$

   c. $\sim (p \to \sim q)$

   d. $(p \to q)$

2. Which of the following statement is a tautology?

   a. $(\sim q \wedge p) \wedge q$

   b. $(\sim q \wedge p) \vee (p \vee \sim p)$

   c. $(p \wedge q) \wedge (\sim (p \wedge q))$

   d. $(\sim q \wedge p) \wedge (p \wedge \sim p)$

3. Which of the following connectives satisfy commutative law? $\wedge, \vee, \to, \leftrightarrow$

   a. $\wedge$

   b. $\wedge, \vee, \leftrightarrow$

   c. $\vee$

   d. $\Leftrightarrow$

4. The logically equivalent propositions of $p \leftrightarrow q$ is

   a. $(p \wedge q) \to (q \vee p)$

   b. $(p \to q) \wedge (q \to p)$

   c. $(p \to q) \vee (q \to p)$

d. $(p \wedge q) \wedge (q \vee p)$

5. Which of the following is logically equivalent to $p \leftrightarrow q$?

   a. $(p \wedge q) \wedge (q \rightarrow p)$

   b. $(p \rightarrow q) \vee (q \rightarrow p)$

   c. $(p \rightarrow q) \wedge (q \rightarrow p)$

   d. $(p \wedge q) \wedge (q \vee p)$

6. Fill in the blanks:

   If two simple statements p and q are connected by the word 'and', then the resulting compound statement "p and q" is called a _____ of p and q.

7. Fill in the blanks:

   The truth value of the statement "Every rectangle is a square and every square is a rectangle" is _____.

8. Check which of the following statements are contrapositive or converse.
   (i) If you do not have winter clothes, then you do not live in Delhi
   (ii) If you have winter clothes, then you live in Delhi.
   (b) If a quadrilateral is a parallelogram then its diagonals bisect each other.
   (i) If the diagonals of a quadrilateral do not bisect each other then the quadrilateral is not a parallelogram.
   (ii) If the diagonals of a quadrilateral bisect each other then it is a parallelogram.

9. Find out this sentence is a statement or not. justify your answer
   Two non-empty sets have always a non-empty intersection.

10. Give three examples of sentences which are not statements. Give reasons for the answers.
    (i) Who are you?
    (ii) May god bless you !
    (iii) How are you?

11. Show that the statement "For any real numbers a and, b, $a^2 = b^2$ implies that a = b" is not true by giving a counter-example.

12. Consider the statement, S: 80 is a multiple of 5 and 4. Check its validity.

13. Which of the following statements are true and which are false? In each case give a valid reason for saying so.
    (i) p: Each radius of a circle is a chord of the circle.
    (ii) q: The centre of a circle bisects each chord of the circle.
    (iii) r: Circle is a particular case of an ellipse.
    (iv) s: If x and y are integers such that x > y then -x < -y.
    (v) $t : \sqrt{11}$ is a rational number.

14. Write the truth value of the following compound statement:

    i. 0 is less than every positive and negative integer.
    ii. 2 is a positive number or negative number.
    iii. If the number R = 57423 is divisible by 3, then the sum of digits forming R is divisible by 3.
    iv. $4 \times 5 = 20$, if and only if $5 \times 4 = 10$.

15. Find the component statements of the following compound statements and check whether they are true or false.
    (i) Number 3 is prime or it is odd.
    (ii) All integers are positive or negative.
    (iii) 100 is divisible by 3, 11, and 5.

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Solution

1. (c) $\sim (p \rightarrow \sim q)$
   Explanation:

   $p \wedge q$ Since $\sim (p \rightarrow q) \equiv p \wedge \sim q$

2. (b) $(\sim q \wedge p) \vee (p \vee \sim p)$
   Explanation:

   $p \vee \sim p \equiv T$

   $T \vee p \equiv T$

3. (b) $\wedge, \vee, \leftrightarrow$
4. (b) $(p \rightarrow q) \wedge (q \rightarrow p)$
   Explanation:

   $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

5. (c) $(p \rightarrow q) \wedge (q \rightarrow p)$
   Explanation:

   By definition $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

6. conjunction

7. False

8. (a) (i) Contra positive (ii) Converse
   (b) (i) Contra positive (ii) Converse

9. A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.
   This sentence is false because there are non-empty sets whose intersection is empty. Hence, it is a statement.

10. (i) This sentence is an interrogative sentence. Hence it is not a statement.
    (ii) This sentence is an exclamatory sentence. Hence it is not a statement.
    (iii) This sentence is an interrogative sentence. Hence it is not a statement.

11. The given compound statement is of the form "if p then q".
    We assume that p is true then
    $a, b \in R$ such that $a^2 = b^2$
    Let us take a =-3 and b = 3
    So when p is true q is false.
    Thus the given compound statement is not true.

12. The given compound statement is
    S : 80 is a multiple of 5 and 4.
    The component statements of the given compound statement are
    p : 80 is a multiple of 5.
    q : 80 is a multiple of 4.
    Then, $S \equiv p \wedge q$
    Now, since we know that 80 is multiple of 5 as well as 4.
    Therefore, the statements p and q are true.
    Hence, the compound statement $S \equiv p \wedge q$ is also true i.e., $S \equiv p \wedge q$ is a valid statement.

13. (i) A chord of a circle is a line whose two end points lie on the circle and all the points on the line lie inside the circle.
    So the radius of a circle is not a chord of the circle.
    Thus the given statement is false.
    (ii) The centre of a circle bisects chord of circle when the chord is diameter of circle.
    When the chord is other than diameter then centre of circle does not lie on the chord.
    Thus the given statement is false.
    (iii) In the equation of an ellipse is we put a = b then we get an equation of circle. Thus the given statement is true.
    (iv) It is given that x, $y \in z$ such that x > y.
    Multiplying both sides by negative sign, we have
    $x, y \in z$ such that -x < -y

Thus the given statement is true.

(v) Since $\sqrt{11}$ can not be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. Thus the given statement is false.

14. i. The connective word "and' is used in the given compound statement.
Here, let p: 0 is less than every positive integer.
q : 0 is less than every negative integer.
Here, the statement p is true, so its truth value is T.
and the statement q is false, so its truth value is F.
Therefore, the compound statement is false because its truth value is F (by conjunction).

ii. The connective word "or" is used in a given compound statement.
Here, p : 2 is a positive number.
q : 2 is a negative number.
Here, the statement p is true, so its truth value is T.
and the statement q is false, so its truth value is F.
Therefore, the compound statement is true because its truth value is T (by disjunction).

iii. The connective word 'If-then' is used in a given compound statement.
Here, let
p : The number R = 57423 is divisible by 3.
q : The sum of digits forming R is divisible by 3.
Here, the statement p is true, so its truth value is T.
And the statement q is also true, so its truth value is T.
Therefore, the compound statement must be true and its truth value is T (by conditional).

iv. The connective word 'If and only if' is used in a given compound statement.
Here, let p : $4 \times 5 = 20$
q : $5 \times 4 = 10$
Here, the statement p is true, so its truth value is T.
And the statement q is false, so its truth value is F.
Therefore, the compound statement is false and its truth value is F (by biconditional).

15. (i) The component statements are

    p : Number 3 is prime

    q : it is odd.

    Both the component statements p and q are true.

    (ii) The component statements are

    p : All integers are positive.

    q : All integers are negative

    Both the component statements p and q are false.

    (iii) The component statements are

    p : 100 is divisible by 3

    q : 100 is divisible by 11

    r : 100 is divisible by 5

    The component statements p and q are false whereas r is true.