

Alternating Current

Alternating Current (AC)

Alternating current is an electric current whose magnitude changes with time continuously and which reverses its direction periodically. It is mathematically represented as $I = I_0 \cos \omega t$ or $I = I_0 \sin \omega t$

Advantages of AC over DC

- Flexibility of converting from one value to other using transformer
- Can be transmitted over long distances economically as well as without much power loss

Mean Value or Average value of AC (I_m or V_m)

It is defined as that value of steady current which sends the same amount of charge through a circuit in the time of half cycle (i.e. $T/2$) as is sent by AC through the same circuit in the same time. Let, in small time dt , charge sent in the circuit due to

AC, $I = I_0 \sin \omega t$, is q .

$$I_m = 2\pi I_0 = 0.637 I_0 \quad I_m = 2\pi I_0 = 0.637 I_0$$

$$V_m = 2\pi V_0 = 0.637 V_0 \quad V_m = 2\pi V_0 = 0.637 V_0$$

Root Mean Square Value (r.m.s.) of AC (I_v or V_v)

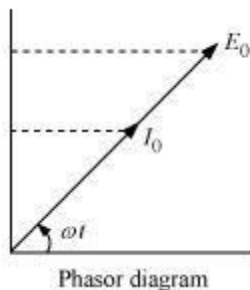
It is defined as that value of steady current which generates the same amount of heat in a resistor in a given time as is done by AC through the same resistor in the same time.

$$I_v = I_0 \sqrt{2} = 0.707 I_0 \quad I_v = I_0 \sqrt{2} = 0.707 I_0$$

$$V_v = V_0 \sqrt{2} = 0.707 V_0 \quad V_v = V_0 \sqrt{2} = 0.707 V_0$$

AC through a resistor:

- When AC flows through a resistor, the voltage and current are in phase with each other.

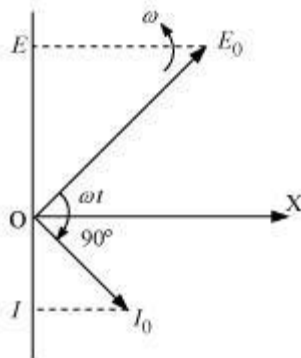


AC through an inductor:

- The alternating emf is ahead of alternating current by a phase angle of $\frac{\pi}{2}$.

Inductive reactance (X_L):

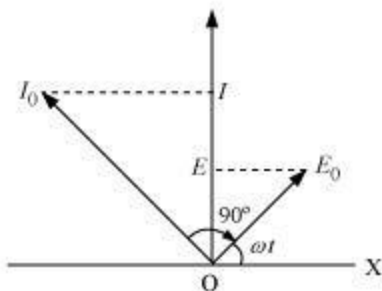
$$X_L = \omega L = 2\pi fL$$



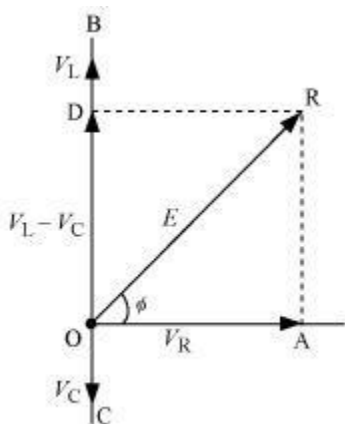
AC through a capacitor:

- The current leads the emf by a phase angle of $\frac{\pi}{2}$.

Capacitive reactance (X_C): $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$



- AC through LCR series circuit:**



$$E = \sqrt{I^2 R^2 + (X_L - X_C)^2} \quad \text{-----} \quad \sqrt{E = I R^2 + (X_L - X_C)^2}$$

$$\tan\Phi = \frac{X_L - X_C}{R} \quad \tan\Phi = \frac{X_L - X_C}{R}$$

Impedance (Z):

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- **Power in LCR circuit:**

$$P_{av} = I_{rms}^2 R \quad P_{av} = I_{rms}^2 R$$

$$\text{Power factor} = \cos\Phi = \frac{R}{Z} \quad \cos\Phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Resonance

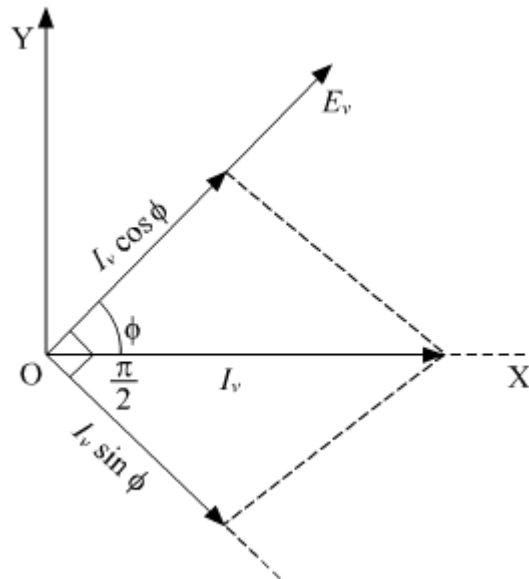
- It is the property exhibited by an LCR circuit.
- At a certain frequency known as resonating frequency (ω_0) the current through the circuit is maximum.
- It occurs at the frequency that can make $X_L = X_C$ or $\omega L = \frac{1}{\omega C}$.
- Impedance of the circuit $Z = R$.
- resonating frequency $\nu = \frac{1}{2\pi LC}$
- Quality factor, $Q = \frac{1}{R} \sqrt{\frac{L}{C}}$ is used to measure the sharpness of the resonance.

Power in AC circuit

- In AC circuit the power is given as $P = VI \cos\phi$
 - where, V = rms value of the voltage
 - I = rms value of the voltage
 - $\cos\phi$ = power factor = $\frac{\text{true power}}{\text{apparent power}}$
 - $\Phi = \tan^{-1} \left(\frac{X_C - X_L}{R} \right)$
- Cases for power factor
 - For purely resistive circuit $\cos\phi = 1$, $\phi = 0$
 - For purely capacitive circuit $\cos\phi = 0$, $\phi = \frac{\pi}{2}$
 - For purely inductive circuit $\cos\phi = 0$, $\phi = -\frac{\pi}{2}$

- **Wattless Current or Idle Current**

It is that current which consumes no power for its maintenance in an electric circuit. In an inductive circuit, the current lags behind the voltage which is shown in the figure below.



The component $I_v \sin \phi$ makes no contribution to the consumption of power in the a.c. circuit. Hence, it is known as wattless current.

LC Oscillations

- When capacitor of capacitance C charged to and inductor of inductance L are connected then:
 - energy stored in C oscillates between L and C .
 - energy of the oscillations is given by $v = \frac{1}{2} \pi LC \sqrt{v} = \frac{1}{2} \pi LC$.
 - Total energy in L and C every instant remains constant.
- A transformer consists of an iron core, on which are wound a primary coil of N_P turns and a secondary coil of N_S turns. If the primary coil is connected to an AC source, the primary and secondary voltages are related by

$$V_s = \left(\frac{N_s}{N_p} \right) V_p$$

And the currents are related by

$$I_s = \left(\frac{N_p}{N_s} \right) I_p$$

If $N_S > N_P \rightarrow$ The voltage is stepped up (step-up transformer)

If $N_S < N_P \rightarrow$ The voltage is stepped-down (step-down transformer)