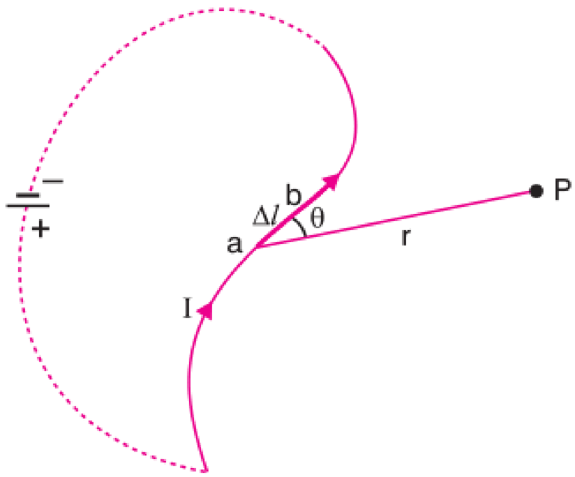
Long Answer Questions

Q. 1. State and explain Biot-Savart law. Use it to derive an expression for the magnetic field produced at a point near a long current carrying wire.



Ans. Biot-Savart law: Suppose the current I is flowing in a conductor and there is a small current element ‘ab’ of length ΔI. According to Biot-Savart the magnetic field (ΔB) produced due to this current element at a point P distant r from the element is given by

$$\Delta B \propto \frac{I \Delta l \sin\theta}{r^2} \text{ or } \Delta B = \frac{\mu}{4\pi}\frac{I\ \Delta l\ \sin\theta}{r^2} \qquad ...(i)$$

Where $\frac{\mu}{4\pi}$ is a constant of proportionality. It depends on the medium between the current element and point of observation (P).μ is called the permeability of medium. Equation (i) is called Biot-Savart law. The product of current (I) and length element (ΔI) (i.e., I ΔI) is called the current element. Current element is a vector quantity, its direction is along the direction of current. If the conductor be placed in vacuum (or air), then μ is replaced by μ0; where μ0 is called the permeability of free space (or air). In S.I. system μ0= 4π ×10^{-7} weber/ampere-metre (or newton/ampere^2).

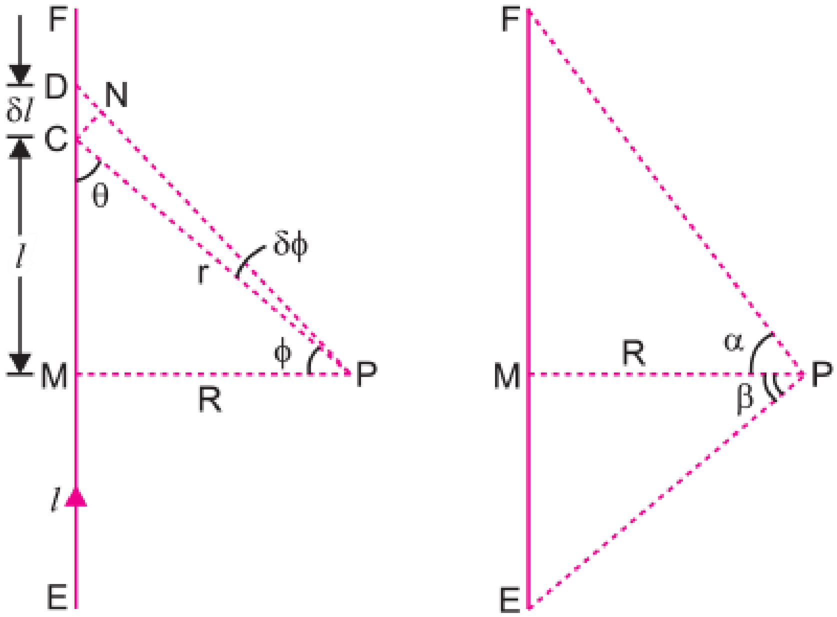
$$\text{Thus } \frac{\mu_0}{4\pi} = 10^{-7} \text{ weber/ampere} \times \text{metre}$$

As in most cases the medium surrounding the conductor is air, therefore, in general, Biot-Savart law is written as

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\Delta l\ \sin\theta}{r^2}$$

The direction of magnetic field is perpendicular to the plane containing current element and the line joining point of observation to current element. So in vector form the expression for magnetic field takes the form

$$\Delta \overrightarrow{B} = \frac{\mu_0}{4\pi} \frac{I\Delta \overrightarrow{l} \times \overrightarrow{r}}{r^3}$$



(a) (b)

Derivation of formula for magnetic field due to a current carrying wire using Biot-Savart law: Consider a wire EF carrying current I in upward direction. The point of observation is P at a finite distance R from the wire. If PM is perpendicular dropped from P on wire; then PM = R. The wire may be supposed to be formed of a large number of small current elements. Consider a small element CD of length δl at a distance l from M.

Let $\angle CPM = \varphi$

And $\angle CPD = \delta\varphi$, $\angle PDM = \theta$

The length δl is very small, so that $\angle PCM$ may also be taken equal to θ.

The perpendicular dropped from C on PD is CN. The angle formed between elements

$I\ \overrightarrow{\delta I}$ and $\vec{r}\ (= \overrightarrow{CP})$ is $(\pi - \theta)$. Therefore according to Biot-Savart law, the magnetic field due to current element $I\,\overrightarrow{\delta I}$ at P is

$$\delta B = \frac{\mu_0}{4\pi}\,\frac{I\ \delta l\ \sin\ (\pi-\theta)}{r^2} = \frac{\mu_0}{4\pi}\,\frac{I\ \delta l\ \sin\ \theta}{r^2} \qquad ...(i)$$

But in $\Delta\ CND$, $\sin\theta = \sin\,(\angle \mathrm{CDN}) = \frac{\mathrm{CN}}{\mathrm{CD}} = \frac{r\ \delta\varphi}{\delta l}$

or $\delta l \sin\theta = r\,\delta\varphi$

$\therefore$ From equation (i)

$$\delta B = \frac{\mu_0}{4\pi}\,\frac{I\ r\ \delta\varphi}{r^2} = \frac{\mu_0}{4\pi}\,\frac{I\ \delta\ \varphi}{r} \qquad ...(ii)$$

Again from fig.

$$\cos\ \varphi = \frac{R}{r} \Rightarrow r = \frac{R}{\cos\ \varphi}$$

From equation (ii)

$$\delta\ B = \frac{\mu_0}{4\pi}\,\frac{I\ \cos\ \varphi\ \delta\varphi}{R} \qquad ...(iii)$$

If the wire is of finite length and its ends make angles α and β with line MP, then net magnetic field (B) at P is obtained by summing over magnetic fields due to all current elements, i.e.,

$$B = \int_{-\beta}^{\alpha} \frac{\mu_0}{4\pi}\,\frac{I\ \cos\ \varphi\ d\varphi}{R} = \frac{\mu_0 I}{4\mu R}\int_{-\beta}^{\alpha}\ \cos\varphi\ d\varphi$$

$$\frac{\mu_0 I}{4\pi R}\,[\sin\ \ \varphi]_{-\beta}^{\alpha} = \frac{\mu_0 I}{4\pi R}\,[\sin\ \ \alpha - \sin\ (-\beta)]$$

$$i.e.,\ \ B = \frac{\mu_0 I}{4\pi R}\,(\sin\ \alpha + \sin\ \beta)$$

This is expression for magnetic field due to current carrying wire of finite length.

If the wir e is of infinite length (or very long), then α = β ⇒ π/2

$$\therefore \quad B = \frac{\mu_0 I}{4\pi R}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right) = \frac{\mu_0 I}{4\pi R}[1+1] \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi R}$$

Q. 2. Answer the following questions.

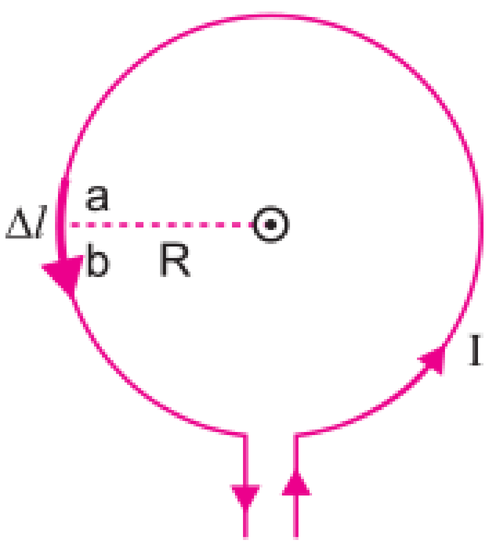
(i) State Biot-Savart Law. Using this law, find an expression for the magnetic field at the centre of a circular coil of N-turns, radius r, carrying current I.

(ii) Sketch the magnetic field for a circular current loop, clearly indicating the direction of the field. [CBSE (F) 2010, Central 2016]

Ans. (i) Biot-Savart Law: Refer to above question

Magnetic field at the centre of circular loop: Consider a circular coil of radius R carrying current I in anticlockwise direction. Say, O is the centre of coil, at which magnetic field is to be computed. The coil may be supposed to be formed of a large number of current elements. Consider a small current element 'ab' of length Δl. According to Biot Savart law the magnetic field due to current element 'ab' at centre O is

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\ \Delta l\ \sin\theta}{R^2}$$



Where θ is angle between current element ab and the line joining the element to the centre O. Here θ =90° because current element at each point of circular path is perpendicular to the radius. Therefore magnetic field produced at O, due to current element ab is

$$\Delta B = \frac{\mu_0}{4\pi} \frac{I \, \Delta l}{R^2}$$

According to Maxwell's right hand rule, the direction of magnetic field at O is upward, perpendicular to the plane of coil. The direction of magnetic field due to all current elements is the same. Therefore the resultant magnetic field at the centre will be the sum of magnetic fields due to all current elements. Thus

$$B = \sum \Delta B = \sum \frac{\mu_0}{4\pi} \frac{I \, \Delta l}{R^2} = \frac{\mu_0}{4\pi} \frac{I}{R^2} \sum \Delta l$$

But $\sum \Delta l$ = total length of circular coil $= 2\pi R$ (for one-turn)

$$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{I}{R^2} . 2\pi R \quad \text{or} \quad B = \frac{\mu_0 I}{2R}$$

If the coil contains N–turns, then $\sum \Delta l = N.\, 2\pi\, R$

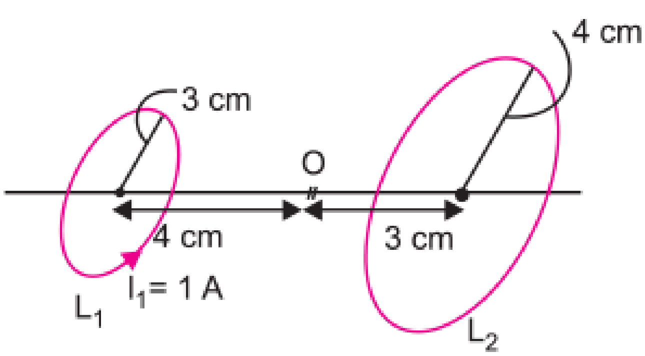
$$B = \frac{\mu_0 I}{4\pi R^2} . N.2\pi R \quad \text{or} \quad B = \frac{\mu_0 \, NI}{2R}$$

Here current in the coil is anticlockwise and the direction of magnetic field is perpendicular to the plane of coil upward; but if the current in the coil is clockwise, then the direction of magnetic field will be perpendicular to the plane of coil downward.

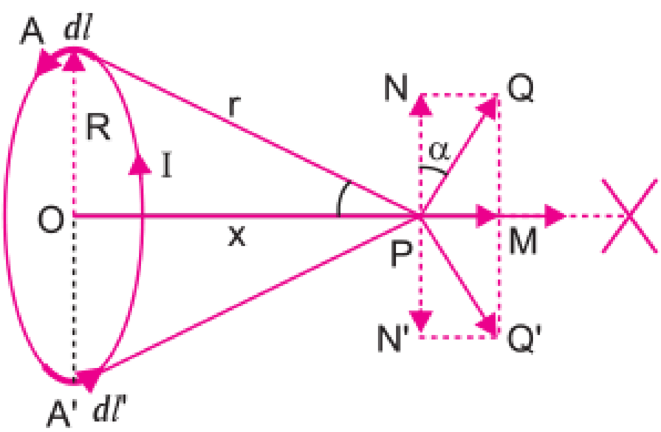
Q. 3. Answer the following questions.

(i) Derive an expression for the magnetic field at a point on the axis of a current carrying circular loop. [CBSE (AI) 2013; (F) 2010]

(ii) Two co-axial circular loops L_1 and L_2 of radii 3 cm and 4 cm are placed as shown. What should be the magnitude and direction of the current in the loop L_2 so that the net magnetic field at the point O be zero?



Ans. (i) Magnetic field at the axis of a circular loop: Consider a circular loop of radius R carrying current I, with its plane perpendicular to the plane of paper. Let P be a point of observation on the axis of this circular loop at a distance x from its centre O. Consider a small element of length dl of the coil at point A. The magnitude of the magnetic induction $\underset{dB}{\longrightarrow}$ at point P due to this element is given by



$$\overrightarrow{\mathrm{dB}} = \frac{\mu_0}{4\pi} \frac{I\delta l \sin\alpha}{r^2} \qquad ...(i)$$

The direction of $\underset{dB}{\longrightarrow}$ is perpendicular to the plane containing $\underset{dI}{\rightarrow}$ and $\underset{r}{\rightarrow}$ and is given by right hand screw rule. As the angle between I $\underset{dI}{\rightarrow}$ and $\underset{r}{\rightarrow}$ and is 90°, the magnitude of the magnetic induction $\underset{dB}{\longrightarrow}$ is given by,

$$\overrightarrow{\mathrm{dB}} = \frac{\mu_0 I}{4\pi} \frac{\mathrm{dl}\ \sin\ 90^o}{r^2} = \frac{\mu_0 I\ \mathrm{dl}}{4\pi r^2}. \qquad ...(ii)$$

If we consider the magnetic induction produced by the whole of the circular coil, then by symmetry the components of magnetic induction perpendicular to the axis will be cancelled out, while those parallel to the axis will be added up. Thus the resultant magnetic induction $\underset{B}{\rightarrow}$ at axial point P is along the axis and may be evaluated as follows:

The component of $\overrightarrow{dB}$ along the axis,

$$\overrightarrow{dB_x} = \frac{\mu_0 I \, dl}{4\pi \, r^2} \sin \alpha \qquad ...(iii)$$

But $\sin \alpha = \frac{R}{r}$ and $r = (R^2 + x^2)^{1/2}$

$$\therefore \quad \overrightarrow{dB_x} = \frac{\mu_0 I \, dl}{4\pi \, r^2} \cdot \frac{R}{r} = \frac{\mu_0 \, IR}{4\pi r^3} dl = \frac{\mu_0 \, IR}{4\pi \, (R^2+x^2)^{3/2}} dl \qquad ..(iv)$$

Therefore the magnitude of resultant magnetic induction at axial point P due to the whole circular coil is given by

$$\overrightarrow{B} = \oint \frac{\mu_0 \, IR}{4\pi(R^2+x^2)^{3/2}} dl = \frac{\mu_0 \, IR}{4\pi(R^2+x^2)^{3/2}} \oint dl$$

But $\oint dl$ = length of the loop = $2\pi R$ ...(v)

$$\text{Therefore,} \quad B = \frac{\mu_0 \, IR}{4\pi(R^2+x^2)^{3/2}} (2\pi R)$$

$$\overrightarrow{B} = B_x \, \hat{i} = \frac{\mu_0 \, IR^2}{2(R^2+x^2)^{3/2}} \, \hat{i}. \left[\text{At centre}, x = 0, \overrightarrow{B} = \frac{\mu_0 I}{2R}\right]$$

If the coil contains N turns, then

$$B = \frac{\mu_0 \, NIR^2}{2(R^2+x^2)^{3/2}} \text{ tesla} \qquad ....(vi)$$

Q. 4. Answer the following questions.

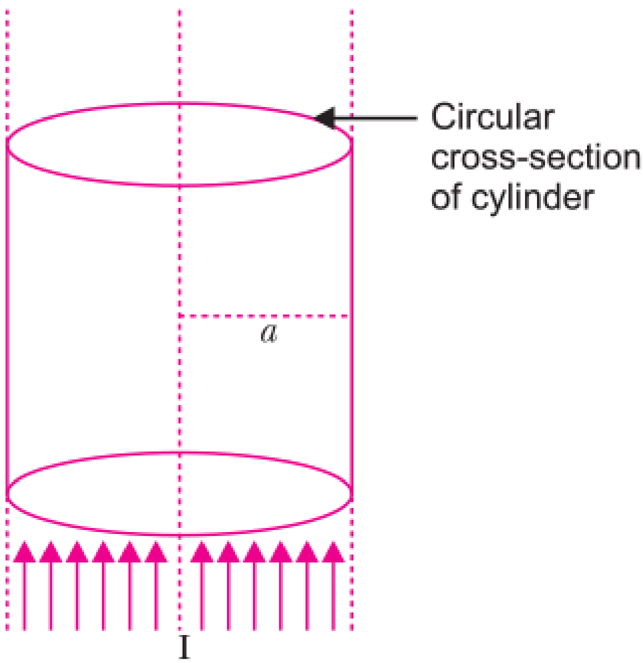
(i) A straight thick long wire of uniform circular cross-section of radius 'a' is carrying a steady current I. The current is uniformly distributed across the cross-section. Use Ampere's circuital law to obtain a relation showing the variation of the magnetic field (Br) inside and outside the wire with distance r, (r ≤ a) and (r > a) of the field point from the centre of its cross-section. What is the magnetic field at the surface of this wire? Plot a graph showing the nature of this variation.

(ii) Calculate the ratio of magnetic field at a point above the surface of the wire to that at a point below its surface. What is the maximum value of the field of this

wire?
[CBSE Delhi 2010; Chennai 2015]

Ans. (i) Magnetic field due to a straight thick wire of uniform cross-section: Consider an infinitely long cylindrical wire of radius a, carrying current I. Suppose that the current is uniformly distributed over whole cross-section of the wire. The cross-section of wire is circular. Current per unit cross-sectional area.

$$i = \frac{I}{\pi a^2} \qquad \ldots(\mathrm{i})$$



Magnetic field at external points (r > a): We consider a circular path of radius r (> a) passing through external point P concentric with circular cross-section of wire. By symmetry the strength of magnetic field at every point of circular path is same and the direction of magnetic field is tangential to path at every point. So line integral of magnetic field $\underset{B}{\rightarrow}$ around the circular path

$$\oint \overrightarrow{B} \,.\, \overrightarrow{\mathrm{dl}} = \oint B \ \mathrm{dl} \ \cos \ 0^o = B \ 2\pi r$$

Current enclosed by path = Total current on circular cross-section of cylinder = I

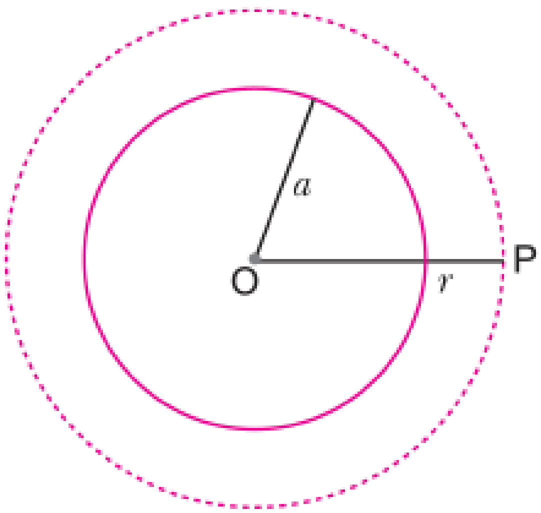
By Ampere’s circuital law

$$\oint \vec{B} \cdot \vec{dl} = \mu \times \text{current enclosed by path}$$

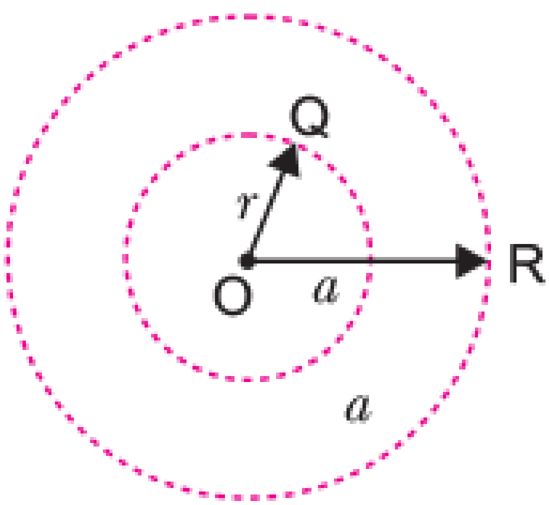
$$\Rightarrow B\, 2\pi r = \mu_0 \times I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

This expression is same as the magnetic field due to a long current carrying straight wire.

This shows that **for external points the current flowing in wire may be supposed to be concerned at the axis of cylinder.**



Magnetic Field at Internal Points (r < a) : Consider a circular path of radius r (<a), passing through internal point Q concentric with circular cross-section of the wire. In this case the assumed circular path encloses only a path of current carrying circular cross-section of the wire.



∴ Current enclosed by path = current per unit cross-section × cross section of assumed circular path

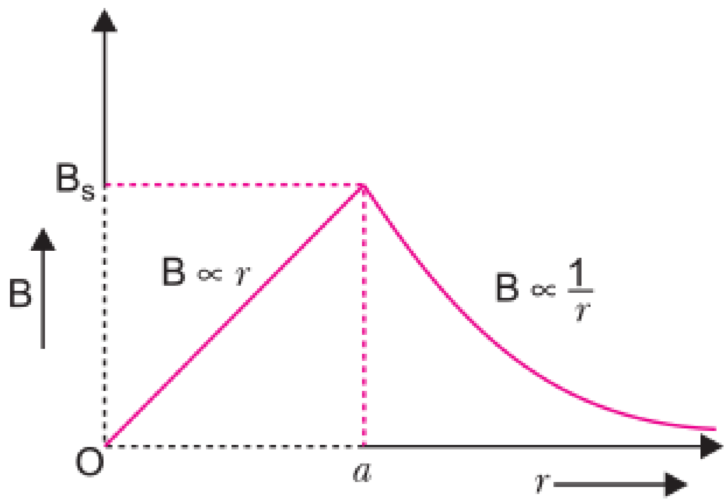
$$= i \times \pi r^2 = \left(\frac{I}{\pi a^2}\right) \times \pi r^2 = \frac{\mathrm{Ir}^2}{a^2}$$

∴ By Ampere's circuital law

$$\oint \vec{B} \cdot \overrightarrow{\mathrm{dl}} = \mu_0 \times \text{ current closed by path}$$

$$\Rightarrow \quad B \cdot 2\pi r = \mu_0 \times \frac{\mathrm{Ir}^2}{a^2} \quad \Rightarrow \quad B = \frac{\mu_0\,\mathrm{Ir}}{2\pi a^2}$$

Clearly, magnetic field strength inside the current carrying wire is directly proportional to distance of the point from the axis of wire.



At surface of cylinder r = a, so magnetic field at surface of wire

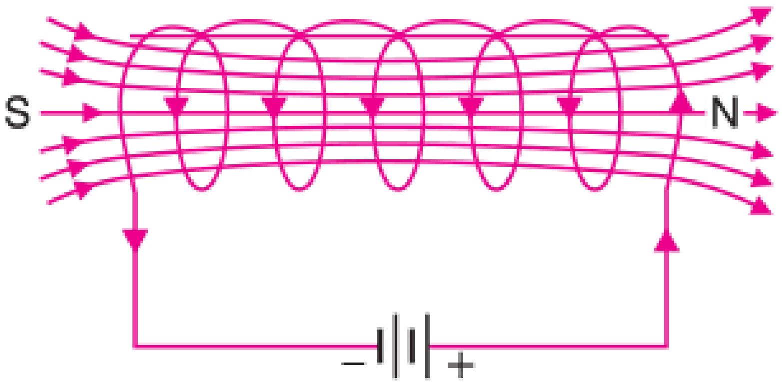
$$B_S = \frac{\mu_0 I}{2\pi a} \text{ (maximum value)}$$

The variation of magnetic field strength (B) with distance (r) from the axis of wire for internal and external points is shown in figure.

Q. 5. Using Ampere's circuital law find an expression for the magnetic field at a point on the axis of a long solenoid with closely wound turns. [CBSE (F) 2010]

Ans. Magnetic field due to a current carrying long solenoid:

A solenoid is a long wire wound in the form of a close-packed helix, carrying current. To construct a solenoid a large number of closely packed turns of insulated copper wire are wound on a cylindrical tube of card-board or china clay. When an electric current is passed through the solenoid, a magnetic field is produced within the solenoid. If the solenoid is long and the successive insulated copper turns have no gaps, then the magnetic field within the solenoid is uniform; with practically no magnetic field outside it. The reason is that the solenoid may be supposed to be formed of a large number of circular current elements. The magnetic field due to a circular loop is along its axis and the current in upper and lower straight parts of solenoid is equal and opposite. Due to this the magnetic field in a direction perpendicular to the axis of solenoid is zero and so the resultant magnetic field is along the axis of the solenoid.



If there are ‘n’ number of turns per metre length of solenoid and I amperes is the current flowing, then magnetic field at axis of long solenoid

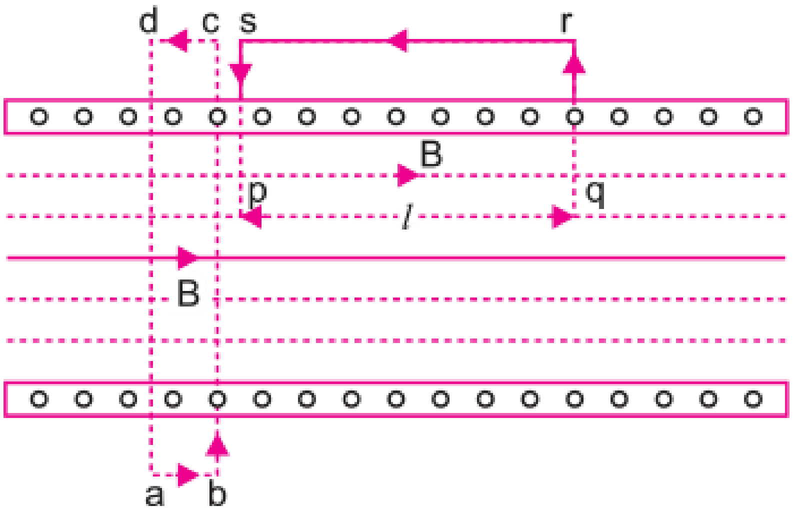
$B = \mu_0 nI$

If there are N turns in length l of wire, then

$$n = \frac{N}{l} \quad \text{or} \quad B = \frac{\mu_0 \, NI}{l}$$

Derivation: Consider a symmetrical long solenoid having number of turns per unit length equal to n.

Let I be the current flowing in the solenoid, then by right hand rule, the magnetic field is parallel to the axis of the solenoid.



Field outside the solenoid: Consider a closed path abcd Applying Ampere's law to this path

$$\oint \vec{B} . \vec{dl} = \mu \times 0 \text{ (since net current enclosed by path is zero)}$$

$$\text{As } dl \neq 0 \therefore B = 0$$

This means that the magnetic field outside the solenoid is zero.

Field inside the solenoid: Consider a closed path pqrs The line integral of magnetic field along path pqrs is

$$\oint_{pqrs} \vec{B}.\vec{dl} = \int_{pq} \vec{B}.\vec{dl} + \int_{qr} \vec{B}.\vec{dl} + \int_{rs} \vec{B}.\vec{dl} + \int_{sp} \vec{B}.\vec{dl} \qquad ...(i)$$

For path pq, $\vec{B}$ and $\vec{dl}$ are along the same direction,

$$\therefore \quad \int_{pq} \vec{B}.\vec{dl} = \int B \ dl = Bl \qquad (pq = l \text{ say})$$

For paths qr and sp, $\vec{B}$ and $d\vec{l}$ are mutually perpendicular.

$$\therefore \quad \int_{qr} \vec{B}.\vec{dl} = \int_{sp} \vec{B}.\ d\vec{l} = \int B \ dl \ \cos 90^o = 0$$

For path rs, $B = 0$ (since field is zero outside a solenoid)

$$\therefore \quad \int_{rs} \vec{B}.\vec{dl} = 0$$

In view of these, equation (i) gives

In view of these, equation (i) gives

$$\therefore \quad \oint_{pqrs} \vec{B}.\vec{dl} = \int_{pq} \vec{B}.\vec{dl} = Bl \qquad ...(ii)$$

By Ampere's law $\oint \vec{B}.\vec{dl} = \mu_0 \times$ net current enclosed by path

$\therefore Bl = \mu_0 \ (nl\ I) \quad \therefore B = \mu_0 \ nI$

This is the well known result.

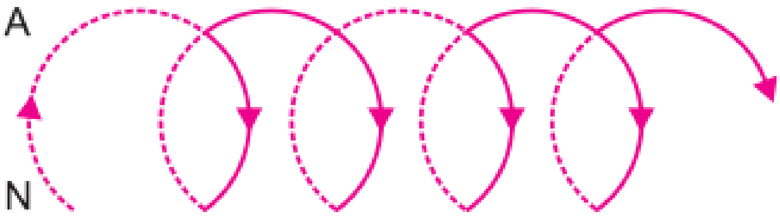
Q. 6. Using Ampere's circuital law, derive an expression for the magnetic field along the axis of a toroidal solenoid. [CBSE (AI) 2013]

OR

(a) State Ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius 'r', having 'n' turns per unit length and carrying a steady current I.

(b) An observer to the left of a solenoid of N turns each of cross section area 'A' observes that a steady current I in it flows in the clockwise direction. Depict the

magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment m = NIA. [CBSE Delhi 2015]



Ans. Magnetic field due to a toroidal solenoid: A long solenoid shaped in the form of closed ring is called a toroidal solenoid (or endless solenoid).

Let n be the number of turns per unit length of toroid and I the current flowing through it. The current causes the magnetic field inside the turns of the solenoid. The magnetic lines of force inside the toroid are in the form of concentric circles. By symmetry the magnetic field has the same magnitude at each point of circle and is along the tangent at every point on the circle.

(i) For points inside the core of toroid

Consider a circle of radius r in the region enclosed by turns of toroid. Now we apply Ampere's circuital law to this circular path, i.e.,

$$\oint \vec{B}.d\vec{l} = \mu_0 I \qquad \text{...(i)}$$

$$\oint \vec{B}.d\vec{l} = \oint \text{Bdl} \ \cos 0 = B.2\pi r$$

Length of toroid $= 2\pi r$

N = Number of turns in toroid = $n(2\pi r)$ and current in one-turn=I

$\therefore$ Current enclosed by circular path $= (n\ 2\pi r).\ I$

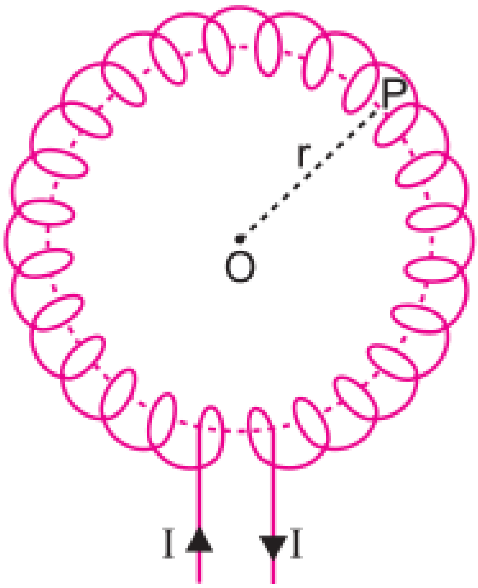
$\therefore$ Equation (i) gives

$$B\ 2\pi r = \mu_0\ (n\ 2\pi r I) \quad \Rightarrow \quad B = \mu_0\ nI$$

(ii) For points in the open space **inside the toroid:** No current flows through the Amperian loop, so I = 0

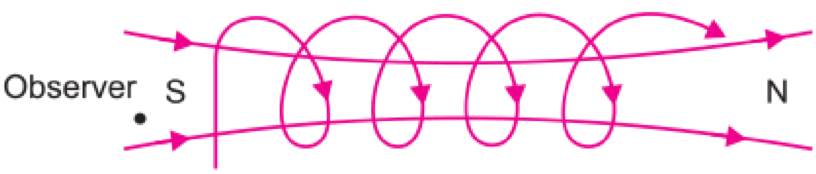
$$\oint \vec{B}.\ \vec{dl} = \mu_0 I = 0 \quad \Rightarrow \quad |B|_{\text{inside}} = 0$$

(iii) For points in the open space **exterior to the toroid :** The net current entering the plane of the toroid is exactly cancelled by the net current leaving the plane of the toroid.

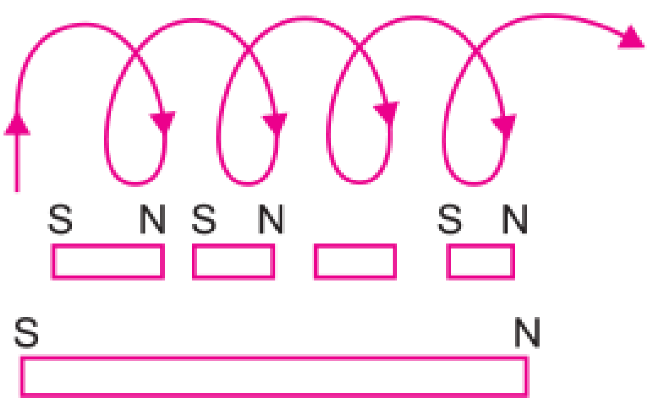


$$\oint \vec{B} \cdot \vec{dl} = 0 \quad \Rightarrow \quad |B|_{exterior} = 0$$

For observer, current is flowing in clockwise direction hence we will see magnetic field lines going towards South Pole.



The solenoid can be regarded as a combination of circular loops placed side by side, each behaving like a magnet of magnetic moment IA, where I is the current and A area of the loop.



These magnets neutralise each other except at the ends where south and north poles appear.

Magnetic moment of bar magnet = NIA

Q. 7. (i) Draw a neat labeled diagram of a cyclotron.

(ii) Show that time period of ions in cyclotron is independent of both the speed of ion and radius of circular path. What is the significance of this property?

(ii) An electron after being accelerated through a potential difference of 100 V enters a uniform magnetic field of 0.004 T perpendicular to its direction of motion. Calculate the radius of the path described by the electron. [CBSE East 2016]

OR

(a) Explain with the help of a labelled diagram construction, principle and working of a cyclotron stating clearly the functions of electric and magnetic fields on a charged particle. Derive an expression for time period of revolution and cyclotron frequency. Show that it is independent of the speed of the charged particles and radius of the circular path. [CBSE (AI) 2009]

(b) What is resonance condition? How is it used to accelerated charged particles? [CBSE (AI) 2009]

(c) Also find the total KE attained by the charged particle.

(d) Is there an upper limit on the energy acquires by the article? Give reason.

OR

With the help of a labelled diagram, state the underlying principle of a cyclotron. Explain clearly how it works to accelerate the charged particles.

Show that cyclotron frequency is independent of energy of the particle. Is there an upper limit on the energy acquired by the particle? Give reason.
[CBSE Delhi 2011, 2014]

OR

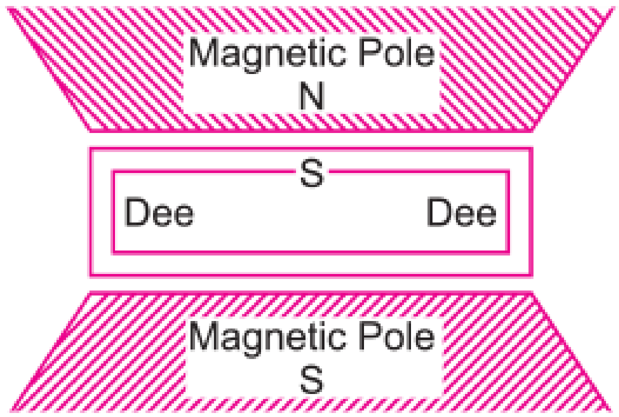
(a) Draw a schematic sketch of a cyclotron. Explain clearly the role of crossed electric and magnetic field in accelerating the charge. Hence derive the expression for the kinetic energy acquired by the particles.

(b) An α–particle and a proton are released from the centre of the cyclotron and made to accelerate.

(i) Can both be accelerated at the same cyclotron frequency? Give reason to justify your answer.

(ii) When they are accelerated in turn, which of the two will have higher velocity at the exit slit of the dees? [CBSE (AI) 2013]

Ans. Cyclotron: The cyclotron, devised by Lawrence and Livingston, is a device for accelerating charged particles to high speed by the repeated application of accelerating potentials.



Construction: The cyclotron consists of two flat semi - circular metal boxes called 'dees' and are arranged with a small gap between them. A source of ions is located near the mid-point of the gap between the dees (fig.). The dees are connected to the terminals of a radio frequency oscillator, so that a high frequency alternating potential of several million cycles per second exists between the dees. Thus dees act as electrodes. The dees are enclosed in an insulated metal box containing gas at low pressure. The whole apparatus is placed between the poles of a strong electromagnet which provides a magnetic field perpendicular to the plane of the dees.

Working: The principle of action of the apparatus is shown in figure. The positive ions produced from a source S at the centre are accelerated by a dee which is at negative potential at that moment. Due to the presence of perpendicular magnetic field the ion will move in a circular path inside the dees. The magnetic field and the frequency of the applied voltages are so chosen that as the ion comes out of a dee, the dees change their polarity (positive becoming negative and vice-versa) and the ion is further accelerated and moves with higher velocity along a circular path of greater radius. The phenomenon is continued till the ion reaches at the periphery of the dees where an auxiliary negative electrode (deflecting plate) deflects the accelerated ion on the target to be bombarded

Role of electric field.

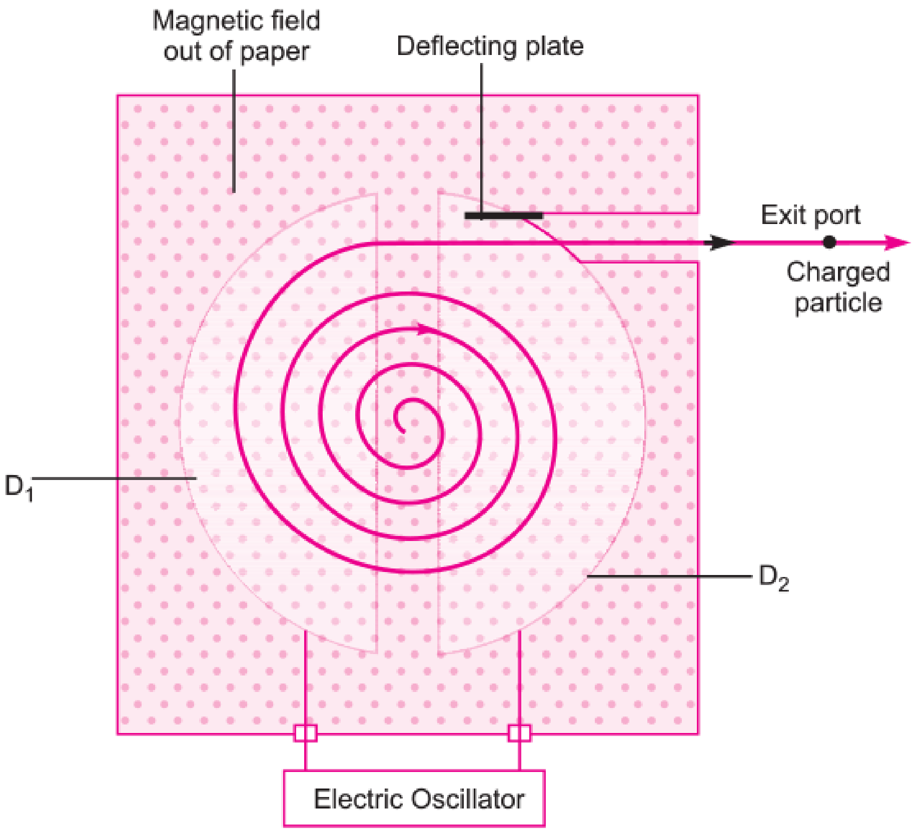
Electric field accelerates the charge particle passing through the gap.

Role of magnetic field

As the accelerated charge particle enters normally to the uniform magnetic field, it exerts a magnetic force in the form of centripetal force and charge particle moves on a semicircular path of increasing radii in each dee (D_1 or D_2) alternatively.

Expression for period of revolution and frequency:

Suppose the positive ion with charge q moves in a dee with a velocity v then,



$$qvB = \frac{mv^2}{r}$$

$$\text{or} \quad r = \frac{mv}{qB} \qquad ...(i)$$

Where m is the mass and r the radius of the path of ion in the dee and B is the strength of the magnetic field.

The angular velocity ω of the ion is given by,

$$\omega = \frac{v}{r} = \frac{qB}{m} \qquad (\text{from } eq.\ i) \qquad ...(ii)$$

The time taken by the ion in describing a semi-circle, i.e., in turning through an angle π is,

$$t = \frac{\pi}{\omega} = \frac{\pi m}{\mathrm{Bq}} \quad ...(iii)$$

Thus the time is independent of the speed of the ion i.e., although the speed of the ion goes on increasing with increase in the radius (from eq. i) when it moves from one dee to the other, yet it takes the same time in each dee.

From eq. (iii) it is clear that for a particular ion, m/q being known, B can be calculated for producing resonance with the high frequency alternating potential.

Significance: The applied voltage is adjusted so that the polarity of dees is reversed in the same time that it takes the ion to complete one half of the revolution.

Resonance condition: The condition of working of cyclotron is that the frequency of radio frequency alternating potential must be equal to the frequency of revolution of charged particles within the dees. This is called resonance condition.

Now for the cyclotron to work, the applied alternating potential should also have the same semi-periodic time (T/2) as that taken by the ion to cross either dee, i.e.,

$$\frac{T}{2} = t = \frac{\pi m}{\mathrm{qB}} \quad ...(iv)$$

$$\text{or} \quad T = \frac{2\pi m}{\mathrm{qB}} \qquad ...(v)$$

This is the expression for period of revolution.

Obviously, period of revolution is independent of speed of charged particle and radius of circular path.

∴ Frequency of revolution of particles

$$f = \frac{1}{T} = \frac{\mathrm{qB}}{2\pi m}$$

This frequency is called the **cyclotron frequency.** Clearly the cyclotron frequency is independent of speed of particle.

Expression for KE attained

If R be the radius of the path and v_{max} the velocity of the ion when it leaves the periphery, then in accordance with eq. (ii)

$$v_{\max} = \frac{\mathrm{qBR}}{m} \qquad \ldots(vi)$$

The kinetic energy of the ion when it leaves the apparatus is,

$$\mathrm{KE} = \frac{1}{2}\mathrm{mv}_{\max} = \frac{q^2B^2R^2}{2m} \qquad \ldots(vii)$$

When charged particle crosses the gap between dees it gains KE = qV

In one revolution, it crosses the gap twice, therefore if it completes n-revolutions before emerging the dees,

The kinetic energy gained = 2nqV ...(viii)

$$\text{Thus } \mathrm{KE} = \frac{q^2B^2R^2}{2m} = 2\,\mathrm{nqV}$$

No, from equation (i) $v = \frac{\mathrm{qBr}}{m}$

$$\Rightarrow \quad v = r\omega = \frac{\mathrm{qBr}}{m} \quad \Rightarrow \quad 2\pi v = \frac{\mathrm{qB}}{m} \quad \Rightarrow \quad v = \frac{\mathrm{qB}}{2\pi m}$$

Cyclotron frequency depends on $\left(\frac{q}{m}\right)$ ratio, since

$$\left(\frac{q}{m}\right)_\alpha < \left(\frac{q}{m}\right)_p$$

or $\quad f_\alpha < f_p$

Kinetic energy, $\mathrm{KE} = \mathrm{eV} = \frac{\mathrm{mv}^2}{2}$ for one revolution

$$v = \sqrt{\frac{2\ \mathrm{eV}}{m}}$$

$$v \propto \frac{e}{m} \qquad \text{for proton} = \frac{e}{m_p} \text{ and for } \alpha-\text{particle}, \frac{2e}{4m} = \frac{e}{2m}$$

Hence, proton acquires higher velocity as compared to a-particle.

$r = \frac{\mathrm{mv}}{\mathrm{qB}}$, here $v \propto r$ for fixed q, B, m.

Hence, the upper limit of energy depends upon the maximum radius of dees of cyclotron.

So, proton acquires higher velocity at the exit slit for fixed radius $r \leq R$, where R is the radius of the dee.

$$r = \frac{\mathrm{mv}}{\mathrm{qB}} = \frac{\sqrt{2\ \mathrm{mqV}}}{\mathrm{qB}}$$

$$r = \frac{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times100}}{1.6\times10^{-19}\times0.004} m$$

$$r = \frac{5.4\times10^{-24}}{6.4\times10^{-22}} m = 8.4\times10^{-3} m$$

Q. 8. Answer the following questions.

(i) Consider a beam of charged particles moving with varying speeds. Show how crossed electric and magnetic fields can be used to select charged particles of a particular velocity?

(ii) Name another device/machine which uses crossed electric and magnetic fields. What does this machine do and what are the functions of magnetic and electric fields in this machine? Where do these field exist in this machine? Write about their natures.
[CBSE South 2016]

Ans. (i) If we adjust the value of $\vec{E}$ and $\vec{B}$ such that magnitude of the two forces are equal, then total force on the charge is zero and the charge will move in the fields undeflected. This happen when

$$qE = Bqv \qquad \text{or} \qquad v = \frac{E}{B}$$

(ii) Name of the device: Cyclotron

It accelerates charged particles or ions.

Electric field accelerates the charged particles.

Magnetic field makes particles to move in circle.

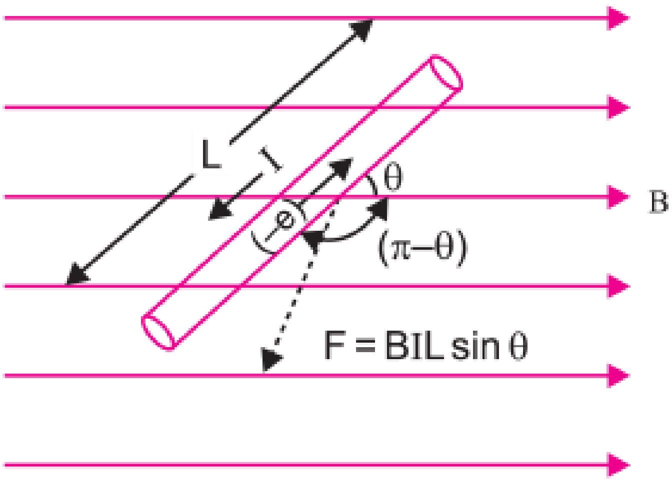
Electric field exists between the Dees.

Magnetic field exists both inside and outside the dees.

Magnetic field is uniform.

Electric field is alternating in nature.

Q. 9. Derive an expression for the force experienced by a current carrying straight conductor placed in a magnetic field. Under what condition is this force maximum?



Ans. Force on a current carrying conductor on the basis of force on a moving charge: Consider a metallic conductor of length L, cross-sectional area A placed in a uniform magnetic field B and its length makes an angle θ with the direction of magnetic field B. The current in the conductor is I.

According to free electron model of metals, the current in a metal is due to the motion of free electrons. When a conductor is placed in a magnetic field, the magnetic field exerts a force on every free-electron. The sum of forces acting on all electrons is the net force acting on the conductor. If vd is the drift velocity of free electrons, then

Current $I = neAv_d$ ...(i)

Where n is number of free electrons per unit volume.

Magnetic force on each electron $= ev_d B \sin \theta$ ...(ii)

Its direction is perpendicular to both $\underset{Vd}{\rightarrow}$ and $\underset{B}{\rightarrow}$

Volume of conductor V = AL

Therefore, the total number of free electrons in the conductor = nAL

Net magnetic force on each conductor

F = (force on one electron) × (number of electrons)

$= (ev_d B \sin \theta) . (nAL) = (neAv_d). BL \sin \theta$

Using equation (i) $F = IBL \sin \theta$ ...(iii)

∴ $F = ILB \sin \theta$

This is the general formula for the force acting on a current carrying conductor.

In vector form $\vec{F} = I \vec{L} \times \vec{B}$ ...(iv)

Force will be maximum when $\sin \theta = 1$ or $\theta = 90°$. That is when length of conductor is perpendicular to magnetic field.

Q. 10. Two long straight parallel conductors carry steady current I_1 and I_2 separated by a distance d. If the currents are flowing in the same direction, show how the magnetic field set up in one produces an attractive force on the other. Obtain the expression for this force. Hence define one ampere. [CBSE Delhi 2016]

OR

Derive an expression for the force per unit length between two long straight parallel current carrying conductors. Hence define SI unit of current (ampere). [CBSE (AI) 2009, 2010, 2012, Patna 2015]

Ans. Suppose two long thin straight conductors (or wires) PQ and RS are placed parallel to each other in vacuum (or air) carrying currents I_1 and I_2 respectively. It has been observed experimentally that when the currents in the wire are in the same direction, they experience an attractive force (fig. a) and when they carry currents in opposite directions, they experience a repulsive force (fig. b).

Let the conductors PQ and RS carry currents I_1 and I_2 in same direction and placed at separation r.

Consider a current–element 'ab' of length ΔL of wire RS. The magnetic field produced by current-carrying conductor PQ at the location of other wire RS

$$B_1 = \frac{\mu_0 I_1}{2\pi\ r} \qquad ...(i)$$

According to Maxwell's right hand rule or right hand palm rule number 1, the direction of B_1 will be perpendicular to the plane of paper and directed downward. Due to this magnetic field, each element of other wire experiences a force. The direction of current element is perpendicular to the magnetic field; therefore the magnetic force on element ab of length ΔL

$$\Delta F = B_1 I_2\ \ \Delta L\ \sin\ 90^o = \frac{\mu_0 I_1}{2\pi\ r} I_2\ \Delta L$$

∴ The total force on conductor of length L will be

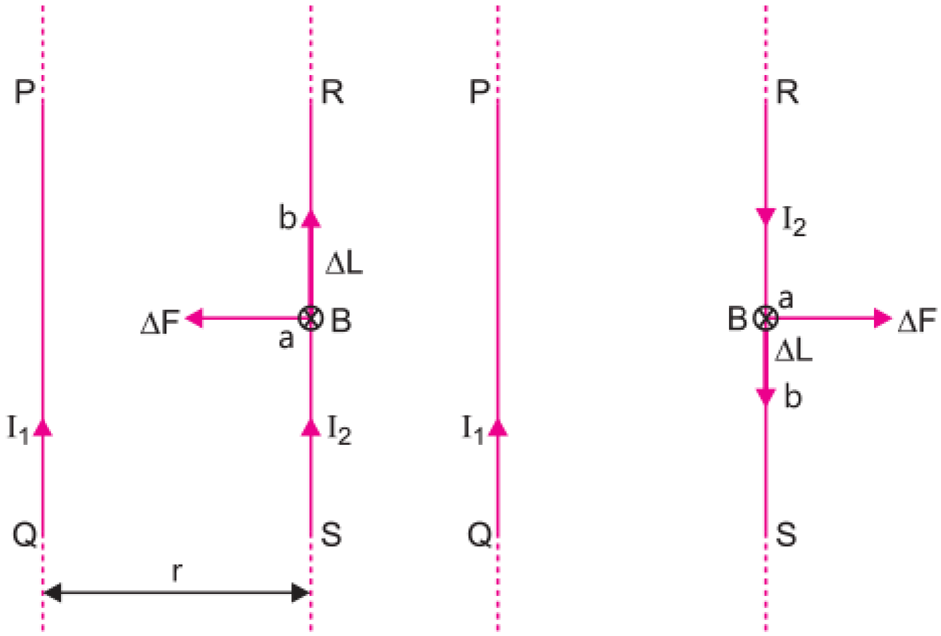
$$F = \frac{\mu_0 I_1 I_2}{2\pi\ r} \sum\ \Delta L = \frac{\mu_0 I_1\ \ I_2}{2\pi\ r} L$$

∴ Force acting per unit length of conductor

$$f = \frac{F}{L} = \frac{\mu_0 I_1\ I_2}{2\pi\ r} N/m \qquad ...(ii)$$

According to Fleming's left hand rule, the direction of magnetic force will be towards PQ i.e. the force will be attractive.

On the other hand if the currents I_1 and I_2 in wires are in opposite directions, the force will be repulsive. The magnitude of force in each case remains the same.



Definition of SI unit of Current (ampere): In SI system of fundamental unit of current 'ampere' is defined assuming the force between the two current carrying wires as standard.

The force between two parallel current carrying conductors of separation r is

$$f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r} N/m$$

If $I_1 = I_2 = 1A$, $r = 1$ m, then

$$f = \frac{\mu_0}{2\pi} = 2 \times 10^{-2} N/m$$

Thus 1 ampere is the current which when flowing in each of parallel conductors placed at separation 1 m in vacuum exert a force of 2×10^{-7} on 1 m length of either wire.

Q. 11. Derive an expression for torque acting on a rectangular current carrying loop kept in a uniform magnetic field B. Indicate the direction of torque acting on the loop.
[CBSE Delhi 2013; (F) 2009]

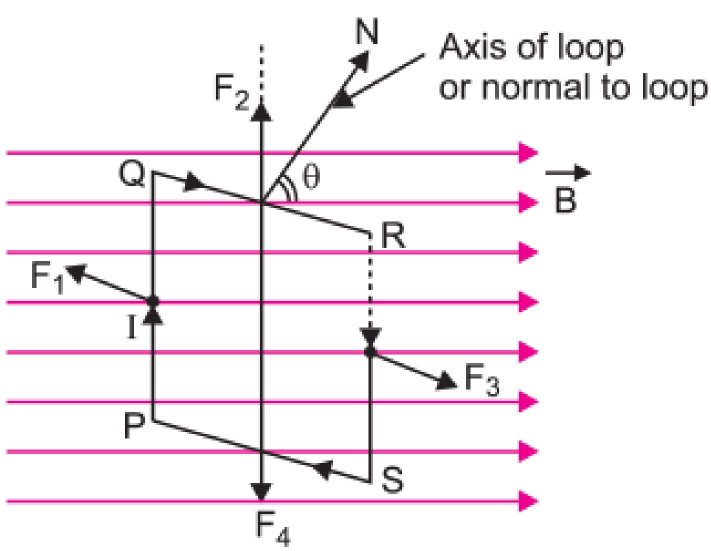
OR

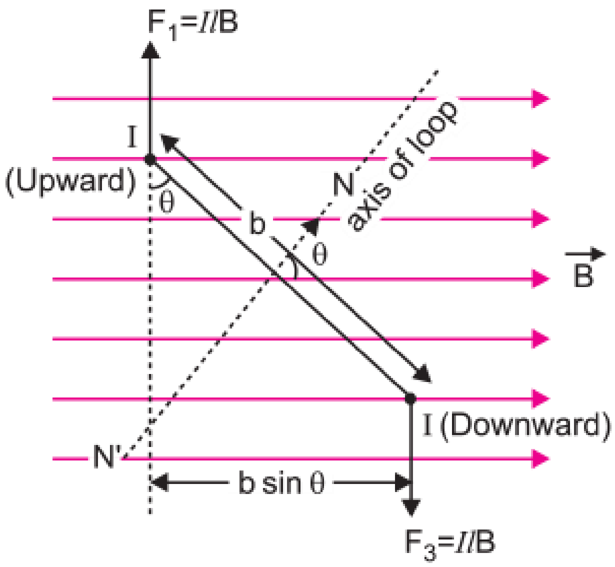
Deduce the expression for the torque $\vec{t}$ acting on a planar loop of area $\vec{A}$ and carrying current I placed in a uniform magnetic field $\vec{B}$

If the loop is free to rotate, what would be its orientation in stable equilibrium? [CBSE Ajmer 2015]

Ans. Torque on a current carrying loop: Consider a rectangular loop PQRS of length l, breadth b suspended in a uniform magnetic field $\vec{B}$ The length of loop = PQ = RS= l and breadth QR = SP = b. Let at any instant the normal to the plane of loop make an angle θ with the direction of magnetic field $\vec{B}$ and I be the current in the loop. We know that a force acts on a current carrying wire placed in a magnetic field. Therefore, each side of the loop will experience a force. The net force and torque acting on the loop will be determined by the forces acting on all sides of the loop. Suppose that the forces on sides PQ, QR, RS and SP are $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ and $\vec{F}_4$ respectively. The sides QR and SP make angle (90°– θ) with the direction of magnetic field. Therefore each of the forces $\vec{F}_2$ and $\vec{F}_4$ acting on these sides has same magnitude F′ = Blb sin (90°– θ) = Blb cos θ. According to Fleming's left hand rule the forces are equal and opposite but their line of action is same. Therefore these forces cancel each other i.e. the resultant of $\vec{F}_2$ and $\vec{F}_4$ is zero.

The sides PQ and RS of current loop are perpendicular to the magnetic field, therefore the magnitude of each of forces $\vec{F}_1$ and $\vec{F}_3$ acting on sides PQ and RS are equal and opposite, but their lines of action are different; therefore the resultant force of $\vec{F}_1$ and $\vec{F}_3$ is zero, but they form a couple called the **deflecting couple.** When the normal to plane of loop makes an angle with the direction of magnetic field the perpendicular distance between F_1 and F_3 is b sin θ.





∴ Moment of couple or Torque,

τ = (Magnitude of one force F) × perpendicular distance = (BIl). (b sin θ)=I (lb) B sin θ

But   lb = area of loop =A (say)

∴. Torque, τ = IAB sin θ

If the loop contains N-turns, then τ = NI AB sin θ

In vector form $\vec{\tau} = \text{NI}\,\vec{A} \times \vec{B}$

The magnetic dipole moment of rectangular current loop = M = NIA

$$\therefore \vec{\tau} = \vec{M} \times \vec{B}$$

Direction of torque is perpendicular to direction of area of loop as well as the direction of magnetic field i.e., along $I\vec{A} \times \vec{B}$.

The current loop would be in stable equilibrium, if magnetic dipole moment is in the direction of the magnetic field ($\underset{B}{\rightarrow}$).

Q. 12. Answer the following questions.

(i) What is the relationship between the current and the magnetic moment of a current carrying circular loop?

(ii) Deduce an expression for magnetic dipole moment of an electron revolving around a nucleus in a circular orbit. Indicate the direction of magnetic dipole moment? Use the expression to derive the relation between the magnetic moment of an electron moving in a circle and its related angular momentum?   [CBSE (AI) 2010; (F) 2009]

(iii) A muon is a particle that has the same charge as an electron but is 200 times heavier than it. If we had an atom in which the muon revolves around a proton instead of an electron, what would be the magnetic moment of the muon in the ground state of such an atom?

Ans. (i) Relation between current and magnetic moment:

Magnetic moment, for a current carrying coil is M = IA

For circular coil of radius r, A = πr^2

$$M = I.\ \pi r^2$$

(ii) Magnetic moment of an electron moving in a circle:

Consider an electron revolving around a nucleus (N) in circular path of radius r with speed v. The revolving electron is equivalent to electric current

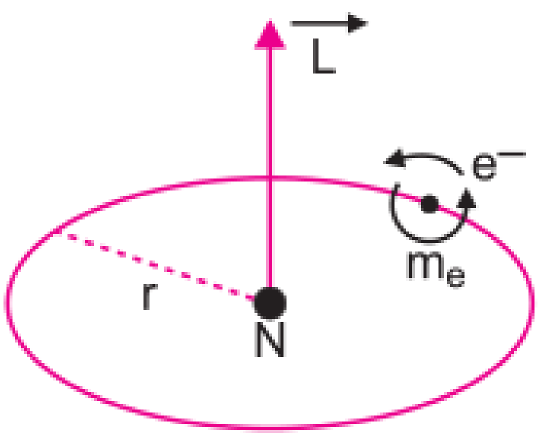
$$I = \frac{e}{T}$$

where T is period of revolution $= \frac{2\pi r}{v}$

$$I = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r} \qquad ...(i)$$

Area of current loop (electron orbit), $A = \pi r^2$

Magnetic moment due to orbital motion,



$$M_l = \text{IA} = \frac{ev}{2\pi r} = (2\pi r^2) = \frac{evr}{2} ...(ii)$$

This equation gives the magnetic dipole moment of a revolving electron. The direction of magnetic moment is along the axis.

Relation between magnetic moment and angular momentum

Orbital angular momentum of electron

$L = m_e vr$ ...(iii)

Where m_e is mass of electron,

Dividing (ii) by (iii), we get

$$\frac{M_l}{L} = \frac{\text{ev } r/2}{m_e v r} = \frac{e}{2m_e}$$

$$\text{Magnetic moment } \overrightarrow{M_l} = -\frac{e}{2m_e}\vec{L} \qquad \ldots(iv)$$

This is expression of magnetic moment of revolving electron in terms of angular momentum of electron.

$$\text{In vector form } \overrightarrow{M_l} = -\frac{e}{2m_e}\vec{L} \qquad \ldots(v)$$

Q. 13. Draw the labelled diagram of a moving coil galvanometer. Prove that in a radial magnetic field, the deflection of the coil is directly proportional to the current flowing in the coil. [CBSE (F) 2012]

(a) Draw a labelled diagram of a moving coil galvanometer. Describe briefly its principle and working.

(b) Answer the following:

(i) Why is it necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer?

(ii) Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity. Explain, giving reason. [CBSE (AI) 2014]

OR

Explain, using a labelled diagram, the principle and working of a moving coil galvanometer. What is the function of (i) uniform radial magnetic field, (ii) soft iron core?

Define the terms (i) current sensitivity and (ii) voltage sensitivity of a galvanometer. Why does increasing the current sensitivity not necessarily increase voltage sensitivity? [CBSE Allahabad 2015]

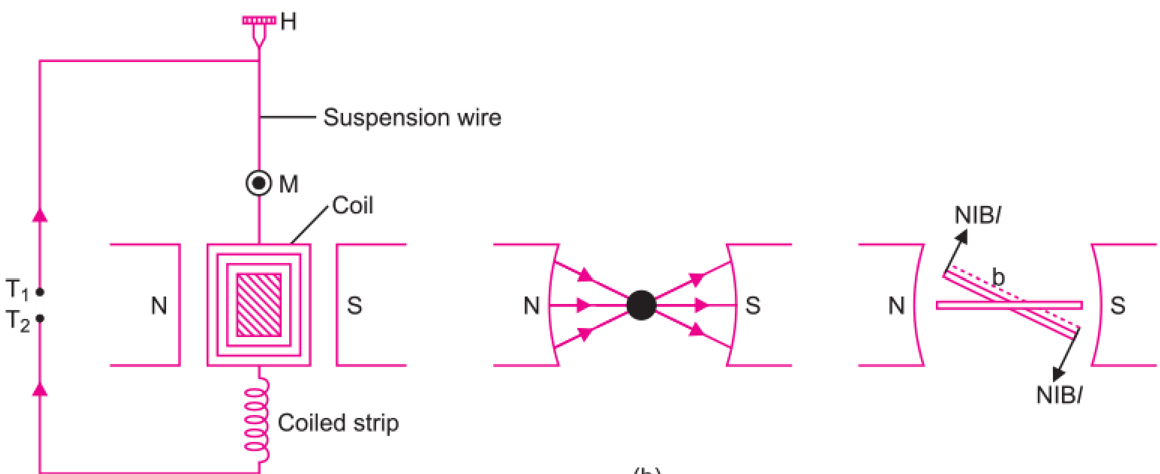
Ans. Moving coil galvanometer: A galvanometer is used to detect current in a circuit.

Construction: It consists of a rectangular coil wound on a non-conducting metallic frame and is suspended by phosphor bronze strip between the pole-pieces (N and S) of a strong permanent magnet.

A soft iron core in cylindrical form is placed between the coil.

One end of coil is attached to suspension wire which also serves as one terminal (T1) of galvanometer. The other end of coil is connected to a loosely coiled strip, which serves as the other terminal (T2). The other end of the suspension is attached to a torsion head which can be rotated to set the coil in zero position. A mirror (M) is fixed on the phosphor bronze strip by means of which the deflection of the coil is measured by the lamp and scale arrangement. The levelling screws are also provided at the base of the instrument.

The pole pieces of the permanent magnet are cylindrical so that the magnetic field is radial at any position of the coil.



(a) (b) Magnetic lines of force of radial magnetic field (c)

Principle and working: When current (I) is passed in the coil, torque τ acts on the coil, given by

$$\tau = NIAB \sin \theta$$

Where θ is the angle between the normal to plane of coil and the magnetic field of strength B, N is the number of turns in a coil.

A current carrying coil, in the presence of a magnetic field, experiences a torque, which produces proportionate deflection.

i.e., Deflection, $\theta \propto \tau$ (Torque)

When the magnetic field is radial, as in the case of cylindrical pole pieces and soft iron core, then in every position of coil the plane of the coil, is parallel to the magnetic field lines, so that θ =90-° and sin 90°=1. The coil experiences a uniform coupler.

Deflecting torque, τ = NIAB

If C is the torsional rigidity of the wire and is the twist of suspension strip, then restoring torque =C θ

For equilibrium, deflecting torque = restoring torque

$$i.e. \qquad NIAB = C\theta$$

$$\therefore \qquad \theta = \frac{\mathrm{NAB}}{C} I \qquad ...(i)$$

$$i.e. \qquad \theta \propto I$$

Deflection of coil is directly proportional to current flowing in the coil and hence we can construct a linear scale.

Importance (or function) of uniform radial magnetic field: Torque for current carrying coil in a magnetic field is τ = NIAB sin θ

In radial magnetic field sinθ = 1, so torque is τ = NIAB

This makes the deflection (θ) proportional to current. In other words, the radial magnetic field makes the **scale linear.**

The cylindrical, soft iron core makes the field radial and increases the strength of the magnetic field, i.e., the magnitude of the torque.

Sensitivity of galvanometer:

Current sensitivity: It is defined as the deflection of coil per unit current flowing in it.

$$\text{Sensitivity, } S_I = \left(\frac{\theta}{I}\right) = \frac{\mathrm{NAB}}{C} \quad ...(i)$$

Voltage sensitivity: It is defined as the deflection of coil per unit potential difference across its ends

$$i.e., \qquad S_V = \frac{\theta}{V} = \frac{\mathrm{NAB}}{R_g.C}, \qquad ...(ii)$$

Where R_g is resistance of galvanometer.

Clearly for greater sensitivity number of turns N, area A and magnetic field strength B should be large and torsional rigidity C of suspension should be small.

Dividing (ii) by (i)

$$\frac{S_V}{S_I} = \frac{1}{G} \quad \Rightarrow \quad S_V = \frac{1}{G} S_I$$

Clearly the voltage sensitivity depends on current sensitivity and the resistance of galvanometer. If we increase current sensitivity then it is not certain that voltage sensitivity will be increased. Thus, the increase of current sensitivity does not imply the increase of voltage sensitivity.

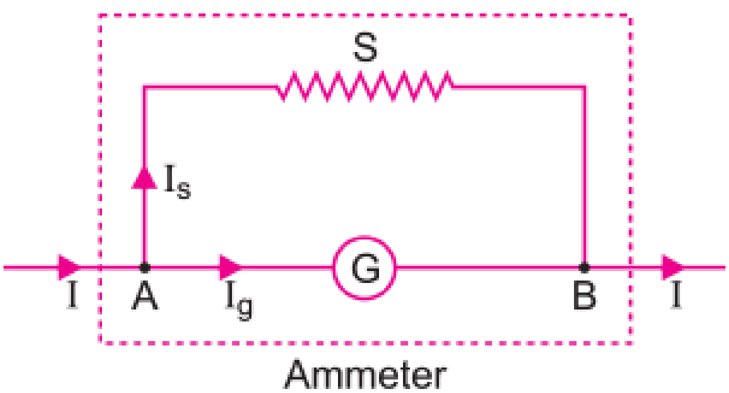
Q. 14. With the help of a circuit, show how a moving coil galvanometer can be converted into an ammeter of a given range. Write the necessary mathematical formula.

Ans. Conversion of galvanometer into ammeter

An ammeter is a low resistance galvanometer and is connected in series in a circuit to read current directly.

The resistance of an ammeter is to be made as low as possible so that it may read current without any appreciable error. Therefore to convert a galvanometer into ammeter a **shunt resistance.** (I.e. small resistance in parallel) is connected across the coil of galvanometer.

Let G be the resistance of galvanometer and I_g the current required for full scale deflection. Suppose this galvanometer is to converted into ammeter of range I ampere and the value of shunt required is S. If It I_s current in shunt, then from fig.



$I = I_g + I_S \quad \Rightarrow I_S = (I - I_g)$ ...(i)

Also potential difference across A and B

$$(V_{AB}) = I_S . S = I_g . G$$

Substituting value of I_S from (i), we get

Or $(I - I_g) S = I_g G$

Or $I_S - I_g S = I_g G$ or $I_S = I_g (S+G)$

or $I_g = \frac{S}{S+G} I$ ...(ii)

i.e. required shunt, $S = \frac{GI_g}{I - I_g}$ ...(iii)

This is the working equation of conversion of galvanometer into ammeter.

The resistance (R_A) of ammeter so formed is given by

$$\frac{1}{R_A} = \frac{1}{S} + \frac{1}{G} \quad \text{or} \quad \frac{1}{R_A} = \frac{S+G}{SG} \quad \Rightarrow \quad R_A = \frac{SG}{S+G}$$

If k is figure of merit of the galvanometer and n is the number of scale divisions, then I_g = nk. Out of the total main current I amperes, only a small permissible value Ig flows through the galvanometer and the rest $I_S = (I - I_g)$ passes through the shunt.

Remark: An ideal ammeter has zero resistance.

Q. 15. A galvanometer of resistance G is converted into a voltmeter to measure upto V volts by connecting a resistance R_1 in series with the coil. If a resistance R_2 is connected in series with it, then it can measure upto V/2 volts. Find the resistance, in terms of R_1 and R_2, required to be connected to convert it into a voltmeter that can read upto 2 V. Also find the resistance G of the galvanometer in terms of R_1 and R_2.
[CBSE Delhi 2015]

Ans. Let I_g be the current through galvanometer at full deflection

To measure V volts, $V = I_g (G + R1)$ ...(i)

$\frac{V}{2}$ volts, $\quad \frac{V}{2} = I_g(G + R_2)$ ...(*ii*)

2 V volts, $\quad 2V = I_g(G + R_3)$ ...(*iii*)

To measure for conversion of range dividing (*i*) by (*ii*),

$$2 = \frac{G+R_1}{G+R_2} \quad \Rightarrow \quad G = R_1 - 2R_2$$

Putting the value of G in (*i*), we have

$$I_g = \frac{V}{R_1 - 2R_2 + R_1} \quad \Rightarrow \quad I_g = \frac{V}{2R_1 - 2R_2}$$

Substituting the value of G and I_g in equation (*iii*), we have

$$2V = \frac{V}{2R_1 - 2R_2}(R_1 - 2R_2 + R_3)$$

$$4R_1 - 4R_2 = R_1 - 2R_2 + R_3$$

$$R_3 = 3R_1 - 2R_2$$