1. કિંમત મેળવો : $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

➡ અહીં $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

$= \tan^{-1}\left(\tan\left(\frac{6\pi - \pi}{6}\right)\right) + \cos^{-1}\left(\cos\left(\frac{12\pi + \pi}{6}\right)\right)$

$= \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{6}\right)\right) + \cos^{-1}\left(\cos\left(2\pi + \frac{\pi}{6}\right)\right)$

$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

($\because \tan(\pi - \theta) = -\tan\theta$ અને $\cos(2\pi + \theta) = \cos\theta$)

$= -\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$= -\frac{\pi}{6} + \frac{\pi}{6}$

$= 0$

આમ, $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = 0$ થાય.

2. કિંમત મેળવો : $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

➡ $\cos\left(\pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right)$

($\because \cos^{-1}(-x) = \pi - \cos^{-1}x$)

$= \cos\left(\pi - \cos^{-1}\left(\cos\frac{\pi}{6}\right) + \frac{\pi}{6}\right)$

$= \cos\left(\pi - \frac{\pi}{6} + \frac{\pi}{6}\right)$ ($\because \cos^{-1}(\cos x) = x,\ x \in [0, \pi]$)

$= \cos(\pi)$

$= -1$

3. સાબિત કરો : $\cot\left(\frac{\pi}{4} - 2\cot^{-1}3\right) = 7$.

➡ ડા.બા. $= \cot\left(\frac{\pi}{4} - 2\cot^{-1}3\right)$

અહીં, $2\cot^{-1}(3)$

$= 2\tan^{-1}\left(\frac{1}{3}\right) \qquad \left(\because \cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right), x > 0\right)$

$= \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{3}\right)$

$$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{3}}{1-\frac{1}{9}}\right)$$

$$= \tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right)$$

$$= \cot^{-1}\left(\frac{4}{3}\right) \qquad \left(\because \tan^{-1}(x) = \cot^{-1}\left(\frac{1}{x}\right), x > 0\right)$$

$\therefore$ ડા.બા. નીચે મુજબ થાય.

$$\cot\left(\frac{\pi}{4} - \cot^{-1}\frac{4}{3}\right)$$

$$= \frac{\cot\left(\frac{\pi}{4}\right)\cot\left(\cot^{-1}\frac{4}{3}\right)+1}{\cot\left(\cot^{-1}\frac{4}{3}\right) - \cot\frac{\pi}{4}}$$

($\because$ $\cot(\alpha - \beta)$ ના સૂત્ર મુજબ)

$$= \frac{(1)\frac{4}{3}+1}{\frac{4}{3}-1} \qquad \left(\because \cot\frac{\pi}{4} = 1 \text{ અને } \cot\left(\cot^{-1}\frac{4}{3}\right) = \frac{4}{3}\right)$$

$$= \frac{4+3}{4-3}$$

$= 7 \qquad =$ જ.બા.

4. **કિંમત મેળવો :** $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right)$

➡ $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right)$

$$= -\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}\left(-\sin\frac{\pi}{2}\right)$$

($\because$ $\tan^{-1}(-x) = -\tan^{-1}x$, $\sin(-\theta) = -\sin\theta$ છે.)

$$= -\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \frac{\pi}{3} - \tan^{-1}(1)$$

$$= -\frac{\pi}{6} + \frac{\pi}{3} - \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{-2\pi + 4\pi - 3\pi}{12}$$

$$= \frac{-5\pi + 4\pi}{12} = -\frac{\pi}{12}$$

5. **કિંમત મેળવો :** $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

➡ $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

$$= \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{3}\right)\right)$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{3}\right) \qquad (\because \tan(\pi - \theta) = -\tan\theta)$$

$= -\tan^{-1}\left(\tan\frac{\pi}{3}\right)$

$= -\left(\frac{\pi}{3}\right)$

6. **સાબિત કરો :** $2\tan^{-1}(-3) = -\frac{\pi}{2} + \tan^{-1}\left(\frac{-4}{3}\right)$

➡ ડા.બા. $= 2\tan^{-1}(-3)$

$= -2\tan^{-1}(3)$

$= -\cos^{-1}\left(\frac{1-3^2}{1+3^2}\right)$

$\left(\because 2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), x \geq 0\right)$

$= -\cos^{-1}\left(\frac{-8}{10}\right)$

$= -\cos^{-1}\left(\frac{-4}{5}\right)$

$= -\left[\pi - \cos^{-1}\frac{4}{5}\right] (\because \cos^{-1}(-x) = \pi - \cos^{-1}x) \quad = -\pi + \cos^{-1}\left(\frac{4}{5}\right)$

$= -\pi + \tan^{-1}\left(\frac{\sqrt{1-\frac{16}{25}}}{\frac{4}{5}}\right)$ (આંતરસંબંધી સૂત્ર મુજબ)

$= -\pi + \tan^{-1}\left(\frac{3}{4}\right)$

$= -\pi + \left[\frac{\pi}{2} - \cot^{-1}\left(\frac{3}{4}\right)\right]$

($\because$ કોટી સંખ્યાના સૂત્ર મુજબ)

$= -\pi + \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$

$= -\frac{\pi}{2} + \tan^{-1}\left(-\frac{4}{3}\right)$

= જ.બા.

7. **જો $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\operatorname{cosec}\theta)$ હોય તો બતાવો કે** $\theta = \frac{\pi}{4}$.

<u>ગણતરી માટેનું સૂચન</u> : $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ સૂત્ર પરથી ગણતરી કરો.

➡ $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\operatorname{cosec}\theta)$

$\therefore \tan^{-1}\left(\frac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}(2\operatorname{cosec}\theta)$

$\therefore \frac{2\cos\theta}{\sin^2\theta} = 2\operatorname{cosec}\theta$

$\therefore \frac{\cos\theta}{\sin^2\theta} = \frac{1}{\sin\theta}$

$\therefore \frac{\cos\theta}{\sin\theta} = 1$

$\therefore \cot\theta = \cot\frac{\pi}{4}$

$\therefore \theta = \frac{\pi}{4}$

8. નીચેના સમીકરણનો ઉકેલ મેળવો : $\tan^{-1}\left(\sqrt{x(x+1)}\right) + \sin^{-1}\left(\sqrt{x^2 + x + 1}\right) = \frac{\pi}{2}$

➡ $\tan^{-1}\left(\sqrt{x(x+1)}\right) + \sin^{-1}\left(\sqrt{x^2 + x + 1}\right) = \frac{\pi}{2}$

હવે ધારો કે, $\sin^{-1}\left(\sqrt{x^2 + x + 1}\right)$

$= \tan^{-1}\left(\frac{\sqrt{x^2 + x + 1}}{1 - \left(\sqrt{x^2 + x + 1}\right)^2}\right)$

($\because$ આંતર સંબંધી સૂત્ર મુજબ)

$= \tan^{-1}\left(\frac{\sqrt{x^2 + x + 1}}{1 - \left(x^2 + x + 1\right)}\right)$

$= \tan^{-1}\left(\frac{\sqrt{x^2 + x + 1}}{\sqrt{-x^2 - x}}\right)$

$\therefore \tan^{-1}\left(\sqrt{x(x+1)}\right) + \tan^{-1}\left(\frac{\sqrt{x^2 + x + 1}}{\sqrt{-x^2 - x}}\right) = \frac{\pi}{2}$

$= \tan^{-1}\left(\sqrt{x(x+1)}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\sqrt{x^2 + x + 1}}{\sqrt{-(x^2 - x)}}\right)$

$\therefore \tan^{-1}\left(\sqrt{x(x+1)}\right) = \cot^{-1}\left(\frac{\sqrt{x^2 + x + 1}}{\sqrt{-(x^2 - x)}}\right)$

$\therefore \tan^{-1}\left(\sqrt{x(x+1)}\right) = \tan^{-1}\left(\frac{\sqrt{-(x^2 + x)}}{\sqrt{x^2 + x + 1}}\right)$

$\therefore \sqrt{x^2 + x} = \frac{\sqrt{-(x^2 + x)}}{\sqrt{x^2 + x + 1}}$

અહીં, સ્પષ્ટ છે કે, $x = 0$ તથા $x = -1$ માટે આ પરિણામની બંને બાજુ સમાન રહે.

$\therefore$ આપેલ સમીકરણનો ઉકેલ $x = 0$ તથા $x = -1$ છે.

9. કિંમત શોધો. $\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos\left(\tan^{-1}2\sqrt{2}\right)$

➡ અહીં, $\tan^{-1}\left(2\sqrt{2}\right)$

$= \cos^{-1}\left(\frac{1}{1 + (2\sqrt{2})^2}\right)$ ($\because$ આંતર સંબંધી સૂત્ર)

$= \cos^{-1}\left(\frac{1}{\sqrt{1 + 8}}\right)$

$= \cos^{-1}\left(\frac{1}{3}\right)$

હવે ધારો કે, $\tan^{-1}\left(\frac{1}{3}\right) = \theta$

$\therefore \frac{1}{3} = \tan\theta$

$\therefore \sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right) + \cos\left(\tan^{-1}(2\sqrt{2})\right)$

$= \sin(2\theta) + \cos\left(\cos^{-1}\frac{1}{3}\right)$

$= \frac{2\tan\theta}{1+\tan^2\theta} + \frac{1}{3}$

$= \frac{2\left(\frac{1}{3}\right)}{1+\frac{1}{9}} + \frac{1}{3}$

$= \frac{\frac{2}{3}}{\frac{10}{9}} + \frac{1}{3}$

$= \frac{3}{5} + \frac{1}{3}$

$= \frac{9+5}{15}$

$= \frac{14}{15}$

10. સમીકરણ ઉકેલો : $\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

➡ $\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

$\therefore \cos\left(\cos^{-1}\frac{1}{\sqrt{x^2+1}}\right) = \sin\left(\tan^{-1}\frac{4}{3}\right)$

(આંતર સંબંધી સૂત્ર મુજબ)

$\therefore \frac{1}{\sqrt{x^2+1}} = \sin\left(\sin^{-1}\frac{\frac{4}{3}}{\sqrt{1+\frac{16}{9}}}\right)$

$= \sin\left(\sin^{-1}\frac{4}{5}\right)$

$\therefore \frac{1}{\sqrt{x^2+1}} = \frac{4}{5}$

$\therefore 5 = 4\sqrt{x^2+1}$

$\therefore 25 = 16(x^2+1)$

$\therefore 25 = 16x^2 + 16$

$\therefore 16x^2 = 9$

$\therefore x^2 = \frac{9}{16}$

$\therefore x = \frac{3}{4}$

($\because$ આંતર સંબંધી સૂત્રમાં $0 < x < 1$ હોય

$\therefore x$ નું મૂલ્ય ઋણ ન હોય.)

$\therefore x = \frac{3}{4}$ આપેલ સમીકરણનો ઉકેલ છે.

11. **સાબિત કરો :** $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$

➥ ડા.બા. $= \cos\left(2\tan^{-1}\frac{1}{7}\right)$

ધારો કે $\tan^{-1}\left(\frac{1}{7}\right) = \alpha$

$\therefore \frac{1}{7} = \tan\alpha$ ....(i)

$\therefore$ ડા.બા. $= \cos\left(2\tan^{-1}\frac{1}{7}\right)$

$= \cos\ (2\alpha)$

$= \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$

$= \frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2}$ ($\because$ પરિણામ (i) પરથી)

$= \frac{1-\frac{1}{49}}{1+\frac{1}{49}}$

$= \frac{48}{50}$

$= \frac{24}{25}$

$\therefore \cos\left(2\tan^{-1}\frac{1}{7}\right) = \frac{24}{25}$ ....(A)

જ.બા. $= \sin\left(4\tan^{-1}\frac{1}{3}\right)$

ધારો કે, $\tan^{-1}\left(\frac{1}{3}\right) = \theta$

$\therefore \tan\theta = \frac{1}{3}$ ....(ii)

હવે જ.બા. $= \sin\left(4\tan^{-1}\frac{1}{3}\right)$

$= \sin\ (4\theta)$

$= 2\sin(2\theta)\ \cos(2\theta)$

➥ ડા.બા. $= \cos\left(2\tan^{-1}\frac{1}{7}\right)$

ધારો કે $\tan^{-1}\left(\frac{1}{7}\right) = \alpha$

$\therefore \frac{1}{7} = \tan\alpha$ ....(i)

$\therefore$ ડા.બા. $= \cos\left(2\tan^{-1}\frac{1}{7}\right)$

$= \cos\ (2\alpha)$

$= \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$

$$= \frac{1 - \left(\frac{1}{7}\right)^2}{1 + \left(\frac{1}{7}\right)^2} \qquad (\because \text{ પરિણામ (i) પરથી})$$

$$= \frac{1 - \frac{1}{49}}{1 + \frac{1}{49}}$$

$$= \frac{48}{50}$$

$$= \frac{24}{25}$$

$$\therefore \cos\left(2\tan^{-1}\frac{1}{7}\right) = \frac{24}{25} \qquad \text{...........(A)}$$

જ.બા. $= \sin\left(4\tan^{-1}\frac{1}{3}\right)$

ધારો કે, $\tan^{-1}\left(\frac{1}{3}\right) = \theta$

$$\therefore \tan\theta = \frac{1}{3} \qquad \text{...........(ii)}$$

હવે જ.બા. $= \sin\left(4\tan^{-1}\frac{1}{3}\right)$

$$= \sin\ (4\theta)$$

$$= 2\sin(2\theta)\ \cos(2\theta)$$