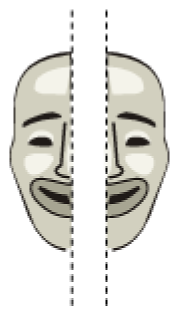
Symmetry and Recognition of Solids

Symmetrical Figures and Lines of Symmetry

Let us consider the following mask.

If we cut this mask exactly from middle, we obtain two halves of the mask as shown below.



We can observe that both left and right half faces are exact copies of each other.

Similarly, if we fold the following pictures from the middle, then the left half of the picture will exactly overlap over the right half of the picture.

India gate

Taj Mahal

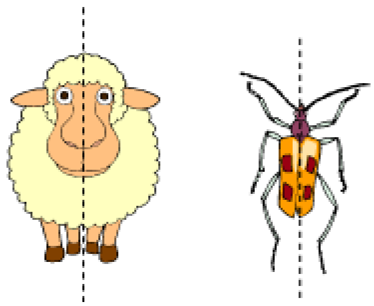
Gateway of India

Such figures are known as **symmetrical figures** or **symmetric.**

If we place a mirror on the line where we folded these pictures, then we will find that the image of one half of the figure is exactly the same as the other half.

Symmetry is something that we observe in many places in our daily lives without even noticing it. It is easily noticeable in various arts, buildings, and monuments. Nature uses symmetry to make things beautiful. For example, consider the pictures of the butterfly and the leaf drawn below.

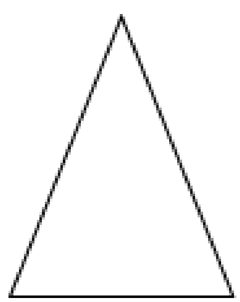
Let us also consider the following pictures.



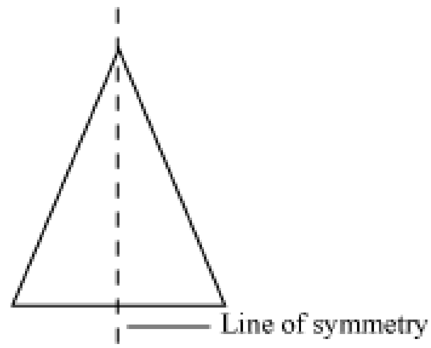
In each of these pictures, the dotted line divides the picture into two parts such that the left half of the picture is exactly same as the right half. This dotted line is known as the **line of symmetry** or **mirror line**. It can be defined as:

The line through which the figure can be folded to form two identical figures is called line of symmetry or axis of symmetry or mirror line.

Let us consider the following isosceles triangle.



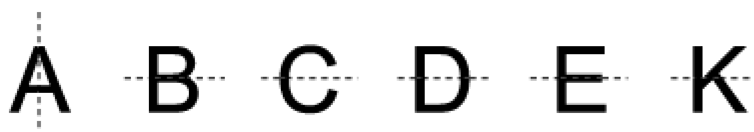
If we draw a dotted line at the middle of the triangle, then we get two parts such that the left part is exactly same as right part.



Here, the dotted line is the line of symmetry. We cannot draw more lines of symmetry for this triangle. Therefore, we can say that an isosceles triangle has only **one line of symmetry**.

Some letters of the English alphabet have only one line of symmetry.

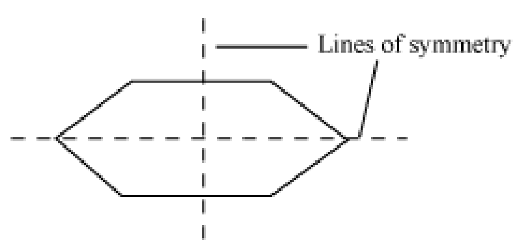
For example, A, B, C, D, E, K, etc.



Here, letter A has a vertical line of symmetry, while each of the letters B, C, D, E, and K has a horizontal line of symmetry.

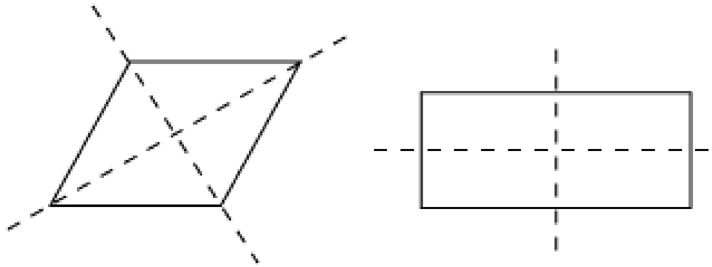
However, it is not necessary that a figure has only one line of symmetry. A figure can have more than one line of symmetry. Let us consider the following figure.

We can draw the lines of symmetry as shown below.



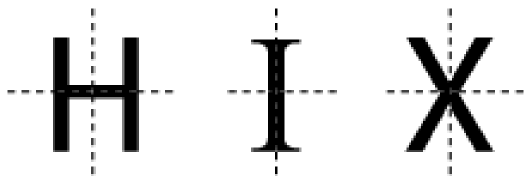
For this figure, we cannot draw more lines of symmetry except these two. Therefore, we can say that this figure has **two lines of symmetry.**

Some more figures with two lines of symmetry are shown below.



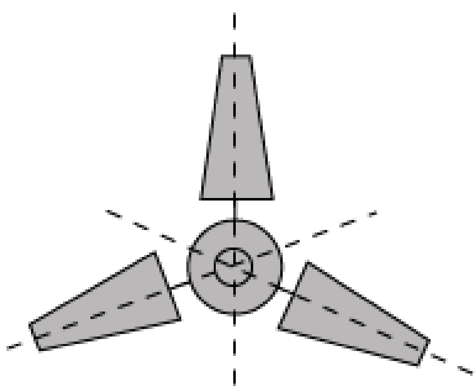
Some letters of the English alphabet contains two lines of symmetry.

For example, H, I, X, etc.

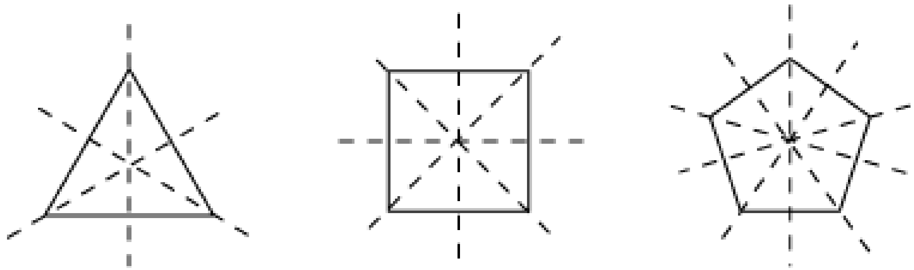


Note that each of the letters H, I, and X has both vertical and horizontal lines of symmetry.

Let us look at the fan drawn in the following figure. It has **three lines of symmetry**. When a figure has more than two lines of symmetry, we say that it has **multiple lines of symmetry**. Thus, we can also say that the fan drawn below has multiple lines of symmetry.



Some geometric figures with multiple lines of symmetry are shown below.

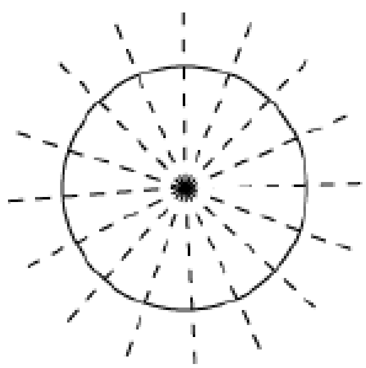


The figures drawn above represent

- an equilateral triangle with three lines of symmetry
- a square with four lines of symmetry
- a regular pentagon with five lines of symmetry

Using the concept of line of symmetry, can we tell how many lines of symmetry are there in a circle?

A circle has infinite number of lines of symmetry as shown in the figure below.

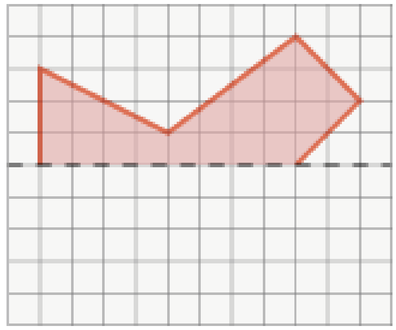


Now, if suppose we are given a part of a figure and its line(s) of symmetry, then can we draw the complete figure?

Yes, we can draw the complete figure by tracing the given part of the figure on the other side of the given line of symmetry.

Let us see this with the help of an example.

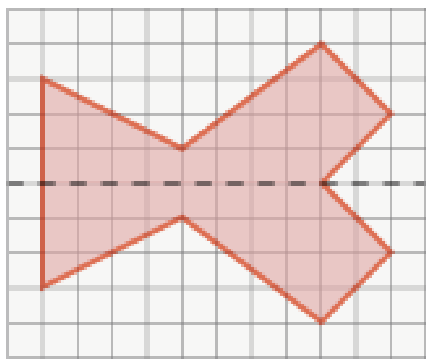
Consider the following figure:



Here, the dotted line is the line of symmetry of the figure.

The complete figure can be obtained by tracing the given figure below its line of symmetry.

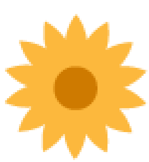
Thus, the complete figure will be represented as:

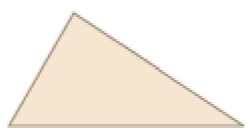


Now, we know how to identify symmetrical figures and their lines of symmetry. Let us discuss some examples based on these concepts.

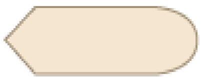
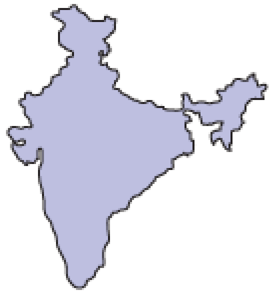
Example 1:

Identify the symmetrical figures out of the following figures. Also draw their lines of symmetry.

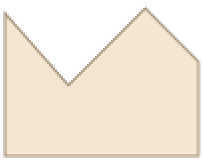
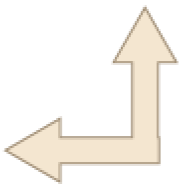




(i) **(ii)** **(iii)**

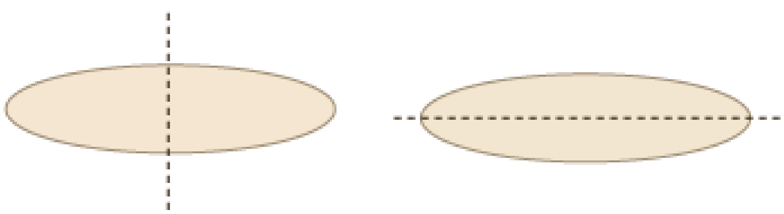
(iv) **(v)** **(vi)**

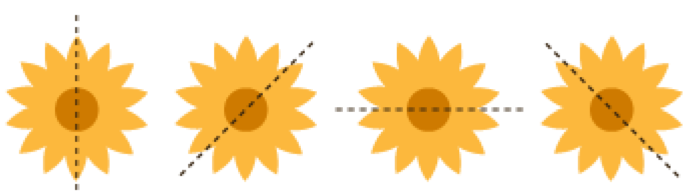
(vii) **(viii)**

Solution:

(i) The given figure is oval (egg-shaped). It has two lines of symmetry. That means the figure is symmetrical.

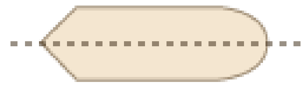


(ii) We can fold the given figure from the centre in any way as it has infinite lines of symmetry. Therefore, the figure is symmetrical. Some of its lines of symmetry are as follows.



(iii) We cannot fold the given triangle to form two identical halves. Therefore, this figure is not symmetrical and does not have any line of symmetry.

(iv) The given figure has a horizontal line of symmetry. Therefore, the figure is symmetrical.

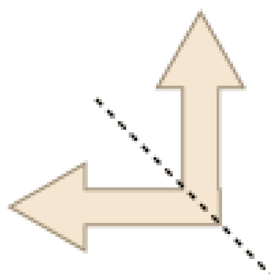


(v) The bottle shown in the given figure is symmetrical, as it has a vertical line of symmetry that divides it into two identical halves.

(vi) The given figure is not symmetrical, as we cannot divide it into two identical halves. Thus, it does not have any line of symmetry.

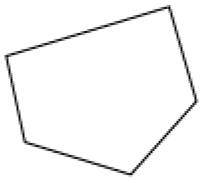
(vii) We cannot fold the figure in any way in order to divide it into two identical halves. Thus, the figure is not symmetrical and does not have any line of symmetry.

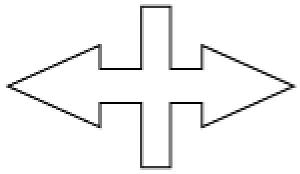
(viii) When we fold the given figure along the dotted line, we obtain two identical halves. Thus, the figure is symmetrical and has one line of symmetry.



Example 2:

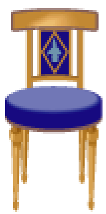
State whether the following figures have single, double, or multiple lines of symmetry. Also, draw their line or lines of symmetry.

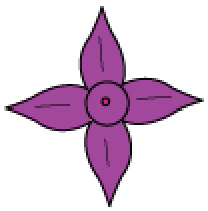


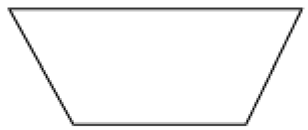


M V

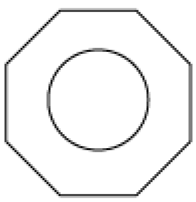
(i) (ii) (iii) (iv)

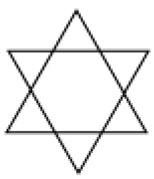






(v) (vi) (vii)

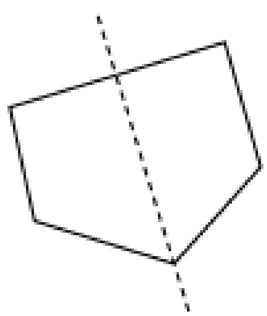




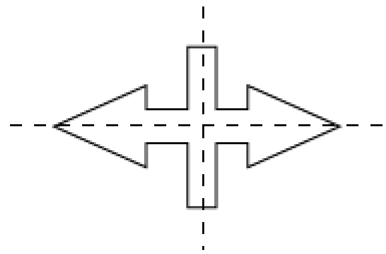
(viii) (ix)

Solution:

(i) The given figure has one line of symmetry.

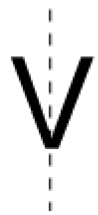


(ii) The given figure has two lines of symmetry.

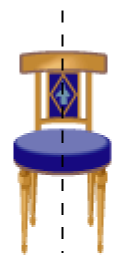


(iii) The given figure has one line of symmetry.

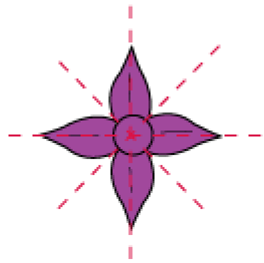
(iv) The given figure has one line of symmetry.



(v) The given figure has one line of symmetry.

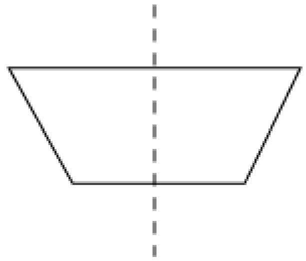


(vi) The given figure has four lines of symmetry, i.e., it has multiple lines of symmetry.

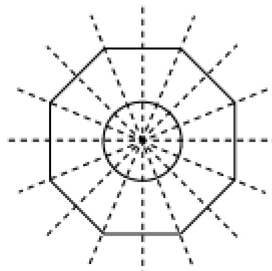


2.

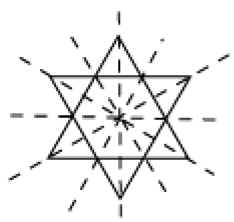
(vii) The given figure has one line of symmetry.



(viii) The given figure has eight lines of symmetry, i.e., it has multiple lines of symmetry.

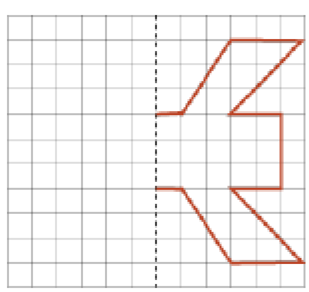


(ix) The given figure has six lines of symmetry, i.e., it has multiple lines of symmetry.

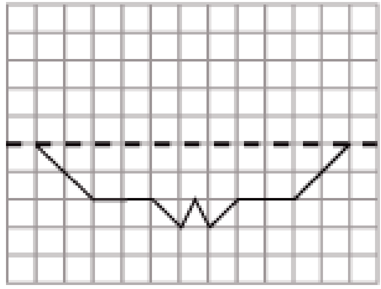


Example 3:
Complete the following figures in which the dotted line shows the line of symmetry.

1.



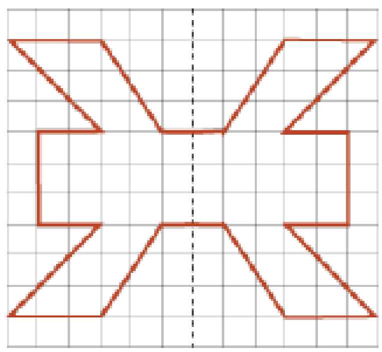
2.



Solution:

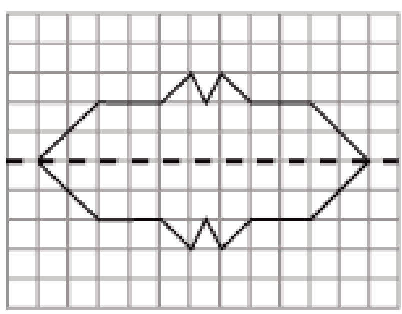
1. The complete figure can be obtained by tracing the given figure to the left of the line of symmetry.

   Thus, the complete figure is represented as:



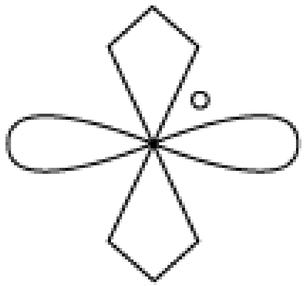
2. The complete figure can be obtained by tracing the given figure above the line of symmetry.

   Thus, the complete figure is represented as:



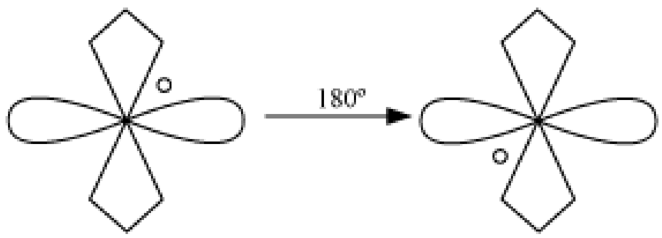
Point Symmetry

Let us look at the following figure.



If we rotate the figure through the point O at an angle of 180°, then what figure do we obtain?

If we rotate the figure through the point O at an angle of 180°, then we obtain the same original figure. This can be shown as:



Such type of symmetry of the given figure is called point symmetry and it can be defined as:

By rotating a figure at an angle of 180°, if it looks the same, then we say that the figure has point symmetry.

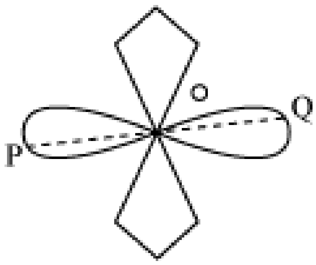
Note

(1) Point symmetry of a figure exists when it is built around a single point, which is called centre of the figure.

For example, in the given figure, O is its centre since the figure is built around O.

(2) For every point in the figure (having point symmetry), there is another point found directly opposite to it on the other side of the centre at the same distance.

For example, for the above figure, there are two points P and Q opposite to O, such that OP = OQ, which is shown below.



In order to understand this concept better, let us look at some examples.

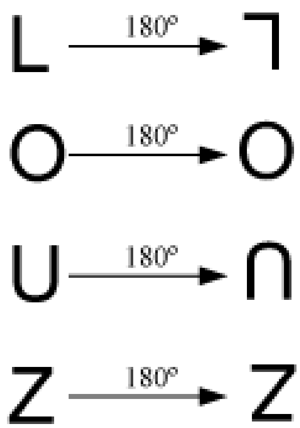
Example 1:

For which of the English alphabets among L, O, U, Z does point symmetry exist?

Also mention the centre of that alphabet using dot marks.

Solution:

By rotating the given English alphabets about 180°, we obtain the following figures.



Clearly, alphabets O and Z have point symmetry.

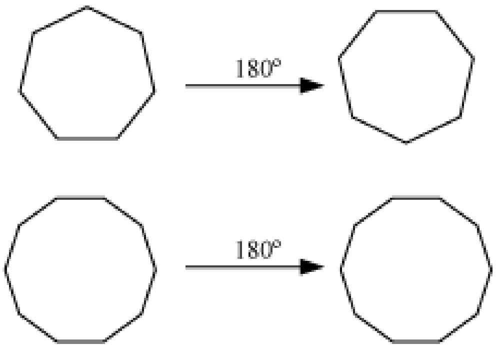
The centre of O and Z can be shown using dot marks as follows.

Example 2:

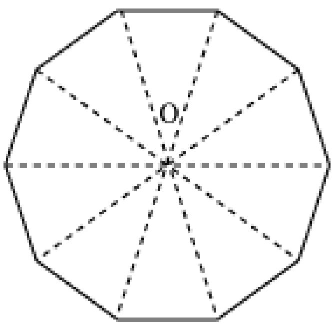
For which of the geometrical figures among a regular heptagon and a regular decagon does point symmetry exist? Mention the centre of that figure.

Solution:

When we rotate a regular heptagon and a regular decagon about 180°, we obtain the following figures.



We may observe that the regular decagon has point symmetry. Its centre is the intersection of the diagonals.



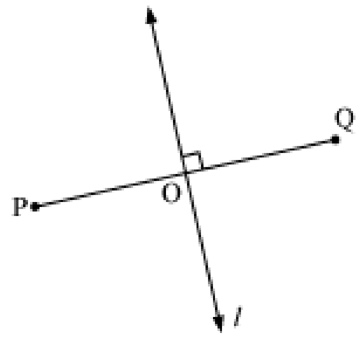
Construction of Line of Symmetry when Two Given Points Are Symmetric with Respect to the Line of Symmetry

Suppose we have a line segment PQ.

Can you say what will be the properties of the line of symmetry of this line PQ?

The line of symmetry of the line PQ will be the perpendicular bisector of PQ.

Note that the points P and Q will be symmetric to each other with respect to the line of symmetry i.e., OP = OQ



Thus, the line of symmetry of two given points is the perpendicular bisector of the line joining the two points.

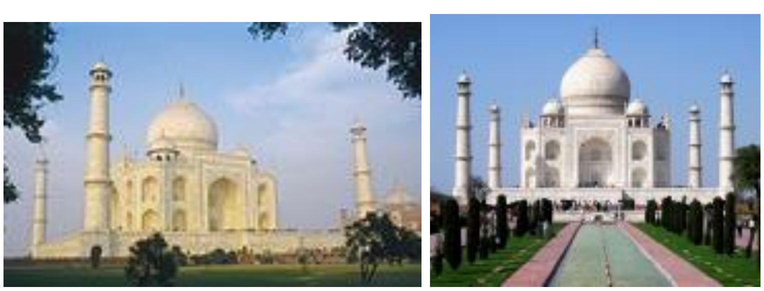
Solid Objects Viewed From Different Angles

Look at the following figure.

This figure shows the top view of Taj Mahal.

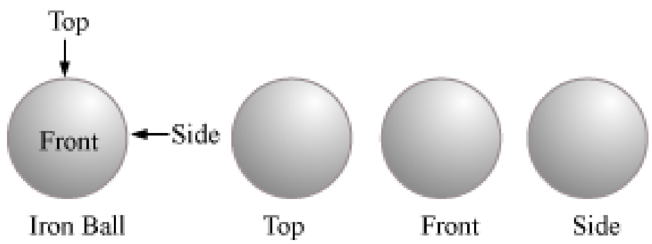
We obtain different views of objects depending upon the angle of looking at the objects. These views help us to know the shape of the object.

The following figures show the side view and front view of Taj Mahal.

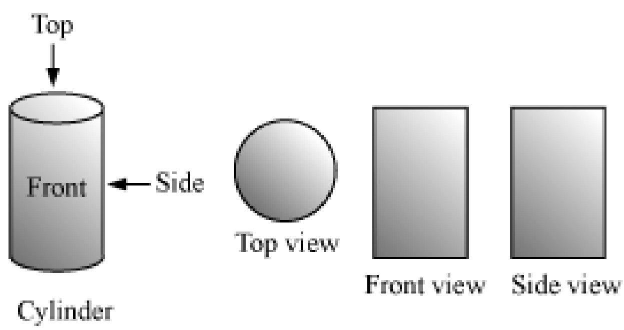


Thus, we can see any object from different angles. It does not change the shape or position of the object. However, it makes a different view of the same object for the observer.

Let us use this concept in some more objects.



Thus, we can say that spherical objects look the same from all angles. It is a special property of such spherical solids.



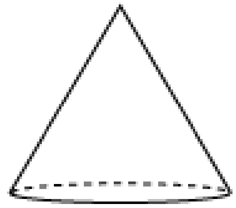
If a cylinder is considered, then its top view will be a circle. The front and side views are the same, which are rectangles.

Now, let us solve some examples to understand the concept better.

Example 1:

Which shape is formed when a cone is seen from

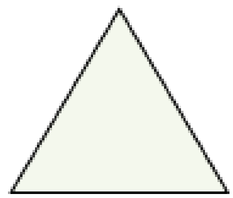
1. **front**
2. **side**
3. **top**



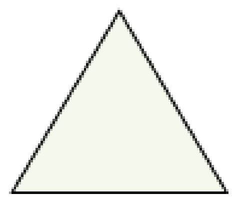
Solution:

When a cone is seen from

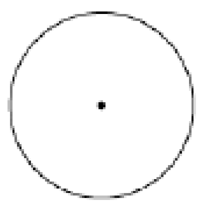
1. front, it looks like a triangle



2. side, it looks like a triangle



3. top, it looks like a circle and the vertex as dot



Identification of Three-dimensional Shapes

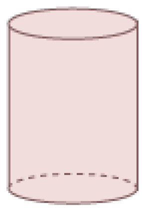
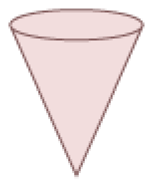
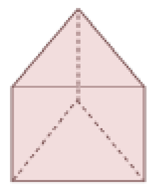
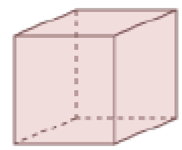
Similar to two-dimensional shapes, we have various types of three-dimensional objects, which are classified on the basis of the nature of arrangement and orientation of various faces of the shape.

Go through the following video to know how to identify various common three-dimensional objects.

Let us now look at some more examples to understand this concept better.

Example 1:

Identify the following shapes.

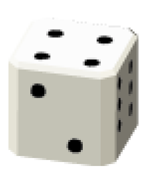
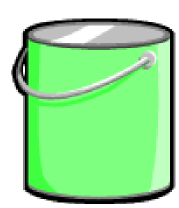
(i) **(ii)** **(iii)** **(iv)**

Solution:

1. Cylinder
2. Cone
3. Prism
4. Cube

Example 2:

Identify the following three-dimensional shapes.

(a) **(b)**

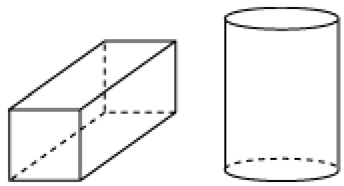
Solution:

1. This figure is cubical in shape. A dice has six sides and all of them are equal. Such types of shapes are known as cubes.

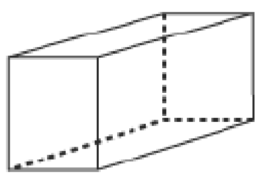
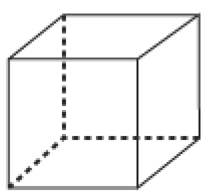
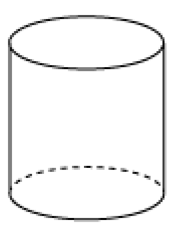
(b) This figure is cylindrical in shape.

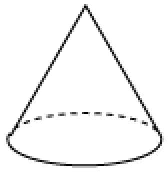
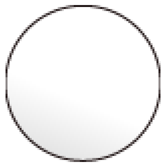
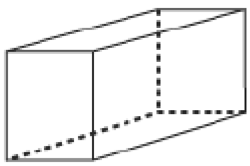
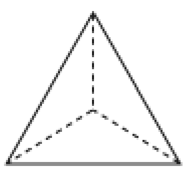
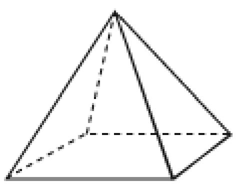
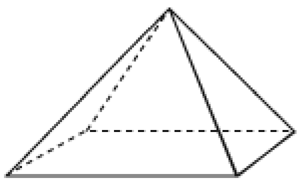
Attributes of Three-dimensional Shapes

Consider the following figures.



The following table helps us to understand the attributes of three-dimensional figures.

Name	Shape	No. of straight edges	No. of faces	No. of Vertices	Example
Cuboid		12	6	8	Pencil box, notebook
Cube		12	6	8	Dice
Cylinder		None	Two flat faces and one curved surface	None	Can, cooking cylinder

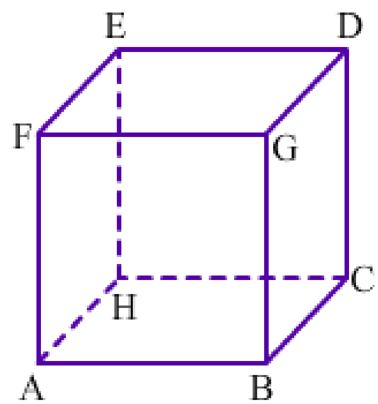
Cone		None	One flat face and one curved surface	1	Softy cone, birthday cap
Sphere		None	None	None	Ball
Triangular prism		9	5	6	Laboratory prisms
Rectangular prism		12	6	8	A rectangular glass slab
Triangular pyramid		6	4	4	
Square pyramid		8	5	5	The great pyramids of Egypt
Rectangular pyramid		8	5	5	

We know about the top and base of the solid, let us learn about its lateral face(s).

The faces that join the bases of a solid are called **lateral faces**.

We know that a cube has six square faces. Any face of the cube can be taken as its base.

Consider the cube shown below.



Here, ABCH is the base of the cube and EFGD is the top of the cube.

Rest four faces of the cube, namely ABGF, BGCD, CDEH and AHEF are the lateral faces of the cube as these faces meet

the base as well as the top of the cube.

Let us now look at some examples.

Example 1:

Find the number of faces and vertices of the following three-dimensional shapes.

(i) Cuboid (ii) Cube (iii) Cylinder (iv) Cone

(v) Sphere

Solution:

1. A cuboid has six faces and eight vertices.
2. A cube has six faces and eight vertices.
3. A cylinder has two flat faces and one curved surface. It has no vertices.
4. A cone has one flat face and one curved surface. It has one vertex.
5. A sphere has no flat face. Also, it has no vertex.

Example 2:

Find the number of faces and edges of the following three-dimensional shapes.

1. **Triangular prism**
2. **Rectangular prism**
3. **Triangular pyramid**
4. **Rectangular pyramid**

Solution:

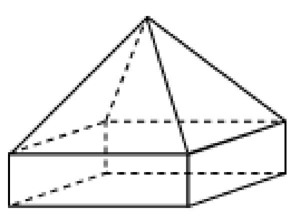
1. A triangular prism has five faces and nine edges.
2. A rectangular prism has six faces and twelve edges.
3. A triangular pyramid has four faces and six edges.
4. A rectangular pyramid has five faces and eight edges.

Euler's Formula

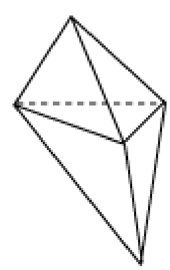
Let us discuss some examples based on Euler's formula.

Example 1:

Verify Euler's formula for the following solids.



(a)



(b)

Solution:

(a) The number of faces (F), edges (E), and vertices (V) for the given solid are 9, 16, and 9 respectively.

Now, $F + V - E = 9 + 9 - 16 = 18 - 16 = 2$

Thus, Euler's formula is verified for the given solid.

(b) The number of faces (F), edges (E), and vertices (V) for the given solid are 6, 9, and 5 respectively.

Now, $F + V - E = 6 + 5 - 9$

$= 11 - 9$

$= 2$

Thus, Euler's formula is verified for the given solid.

Example 2:

Is a polyhedron with 12 faces, 21 edges, and 13 vertices possible?

Solution:

The number of faces (F), edges (E), and vertices (V) is given as 12, 21, and 13 respectively.

Now, $F + V - E = 12 + 13 - 21 = 25 - 21 = 4$

However, according to Euler's formula, the relation between the number of faces (F), the number of edges (E), and the number of vertices (V) for any polyhedron is $F + V - E = 2$

Thus, a polyhedron with 12 faces, 21 edges, and 13 vertices is not possible.

Example 3:

Find the unknown values for polyhedrons in the following table.

Face	Edge	Vertex
8	12	?
12	?	20
?	18	12

Solution:

Let the number of vertices for the first polyhedron be V.

It is given that the number of faces (F) and edges (E) for this polyhedron are 8 and 12 respectively.

Using Euler's formula, we obtain

$F + V - E = 2$

$8 + V - 12 = 2$

$V = 2 + 12 - 8$

$V = 6$

Thus, the number of vertices for the first polyhedron is 6.

Let the number of edges for the second polyhedron be E.

It is given that the number of faces (F) and vertices (V) for this polyhedron are 12 and 20 respectively.

Using Euler's formula, we obtain

$F + V - E = 2$

$12 + 20 - E = 2$

$E = 12 + 20 - 2$

$E = 30$

Thus, the number of edges for the second polyhedron is 30.

Let the number of faces for the third polyhedron be F.

It is given that the number of edges (E) and vertices (V) for this polyhedron are 18 and 12 respectively.

Using Euler's formula, we obtain

$F + V - E = 2$

$F + 12 - 18 = 2$

$F = 2 + 18 - 12$

$F = 8$

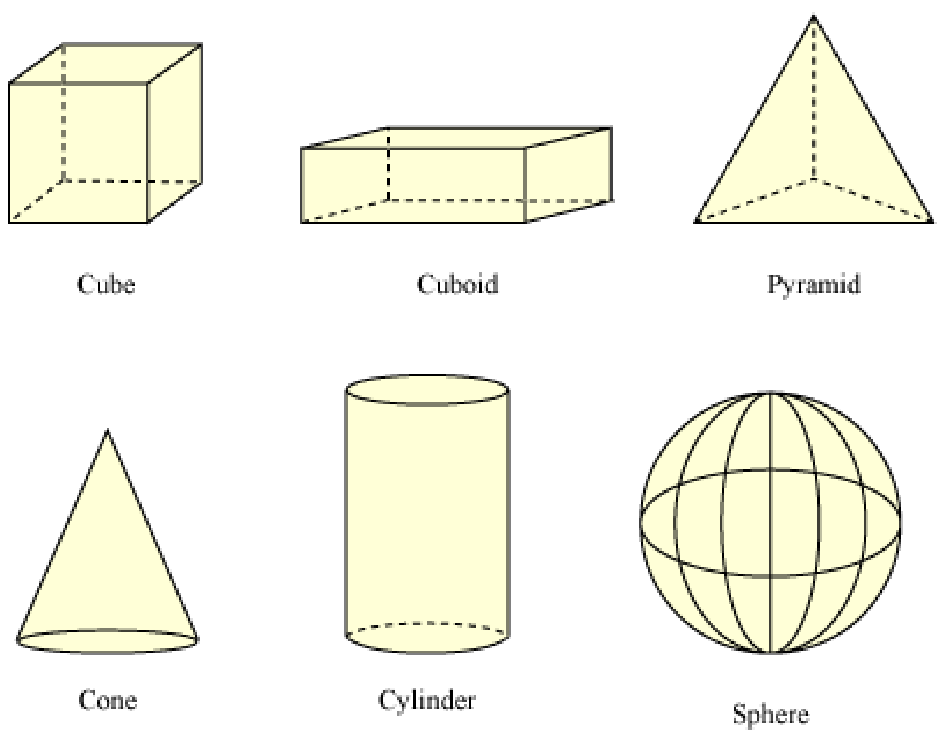
Thus, the number of faces for the third polyhedron is 8.

Nets of Three-Dimensional Figures

Look at the following figures.

What do you think about the shapes of the objects shown in the figure above?

These figures are three-dimensional in shape. Let us see some more three-dimensional figures.

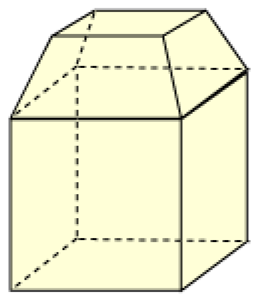


As the paper is two-dimensional, we cannot draw these figures on paper very easily. If we try to draw them on paper, then the back edges will not be shown. But we can draw the net of the three-dimensional solids.

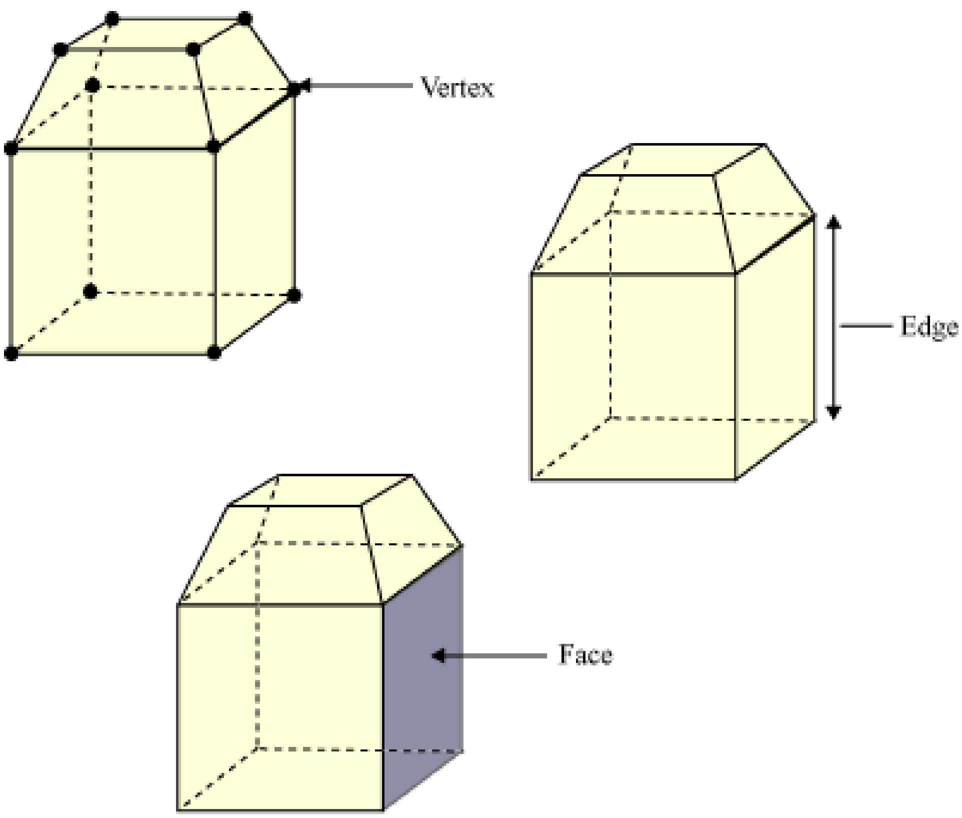
Let us learn more about three-dimensional shapes through various illustrative examples.

Example 1:

Find the number of vertices, edges, and faces in the following figure.



Solution:

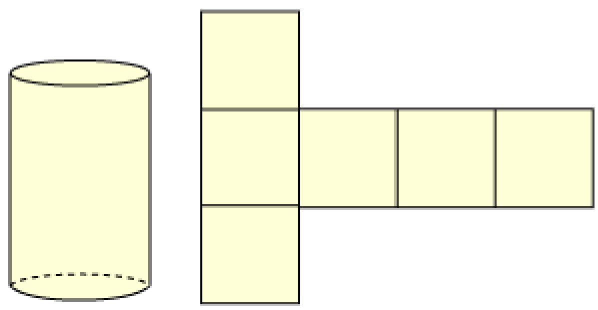


The given figure has 12 vertices, 20 edges, and 10 faces.

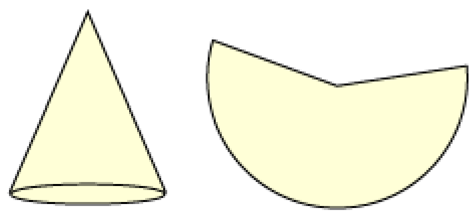
Example 2:

Match the following shapes with their appropriate nets.

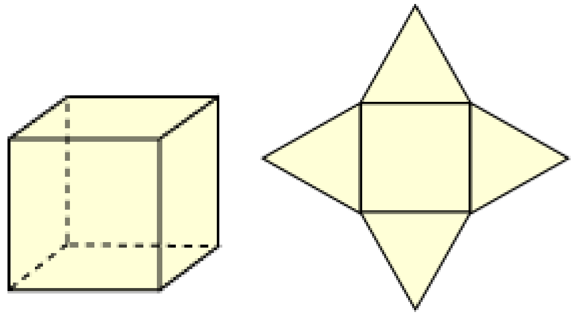
(i) (a)



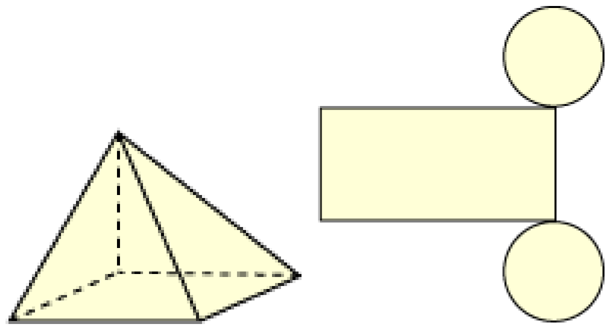
(ii) (b)



(iii) (c)

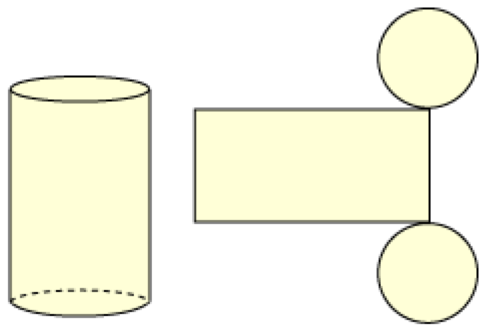


4. **(d)**

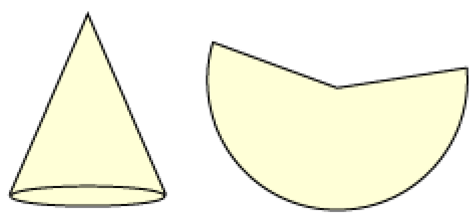


Solution:

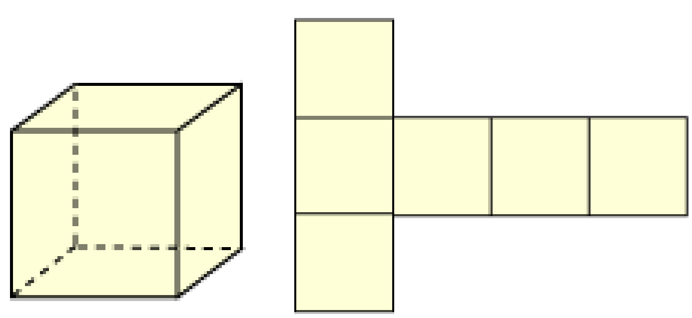
(i) (d)



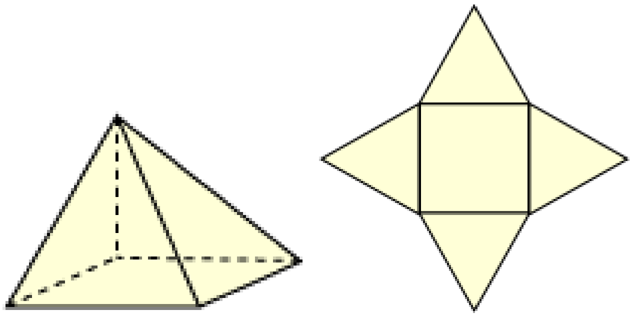
(ii) (b)



(iii) (a)



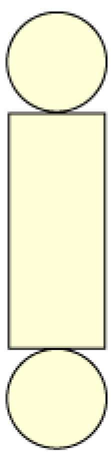
(iv) (c)



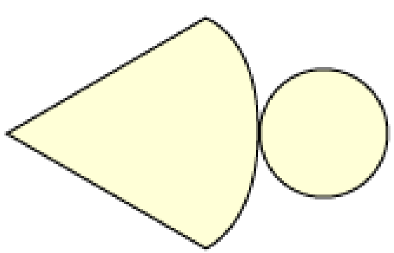
Example 3:

Draw the 3-D shapes that can be obtained from the following 2-D nets.

(i)

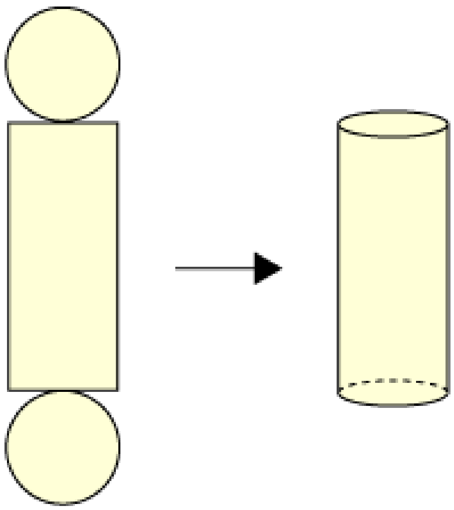


(ii)



Solution:

(i) From the given net, a cylinder will be obtained.



(ii) From the given net, a cone will be obtained.