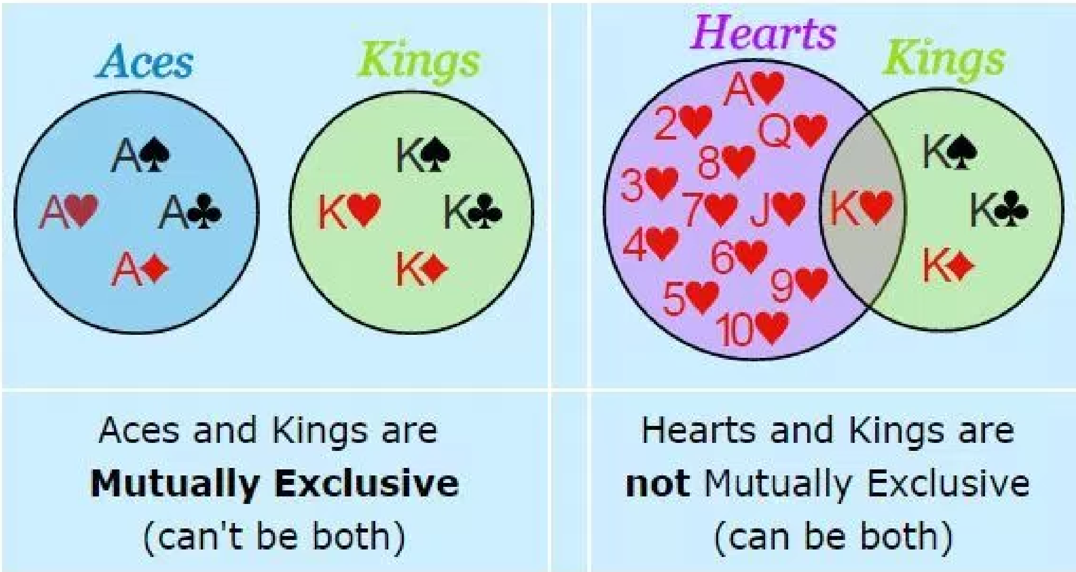
13. Probability

Probability – Types of Events

Event: An event is a subset of a sample space.

1. **Simple event:** An event containing only a single sample point is called an elementary or simple event.
2. **Compound events:** Events obtained by combining together two or more elementary events are known as the compound events or decomposable events.
3. **Equally likely events:** Events are equally likely if there is no reason for an event to occur in preference to any other event.
4. **Mutually exclusive or disjoint events:** Events are said to be mutually exclusive or disjoint or incompatible if the occurrence of any one of them prevents the occurrence of all the others.
5. **Mutually non-exclusive events:** The events which are not mutually exclusive are known as compatible events or mutually non exclusive events.



6. **Independent events:** Events are said to be independent if the happening (or non-happening) of one event is not affected by the happening (or non-happening) of others.
7. **Dependent events:** Two or more events are said to be dependent if the happening of one event affects (partially or totally) other event.

Mutually exclusive and exhaustive system of events:

Let S be the sample space associated with a random experiment. Let A_1, A_2, ……….. A_n be subsets of S such that
(i) $A_i \cap A_j = \phi$ for $i \neq j$ and (ii) $A_1 \cup A_2 \cup \ldots . \cup A_n = S$
Then the collection of events is said to form a mutually exclusive and exhaustive system of events.
If E_1, E_2, ……….. E_n are elementary events associated with a random experiment, then
(i) $E_i \cap E_j = \phi$ for $i \neq j$ and (ii) $E_1 \cup E_2 \cup \ldots . \cup E_n = S$
So, the collection of elementary events associated with a random experiment always form a system of mutually exclusive and exhaustive system of events.
In this system, $P(A_1 \cup A_2 \ldots \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots \ldots + P(A_n) = 1$

Probability and Permutations

Things to remember:

• When dealing with probability and permutations, it is important to know if the problem deals with replacement, or without replacement. For example, “with replacement” would be drawing an ace from a deck of cards and then replacing the ace in the deck before drawing a second card. “Without replacement” would be drawing the ace and not replacing it in the deck before drawing the second card.
• Don’t forget to use the counting principle for many compound events. It is fast and easy.

- **Probability formula:** $P(E) = \dfrac{n(E)}{n(S)}$ $\quad \dfrac{\text{event}}{\text{total}}$ Where $n(S)$ is the number of elements in the space and $n(E)$ is the number of outcomes in the event.

Examples:

1. Two cards are drawn at random from a standard deck of 52 cards, without replacement. What is the probability that both cards drawn are queens?

$\dfrac{\text{event}}{\text{total}}$ $\quad \dfrac{\text{the way to draw 2 cards out of a possible 4 queens}}{\text{the way to draw 2 cards from a deck of 52 cards}}$ $\quad \dfrac{{}_4P_2}{{}_{52}P_2}$

$$\frac{{}_4P_2}{{}_{52}P_2} = \frac{4\cdot 3}{52\cdot 51} = \frac{12}{2652} = \frac{1}{221}$$

2. Mrs. Schultzkie has to correct papers for three different classes: Algebra, Geometry, and Trig. If Mrs. Schultzkie corrects the papers for each class at random, what is the probability she corrects Algebra papers first?

There is only one way to correct Algebra papers first.
Then, there are ${}_2P_2$ ways to correct the other two sets of papers.
The "total" - three class sets of papers ${}_3P_3$.

$$\frac{1 \cdot {}_2P_2}{{}_3P_3} = \frac{1\cdot 2\cdot 1}{3\cdot 2\cdot 1} = \frac{2}{6} = \frac{1}{3}$$

3. A card is drawn from a deck of standard cards and then replaced in the deck. A second card is then drawn and replaced. What is the probability that a queen is drawn each time?

Solution :

On the first draw, the probability of getting one of the four queens in the deck is 4 out of 52 cards. Because the queen is replaced into the deck, the probability of getting a queen on the second draw remains the same. Using the counting principle we have:

$$P(\text{draw 2 queens}) = P(\text{queen on first draw}) \bullet P(\text{queen on second draw})$$

$$= \frac{4}{52}\bullet\frac{4}{52} = \frac{16}{2704} = \frac{1}{169}$$