CHAPTER : 1

UNITS, DIMENSIONS AND VECTORS

In science, particularly in physics, we try to make measurements as precisely as possible. Several times in the history of science, precise measurements have led to new discoveries or important developments. Obviously, every measurement must be expressed in some units. For example, if you measure the length of your room, it is expressed in suitable units. Similarly, if you measure the interval between two events, it is expressed in some other units. The unit of a physical quantity is derived, by expressing it in base units fixed by international agreement. The idea of base units leads us to the concept of **dimensions**, which as we shall see, has important applications in physics.

You will learn that physical quantities can generally be divided in two groups: **scalars** and **vectors**. Scalars have only magnitudes while vectors have both magnitude and direction. The mathematical operations with vectors are somewhat different from those which you have learnt so far and which apply to scalars. The concepts of vectors and scalars help us in understanding physics of different natural phenomena. You will experience it in this course.

OBJECTIVES

After studying this lesson, you should be able to:

- *describe the scope of physics, nature of its laws and applications of the principles of physics in our life;*
- *identify the number of significant figures in measurements and give their importance;*
- *distinguish between the fundamental and derived quantities and give their SI units;*
- *write the dimensions of various physical quantities;*

- *apply dimensional analysis to check the correctness of an equation and determine the dimensional nature of 'unknown' quantities;*
- *differentiate between scalar and vector quantities and give examples of each;*
- *add and subtract two vectors and resolve a vector into its components; and*
- *calculate the product of two vectors.*

1.1 PHYSICAL WORLD AND MEASUREMENTS

1.1.1 Physics: Scope and Excitement

The scope of Physics is very wide. It covers a vast variety of natural phenomena. It includes the study of mechanics; heat and thermodynamics; optics; waves and oscillations; electricity and magnetism; atomic and nuclear physics; electronics and computers etc. Of late, need for solutions of quite a few problems has led to the development of subjects like biophysics, chemical physics, astrophysics, soil physics, geophysics etc., thus widening the scope of physics further. In physics, we study large objects such as stars, planets etc.; and tiny objects like elementary particles; large distances such as 10^{26} m (size of the universe) as well as small distances such as 10^{-14} m (size of the nucleus of an atom); large masses such as 10^{55} kg (mass of universe) as well as tiny masses of 10^{-30} kg (mass of an electron).

Physics is perhaps the most basic of all sciences. All developments in engineering or technology are nothing but the applications of Physics.

The study of Physics has led to many exciting discoveries, inventions and their applications for example:

(i) A falling apple led to the understanding of gravitation.

(ii) Production of electrical energy by hydro, thermal or nuclear power plants (imagine the life and the world without electricity).

(iii) Receiving messages and visuals from anywhere on the globe by telephone and television,

(iv) Landing on the moon and the study of planets like Mars and other astronomical objects with robotic control from the ground,

(v) The study of the outer space with the help of artificial satellites, and satellite mounted telescopes,

(vi) Lasers and its numerous applications

(vii) High speed computers, and many more.

1.1.2 Nature of Physical Laws

Physicists explore the universe. Their investigations based on scientific process range from sub-atomic particles to big stars.

Physical laws are typical conclusions based on repeated scientific experiments and observations over many years and which have been accepted universally within the scientific community. Physical laws are:

- True at least within their regime of validity.
- Universal. They appear to apply everywhere in the universe.
- Simple. They are typically expressed in terms of a single mathematical equation.
- Absolute. Nothing in the universe appears to affect them.
- Stable. Unchanged since discovered (although they may have some approximations and/or exceptions).
- Omnipotent. Everything in the universe apparently must comply with them.

1.1.3 Physics, Technology and Society

Technology is the application of the principles of physics for the manufacture of machines, gadgets etc. and improvements in them, which leads to better quality of our physical life. For example:

(i) Different types of Engines (steam, petrol, diesel etc.) are based on the laws of thermodynamics.

(ii) Means of communication e.g. radio, telephone, television etc. are based on the propagation of electromagnetic waves.

(iii) Generation of electricity is based on the principle of electromagnetic induction.

(iv) Nuclear reactors – are based on the phenomenon of controlled nuclear fission.

(v) Jet aeroplanes and rockets are based on Newton's second and third laws of motion.

(vi) X–rays, ultraviolet rays and infrared rays are used in medical science for diagnostic and healing purposes.

(vii) Mobile phones, calculators and computers are based on the principles of electronics.

(viii) Lasers are based on the phenomenon of population inversion, and so on.

1.1.4 Need of Measurement

Every new discovery brings in revolutionary change in the structure of society and life of its people. Can you illustrate this fact with the help of some examples?

Physics, as we know, is a branch of science which deals with nature and natural phenomena. For complete and proper study of any phenomenon, measurement of quantities involved is essential. For example, to study the motion of a particle, measurement of its displacement, velocity, and acceleration at any instant has to be made accurately. For this, measurement of time and distance has to be done. Similarly, measurement of volume, pressure and temperature is necessary to study the state of a gas fully. Measurement of mass, volume and temperature of a liquid has to be made to study the effect of heat on it. Thus, we find that measurement of quanties, such as, distance, time, temperature, mass, force etc. has to be made to study every natural phenomena. This explains the need for measurement.

1.2 UNIT OF MEASUREMENT

The laws of physics are expressed in terms of physical quantities such as distance, speed, time, force, volume, electric current, etc. For measurement, each physical quantity is assigned a unit. For example, time could be measured in minutes, hours or days. But for the purpose of useful communication among different people, this unit must be compared with a standard unit acceptable to all. As another example, when we say that the distance between Mumbai and Kolkata is nearly 2000 kilometres, we have for comparison a basic unit in mind, called a kilometre. Some other units that you may be familiar with are a kilogram for mass and a second for time. It is essential that all agree on the standard units, so that when we say 100 kilometres, or 10 kilograms, or 10 hours, others understand what we mean by them. In science, international agreement on the basic units is absolutely essential; otherwise scientists in one part of the world would not understand the results of an investigation conducted in another part.

Suppose you undertake an investigation on the solubility of a chemical in water. You weigh the chemical in tolas and measure the volume of water in cupfuls. You communicate the results of your investigation to a scientist friend in Japan. Would your friend understand your results?

It is very unlikely that your friend would understand your results because he/she may not be familiar with tola and the cup used in your measerments, as they are not standard units.

Do you now realize the need for agreed standards and units?

Remember that in science, the results of an investigation are considered established only if they can be reproduced by investigations conducted elsewhere under identical conditions.

Measurements in Indian Traditions

Practices of systematic measurement are very old in India. The following quote from Manusmriti amply illustrates this point :

"The king should examine the weights and balance every six months to ensure true measurements and to mark them with royal stamp." – Manusmriti, 8th Chapter, sloka–403.

In **Harappan Era**, signs of systematic use of measurement are found in abundance : the equally wide roads, bricks having dimensions in the ratio 4 : 2 : 1, Ivory scale in Lothal with smallest division of 1.70 mm, Hexahedral weights of 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200 and 500 units (1 unit = 20 g)

In Mauriyan Period, the following units of length were prevalent

8 Parmanu	= 1 Rajahkan
8 Rajahkan	= 1 Liksha
8 Liksha	= 1 Yookamadhya
8 Yookamadhya	= 1 Yavamadhya
8 Yavamadhya	= 1 Angul
8 Angul	= 1 Dhanurmushthi

In **Mughal Period**, Shershah and Akbar tried to re-establish uniformity of weights and measures. Akbar introduced gaz of 41 digits for measuring length. For measuring area of land, bigha was the unit. 1 bigha was 60 gaz × 60 gaz.

Units of mass and volume were also well obtained in Ayurveda.

1.2.1 The SI Units

With the need of agreed units in mind, the 14th General Conference on Weights and Measures held in 1971, adopted seven **base** or **fundamental units**. These units form the SI system. The name SI is abbreviation for **Système International d'Unités** for the International System of units. The system is popularly known as the metric system. The SI units along with their symbols are given in Table 1.1.

Table 1.1 : Base SI Units

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Luminous Intensity	candela	cd
Amount of Substance	mole	mol

The mile, yard and foot as units of length are still used for some purposes in India as well in some other countries. **However, in scientific work we always use SI units.**

As may be noted, the SI system is a metric system. It is quite easy to handle because the smaller and larger units of the base units are always submultiples or multiples of ten. These multiples or submultiples are given special names. These are listed in Table 1.2.

Table 1.2 : Prefixes for powers of ten

Power of ten	Prefix	Symbol	Example	
10^{-18}	atto	a	attometre	(am)
10^{-15}	femto	f	femtometre	(fm)
10^{-12}	pico	p	picofarad	(pF)
10^{-9}	nano	n	nanometre	(nm)
10^{-6}	micro	μ	micron	(μm)
10^{-3}	milli	m	milligram	(mg)
10^{-2}	centi	c	centimetre	(cm)
10^{-1}	deci	d	decimetre	(dm)
10^{1}	deca	da	decagram	(dag)
10^{2}	hecto	h	hectometre	(hm)
10^{3}	kilo	k	kilogram	(kg)
10^{6}	mega	M	megawatt	(MW)
10^{9}	giga	G	gigahertz	(GHz)
10^{12}	tera	T	terahertz	(THz)
10^{15}	peta	P	peta kilogram	(Pkg)
10^{18}	exa	E	exa kilogram	(Ekg)

Just to get an idea of the masses and sizes of various objects in the universe, see Table 1.3 and 1.4. Similarly, Table 1.5 gives you an idea of the time scales involved in the universe.

Table 1.3 : Order of magnitude of some masses

Mass	kg
Electron	10^{-30}
Proton	10^{-27}
Amino acid	10^{-25}
Hemoglobin	10^{-22}
Flu virus	10^{-19}
Giant amoeba	10^{-8}
Raindrop	10^{-6}
Ant	10^{-2}
Human being	10^{2}
Saturn 5 rocket	10^{6}
Pyramid	10^{10}
Earth	10^{24}
Sun	10^{30}
Milky Way galaxy	10^{41}
Universe	10^{55}

Table 1.4 : Order of magnitude of some lengths

Length	m
Radius of proton	10^{-15}
Radius of atom	10^{-10}
Radius of virus	10^{-7}
Radius of giant amoeba	10^{-4}
Radius of walnut	10^{-2}
Height of human being	10^{0}
Height of highest mountain	10^{4}
Radius of earth	10^{7}
Radius of sun	10^{9}
Earth-sun distance	10^{11}
Radius of solar system	10^{13}
Distance to nearest star	10^{16}
Radius of Milky Way galaxy	10^{21}
Radius of visible universe	10^{26}

Table 1.5 : Order of magnitude of some time intervals

Interval	s
Time for light to cross nucleus	10^{-23}
Period of visible light	10^{-15}
Period of microwaves	10^{-10}
Half-life of muon	10^{-6}
Period of highest audible sound	10^{-4}
Period of human heartbeat	10^{0}
Half-life of free neutron	10^{3}
Period of the Earth's rotation (day)	10^{5}
Period of revolution of the Earth (year)	10^{7}
Lifetime of human beings	10^{9}
Half-life of plutonium-239	10^{12}
Lifetime of a mountain range	10^{15}
Age of the Earth	10^{17}
Age of the universe	10^{18}

1.2.2 Standards of Mass, Length and Time

Once we have chosen to use the SI system of units, we must decide on the set of standards against which these units will be measured. We define here standards of mass, length and time.

(i) Mass : The SI unit of mass is **kilogram**. The standard kilogram was established in 1887. **It is the mass of a particular cylinder made of platinum-iridium alloy**, which is an unusually stable alloy. The standard is kept in the International Bureau of Weights and Measures in Paris, France. The prototype kilograms made of the same alloy have been distributed to all countries the world over. For India, the national prototype is the kilogram no. 57. This is maintained by the National Physical Laboratory, New Delhi (Fig. 1.1).

Fig. 1.1 : Prototype of kilogram

(ii) Length : The SI unit of length is metre. It is defined in terms of a natural phenomenon: **One metre is defined as the distance travelled by light in vacuum in a time interval of 1/299792458 second.**

This definition of metre is based on the adoption of the speed of light in vacuum as 299792458 ms^{-1}

(iii) **Time :** One second is defined as **the time required for a Cesium - 133 (^{133}Cs) atom to undergo 9192631770 vibrations between two hyperfine levels of its ground state.**

This definition of a second has helped in the development of a device called atomic clock (Fig. 1.2). The cesium clock maintained by the National Physical Laboratory (NPL) in India has an uncertainty of $\pm 1 \times 10^{-12}$ s, which corresponds to an accuracy of one picosecond in a time interval of one second.

Cesium Atomic Clock
(S60,000)
Cesium Beam Tube

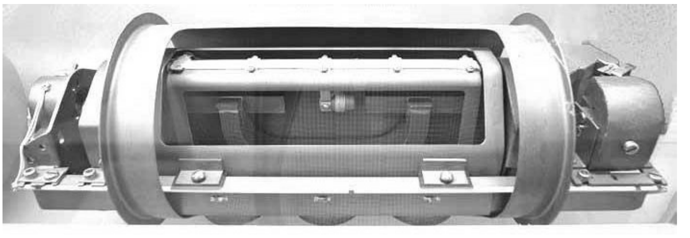


Fig. 1.2 : Atomic Clock

As of now, clock with an uncertainty of 5 parts in 10^{15} have been developed. This means that if this clock runs for 10^{15} seconds, it will gain or lose less than 5 seconds. You can convert 10^{15} s to years and get the astonishing result that this clock could run for 6 million years and lose or gain less than a second. This is not all. Researches are being conducted today to improve upon this accuracy constantly. Ultimately, we expect to have a clock which would run for 10^{18} second before it could gain or lose a second. To give you an idea of this technological achievement, if this clock were started at the time of the birth of the universe, an event called the Big Bang, it would have lost or gained only two seconds till now.

Role of Precise Measurements in New Discoveries

A classical example of the fact that precise measurements may lead to new discoveries are the experiments conducted by Lord Rayleigh to determine density of nitrogen.

In one experiment, he passed the air bubbles through liquid ammonia over red hot copper contained in a tube and measured the density of pure nitrogen so obtained. In another experiment, he passed air directly over red hot copper and measured the density of pure nitrogen. The density of nitrogen obtained in second experiment was found to be 0.1% higher than that obtained in the first case. The experiment suggested that air has some other gas heavier than nitrogen present in it. Later he discovered this gas – Argon, and got Nobel Prize for this discovery.

Another example is the failed experiment of Michelson and Morley. Using Michelson's interferometer, they were expecting a shift of 0.4 fringe width in the interference pattern obtained by the superposition of light waves travelling in the direction of motion of the earth and those travelling in a transverse direction. The instrument was hundred times more sensitive to detect the shift than the expected shift. Thus they were expecting to measure the speed of earth with respect to ether and conclusively prove that ether existed. But when they detected no shift, the world of science entered into long discussions to explain the negative results. This led to the concepts of length contraction, time dilation etc and ultimately to the theory of relativity.

Several discoveries in nuclear physics became possible due to the new technique of spectroscopy which enabled detection, with precision, of the traces of new atoms formed in a reaction.

1.2.3 Significant Figures

When a student measures the length of a line as 6.8 cm, the digit 6 is certain, while 8 is uncertain as a little less or more than 0.8 cm is reported as 0.8 cm. Normally those digits in measurement that are known with certainly plus the first uncertain digit, are called significant figures.

Thus, there are two significant figures in 1.4 cm. The number of significant figures in any quantity depends upon the accuracy of the measuring instrument. More the number of significant number of figures, less is the percentage of error in the measurement of the quantity. If there are lesser number of significant figures (in a measurement) more will be the percentage error in the measurement.

The number of significant figures of a quantity may be found by the following rules:

(i) All non-zero digits are significant. For example, 315.58 has five significant figures.

(ii) All zeros between two non-zero digits are significant. For example, 5300405.003 has ten significant figures.

(iii) All zeros which are to the right of a decimal point and also to the right of a non-zero digit are significant. For example, 50.00 has four significant figures, and so has .04050. It should be noted that in .04050, the first zero to the right of the decimal is not significant but, the last zero is significant.

(iv) All zeros to the right of a decimal point and to the left of a non-zero digit in a decimal fraction are not significant. For example, .00043 has only two significant figures but 2.00023 has 6 significant figures. It is also to be noted that zero conventionally placed to the left of a decimal point is not significant.

(v) All zero to the right of last of non-zero digit are significant, if they come from some measurement. For example, if the distance between two objects is 4050 m (measured to the nearest metre), then 4050 m contains 4 significant figures.

(vi) The number of significant figures does not vary with the change in unit. For example, if the length of an object is 348.6 cm, it has 4 significant figures. If the length is expressed in metre, then it is equal to 3.486 m. It still has 4 significant figures.

(vii) In a whole number all zeros to the right of the last non zero number are not significant, for example 5000 has only one significant figure.

Importance of significant figures in measurement.

As stated earlier, the accuracy of the measurement determines the number of significant figures in the quantity. Suppose the diameter of a coin is 2 cm. If a student measures the diameter with a metre scale which can read up to .1 cm only (i.e. cannot read less than 0.1 cm) the student will report the diameter to be 2.0 cm i.e. upto 2 significant figures only. If the diameter is measured by an instrument which can read upto .01 cm only (or which cannot measure less than .01cm), viz a Vernier Callipers, he will report the diameter as 2.00 cm i.e. upto 3 significant figure. Similarly if the measurement is made by an instrument like a screw gauge which can measure upto .001 cm only (i.e. cannot measure less than .001 cm), the diameter will be recorded as 2.000 cm i.e. upto 4 significant figures. Thus any measurement should be recorded keeping in view the accuracy of the measuring instrument.

Importance of significant figures in expressing the result of calculations

Suppose a student measures the side of a cube with the help of a metre scale which comes to be 3.2 cm. He calculates the volume of this cube mathematically and reports it to be $(3.2 \times 3.2 \times 3.2)$ cubic centimetre or 32.768 cm^3. The reported result is mathematically correct but is not correct in scientific measurement. The correct volume should be recorded as 33 cm^3. This is because there are only two significant figures in the length of the side of the cube, hence the volume should also have two significant figures, whereas there are 5 significant figures in 32.768 which is not correct.

Significant figures in addition, subtraction, multiplication and division

(i) Addition and subtraction – Suppose we have to add three quantities, 2.7 m, 3.68 m and 0.486 m. In these quantities, the first measurement is known upto one decimal place only, hence the sum of these numbers will be definite upto one decimal place only. Therefore, the correct sum of these numbers should not be written as 6.848 m but 6.8 m.

Similarly, to find the sum of quantities like 2.65×10^3 cm and 2.63×10^2 cm, all quantities should be converted to the same power of 10. These quantities will then be, 2.65×10^3 cm and $.263 \times 10^3$ cm. Since, the first number is known upto 2 decimal places, their sum will also be upto 2 decimal places. Hence 2.65×10^3cm + $.263 \times 10^3$ cm = 2.91×10^3 cm.

The same is done with subtraction. For example the result of subtracting 2.38 cm from 4.6 cm will be 2.2 cm, not 2.22 cm.

(ii) Multiplication and division – Suppose the length of a plate is measured as 3.003 m and its width as 2.26 m. According to Mathematical Calculation, the area of the plate will be 6.78678 m^2. But, it is not correct in scientific measurement. There are six significant figures in this result. But, the least number of significant figures (in the width) are only 3. Hence, the multiplication should also be writen upto 3 significant figures. Therefore, the correct area would be 6.79 m^2.

The same method is applied for division also. For example, dividing 248.57 by 56.9 gives 4.3685413. But, the result should be recorded upto 3 significant figures only as the least number of significant figures in the given numbers is only 3. Hence, the result will be 4.37.

Similarly, if a body travels a distance of 1452 m in 142 seconds, its speed according to mathematical calculations will be $\frac{1452}{142}$ m per second or 10.225352 m s^{-1}, but in scientific measurements it should be 10.2 m s^{-1}, as there are only 3 significant figures in the number for time.

(iii) Value of constants used in Calculation

If the radius (r) of a circle is 3.35 cm, to calculate its area (πr^2) the value of π should be taken upto two places of decimal (i.e $\pi = 3.14$, not 3.1416). Hence, the area of this circle $\pi r^2 = (3.14 \times 3.35 \times 3.35)$ cm^2 = 35.2 cm^2, not 35.23865 cm^2.

(iv) If a measured quantity is multiplied by a constant, all the digits in the product are significant that are obtained by multiplication. For example, if the mass of a ball is 32.59 g the mass of 10 similar balls will be 32.59×10 = 325.90 g. Note that there are five significant figures in the number.

1.2.4. Derived Units

We have so far defined three basic units for the measurement of mass, length and time. For many quantities, we need units which we get by combining the basic units. These units are called derived units. For example, combination of the units of length and time gives us the derived unit of speed or velocity, $m\ s^{-1}$. Another example is the interaction of the unit of length with itself. We get derived units of area and volume as m^2 and m^3, respectively.

Now are would like you to list all the physical quantities that you are familiar with and the units in which they are expressed.

Some derived units have been given special names. Examples of most common of such units are given in Table 1.6.

Table 1.6 : Examples of derived units with special names

Quantity	Name	Symbol	Unit Symbol
Force	newton	N	$kg\ m\ s^{-2}$
Pressure	pascal	Pa	$N\ m^{-2}$
Energy/work	joule	J	N m
Power	watt	W	$J\ s^{-1}$

One of the advantages of the SI system of units is that they form a coherent set in the sense that the product or division of the SI units gives a unit which is also the SI unit of some other derived quantity. For example, product of the SI units of force and length gives directly the SI unit of work, namely, newton-metre (Nm) which has been given a special name joule. **Some care should be exercised in the order in which the units are written**. For example, Nm should be written in this order. If by mistake we write it as mN, it becomes millinewton, which is something entirely different.

Remember that in physics, a quantity must be written with correct units. Otherwise, it is meaningless and, therefore, of no significance.

Example 1.1 : Anand, Rina and Kaif were asked by their teacher to measure the volume of water in a beaker.

Anand wrote : 200; Rina wrote : 200 mL; Kaif wrote : 200 Lm

Which one of these answers is correct?

Solution : The first one has no units. Therefore, we do not know what it means. The third is also not correct because there is no unit like Lm. The second one is the only correct answer. It denotes millilitre.

Note that the mass of a book, for example, can be expressed in kg or g. **You should not use gm for gram because the correct symbol is g and not gm.**

Nomenclature and Symbols

(i) Symbols for units should not contain a full stop and should remain the same in the plural. For example, the length of a pencil should be expressed as 7cm and not 7cm. or 7cms.

(ii) Double prefixes should be avoided when single prefixes are available, e.g., for nanosecond, we should write ns and not mμs; for pico farad we write pF and not μμf.

(iii) When a prefix is placed before the symbol of a unit, the combination of prefix and symbol should be considered as one symbol, which can be raised to a positive or a negative power without using brackets, e.g., μs^{-1}, cm^2, mA^2.

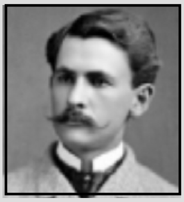
$\mu s^{-1} = (10^{-6} s)^{-1}$ (and not $10^{-6} s^{-1}$)

(iv) Do not write cm/s/s for cm s^{-2}. Similarly 1 poise = 1 g $s^{-1} cm^{-1}$ and not 1 g/s/cm.

(v) When writing a unit in full in a sentence, the word should be spelt with the letter in lower case and not capital, e.g., 6 hertz and not 6 Hertz.

(vi) For convenience in reading of large numbers, the digits should be written in groups of three starting from the right but no comma should be used: 1 532; 1 568 320.

Albert Abraham Michelson
(1852-1931)



German-American Physicst, inventor and experimenter devised Michelson's interferometer with the help of which, in association with Morley, he tried to detect the motion of earth with respect to ether but failed. However, the failed experiment stirred the scientific world to reconsider all old theories and led to a new world of physics.

He developed a technique for increasing the resolving power of telescopes by adding external mirrors. Through his stellar interferometer along with 100″ Hookes telescope, he made some precise measurements about stars.

Now, it is time to check your progress. Solve the following questions. In case you have any problem, check answers given at the end of the lesson.

INTEXT QUESTIONS 1.1

1. Discuss the nature of laws of physics.
2. How has the application of the laws of physics led to better quality of life?
3. What is meant by significant figures in measurement?
4. Find the number of significant figures in the following quantity, quoting the relevant laws:

   (i) 426.69 (ii) 4200304.002 (iii) 0.3040 (iv) 4050 m (v) 5000
5. The length of an object is 3.486 m, if it is expressed in centimetre (i.e. 348.6 cm) will there be any change in number of significant figures in the two cases.
6. What are the four applications of the principles of dimensions? On what principle are the above based?
7. The mass of the sun is 2×10^{30} kg. The mass of a proton is 2×10^{-27} kg. If the sun was made only of protons, calculate the number of protons in the sun?
8. Earlier the wavelength of light was expressed in angstroms. One angstrom equals 10^{-8} cm. Now the wavelength is expressed in nanometers. How many angstroms make one nanometre?
9. A radio station operates at a frequency of 1370 kHz. Express this frequency in GHz.
10. How many decimetres are there in a decametre? How many MW are there in one GW?

1.3 DIMENSIONS OF PHYSICAL QUANTITIES

Most physical quantities you would come across in this course can be expressed in terms of five basic dimensions : mass (M), length (L), time (T), electrical current (I) and temperature (θ). Since all quantities in mechanics can be expressed in terms of mass, length and time, it is sufficient for our present purpose to deal with only these three dimensions. Following examples show how dimensions of the physical quantities are combinations of the powers of M, L and T :

(i) Volume requires 3 measurements in length. So it has 3 dimensions in length (L^3).

(ii) Density is mass divided by volume. Its dimensional formula is ML^{-3}.

(iii) Speed is distance travelled in unit time or length divided by time. Its dimensional formula is LT^{-1}.

(iv) Acceleration is change in velocity per unit time, i.e., length per unit time per unit time. Its dimensionsal formula is LT^{-2}.

(v) Force is mass multiplied by acceleration. Its dimensions are given by the formula MLT^{-2}.

Similar considerations enable us to write dimensions of other physical quantities.

Note that numbers associated with physical quantities have no significance in dimensional considerations. Thus if dimension of x is L, then dimension of $3x$ will also be L.

Write down the dimensions of momentum, which is product of mass and velocity and work which is product of force and displacement.

Remember that dimensions are not the same as the units. For example, speed can be measured in m s^{-1} or kilometre per hour, but its dimensions are always given by length divided by time, or simply LT^{-1}.

Dimensional analysis is the process of checking the dimensions of a quantity, or a combination of quantities. One of the important principles of dimensional analysis is that **each physical quantity on the two side of an equation must have the same dimensions**. Thus if $x = p + q$, then p and q will have the same dimensions as x. This helps us in checking the accuracy of equations, or getting the dimensions of a quantity using an equation. The following examples illustrate the use of dimensional analysis.

1.3.1 Applications of Dimensions (or dimensional equations)

There are four applications of dimensions (or dimensional equations)

(i) Derivation of a relationship between different physical quantities (or formula);

(ii) Checking up of accuracy of a formula (or relationship between different physical quantities);

(iii) Conversion of one system of units into another; and

(iv) Derivation of units of a physical quantity

The above applications are based on the principle that the dimensions of physical quantities on the two sides of a relation/equation/formula must be the same. This is called 'the Principle of Homogeneity of Dimensions'.

Example 1.2 : You know that the kinetic energy of a particle of mass m is $\frac{1}{2}mv^2$ while its potential energy is mgh, where v is the velocity of the particle, h is its height from the ground and g is the acceleration due to gravity. Since the two expressions represent the same physical quantity i.e, energy, their dimensions must be the same. Let us prove this by actually writing the dimensions of the two expressions.

Solution : The dimensions of $\frac{1}{2}mv^2$ are $M.(LT^{-1})^2$, or ML^2T^{-2}. (Remember that the numerical factors have no dimensions.) The dimensions of mgh are $M.LT^{-2}.L$, or ML^2T^{-2}. Clearly, the two expressions are the same and hence represent the same physical quantity.

Let us take another example to find an expression for a physical quantity in terms of other quantities.

Example 1.3 : Experience tells us that the distance covered by a car, say x, starting from rest and having uniform acceleration depends on time t and acceleration a. Let us use dimensional analysis to find expression for the distance covered.

Solution : Suppose x depends on the mth power of t and nth power of a. Then we may write

$$x \propto t^m . a^n$$

Expressing the two sides now in terms of dimensions, we get

$$L^1 \propto T^m (LT^{-2})^n,$$

or,
$$L^1 \propto T^{m-2n} L^n.$$

Comparing the powers of L and T on both sides, you will easily get $n = 1$, and $m = 2$. Hence, we have

$$x \propto t^2 a^1, \text{ or } x \propto at^2.$$

This is as far as we can go with dimensional analysis. It does not help us in getting the numerical factors, since they have no dimensions. To get the numerical factors, we have to get input from experiment or theory. In this particular case, of course, we know that the complete relation is $x = (1/2)at^2$.

Besides numerical factors, other quantities which do not have dimensions are angles and arguments of trigonometric functions (sine, cosine, etc), exponential and logarithmic functions. In sin x, x is said to be the argument of sine function. In e^x, x is said to be the argument of the exponential function.

Now take a pause and attempt the following questions to check your progress.

INTEXT QUESTIONS 1.2

1. Experiments with a simple pendulum show that its time period depends on its length (l) and the acceleration due to gravity (g). Use dimensional analysis to obtain the dependence of the time period on l and g .

2. Consider a particle moving in a circular orbit of radius r with velocity υ and acceleration a towards the centre of the orbit. Using dimensional analysis, show that $a \propto \upsilon^2/r$.

3. You are given an equation: $m\upsilon = Ft$, where m is mass, υ is speed, F is force and t is time. Check the equation for dimensional correctness.

1.4 VECTORS AND SCALARS

1.4.1 Scalar and Vector Quantities

In physics we classify physical quantities in two categories. In one case, we need only to state their magnitude with proper units and that gives their complete description. Take, for example, mass. If we say that the mass of a ball is 50 g, we do not have to add anything to the description of mass. Similarly, the statement that the density of water is 1000 kg m^{-3} is a complete description of density. Such quantities are called scalars. **A scalar quantity has only magnitude; no direction.**

On the other hand, there are quantities which require both magnitude and direction for their complete description. A simple example is velocity. The statement that the velocity of a train is 100 km h^{-1} does not make much sense unless we also tell the direction in which the train is moving. Force is another such quantity. We must specify not only the magnitude of the force but also the direction in which the force is applied. Such quantities are called vectors. **A vector quantity has both magnitude and direction.**

Some examples of vector quantities which you come across in mechanics are: displacement (Fig. 1.3), acceleration, momentum, angular momentum and torque etc.

What is about energy? Is it a scalar or a vector?

To get the answer, think if there is a direction associated with energy. If not, it is a scalar.

1.4.2 Representation of Vectors

A vector is represented by a line with an arrow indicating its direction. Take vector AB in Fig. 1.4. The length of the line represents its magnitude on some scale. The arrow indicates its direction. Vector CD is a vector in the same direction but its magnitude is smaller. Vector EF is a vector whose magnitude is the same as that of vector CD, but its direction is different. In any vector, the initial point, (point A in **AB),** is called the **tail** of the vector and the final point, (point B in **AB)** with the arrow mark is called its **tip** (or **head**).

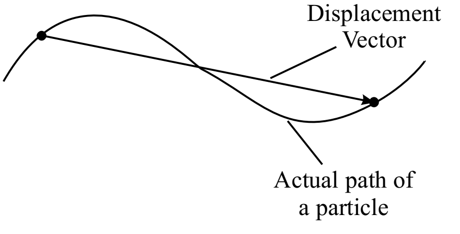


Fig. 1.3 : Displacement vector

A vector is written with an arrow over the letter representing the vector, for example, $\vec{A}$. The magnitude of vector $\vec{A}$ is simply A or $|\vec{A}|$. In print, a vector is indicated by a bold letter as **A**.

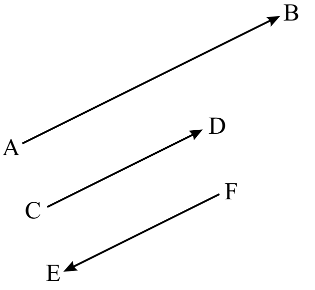


Fig. 1.4 : Directions and magnitudes of vectors

Two vectors are said to be equal if their magnitudes are equal and they point in the same direction. This means that all vectors which are parallel to each other have the same magnitude and point in the same direction are equal. Three vectors **A, B** and **C** shown in Fig. 1.5 are equal. We say **A = B = C**. But **D** is not equal to **A**.

A vector (here **D**) which has the same magnitude as A but has opposite direction, is called **negative** of **A**, or **–A**. Thus, **D** = –**A**.

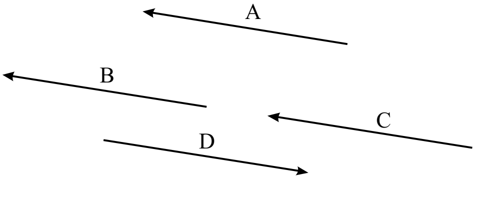


Fig. 1.5 : Three vectors are equal but fourth vector D is not equal.

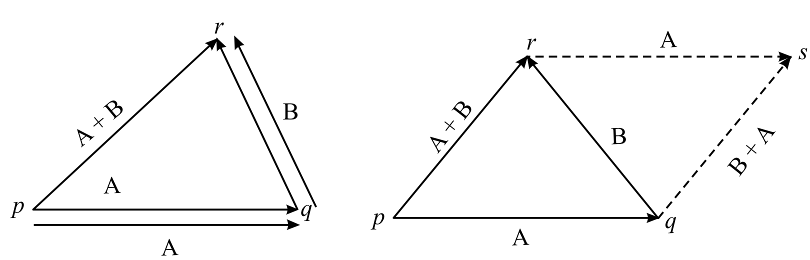
For respresenting a physical vector quantitatively, we have to invariably choose a proportionality scale. For instance, the vector displacement between Delhi and Agra, which is about 300 km, is represented by choosing a scale 100 km = 1 cm, say. Similarly, we can represent a force of 30 N by a vector of length 3cm by choosing a scale 10N = 1cm.

From the above we can say that if we translate a vector parallel to itself, it remains unchanged. This important result is used in addition of vectors. Let us sec how.

1.4.3 Addition of Vectors

You should remember that only **vectors of the same kind can be added**. For example, two forces or two velocities can be added. But a force and a velocity cannot be added.

Suppose we wish to add vectors **A** and **B.** First redraw vector **A** [Fig. 1.6 (a)]. For this draw a line (say *pq*) parallel to vector **A.** The length of the line i.e. *pq* should be equal to the magnitude of the vector. Next draw vector **B** such that its tail coincides with the tip of vector **A**. For this, draw a line *qr* from the tip of **A** (i.e., from the point *q*) parallel to the direction of vector **B.** The sum of two vectors then is the vector from the tail of **A** to the tip of **B,** i.e. the resultant will be represented in magnitude and direction by line *pr.* You can now easily prove that **vector addition is commutative. That is, A + B = B + A,** as shown in Fig. 1.6 (b). In Fig. 1.6(b) we observe that *pqr is* a triangle and its two sides *pq* and *qr* respectively represent the vectors **A** and **B** in magnitude and direction, and the third side *pr*, of the triangle represents the resultant vector with its direction from *p* to *r*. This gives us a rule for finding the resultant of two vectors :



(a) (b)

Fig. 1.6 : Addition of vectors A and B

If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, the resultant is represented by the third side of the triangle taken in the opposite order. This is called triangle law of vectors.

The sum of two or more vectors is called the **resultant** vector. In Fig. 1.6(b), **pr** is the resultant of **A** and **B**. What will be the resultant of three forces acting along the three sides of a triangle in the same order? If you think that it is zero, you are right.

Let us now learn to calculate resultant of more than two vectors.

The resultant of more than two vectors, say **A**, **B** and **C**, can be found in the same manner as the sum of two vectors. First we obtain the sum of **A** and **B**, and then add the resultant of the two vectors, (**A + B**), to **C**. Alternatively, you could add **B** and **C**, and then add **A** to (**B + C**) (Fig. 1.7). In both cases you get the same vector. Thus, vector addition is associative. That is, **A + (B + C) = (A + B) + C.**

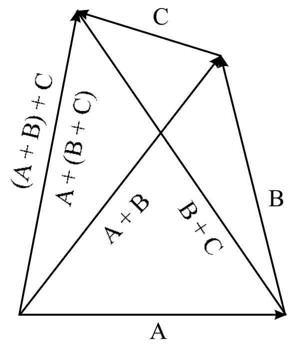


Fig. 1.7 : Addition of three vectors in two different orders

If you add more than three vectors, you will discover that **the resultant vector is the vector from the tail of the first vector to the tip of the last vector.**

Many a time, the point of application of vectors is the same. In such situations, it is more convenient to use parallelogram law of vector addition. Let us now learn about it.

1.4.4 Parallelogram Law of Vector Addition

Let **A** and **B** be the two vectors and let θ be the angle between them as shown in Fig. 1.8. To calculate the vector sum, we complete the parallelogram. Here side PQ represents vector **A**, side PS represents **B** and the diagonal PR represents the resultant vector **R**. Can you recognize that the diagonal PR is the sum vector **A + B**? It is called the **resultant** of vectors **A** and **B**. The resultant makes an angle α with the direction of vector **A**. Remember that vectors **PQ** and **SR** are equal to **A**, and vectors **PS** and **QR** are equal, to **B**. To get the magnitude of the resultant vector **R**, drop a perpendicular RT as shown. Then in terms of magnitudes

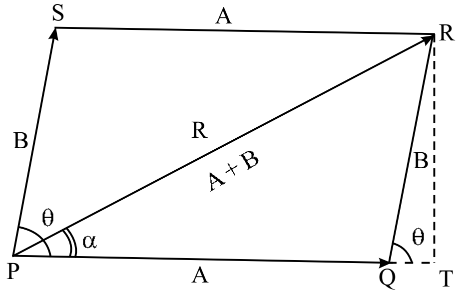


Fig. 1.8: Parallelogram law of addition of vectors

$$
\begin{aligned}
(PR)^2 &= (PT)^2 + (RT)^2 \\
&= (PQ + QT)^2 + (RT)^2 \\
&= (PQ)^2 + (QT)^2 + 2PQ.QT + (RT)^2 \\
&= (PQ)^2 + [(QT)^2 + (RT)^2] + 2PQ.QT \qquad (1.1) \\
&= (PQ)^2 + (QR)^2 + 2PQ.QT \\
&= (PQ)^2 + (QR)^2 + 2PQ.QR\ (QT / QR) \\
R^2 &= A^2 + B^2 + 2AB.\cos\theta
\end{aligned}
$$

Therefore, the magnitude of **R** is

$$|R| = \sqrt{A^2 + B^2 + 2AB.\cos\theta} \qquad (1.2)$$

For the direction of the vector **R**, we observe that

$$\tan\alpha = \frac{RT}{PT} = \frac{RT}{PQ + QT} = \frac{B\sin\theta}{A + B\cos\theta} \qquad (1.3)$$

So, the direction of the resultant can be expressed in terms of the angle it makes with base vector.

Special Cases

Now, let us consider as to what would be the resultant of two vectors when they are parallel?

To find answer to this question, note that the angle between the two parallel vectors is zero and the resultant is equal to the sum of their magnitudes and in the direction of these vectors.

Suppose that two vectors are perpendicular to each other. What would be the magnitude of the resultant? In this case, $\theta = 90^\circ$ and $\cos\theta = 0$.

Suppose further that their magnitudes are equal. What would be the direction of the resultant?

Notice that $\tan\alpha = B/A = 1$. So what is α?

Also note that when $\theta = \pi$, the vectors become anti-parallel. In this case $\alpha = 0$. The resultant vector will be along **A** or **B**, depending upon which of these vectors has larger magnitude.

Example 1.4: A cart is being pulled by Ahmed north-ward with a force of magnitude 70 N. Hamid is pulling the same cart in the south-west direction with a force of magnitude 50 N. Calculate the magnitude and direction of the resulting force on the cart.

Solution :

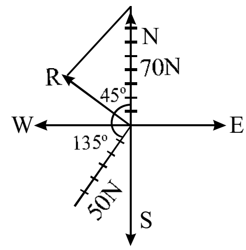


Fig. 1.9: Resultant of forces inclined at an angle

Here, magnitude of first force, say, A = 70 N.

The magnitude of the second force, say, B = 50 N.

Angle θ between the two forces = 135 degrees.

So, the magnitude of the resultant is given by Eqn. (1.2) :

$$R = \sqrt{(70)^2 + (50)^2 + 2 \times 70 \times 50 \times \cos(135)}$$

$$= \sqrt{4900 + 2500 - 7000 \times \sin 45}$$

$$= 49.5 \text{ N}$$

The magnitude of **R** = 49.5 N.

The direction is given by Eqn. (1.3):

$$\tan\alpha = \frac{B \sin\theta}{A + B\cos\theta} = \frac{50 \times \sin(135)}{70 + 50\cos(135)} = \frac{50 \times \cos 45}{70 - 50 \sin 45} = 1.00$$

Therefore, $\alpha = 45.0°$ (from the tables). Thus **R** makes an angle of 45° with 70 N force. That is, **R** is in North-west direction as shown in Fig. 1.9.

1.4.5 Subtraction of Vectors

How do we subtract one vector from another? If you recall that the difference of two vectors, **A – B**, is actually equal to **A + (–B)**, then you can adopt the same method as for addition of two vectors. It is explained in Fig. 1.10. Draw vector **–B** from the tip of **A**. Join the tail of **A** with the tip of **–B**. The resulting vector is the difference **(A – B)**.

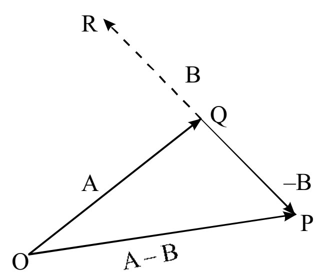
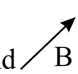


Fig. 1.10 : Subtraction of vector B from vector A

You may now like to check your progress.

INTEXT QUESTIONS 1.3

Given vectors 

and 

1. Make diagrams to show how to find the following vectors:

   (a) **B – A**, (b) **A + 2B**, (c) **A – 2B** and (d) **B – 2A**.

2. Two vectors **A** and **B** of magnitudes 10 units and 12 units are anti-parallel. Determine **A + B** and **A – B**.
3. Two vectors **A** and **B** of magnitudes A = 30 units and B = 60 units respectively are inclined to each other at angle of 60 degrees. Find the resultant vector.

1.5 MULTIPLICATION OF VECTORS

1.5.1 Multiplication of a Vector by a Scalar

If we multiply a vector **A** by a scalar k, the product is a vector whose magnitude is the absolute value of k times the magnitude of **A**. This means that the magnitude of the resultant vector is $k|\mathbf{A}|$. The direction of the new vector remains unchanged if k is positive. If k is negative, the direction of the new vector is opposite to its original direction. For example, vector 3**A** is thrice the magnitude of vector **A**, and it is in the same direction as **A**. But vector –3**A** is in a direction opposite to vector **A**, although its magnitude is thrice that of vector **A**.

1.5.2 Scalar Product of Vectors

The **scalar product** of two vectors **A** and **B** is written as **A.B** and is equal to AB cosθ, where θ is the angle between the vectors. If you look carefully at Fig. 1.11, you would notice that B cosθ is the projection of vector **B** along vector **A**. Therefore, the scalar product of **A** and **B** is the product of magnitude of **A** with the length of the projection of **B** along **A**. Another thing to note is that even if we take the angle between the two vectors as 360 – θ, it does not matter because the cosine of both angles is the same. Since a dot between the two vectors indicates the scalar product, it is also called the **dot product. Remember that the scalar product of two vectors is a scalar quantity.**

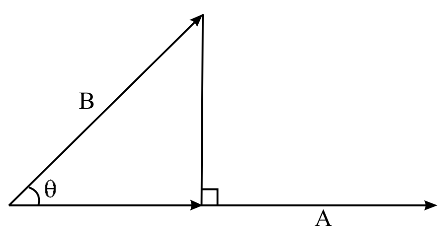


Fig. 1.11: Projection of B on A

A familiar example of the scalar product is the work done when a force **F** acts on a body moving at an angle to the direction of the force. If **d** is the displacement of the body and θ is the angle between **F** and **d**, then the work done by the force is $Fd\cos\theta$.

Since dot product is a scalar, it is commutative: **A.B** = **B.A** = ABcosθ. It is also distributive: **A.(B + C) = A.B + A.C**.

1.5.3 Vector Product of Vectors

Suppose we have two vectors **A** and **B** inclined at an angle θ. We can draw a plane which contains these two vectors. Let that plane be called Ω ((Fig. 1.12 a)

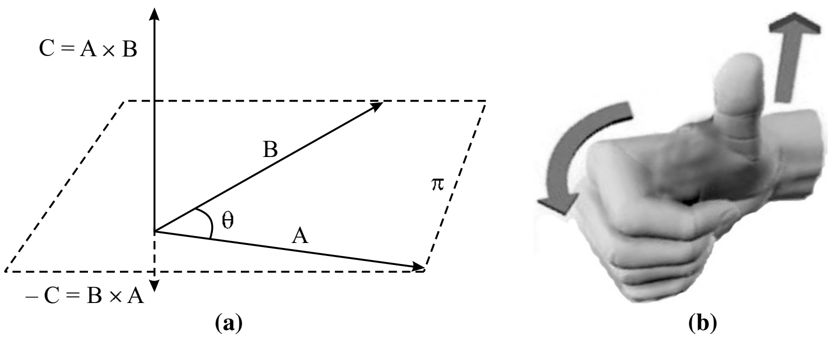


Fig.1.12 (a) : Vector product of Vectors; (b) Direction of the product vector C = A × B is given by the right hand rule. If the right hand is held so that the curling fingers point from A to B through the smaller angle between the two, then the thumb strectched at right angles to fingers will point in the direction of C.

which is perpendicular to the plane of paper here. Then the vector product of these vectors, written as **A × B**, is a vector, say **C**, whose magnitude is AB $\sin\theta$ and whose direction is perpendicular to the plane Ω. The direction of the vector **C** can be found by **right-hand rule** (Fig. 1.12 b). Imagine the fingers of your right hand curling from **A** to **B** along the smaller angle between them. Then the direction of the thumb gives the direction of the product vector **C**. If you follow this rule, you can easily see that direction of vector **B × A** is opposite to that of the vector **A × B**. This means that **the vector product is not commutative**. Since a cross is inserted between the two vectors to indicate their vector product, the vector product is also called the cross product.

A familiar example of vector product is the angular momentum possessed by a rotating body.

To check your progress, try the following questions.

INTEXT QUESTIONS 1.4

1. Suppose vector **A** is parallel to vector **B**. What is their vector product? What will be the vector product if **B** is anti-parallel to **A**?
2. Suppose we have a vector **A** and a vector **C** = $\frac{1}{2}$ **B.** How is the direction of vector **A × B** related to the direction of vector **A × C**.
3. Suppose vectors **A** and **B** are rotated in the plane which contains them. What happens to the direction of vector **C = A × B**.
4. Suppose you were free to rotate vectors **A** and **B** through arbitrary amounts keeping them confined to the same plane. Can you make vector **C = A × B** to point in exactly opposite direction?

5. If vector **A** is along the x-axis and vector **B** is along the y-axis, what is the direction of vector $\mathbf{C} = \mathbf{A} \times \mathbf{B}$? What happens to **C** if **A** is along the y-axis and **B** is along the x-axis?

6. **A** and **B** are two mutually perpendicular vectors. Calculate (a) **A . B** and (b) **A × B.**

1.6 RESOLUTION OF VECTORS

Resolution of vectors is converse of addition of vectors. Here we calculate components of a given vector along any set of coordinate axes. Suppose we have vector **A** as shown in Fig. 1.13 and we need to find its components along x and y-axes. Let these components be called A_x and A_y respectively. Simple trigonometry shows that

$$A_x = A \cos \theta \qquad (1.4)$$

and

$$A_y = A \sin \theta, \qquad (1.5)$$

where θ is the angle that **A** makes with the x - axis. What about the components of vector **A** along X and Y-axes (Fig. 1.13)? If the angle between the X-axis and **A** is ϕ, then

$$A_X = A \cos \phi$$

and

$$A_Y = A \sin \phi.$$

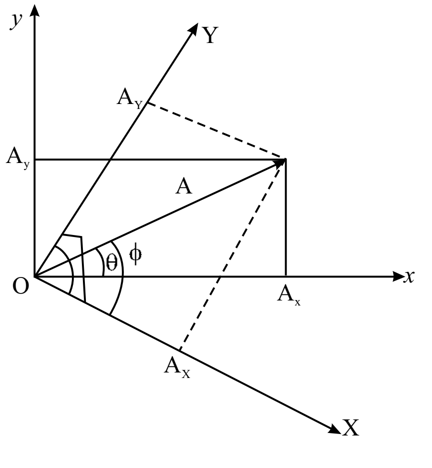


Fig. 1.13 : Resolution of vector A along two sets of coordinates (x, y) and (X, Y)

It must now be clear that the components of a vector are not fixed quantities; they depend on the particular set of axes along which components are required. Note also that the magnitude of vector **A** and its direction in terms of its components are given by

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{A_X^2 + A_Y^2} \qquad (1.6)$$

and $\tan\theta = A_y / A_x$, $\tan\phi = A_Y / A_X$. (1.7)

So, if we are given the components of a vector, we can combine them as in these equations to get the vector.

1.7 UNIT VECTOR

At this stage we introduce the concept of a **unit vector**. As the name suggests, a unit vector has unitary magnitude and has a specified direction. It has no units and no dimensions. As an example, we can write vector **A** as A $\hat{\mathbf{n}}$ where a cap on **n** (i.e. $\hat{\mathbf{n}}$) denotes a unit vector in the direction of **A**. Notice that a unit vector has been introduced to take care of the direction of the vector; the magnitude has been taken care of by A. Of particular importance are the unit vectors along coordinate axes. Unit vector along *x*-axis is denoted by $\hat{\mathbf{i}}$, along *y*-axis by $\hat{\mathbf{j}}$ and along *z*-axis by $\hat{\mathbf{k}}$. Using this notation, vector **A**, whose components along *x* and *y* axes are respectively A_x and A_y, can be written as

$$\mathbf{A} = A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}}\,. \tag{1.8}$$

Another vector **B** can similarly be written as

$$\mathbf{B} = B_x\,\hat{\mathbf{i}} + B_y\,\hat{\mathbf{j}}\,. \tag{1.9}$$

The sum of these two vectors can now be written as

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\,\hat{\mathbf{i}} + (A_y + B_y)\,\hat{\mathbf{j}} \tag{1.10}$$

By the rules of scalar product you can show that

$$\hat{\mathbf{i}}.\,\hat{\mathbf{i}}=1,\ \hat{\mathbf{j}}.\hat{\mathbf{j}}=1,\ \hat{\mathbf{k}}.\hat{\mathbf{k}}=1,\ \hat{\mathbf{i}}.\hat{\mathbf{j}}=0,\ \hat{\mathbf{i}}.\hat{\mathbf{k}}=0, \text{ and } \hat{\mathbf{j}}.\hat{\mathbf{k}}=0 \tag{1.11}$$

The dot product between two vectors **A** and **B** can now be written as

$$\begin{aligned}\mathbf{A}\,.\,\mathbf{B} &= (A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}}).\,(B_x\,\hat{\mathbf{i}} + B_y\,\hat{\mathbf{j}})\\ &= A_xB_x\,(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}) + A_xB_y\,(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}) + A_yB_x\,(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}) + A_yB_y\,(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}})\\ &= A_xB_x + A_yB_y,\end{aligned} \tag{1.12}$$

Here, we have used the results contained in Eqn. (1.11).

Example 1.4: On a coordinate system (showing all the four quadrants) show the following vectors:

$$\mathbf{A} = 4\hat{\mathbf{i}} + 0\hat{\mathbf{j}},\ \mathbf{B} = 0\hat{\mathbf{i}} + 5\hat{\mathbf{j}},\ \mathbf{C} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}},$$

$$\mathbf{D} = 6\hat{\mathbf{i}} - 4\hat{\mathbf{j}}.$$

Find their magnitudes and directions.

Solution : The vectors are given in component form. The factor multiplying $\hat{\mathbf{i}}$ is the x component and the factor multiplying $\hat{\mathbf{j}}$ is the y component. All the vectors are shown on the coordinate grid (Fig. 1.14).

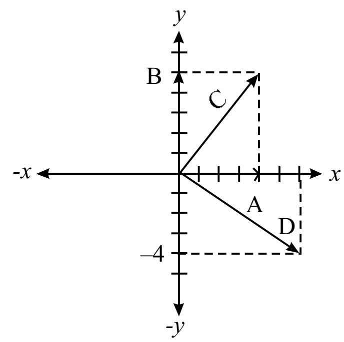


Fig. 1.14

The components of **A** are $\mathbf{A}_x = 4$, $\mathbf{A}_y = 0$. So, the magnitude of $\mathbf{A} = 4$. Its direction is $\tan^{-1}\left(\frac{A_y}{A_x}\right)$ in accordance with Eqn. (1.7). This quantity is zero, since $A_y = 0$. This makes it to be along the x-axis, as it is. Vector **B** has x-component = 0, so it lies along the y-axis and its magnitude is 5.

Let us consider vector **C**. Here, $\mathbf{C}_x = 4$ and $\mathbf{C}_y = 5$. Therefore, the magnitude of **C** is $C = \sqrt{4^2 + 5^2} = \sqrt{41}$. The angle that it makes with the x-axis is $\tan^{-1}(\mathbf{C}_y/\mathbf{C}_x) =$ 51.3 degrees. Similarly, the magnitude of **D** is $D = \sqrt{60}$. Its direction is $\tan^{-1}(\mathbf{D}_y/\mathbf{D}_x) = \tan^{-1}(0.666) = -33.7°$ (in the fourth quadrant).

Example 1.5: Calculate the product **C . D** for the vectors given in Example 1.4.

Solution : The dot product of **C** with **D** can be found using Eqn. (1.12):

$$\mathbf{C} \cdot \mathbf{D} = C_x D_x + C_y D_y = 4 \times 6 + 5 \times (-4) = 24 - 20 = 4.$$

The cross product of two vectors can also be written in terms of the unit vectors. For this we first need the cross product of unit vectors. For this remember that the angle between the unit vectors is a right angle. Consider, for example, $\hat{\mathbf{i}} \times \hat{\mathbf{j}}$. Sine of the angle between them is one. The magnitude of the product vector is also 1. Its direction is perpendicular to the xy - plane containing $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, which is the z-axis. By the right hand rule, we also find that this must be the positive z-axis. And what is the unit vector in the positive z - direction. The unit vector $\hat{\mathbf{k}}$. Therefore,

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}. \qquad (1.13)$$

Using similar arguments, we can show,

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}},\ \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}},\ \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}},\ \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}},\ \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}, \qquad (1.14)$$

and $$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0. \qquad (1.15)$$

Example 1.6: Calculate the cross product of vectors **C** and **D** given in Example (1.4).

Solution : We have

$$\mathbf{C} \times \mathbf{D} = (4\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}}) \times (6\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}})$$

$$= 24\,(\hat{\mathbf{i}} \times \hat{\mathbf{i}}) - 16\,(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) + 30\,(\hat{\mathbf{j}} \times \hat{\mathbf{i}}) - 20\,(\hat{\mathbf{j}} \times \hat{\mathbf{j}})$$

Using the results contained in Eqns. (1.13 – 1.15), we can write

$$\mathbf{C} \times \mathbf{D} = -16\ \hat{\mathbf{k}} - 30\ \hat{\mathbf{k}} = -46\ \hat{\mathbf{k}}$$

So, the cross product of **C** and **D** is a vector of magnitude 46 and in the negative *z* direction. Since **C** and **D** are in the *xy*-plane, it is obvious that the cross product must be perpendicular to this plane, that is, it must be in the *z*-direction.

INTEXT QUESTIONS 1.5

1. A vector **A** makes an angle of 60 degrees with the *x*-axis of the *xy*-system of coordinates. If its magnitude is 50 units, find its components in *x*, *y* directions. If another vector **B** of the same magnitude makes an angle of 30 degrees with the X-axis of the XY- system of coordinates. Find its components now. Are they same as before?
2. Two vectors **A** and **B** are given respectively as $3\ \hat{\mathbf{i}} - 4\hat{\mathbf{j}}$ and $-2\ \hat{\mathbf{i}} + 6\hat{\mathbf{j}}$. Sketch them on the coordinate grid. Find their magnitudes and the angles that they make with the *x*-axis (see Fig. 1.14).
3. Calculate the dot and cross product of the vectors given in the above question.

You now know that each term in an equation must have the same dimensions. Having learnt vectors, we must now add this: **For an equation to be correct, each term in it must have the same character: either all of them be vectors or all of them be scalars.**

WHAT YOU HAVE LEARNT

- The number of significant figures determines the accuracy of a measurement.
- Every physical quantity must be measured in some unit and also expressed in this unit. The SI system has been accepted and followed universally for scientific reporting.
- Base SI units for mass, length and time are respectively kg, m and s. In addition to base units, there are derived units.
- Every physical quantity has dimensions. Dimensional analysis is a useful tool for checking correctness of equations.
- In physics, we deal generally with two kinds of quantities, scalars and vectors. A scalar has only magnitude. A vector has both direction and magnitude.
- Vectors are added according to the parallelogram rule.
- The scalar product of two vectors is a scalar.
- The vector product of two vectors is a vector perpendicular to the plane containing the two vectors.
- Vectors can be resolved into components along a specified set of coordinates axes.

ANSWERS TO INTEXT QUESTIONS

1.1

4. (i) 5 (ii) 10 (iii) 4 (iv) 4 (v) 1

5. No, in both cases, the number of significant figures will be 4.

7. Mass of the sun = 2×10^{30} kg

   Mass of a proton = 2×10^{-27} kg

   (No of protons in the sun = $\dfrac{2\times10^{30}\ \text{kg}}{2\times10^{-27}\ \text{kg}} = 10^{57}$.

8. 1 angstrom = 10^{-8} cm = 10^{-10} m

   1 nanometer (nm) = 10^{-9} m

   $\therefore$ 1 nm/1 angstrom = 10^{-9} m /10^{-10} m = 10 so 1 nm = 10 Å

9. 1370 kHz = 1370×10^3 Hz = $(1370 \times 10^3)/10^9$ GHz = 1.370×10^{-3} GHz

10. 1 decameter (dam) = 10 m

    1 decimeter (dm) = 10^{-1} m

    $\therefore$ 1 dam = 100 dm

    1 MW = 10^6 W

    1 GW = 10^9 W

    $\therefore$ 1 GW = 10^3 MW

1.2

1. Dimension of length = L

   Dimension of time = T

   Dimensions of $g = LT^{-2}$

   Let time period t be proportional to l^{α} and g^{β}

   Then, writing dimensions on both sides $T = L^{\alpha} (LT^{-2})^{\beta} = L^{\alpha+\beta} T^{-2\beta}$

   Equating powers of L and T,

   $\alpha + \beta = 0,\ 2\beta = -1 \Rightarrow \beta = -1/2$ and $\alpha = 1/2$

   So, $t \propto \sqrt{\dfrac{l}{g}}$.

2. Dimension of $a = LT^{-2}$

Dimension of $\upsilon = LT^{-1}$

Dimension of $r = L$

Let a be proportional to υ^{α} and r^{β}

Then dimensionally,

$$LT^{-2} = (LT^{-1})^{\alpha}\, L^{\beta} = L^{\alpha+\beta}\, T^{-\alpha}$$

Equating powers of L and T,

$$\alpha + \beta = 1,\ \alpha = 2,\ \Rightarrow \alpha = -1$$

So, $\alpha \propto \upsilon^2/r$

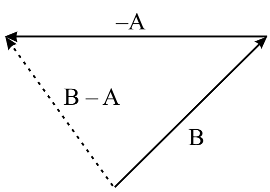
3. Dimensions of $m\upsilon = MLT^{-1}$

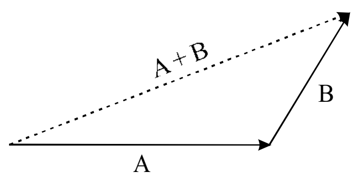
Dimensions of $Ft = MLT^{-2}\, T^{1} = MLT^{-1}$

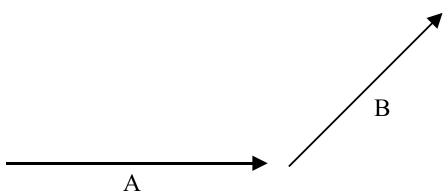
Dimensions of both the sides are the same, therefore, the equation is dimensionally correct.

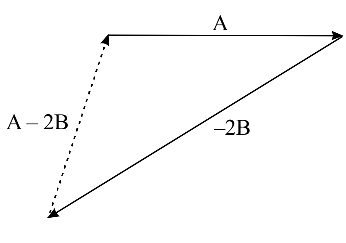
1.3

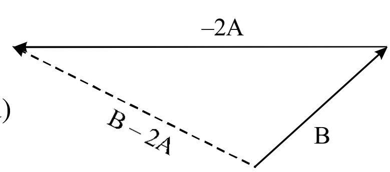
1. Suppose

(a) 

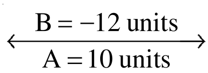
(b) 



(c) 

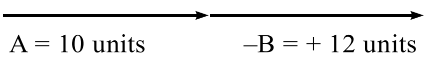
(d) 

2. 

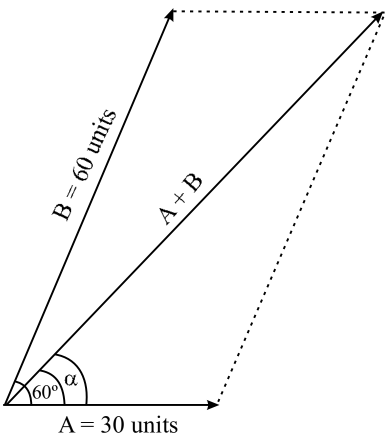


$A + B = 10 + (-12)$

$= -2$ units

also 

$\mathbf{A} - \mathbf{B} = 22$ units

3. 

$|A + B| = 77$ units

1.4

1. If **A** and **B** are parallel, the angle θ between them is zero. So, their cross product

   $\mathbf{A} \times \mathbf{B} = AB \sin\theta = 0.$

   If they are antiparallel then the angle between them is 180°. Therefore,

   $\mathbf{A} \times \mathbf{B} = AB \sin\theta = 0$, because $\sin 180^\circ = 0$.

2. If magnitude of **B** is halved, but it remains in the same plane as before, then the direction of the vector product $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ remains unchanged. Its magnitude may change.

3. Since vectors **A** and **B** rotate without change in the plane containing them, the direction of $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ will not change.

4. Suppose initially the angle between **A** and **B** is between zero and 180°. Then $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ will be directed upward perpendicular to the plane. After rotation through arbitrary amounts, if the angle between them becomes > 180°, then **C** will drop underneath but perpendicular to the plane.

5. If **A** is along x-axis and **B** is along y-axis, then they are both in the xy plane. The vector product $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ will be along z-direction. If **A** is along y-axis and **B** is along x-axis, then **C** is along the negative z-axis.

6. (a) $\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \cos\theta = 0$ when $\theta = 90°$

   (b) $\mathbf{A} \times \mathbf{B} = |\mathbf{A}|\,|\mathbf{B}| \sin\theta = |\mathbf{A}|\,|\mathbf{B}|$ as $\sin\theta = 1$ at $\theta = 90°$

1.5

1. When **A** makes an angle of 60° with the x-axis:

   $A_x = A \cos 60 = 50 \,.\, ½ = 25$ units

   $A_y = A \sin 60 = 50.\sqrt{3}/2 = 50 \,.\, 0.866$

   $= 43.3$ units

   When **A** makes an angle of 30° with the x-axis

   $A_x = 50 \cos 30 = 50 \,.\, 0.866 = 43.3$ units

   $A_y = 50 \sin 30 = 50 \,.\, ½ = 25$ units

   The components in the two cases are obviously not the same.

2. The position of vectors on the coordinate grid is shown in Fig. 1.14.

   Suppose **A** makes an angle θ with the x-axis, then

   $\tan\theta = -4/3 \Rightarrow \theta = \tan^{-1}(-4/3)$

   $= -53°\ 6'$ or $306°\ 54'$

   after taking account of the quadrant in which the angle lies.

   If B makes an angle ϕ with the x-axis, then

   $\tan\phi = 6/-2 = -3 \Rightarrow \phi = \tan^{-1}(-3)$

   $= 108°\ 24'$

3. The dot product of **A** and **B**:

   $$\mathbf{A} \cdot \mathbf{B} = (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}).(-2\hat{\mathbf{i}} + 6\hat{\mathbf{j}})$$

   $$= -6(\hat{\mathbf{i}}.\hat{\mathbf{i}}) - 24(\hat{\mathbf{j}}.\hat{\mathbf{j}}) = -30$$

   because $\hat{\mathbf{i}}.\hat{\mathbf{j}} = \hat{\mathbf{j}}.\hat{\mathbf{i}} = 0$, and $\hat{\mathbf{i}}.\hat{\mathbf{i}} = \hat{\mathbf{j}}.\hat{\mathbf{j}} = 1$

   The cross product of **A** and **B**:

$$\mathbf{A} \times \mathbf{B} = (3\hat{\mathbf{i}} - 4\hat{\ }) \times (-2\hat{\mathbf{i}} + 6\hat{\ })$$

$$= 18\,(\hat{\mathbf{i}} \times \hat{\ }) + 8\,(\hat{\ } \times \hat{\mathbf{i}}) = 18\,\hat{\mathbf{k}} - 8\,\hat{\mathbf{k}} = 10\,\hat{\mathbf{k}}$$

on using Eqs.(1.14) and (1.15). So, the cross product is in the direction of z-axis, since **A** and **B** lie in the *xy* plane.