SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	2(2)	–	1(3)	–	**3(5)**
2.	Inverse Trigonometric Functions	1(1)	1(2)	–	–	**2(3)**
3.	Matrices	2(2)	–	–	–	**2(2)**
4.	Determinants	1(1)*	1(2)	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	1(1)	1(2)	2(6)#	–	**4(9)**
6.	Application of Derivatives	1(4)	1(2)	1(3)	–	**3(9)**
7.	Integrals	1(1)*	1(2)*	2(6)#	–	**4(9)**
8.	Application of Integrals	–	1(2)	–	–	**1(2)**
9.	Differential Equations	1(1)*	1(2)*	1(3)	–	**3(6)**
10.	Vector Algebra	1(1)	1(2)*	–	–	**2(3)**
11.	Three Dimensional Geometry	2(2)# + 1(4)	–	–	1(5)*	**4(11)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	4(4)#	2(4)	–	–	**6(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Evaluate : $\int \frac{(a^x + b^x)^2}{a^x b^x} dx$

OR

Evaluate : $\int \frac{dx}{\sqrt{1-2x-x^2}}$

2. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I = O$, then find the value of k.

3. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

OR

If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B \mid A) = 0.6$, then find $P(A \cup B)$.

4. Differentiate the function $\left(\frac{2 \tan x}{\tan x + \cos x}\right)^2$ w.r.t. x.

5. Find the cofactors of the element of third row and second column of the following determinant $\begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix}$.

OR

If A is a matrix of order 3×3 and $|A| = 5$, then find the value of $|\text{adj } A|$.

6. Set A has three elements and set B has four elements. Find the number of injections that can be defined from A to B.

7. Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$.

OR

Find the solution of $y' = y \cot 2x$.

8. Find the principal value of $\cot^{-1}(-\sqrt{3})$.

9. Find the direction cosines of a line, for which $\alpha = \beta$ and $\gamma = 45°$.

OR

If $P = (-2, 3, 6)$, then find the d.c.'s of OP.

10. How many equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) are there in all?

11. If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with x-axis, then find the value of α.

12. If A and B are two independent events such that $P(A \cup B) = 0.6$ and $P(A) = 0.2$, then find $P(B)$.

13. If $\begin{pmatrix} 2x+y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 6 & 4 \end{pmatrix}'$, then $x =$ ______.

14. If A and B are events such that $P(A) > 0$ and $P(B) \neq 1$, then prove that $P(A' \mid B') = \frac{1 - P(A \cup B)}{P(B')}$.

15. Find the value of k in the following probability distribution.

$X = x$	0.5	1	1.5	2
$P(X = x)$	k	k^2	$2k^2$	k

16. If the angle between $\hat{i} + \hat{k}$ and $\hat{i} + \hat{j} + a\hat{k}$ is $\frac{\pi}{3}$, then find the value of a.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. A poster is to be formed for a company advertisement. The top and bottom margins of poster should be 4 cm and the side margins should be 6 cm. Also, the area for printing the advertisement should be 384 cm^2. Based on the above answer the following :

(i) If a be the width and b be the height of poster, then the area of poster, expressed in terms of a and b, is given by

(a) $288 + 8a + 12b$ (b) $8a + 12b$ (c) $384 + 8a + 12b$ (d) none of these

(ii) The relation between a and b is given by

(a) $a = \frac{288 + 12b}{b - 8}$ (b) $a = \frac{12b}{b - 8}$ (c) $a = \frac{12b}{b + 8}$ (d) none of these

(iii) Area of poster in terms of b is

(a) $\frac{12b^2}{b-8}$ (b) $\frac{288b+12b^2}{b-8}$ (c) $\frac{288b+12b^2}{b+8}$ (d) $\frac{12b^2}{b+8}$

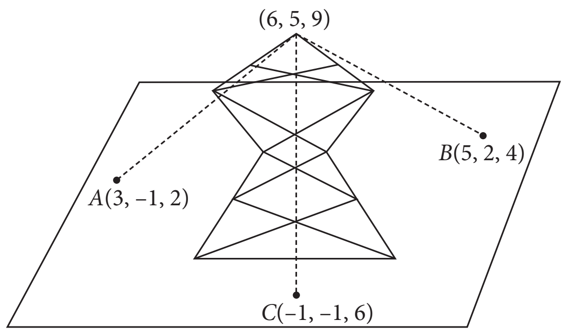
(iv) The value of b, so that area of the poster is minimized, is

(a) 24 (b) 36 (c) 18 (d) 22

(v) The value of a, so that area of the poster is minimized, is

(a) 24 (b) 36 (c) 18 (d) 22

18. Consider the earth as a plane having points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$ on it. A mobile tower is tied with 3 cables from the point A, B and C such that it stand vertically on the ground. The peak of the tower is at the point (6, 5, 9), as shown in the figure.



Based on the above answer the following :

(i) The equation of plane passing through the points A, B and C is

(a) $3x - 4y + 3z = 0$ (b) $3x - 4y + 3z = 19$ (c) $4x - 3y + 3z = 0$ (d) $4x - 3y + 3z = 19$

(ii) The height of the tower from the ground is

(a) 6 units (b) 5 units (c) $\frac{6}{\sqrt{34}}$ units (d) $\frac{5}{\sqrt{34}}$ units

(iii) The equation of line of perpendicular drawn from its peak to the ground is

(a) $\frac{x-6}{3} = \frac{y-4}{-5} = \frac{z-9}{3}$ (b) $\frac{x-6}{3} = \frac{y-5}{-4} = \frac{z-9}{3}$

(c) $\frac{x-6}{3} = \frac{y-4}{5} = \frac{z-9}{3}$ (d) $\frac{x-6}{3} = \frac{y-5}{4} = \frac{z-9}{3}$

(iv) The coordinates of foot of perpendicular are

(a) $\left(\frac{93}{17}, \frac{97}{17}, \frac{144}{17}\right)$ (b) $\left(\frac{144}{17}, \frac{97}{17}, \frac{93}{17}\right)$ (c) $\left(\frac{91}{17}, \frac{93}{17}, \frac{144}{17}\right)$ (d) none of these

(v) The area of ΔABC is

(a) $\sqrt{34}$ sq. units (b) $2\sqrt{34}$ sq. units (c) $\sqrt{17}$ sq. units (d) $2\sqrt{7}$ sq. units

PART – B

Section III

19. Find the derivative of the function $\sqrt{a + \sqrt{a + x}}$ w.r.t. x.

20. Evaluate : $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx$

OR

Evaluate : $\int \frac{1}{\sin x + \sqrt{3} \cos x} dx$

21. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Determine:

(i) K (ii) $P(X < 3)$

22. If $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1}x)$, then find x.

23. Solve the differential equation $\cos^2(x-2y) = 1 - 2\dfrac{dy}{dx}$.

OR

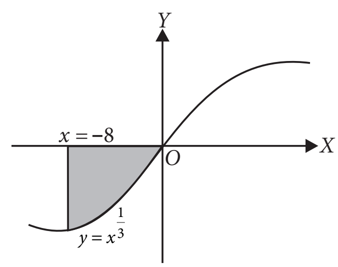
Find the solution of the differential equation $x + y\dfrac{dy}{dx} = \sec(x^2+y^2)$.

24. Find the equation of normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at (1, 1).

25. If $P(\text{not } A) = 0.7$, $P(B) = 0.7$ and $P(B \mid A) = 0.5$, then find $P(A \mid B)$ and $P(A \cup B)$.

26. Find the inverse of the matrix $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$.

27. Compute the shaded area shown in the given figure.



28. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{k}$.

OR

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ having the same length $\sqrt{2}$ and their scalar product is -1.

Section - IV

29. Let a relation R on the set A of real numbers be defined as $(a, b) \in R \Rightarrow 1 + ab > 0$ for all $a, b \in A$. Show that R is reflexive and symmetric but not transitive.

30. Sketch the graph $y = |x + 1|$. Evaluate $\int_{-4}^{2} |x + 1|\, dx$.

31. Evaluate : $\int \dfrac{x^2+9}{x^4+81}\, dx$

OR

Evaluate : $\int x^2 \sin 2x\, dx$

32. Solve : $\sin^{-1}\left(\dfrac{dy}{dx}\right) = x + y$

33. If $f(x)=\begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{for } x \neq 0 \\ 1, & \text{for } x = 0 \end{cases}$, then show that the function is discontinuous at $x = 0$.

34. If $(ax + b)\, e^{y/x} = x$, then show that $x^3 \dfrac{d^2y}{dx^2} = \left(x\dfrac{dy}{dx} - y\right)^2$.

OR

Find $\dfrac{dy}{dx}$, when $x = a\left\{\cos t + \dfrac{1}{2}\log \tan^2 \dfrac{t}{2}\right\}$ and $y = a \sin t$.

35. Show that the condition that the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ should intersect orthogonally is $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{a'} - \dfrac{1}{b'}$.

Section-V

36. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{bmatrix}$, then find A^{-1}. Hence find $|\text{adj } A|$ and $|A^{-1}|$.

OR

Find the inverse of $A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$. Hence find $(A^{-1})^2$.

37. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i} - 7\hat{j} + 4\hat{k}) = 3$ and $\vec{r} \cdot (3\hat{i} - 5\hat{j} + 4\hat{k}) + 11 = 0$ and passing through the point $(-2, 1, 3)$.

OR

Find the co-ordinates of the points on the line $x - 2 = \dfrac{y+3}{-2} = \dfrac{z+5}{2}$, which are on either side of the point $A(2, -3, -5)$ at a distance of 3 units from it.

38. Solve the following LPP graphically :

Maximize $Z = 600x + 400y$

subject to the constraints :

$x + 2y \leq 12,\ 2x + y \leq 12$

$x + \dfrac{5}{4}y \geq 5$ and $x, y \geq 0$.

OR

Find the number of points at which the objective function $z = 3x + 2y$ can be maximized subject to $3x + 5y \leq 15$, $5x + 2y \leq 20$, $x \geq 0$, $y \geq 0$.

SOLUTIONS

1. We have, $\int \frac{(a^x+b^x)^2}{a^xb^x}dx = \int \frac{a^{2x}+b^{2x}+2a^xb^x}{a^xb^x}dx$

$$= \int\left(\left(\frac{a}{b}\right)^x + \left(\frac{b}{a}\right)^x + 2\right)dx = \frac{\left(\frac{a}{b}\right)^x}{\log\frac{a}{b}} + \frac{\left(\frac{b}{a}\right)^x}{\log\frac{b}{a}} + 2x + C, a \neq b$$

OR

Let $I = \int \frac{dx}{\sqrt{1-(x^2+2x)}} = \int \frac{dx}{\sqrt{2-(x^2+2x+1)}}$

$$= \int \frac{dx}{\sqrt{2-(1+x)^2}} = \int \frac{dx}{\sqrt{(\sqrt{2})^2-(1+x)^2}}$$

Let $1 + x = z \Rightarrow dx = dz$

$$\therefore\ I = \int \frac{dz}{\sqrt{(\sqrt{2})^2 - z^2}} = \sin^{-1}\frac{z}{\sqrt{2}} + c = \sin^{-1}\left(\frac{1+x}{\sqrt{2}}\right) + c$$

2. Given, $A^2 - kA - 5I = O$

$\Rightarrow kA = A^2 - 5I$

$$\Rightarrow kA = \begin{bmatrix}1 & 3\\3 & 4\end{bmatrix}\begin{bmatrix}1 & 3\\3 & 4\end{bmatrix} - 5\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}10 & 15\\15 & 25\end{bmatrix} - \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix} = \begin{bmatrix}5 & 15\\15 & 20\end{bmatrix} = 5\begin{bmatrix}1 & 3\\3 & 4\end{bmatrix} = 5A$$

$\Rightarrow kA = 5A \quad \therefore \quad k = 5$

3. Let E : 'a total of 8' and F : 'red die resulted in a number less than 4'

i.e., $E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

and $F = \{(x, y) : x \in \{1, 2, 3, 4, 5, 6\}, y \in \{1, 2, 3\}\}$

i.e., $F = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$

Hence, $E \cap F = \{(5, 3), (6, 2)\}$

$P(E) = 5/36$,

$P(F) = 18/36$, $P(E \cap F) = 2/36$

$\therefore$ Required probability $= P(E|F)$

$$= \frac{P(E\cap F)}{P(F)} = \frac{2/36}{18/36} = \frac{2}{18} = \frac{1}{9}.$$

OR

Given, $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$

Clearly, $P(A \cap B) = P(B|A)P(A) = 0.6 \times 0.4 = 0.24$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.8 - 0.24 = 0.96$

4. Let $y = \left(\frac{2\tan x}{\tan x + \cos x}\right)^2$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 2\left(\frac{2\tan x}{\tan x + \cos x}\right)\cdot\frac{(\tan x + \cos x)\cdot 2\sec^2 x - 2\tan x\cdot(\sec^2 x - \sin x)}{(\tan x + \cos x)^2}$$

$$= \frac{8\tan x\,(\cos x\sec^2 x + \tan x\sin x)}{(\tan x + \cos x)^3}$$

$$= \frac{8\tan x\,(\sec x + \tan x\sin x)}{(\tan x + \cos x)^3}$$

5. $M_{32} = \begin{vmatrix}1 & y+z\\1 & z+x\end{vmatrix} = z + x - y - z = x - y$

$\Rightarrow c_{32} = -M_{32} = y - x$

OR

$|\text{adj } A| = |A|^{n-1}$

$= 5^{(3-1)} = 5^2 = 25$

6. Since $3 < 4$, injective functions from A to B are defined and the total number of such functions is ${}^4P_3$

$$= \frac{4!}{(4-3)!} = 4 \times 3 \times 2 \times 1 = 24.$$

7. We have, $\frac{dy}{dx} = x^3e^{-2y} \Rightarrow e^{2y}dy = x^3dx$

On integrating, we get $\frac{e^{2y}}{2} = \frac{x^4}{4} + C'$

$\Rightarrow 2e^{2y} = x^4 + C$, where $C = 4C'$

OR

We have, $y' = y\cot 2x \Rightarrow \frac{dy}{dx} = y\cot 2x$

$\Rightarrow \frac{dy}{y} = \cot 2x\,dx$

Integrating both sides, we get

$$\int\frac{dy}{y} = \int\cot 2x\,dx$$

$\Rightarrow \log|y| = \frac{1}{2}\log|\sin 2x| + \log c$

$\Rightarrow \log|y| = \log\left|\sqrt{\sin 2x}\right| + \log c$

$\Rightarrow \log|y| = \log\left|c\sqrt{\sin 2x}\right| \Rightarrow y = c\sqrt{\sin 2x}$

8. Let $\cot^{-1}\left(-\sqrt{3}\right) = \theta \Rightarrow \cot\theta = -\sqrt{3} = -\cot\dfrac{\pi}{6}$

$$= \cot\left(\pi - \frac{\pi}{6}\right) = \cot\frac{5\pi}{6} \;\Rightarrow\; \theta = \frac{5\pi}{6} \in (0, \pi)$$

$\therefore$ Principal value of $\cot^{-1}\left(-\sqrt{3}\right)$ is $\dfrac{5\pi}{6}$.

9. Since, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 2\cos^2\alpha + \cos^2 45° = 1$ $\quad (\because \alpha = \beta)$

$\Rightarrow 2\cos^2\alpha = 1 - \dfrac{1}{2} = \dfrac{1}{2} \Rightarrow \cos^2\alpha = \dfrac{1}{4}$

$\Rightarrow \cos\alpha = \pm\dfrac{1}{2}$

So, dc's are $\left(\pm\dfrac{1}{2}, \pm\dfrac{1}{2}, \dfrac{1}{2}\right)$

OR

Here, $O \equiv (0, 0, 0)$ and $P \equiv (-2, 3, 6)$

Direction ratios of OP are $-2 - 0, 3 - 0, 6 - 0$ *i.e.*, $-2, 3, 6$

$\therefore$ Direction cosines of OP are

$$< \frac{-2}{\sqrt{(-2)^2 + 3^2 + 6^2}}, \frac{3}{\sqrt{(-2)^2 + 3^2 + 6^2}}, \frac{6}{\sqrt{(-2)^2 + 3^2 + 6^2}} >$$

i.e., $< \dfrac{-2}{7}, \dfrac{3}{7}, \dfrac{6}{7} >$

10. Possible equivalence relations are {(1, 2), (2, 1), (1, 1), (2, 2), (3, 3)} and {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2)}

Hence, there are two possible equivalence relations.

11. Direction ratios of x-axis is (1, 0, 0) and direction ratios of the normal to the plane $2x - 3y + 6z = 11$ is $(2, -3, 6)$.

Then, $\sin(\sin^{-1}\alpha) = \dfrac{2 + 0 + 0}{\sqrt{0^2 + 0^2 + 1^2}\sqrt{2^2 + (-3)^2 + 6^2}}$

$\Rightarrow \alpha = \left(\dfrac{2}{7}\right)$

12. If A and B are two independent events, then

$P(A \cap B) = P(A) \times P(B)$

It is given that $P(A \cup B) = 0.6$, $P(A) = 0.2$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow$ $P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

$\Rightarrow$ $0.6 = 0.2 + P(B)(1 - 0.2)$

$\Rightarrow$ $0.4 = P(B)\,(0.8)$

$\Rightarrow$ $P(B) = \dfrac{0.4}{0.8} \Rightarrow P(B) = \dfrac{1}{2} = 0.5$

13. We have, $\begin{pmatrix} 2x + y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 6 & 4 \end{pmatrix}'$

$\Rightarrow \begin{pmatrix} 2x + y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 6 \\ 0 & 4 \end{pmatrix}$

By equality of two matrices, we have

$2x + y = 6$ and $3y = 6 \Rightarrow y = 2$.

Putting the value of y, we get

$2x + 2 = 6 \Rightarrow 2x = 4 \Rightarrow x = 2$.

14. By definition, $P(A' \mid B') = \dfrac{P(A' \cap B')}{P(B')}$

$$= \frac{P((A \cup B)')}{P(B')} = \frac{1 - P(A \cup B)}{P(B')}$$

15. Since $P(X)$ is a probability distribution of X,

$\therefore \displaystyle\sum_{x_i = 0.5}^{2} P(X = x) = 1$

$\Rightarrow$ $P(X = 0.5) + P(X = 1) + P(X = 1.5) + P(X = 2) = 1$

$\Rightarrow$ $k + k^2 + 2k^2 + k = 1 \Rightarrow 3k^2 + 2k - 1 = 0$

$\Rightarrow$ $(3k - 1)(k + 1) = 0$

$\Rightarrow$ $k = \dfrac{1}{3}$ or -1

But $P(X = 0.5) = k = -1$, which is not possible

$\therefore$ $k = \dfrac{1}{3}$

16. We have, $\cos\dfrac{\pi}{3} = \dfrac{(\hat{i} + \hat{k}) \cdot (\hat{i} + \hat{j} + a\hat{k})}{\sqrt{2}\sqrt{1 + 1 + a^2}}$

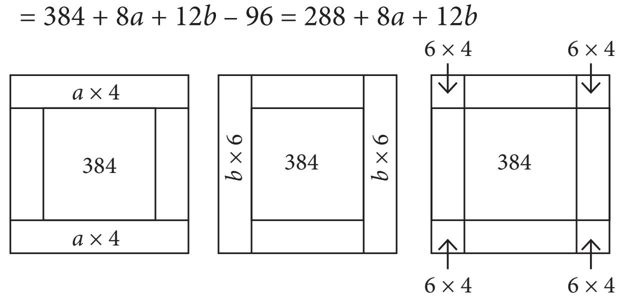
$\Rightarrow \dfrac{1}{2} = \dfrac{1 + a}{\sqrt{2}\sqrt{2 + a^2}} \Rightarrow \dfrac{1}{4} = \dfrac{(1 + a)^2}{2(2 + a^2)}$

$\Rightarrow 2 + a^2 = 2(1 + a^2 + 2a) \Rightarrow a^2 + 4a = 0 \Rightarrow a = 0, -4$

17. (i) (a) : Let A be the area of the poster, then

$A = 384 + 2(a \cdot 4) + 2(b \cdot 6) - 4(6 \cdot 4)$

$= 384 + 8a + 12b - 96 = 288 + 8a + 12b$



(ii) (a) : Clearly, $A = a \cdot b$

$\therefore$ $288 + 8a + 12b = ab$

$\Rightarrow$ $ab - 8a = 288 + 12b \Rightarrow a(b - 8) = 288 + 12b$

$\Rightarrow$ $a = \dfrac{288 + 12b}{b - 8}$

(iii) (b) : Since, $A = a \cdot b$, therefore

$A = \left(\dfrac{288 + 12b}{b - 8}\right) \cdot b = \dfrac{288b + 12b^2}{b - 8}$ $\quad \left[\because a = \dfrac{288 + 12b}{b - 8}\right]$

(iv) (a) : Clearly,

$$A'(b)=\frac{(b-8)(288+24b)-(288b+12b^2)}{(b-8)^2}$$

$$=\frac{12[b^2-16b-192]}{(b-8)^2}$$

For minimum, consider $A'(b) = 0$

$\Rightarrow b^2 - 16b - 192 = 0$

$\Rightarrow b^2 - 24b + 8b - 192 = 0$

$\Rightarrow b(b - 24) + 8(b - 24) = 0$

$\Rightarrow b = -8$ or $b = 24$

$\because$ b is height, therefore can't be negative.

So, $b = 24$.

(v) (b) : Since, $a=\frac{288+12b}{b-8}$

$$\therefore\ a=\frac{288+12\times24}{24-8}=\frac{288+288}{16}=36$$

18. (i) (b) : The equation of plane passing through three non-collinear points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} x-3 & y+1 & z-2 \\ 5-3 & 2+1 & 4-2 \\ -1-3 & -1+1 & 6-2 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix}=0$$

$\Rightarrow (x - 3)\ 12 - (y + 1)\ [8 + 8] + (z - 2)\ (12) = 0$

$\Rightarrow 12x - 16y + 12z - 36 - 16 - 24 = 0$

$\Rightarrow 12x - 16y + 12z = 76$

$\Rightarrow 3x - 4y + 3z = 19$

(ii) (c) : Height of tower = Perpendicular distance from the points (6, 5, 9) to the plane $3x - 4y + 3z = 19$

$$=\left|\frac{18-20+27-19}{\sqrt{3^2+(-4)^2+3^2}}\right|=\frac{6}{\sqrt{34}}\text{ units}$$

(iii) (b) : dr's of perpendicular are < 3, –4, 3 >

[$\because$ Perpendicular is parallel to the normal to the plane]

Since, perpendicular is passing through the point (6, 5, 9), therefore its equation is

$$\frac{x-6}{3}=\frac{y-5}{-4}=\frac{z-9}{3}$$

(iv) (a) : Let the coordinates of foot of perpendicular are $(3\lambda + 6, -4\lambda + 5, 3\lambda + 9)$

Since, this point lie on the plane $3x - 4y + 3z = 19$, therefore we get

$3(3\lambda + 6) - 4(-4\lambda + 5) + 3(3\lambda + 9) - 19 = 0$

$\Rightarrow 9\lambda + 16\lambda + 9\lambda + 18 - 20 + 27 - 19 = 0$

$\Rightarrow 34\lambda = -6$

$\Rightarrow \lambda=\frac{-6}{34}=\frac{-3}{17}$

Thus, the coordinates of foot of perpendicular are

$$\left(\frac{-9}{17}+6, \frac{12}{17}+5, \frac{-9}{17}+9\right)$$

i.e., $\left(\frac{93}{17}, \frac{97}{17}, \frac{144}{17}\right)$

(v) (b) : Clearly, Area of $ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$$=\frac{1}{2}\left|(2\hat{i}+3\hat{j}+2\hat{k})\times(-4\hat{i}+4\hat{k})\right|$$

$$=\frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{matrix}\right\|$$

$$=\frac{1}{2}|12\hat{i}-16\hat{j}+12\hat{k}|$$

$$=\frac{1}{2}\sqrt{12^2+16^2+12^2}$$

$$=\frac{1}{2}\sqrt{544}=2\sqrt{34}\text{ sq. units}$$

19. Let $y=\sqrt{a+\sqrt{a+x}}=\left(a+\sqrt{a+x}\right)^{1/2}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{2}(a+\sqrt{a+x})^{\frac{1}{2}-1}\frac{d}{dx}(a+\sqrt{a+x})$$

$$=\frac{1}{2\sqrt{a+\sqrt{a+x}}}\left\{\frac{1}{2}(a+x)^{\frac{1}{2}-1}\frac{d}{dx}(a+x)\right\}$$

$$=\frac{1}{4\sqrt{a+x}\sqrt{a+\sqrt{a+x}}}(0+1)=\frac{1}{4\sqrt{a+x}\sqrt{a+\sqrt{a+x}}}$$

20. Let $I=\int\frac{10x^9+10^x\log_e 10}{10^x+x^{10}}dx$

Put $10^x + x^{10} = t$

$\Rightarrow (10^x \log_e 10 + 10x^9)dx = dt$

$\therefore\ I=\int\frac{dt}{t}$

$= \log_e t + c = \log_e(10^x + x^{10}) + c$

OR

Let $I=\int\frac{1}{\sin x+\sqrt{3}\cos x}dx=\frac{1}{2}\int\frac{dx}{\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x}$

$$\Rightarrow I = \frac{1}{2}\int \frac{1}{\sin\left(x+\frac{\pi}{3}\right)}dx = \frac{1}{2}\int \operatorname{cosec}\left(x+\frac{\pi}{3}\right)dx$$

$$\Rightarrow I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)\right| + C$$

21. (i) Since $\Sigma P(X) = 1$

$\therefore\ 0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$\Rightarrow 10K^2 + 9K - 1 = 0$

$$\Rightarrow K = \frac{-9 \pm \sqrt{81+40}}{20} = \frac{-9\pm 11}{20} = \frac{1}{10}, -1$$

Since the probability of the event lies between 0 and 1.

So, $K = \frac{1}{10}$.

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0 + K + 2K = 3K = \frac{3}{10}$ $\left(\because K = \frac{1}{10}\right)$

22. We have, $\sin[\cot^{-1}(x + 1)] = \cos(\tan^{-1}x)$ ... (i)

Let $\cot^{-1}(x + 1) = A$ and $\tan^{-1} x = B$

$$\Rightarrow x + 1 = \cot A \Rightarrow \sin A = \frac{1}{\sqrt{(x+1)^2+1}}$$

Also, $x = \tan B \Rightarrow \cos B = \frac{1}{\sqrt{x^2+1}}$

Now, $\sin A = \cos B$ [From (i)]

$$\Rightarrow \frac{1}{\sqrt{(x+1)^2+1}} = \frac{1}{\sqrt{x^2+1}} \Rightarrow (x + 1)^2 + 1 = x^2 + 1$$

$$\Rightarrow 1 + 2x = 0 \Rightarrow x = -\frac{1}{2}$$

23. Given, $\cos^2(x - 2y) = 1 - 2\frac{dy}{dx}$ ...(i)

Let, $x - 2y = u \Rightarrow 1 - \frac{2dy}{dx} = \frac{du}{dx}$

$\therefore$ equation (i) becomes $\cos^2 u = \frac{du}{dx}$

$\Rightarrow \int dx = \int \sec^2 u\, du$

$\Rightarrow x = \tan u + c \Rightarrow x = \tan(x - 2y) + c$

OR

We have $x + y\frac{dy}{dx} = \sec(x^2 + y^2)$

Put $x^2 + y^2 = u \Rightarrow x + y\frac{dy}{dx} = \frac{1}{2}\frac{du}{dx}$

$\therefore \frac{1}{2}\frac{du}{dx} = \sec u \Rightarrow \int \cos u\, du = 2\int dx$

$\Rightarrow \sin u = 2x + c \Rightarrow \sin(x^2 + y^2) = 2x + c$

24. Differentiating $x^{2/3} + y^{2/3} = 2$ with respect to x, we get

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}$$

$\therefore$ Slope of the tangent at (1, 1) = – 1

Also, the slope of the normal at (1, 1) is given by

$$\frac{-1}{\text{slope of the tangent at (1, 1)}} = 1$$

Therefore, the equation of the normal at (1, 1) is

$y - 1 = 1(x - 1) \Rightarrow y - x = 0$

25. We have, $P(\text{not } A) = 0.7$ or $P(\bar{A}) = 0.7$

$\Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$ $[\because P(A) + P(\bar{A}) = 1]$

Now, $P(B|A) = \frac{P(A\cap B)}{P(A)}$

$\Rightarrow 0.5 = \frac{P(A\cap B)}{0.3} \Rightarrow P(A \cap B) = 0.15$

$\therefore P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{3}{14}$

and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.7 - 0.15 = 0.85$

26. We have, $|A| = \begin{vmatrix} 2 & -3 \\ -4 & 7 \end{vmatrix} = 14 - 12 = 2 \neq 0$

So, A^{-1} exists

$\therefore$ adj $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$$= \frac{1}{2}\cdot\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7/2 & 3/2 \\ 2 & 1 \end{bmatrix}$$

27. Required area

$$= \left|\int_{-8}^{0} x^{1/3}\, dx\right| = \left|\left[\frac{x^{4/3}}{4/3}\right]_{-8}^{0}\right| = \left|\frac{3}{4}[0-(-8)^{4/3}]\right|$$

$$= \left|\frac{3}{4}[-(-2)^4]\right| = \frac{3}{4}\times 16 = 12 \text{ sq. units}$$

28. We are given, $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{k}$

$$\therefore \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix}$$

$$= (9-0)\hat{i} - (3-2)\hat{j} + (0+3)\hat{k} = 9\hat{i} - \hat{j} + 3\hat{k}$$

$$\therefore |\vec{a}\times\vec{b}| = \sqrt{9^2 + (-1)^2 + 3^2} = \sqrt{81+1+9} = \sqrt{91}$$

OR

Let θ be the angle between vectors $\vec{a}$ and $\vec{b}$.

We have, $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a}\cdot\vec{b} = -1$

$$\therefore \quad \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \Rightarrow \cos\theta = \frac{-1}{\sqrt{2}\times\sqrt{2}} = -\frac{1}{2}$$

$$\Rightarrow \cos\theta = \cos\frac{2\pi}{3} \Rightarrow \theta = \frac{2\pi}{3}$$

Hence, the angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$.

29. Reflexive : Let a be any real number, then

$1 + aa = 1 + a^2 > 0$ $\quad(\because a^2 > 0$ for all $a \in A)$

So, R is reflexive.

Symmetric : Let $(a, b) \in R$, then

$1 + ab > 0 = 1 + ba > 0$ $\quad(\because ab = ba$ for all $a, b \in A)$

$\Rightarrow (b, a) \in R$

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$.

Hence, R is symmetric.

Transitive : We observe that

$\left(1, \frac{1}{2}\right) \in R$ and $\left(\frac{1}{2}, -1\right) \in R$ but $(1, -1) \notin R$ because

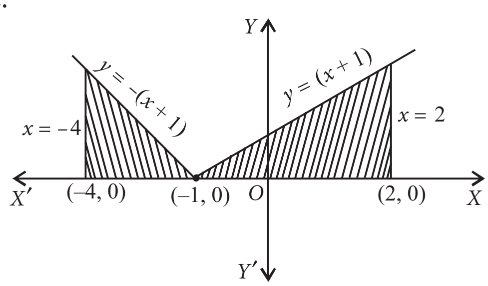
$1 + 1 \times (-1) = 0 \not> 0$

Hence, R is not transitive on A.

30. We have, $y = |x + 1|$

$$\therefore \quad y = \begin{cases} -(x+1) & x < -1 \\ (x+1) & x \geq -1 \end{cases}$$

The rough sketch of the curve $y = |x + 1|$ is shown in figure.



$$\therefore \int_{-4}^{2} |x+1|dx = \int_{-4}^{-1} -(x+1)dx + \int_{-1}^{2} (x+1)dx$$

$$= -\left[\frac{x^2}{2} + x\right]_{-4}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^{2}$$

$$= -\left[\left(\frac{1}{2} - 1\right) - \left(\frac{16}{2} - 4\right)\right] + \left[\left(\frac{4}{2} + 2\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= -\left[-\frac{1}{2} - 4\right] + \left[4 + \frac{1}{2}\right] = \frac{9}{2} + \frac{9}{2} = 9$$

31. Let $I = \int \frac{x^2+9}{x^4+81}dx \Rightarrow I = \int \frac{1+9/x^2}{x^2+\frac{81}{x^2}}dx$

$$\Rightarrow I = \int \frac{1+9/x^2}{x^2+\left(\frac{9}{x}\right)^2 - 18 + 18}dx = \int \frac{1+9/x^2}{\left(x-\frac{9}{x}\right)^2 + 18}dx$$

Let $x - \frac{9}{x} = t \Rightarrow \left(1 + \frac{9}{x^2}\right)dx = dt$

$$\therefore \quad I = \int \frac{dt}{t^2+18} \Rightarrow I = \int \frac{dt}{t^2 + (3\sqrt{2})^2}$$

$$\Rightarrow I = \frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{t}{3\sqrt{2}}\right) + c$$

$$\Rightarrow I = \frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{x^2-9}{3\sqrt{2}x}\right) + c$$

OR

Let $I = \int \underset{\text{I}}{x^2} \underset{\text{II}}{\sin 2x}\, dx$

$$= x^2\left(\frac{-\cos 2x}{2}\right) - \int 2x\cdot\left(\frac{-\cos 2x}{2}\right)dx$$

$$= \frac{-1}{2}x^2\cos 2x + \int \underset{\text{I}}{x}\,\underset{\text{II}}{\cos 2x}\, dx$$

$$= \frac{-1}{2}x^2\cos 2x + \left[x\left(\frac{\sin 2x}{2}\right) - \int \frac{\sin 2x}{2}dx\right]$$

$$= \frac{-1}{2}x^2\cos 2x + \frac{x\sin 2x}{2} + \frac{1}{4}\cos 2x + c$$

$$\therefore \quad I = \frac{-x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{\cos 2x}{4} + c$$

32. We are given that $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

$$\Rightarrow \frac{dy}{dx} = \sin(x+y) \qquad ...(i)$$

Let $x + y = v$. Then, $1 + \frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 1$

$\therefore$ From (i), $\frac{dv}{dx} - 1 = \sin v$

$$\Rightarrow \frac{dv}{dx} = 1 + \sin v \Rightarrow \frac{dv}{1+\sin v} = dx$$

$$\Rightarrow \int \frac{1}{1+\sin v}dv = \int dx \qquad \text{[Integrating both sides]}$$

$$\Rightarrow \int dx = \int \frac{1-\sin v}{1-\sin^2 v}dv \Rightarrow \int dx = \int \frac{1-\sin v}{\cos^2 v}dv$$

$$\Rightarrow \int dx = \int (\sec^2 v - \tan v \sec v)dv$$

$\Rightarrow x = \tan v - \sec v + C$

$\Rightarrow x = \tan(x+y) - \sec(x+y) + C$, which is the required solution.

33. We have, $f(0) = 1$ ...(1)

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$$

We have, $-1 \le \sin\frac{1}{x} \le 1 \Rightarrow -x^2 \le x^2 \sin\frac{1}{x} \le x^2$

$$\Rightarrow \lim_{x\to 0}(-x^2) \le \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) \le \lim_{x\to 0} x^2$$

$$\Rightarrow 0 \le \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) \le 0$$

$$\Rightarrow \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$

$$\Rightarrow \lim_{x\to 0} f(x) = 0 \quad \text{...(2)}$$

From (1) & (2), $\lim_{x\to 0} f(x) \ne f(0)$

$\therefore$ f is discontinuous at $x = 0$

34. Given, $(ax + b)e^{y/x} = x$

$$\Rightarrow e^{y/x} = \frac{x}{ax+b}$$

Taking log on both sides, we get

$$\frac{y}{x}\cdot \log e = \log\frac{x}{ax+b}$$

$$\Rightarrow \frac{y}{x} = \log x - \log(ax+b) \quad (\because \log e = 1)$$

Differentiating w.r.t. x, we get

$$\frac{x\cdot\frac{dy}{dx} - y\cdot 1}{x^2} = \frac{1}{x} - \frac{1}{ax+b}\cdot a$$

$$\Rightarrow x\frac{dy}{dx} - y = x^2\cdot\frac{ax+b-ax}{x(ax+b)}$$

$$\Rightarrow x\frac{dy}{dx} - y = \frac{bx}{ax+b} \quad \text{...(i)}$$

Differentiating again w.r.t. x, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 1 - \frac{dy}{dx} = \frac{(ax+b)\cdot b - bx\cdot a}{(ax+b)^2}$$

$$\Rightarrow x\frac{d^2y}{dx^2} = \frac{b^2}{(ax+b)^2} \Rightarrow x^3\frac{d^2y}{dx^2} = \left(\frac{bx}{ax+b}\right)^2$$

$$\Rightarrow x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2 \quad \text{(Using (i))}$$

OR

We have,

$$x = a\left\{\cos t + \frac{1}{2}\log\tan^2\frac{t}{2}\right\} \text{ and } y = a\sin t$$

$$\Rightarrow x = a\left\{\cos t + \frac{1}{2}\cdot 2\log\tan\frac{t}{2}\right\}$$

$$\Rightarrow x = a\left\{\cos t + \log\tan\frac{t}{2}\right\}$$

Differentiating w.r.t. t, we get

$$\frac{dx}{dt} = a\left\{-\sin t + \frac{1}{\tan t/2}\sec^2\frac{t}{2}\cdot\frac{1}{2}\right\} \text{ and } \frac{dy}{dt} = a\cos t$$

$$\Rightarrow \frac{dx}{dt} = a\left\{-\sin t + \frac{1}{2\sin(t/2)\cos(t/2)}\right\}$$

$$\Rightarrow \frac{dx}{dt} = a\left\{-\sin t + \frac{1}{\sin t}\right\} \Rightarrow \frac{dx}{dt} = a\left\{\frac{-\sin^2 t + 1}{\sin t}\right\}$$

$$\Rightarrow \frac{dx}{dt} = \frac{a\cos^2 t}{\sin t}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\cos t}{\frac{a\cos^2 t}{\sin t}} = \tan t$$

35. We have, $ax^2 + by^2 = 1$ ...(i)

and $a'x^2 + b'y^2 = 1$ ...(ii)

Let (x_1, y_1) be the point of intersection of the given curves. Then,

$ax_1^2 + by_1^2 = 1$ ...(iii)

$a'x_1^2 + b'y_1^2 = 1$ ...(iv)

Differentiating (i) w.r.t. x, we get

$$2ax + 2by\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{ax}{by}$$

$$\Rightarrow m_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{ax_1}{by_1} \quad \text{...(v)}$$

Differentiating (ii) w.r.t. x, we get

$$2a'x + 2b'y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{a'x}{b'y}$$

$$\Rightarrow m_2 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{a'x_1}{b'y_1} \quad \text{...(vi)}$$

The two curves will intersect orthogonally, if

$m_1 m_2 = -1$

$$\Rightarrow \left(-\frac{ax_1}{by_1}\right)\times\left(-\frac{a'x_1}{b'y_1}\right) = -1 \Rightarrow aa'x_1^2 = -bb'y_1^2 \quad \text{...(vii)}$$

Subtracting (iv) from (iii), we get

$(a - a')x_1^2 = -(b - b')y_1^2$ ...(viii)

Dividing (viii) by (vii), we get

$$\frac{a-a'}{aa'} = \frac{b-b'}{bb'} \Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{1}{b'}.$$

36. We have, $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{bmatrix}$

$|A| = 1(5 - 6) + 1(2 - 0) + 0(4 - 0)$

$= -1 + 2 + 0 = 1 \ne 0$

$\therefore$ A^{-1} exists

Now, $\text{adj} A = \begin{bmatrix} -1 & -2 & 4 \\ 1 & 1 & -2 \\ -3 & -3 & 7 \end{bmatrix}' = \begin{bmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{bmatrix}$

So, $A^{-1} = \frac{1}{|A|} \text{adj } A = \begin{bmatrix} -1 & 1 & 3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{bmatrix}$

Now, $|\text{adj } A| = \begin{vmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{vmatrix}$

$= -1(7-6)-1(-14+12)-3(4-4) = -1 + 2 = 1$

Also, $|A^{-1}| = \begin{vmatrix} -1 & 1 & 3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{vmatrix} = |\text{adj } A| = 1$

OR

We have, $A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$

$\Rightarrow |A| = 3(-16 + 8) + 10(4 - 4) -1(8 - 16) = -24 + 8 = -16 \neq 0$. So, A^{-1} exists

$\therefore \text{ adj } A = \begin{bmatrix} -8 & -16 & -12 \\ 0 & -4 & -4 \\ -8 & -8 & 4 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{|A|}.(\text{adj } A)$

$= \frac{-1}{16} \cdot \begin{bmatrix} -8 & -16 & -12 \\ 0 & -4 & -4 \\ -8 & -8 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 & \frac{3}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$

Now, $(A^{-1})^2 \begin{bmatrix} \frac{1}{2} & 1 & \frac{3}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 & \frac{3}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{4} \end{bmatrix}$

$= \begin{bmatrix} \frac{5}{8} & \frac{9}{8} & \frac{7}{16} \\ \frac{1}{8} & \frac{3}{16} & 0 \\ \frac{1}{8} & \frac{1}{2} & \frac{9}{16} \end{bmatrix}$

37. Vector equation of given planes are

$\vec{r} \cdot (2\hat{i} - 7\hat{j} + 4\hat{k}) = 3$ and $\vec{r} \cdot (3\hat{i} - 5\hat{j} + 4\hat{k}) + 11 = 0$

So, equation of a plane passing through intersection of both planes is

$\vec{r} \cdot (2\hat{i} - 7\hat{j} + 4\hat{k}) - 3 + \lambda\left[\vec{r} \cdot (3\hat{i} - 5\hat{j} + 4\hat{k}) + 11\right] = 0$

$\Rightarrow \vec{r} \cdot \left[(2\hat{i} - 7\hat{j} + 4\hat{k}) + \lambda(3\hat{i} - 5\hat{j} + 4\hat{k})\right] = 3 - 11\lambda$ ...(i)

Since it passes through (– 2, 1, 3) *i.e.*, $-2\hat{i} + \hat{j} + 3\hat{k}$

$\therefore (-2\hat{i} + \hat{j} + 3\hat{k}) \cdot \left[(2\hat{i} - 7\hat{j} + 4\hat{k}) + \lambda(3\hat{i} - 5\hat{j} + 4\hat{k})\right] = 3 - 11\lambda$

$\Rightarrow -4 - 7 + 12 + \lambda(-6 - 5 + 12) = 3 - 11\lambda$

$\Rightarrow 1 + \lambda = 3 - 11\lambda \Rightarrow 12\lambda = 2 \Rightarrow \lambda = 1/6$

Putting value of λ in (i), we get

$\vec{r} \cdot \left[2\hat{i} - 7\hat{j} + 4\hat{k} + \frac{3\hat{i} - 5\hat{j} + 4\hat{k}}{6}\right] = 3 - \frac{11}{6}$

$\Rightarrow \vec{r} \cdot \left[\frac{(12+3)\hat{i} - (42+5)\hat{j} + (24+4)\hat{k}}{6}\right] = \frac{18-11}{6}$

$\Rightarrow \vec{r} \cdot \left(\frac{15\hat{i} - 47\hat{j} + 28\hat{k}}{6}\right) = \frac{7}{6}$

$\Rightarrow \vec{r} \cdot (15\hat{i} - 47\hat{j} + 28\hat{k}) = 7$

OR

$\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z+5}{2}$ is the given line ...(i)

Let $A(2, -3, -5)$ lies on the line.

Direction ratios of line (i) are 1, –2, 2

$\therefore$ Direction cosines of line are $\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}$

$\therefore$ (i) may be written as

$\frac{x-2}{\frac{1}{3}} = \frac{y+3}{-\frac{2}{3}} = \frac{z+5}{\frac{2}{3}}$ ...(ii)

Coordinates of any point on the line (ii), may be taken as

$\left(\frac{1}{3}r + 2, \frac{-2}{3}r - 3, \frac{2}{3}r - 5\right)$

Let $Q = \left(\frac{1}{3}r + 2, \frac{-2}{3}r - 3, \frac{2}{3}r - 5\right)$

Given $|r| = 3$, $\therefore r = \pm 3$

Putting the values of r, we have

$Q \equiv (3, -5, -3)$ or $Q \equiv (1, -1, -7)$

38. Maximize, $Z = 600x + 400y$

subject to the constraints :

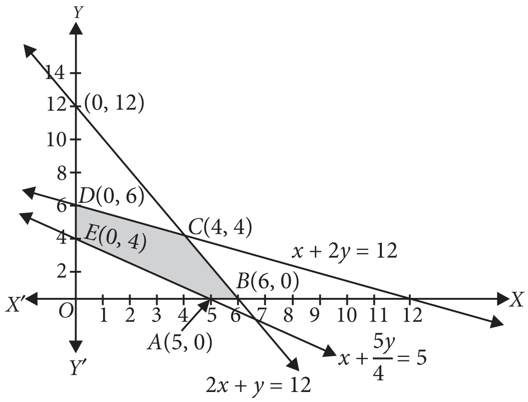
$x + 2y \leq 12$ ...(i)

$$2x + y \le 12 \qquad \text{...(ii)}$$

$$x + \frac{5}{4}y \ge 5 \qquad \text{...(iii)}$$

$$x, y \ge 0 \qquad \text{...(iv)}$$

Let us draw the graph of constraints (i) to (iv). *ABCDEA* is the feasible region (shaded) as shown in the figure. Observe that the feasible region is bounded, and coordinates of the corner points *A*, *B*, *C*, *D* and *E* are (5, 0), (6, 0), (4, 4), (0, 6) and (0, 4) respectively.



Let us evaluate $Z = 600x + 400y$ at these corner points.

Corner Points	$Z = 600x + 400y$	
$A(5, 0)$	3000	
$B(6, 0)$	3600	
$C(4, 4)$	4000	←(Maximum)
$D(0, 6)$	2400	
$E(0, 4)$	1600	

We clearly see that the point (4, 4) is giving the maximum value of *Z*.

OR

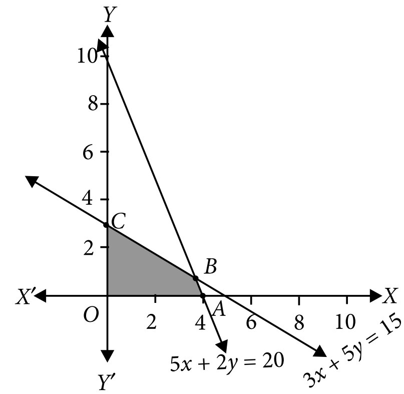
Converting inequations into equations and drawing the corresponding lines.

$3x + 5y = 15,\ 5x + 2y = 20$

i.e. $\frac{x}{5} + \frac{y}{3} = 1,\ \frac{x}{4} + \frac{y}{10} = 1$

As $x \ge 0, y \ge 0$ solution lies in first quadrant.

Let us draw the graph of the above equations.



B is the point of intersection of the lines $3x + 5y = 15$ and $5x + 2y = 20$, *i.e.* $B = \left(\frac{70}{19}, \frac{15}{19}\right)$

We have points $O(0, 0)$ $A(4, 0)$, $B\left(\frac{70}{19}, \frac{15}{19}\right)$ and $C(0, 3)$

Now $z = 3x + 2y$

$\therefore\ z(O) = 3(0) + 2(0) = 0$

$z(A) = 3(4) + 2(0) = 12$

$$z(B) = 3\left(\frac{70}{19}\right) + 2\left(\frac{15}{19}\right) = 12.63$$

$z(C) = 3(0) + 2(3) = 6$

$\therefore$ z has maximum value 12.63 at only one point *i.e.* $B\left(\frac{70}{19}, \frac{15}{19}\right)$

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