

# Geometry

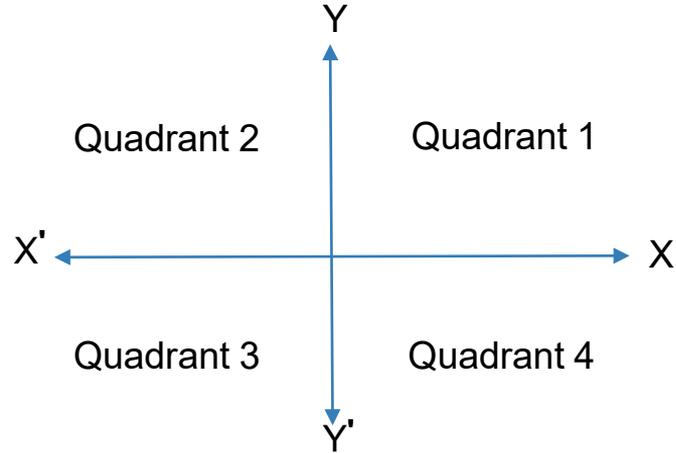
## Tip 1

- Geometry is one of the hardest sections to crack without preparation and one of the easiest with preparation.
- With so many formulas to learn and remember, this section is going to take a lot of time to master.
- Remember, read a formula, try to visualize the formula and solve as many questions related to the formula as you can.

## Tip 2

- Knowing a formula and knowing when to apply it are two different abilities.
- The first will come through reading the formulae list and theory but the latter can come only through solving many different problems.
- So in this document we are going to provide an exhaustive list of formulas and tips for making geometry section a lot easier.
- Try to remember all of them and don't forget to share.

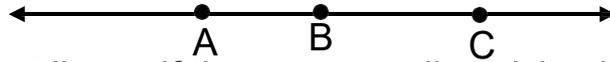
## Tip 3



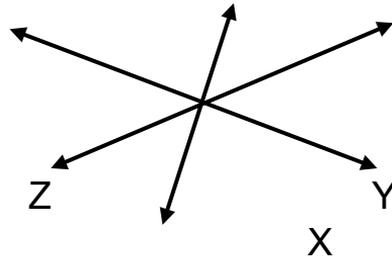
Quadrant I	X is Positive	Y is Positive
Quadrant II	X is Negative	Y is Positive
Quadrant III	X is Negative	Y is Negative
Quadrant IV	X is Positive	Y is Negative

## Tip 4

- Collinear points : Three or more points lying on the single straight line. In this diagram the three points A,B and C are collinear



- Concurrent lines : If three or more lines lying in a same plane intersect at a single point then that lines are called concurrent lines. The three lines X, Y and Z are here concurrent lines.



## Tip 5

- The distance between two points with coordinates  $(X_1, Y_1)$ ,  $(X_2, Y_2)$  is given by  $D = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$
- Slope,  $m = \frac{y_2 - y_1}{x_2 - x_1}$  (If  $x_2 = x_1$  then the lines are perpendicular to each other)
- Mid point between two points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  is  $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$
- When two lines are parallel, their slopes are equal i.e.  $m_1 = m_2$
- When two lines are perpendicular, product of their slopes = -1 i.e.  $m_1 * m_2 = -1$

## Tip 6

- If two intersecting lines have slopes  $m_1$  and  $m_2$  then the angle between two lines will be  $\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$  (where  $\theta$  is the angle between the lines)

- The length of perpendicular from a point  $(X_1, Y_1)$  on the line  $AX + BY + C = 0$  is

$$P = \frac{AX_1 + BY_1 + C}{\sqrt{A^2 + B^2}}$$

- The distance between two parallel lines  $Ax + By + C_1 = 0$  and  $Ax + By + C_2 = 0$  is

$$D = \left| \frac{C_1 - C_2}{\sqrt{A^2 + B^2}} \right|$$

## Tip 6

- Coordinates of a point P that divides the line joining A  $(x_1, y_1)$  and B  $(x_2, y_2)$  internally in the ratio  $l:m$  :  $\left( \frac{(lx_2 + mx_1)}{l + m}, \frac{(ly_2 + my_1)}{l + m} \right)$
- Coordinates of a point P that divides the line joining A  $(x_1, y_1)$  and B  $(x_2, y_2)$  externally in the ratio  $l:m$  :  $\left( \frac{(lx_2 - mx_1)}{l - m}, \frac{(ly_2 - my_1)}{l - m} \right)$
- For a triangle ABC, A  $(x_1, y_1)$ , B  $(x_2, y_2)$ , C  $(x_3, y_3)$ :
  - Centroid =  $\left( \frac{(x_1 + x_2 + x_3)}{3}, \frac{(y_1 + y_2 + y_3)}{3} \right)$
  - Incentre =  $\left( \frac{(ax_1 + bx_2 + cx_3)}{a + b + c}, \frac{(ay_1 + by_2 + cy_3)}{a + b + c} \right)$ ; where a, b and c are the lengths of the BC, AC and AB respectively.

# Tip 7

Equations of a lines :

General equation of a line	$Ax + By = C$
Slope intercept form	$y = mx + c$ (c is y intercept)
Point-slope form	$y - y_1 = m(x - x_1)$
Intercept form	$\frac{x}{a} + \frac{y}{b} = 1$
Two point form	$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

## Tip 8

### General equation of a circle :

The general equation of a circle is  $x^2+y^2+2gx+2fy+c=0$

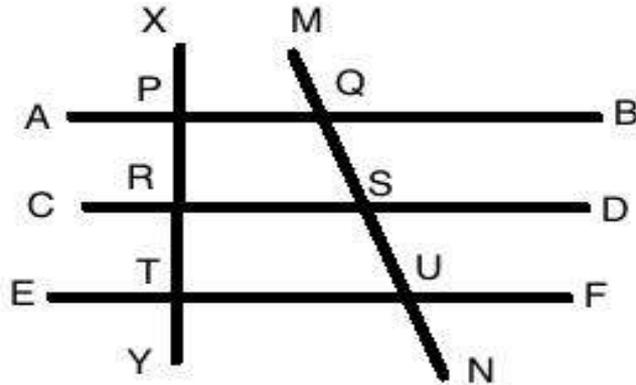
- Centre of the circle is  $(-g,-f)$
- Radius of the circle =  $\sqrt{g^2 + f^2 - c}$
- If the origin is the centre of the circle then equation of the circle is  $x^2+y^2=r^2$

## Tip 9

- When two angles A and B are complementary, sum of A and B is  $90^\circ$
- When two angles A and B are supplementary, sum of A and B is  $180^\circ$
- When two lines intersect, opposite angles are equal. Adjacent angles are supplementary
- When any number of lines intersect at a point, the sum of all the angles formed =  $360^\circ$

## Tip 10

- Consider parallel lines AB, CD and EF as shown in the figure.



- XY and MN are known as transversals
- $\angle XPQ = \angle PRS = \angle RTU$  as corresponding angles are equal

# Tip 11

- Interior angles on the side of the transversal are supplementary.  
i.e.  $\angle PQS + \angle QSR = 180^\circ$
- Exterior angles on the same side of the transversal are supplementary. i.e.  $\angle MQB + \angle DSU = 180^\circ$
- Two transversals are cut by three parallel lines in the same ratio i.e.  
$$\frac{PR}{RT} = \frac{QS}{SU}$$
- Interior angles on the side of the transversal are supplementary.  
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- Two transversals are cut by three parallel lines in the same ratio i.e.  
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## Tip 12

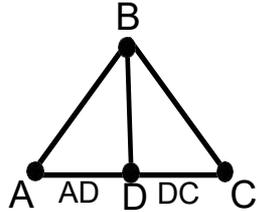
- Sum of all angles in a triangle is  $180^\circ$
- An angle less than  $90^\circ$  is called an acute angle. An angle greater than  $90^\circ$  is called an obtuse angle.
- A triangle with all sides unequal is called scalene triangle
- A triangle with two sides equal is called isosceles triangle. The two angles of an isosceles triangle that are not contained between the equal sides are equal
- A triangle with all sides equal is called equilateral triangle. All angles of an equilateral triangle equal  $60^\circ$ .

## Tip 13

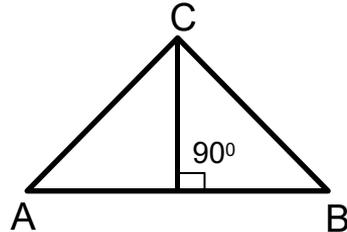
- If in a triangle all of its angles are less than  $90^\circ$  then that triangle is called as acute angled triangle
- A triangle with one of its angle equal to  $90^\circ$  then that triangle is called as Right angled triangle
- A triangle with one of its angle greater than  $90^\circ$  then that triangle is called as Obtuse angled triangle
- If one side of a triangle is produced then that exterior angle formed is equal to the sum of opposite remote interior angles

## Tip 14

- A line joining the mid point of a side with the opposite vertex is called a median. (Here D is the midpoint of AC side or  $AD = DC$ ). BD is the median of this triangle ABC.



- A perpendicular drawn from a vertex to the opposite side is called the altitude



## Tip 15

- A line that bisects and also makes right angle with the same side of the triangle is called perpendicular bisector
- A line that divides the angle at one of the vertices into two parts is called angular bisector
- All points on an angular bisector are equidistant from both arms of the angle.
- All points on a perpendicular bisector of a line are equidistant from both ends of the line.
- In an equilateral triangle, the perpendicular bisector, median, angle bisector and altitude (drawn from a vertex to a side) coincide.

## Tip 16

- The point of intersection of the three altitudes is the Orthocentre
- The point of intersection of the three medians is the centroid.
- The three perpendicular bisectors of a triangle meet at a point called the Circumcentre. A circle drawn from this point with the circumradius would pass through all the vertices of the triangle.
- The three angle bisectors of a triangle meet at a point called the incentre of a triangle. The incentre is equidistant from the three sides and a circle drawn from this point with the inradius would touch all the sides of the triangle.

# Tip 17

- Sum of any two sides of a triangle is always greater than its third side
- Difference of any two sides of a triangle is always lesser than its third side

## **Pythagoras theorem :**

In a right angled triangle ABC where  $\angle B=90^\circ$ ,  $AC^2 = AB^2 + BC^2$

## **Apollonius theorem**

In a triangle ABC, if AD is the median to side BC then by Apollonius theorem,  
$$2*(AD^2+BD^2) = AC^2 + AB^2$$

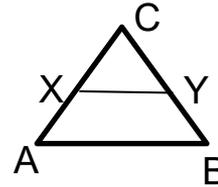
# Tip 18

## Mid Point Theorem :

The line joining the midpoint of any two sides in a triangle is parallel to the third side and is half the length of the third side.

If X is the midpoint of CA and Y is the midpoint of CB

Then XY will be parallel to AB and  $XY = \frac{1}{2} * AB$

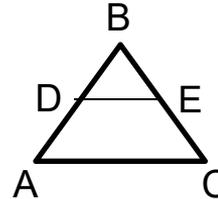


## Basic proportionality theorem :

If a line is drawn parallel to one side of a triangle and it intersects the other two sides at two distinct points then it divides the two sides in the ratio of respective sides

If in a triangle ABC, D and E are the points lying on AB and BC respectively and DE is parallel to AC then

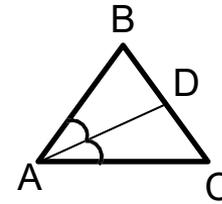
$AD/DB = EC/BE$



# Tip 19

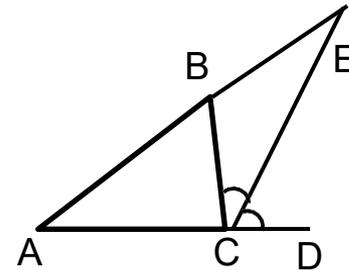
## Interior Angular Bisector theorem :

In a triangle the angular bisector of an angle divides the side opposite to the angle, in the ratio of the remaining two sides. In a triangle ABC if AD is the angle bisector of angle A then AD divides the side BC in the same ratio as the other two sides of the triangle.  
i.e.  $BD/CD = AB/AC$ .



## Exterior Angular Bisector theorem :

The angular bisector of exterior angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle. In a triangle ABC, if CE is the angular bisector of exterior angle BCD of a triangle, then  $AE/BE = AC/BC$

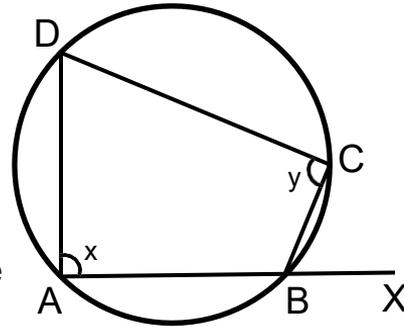


# Tip 20

## Cyclic Quadrilateral :

If a quadrilateral has all its vertices on the circle and its opposite angles are supplementary (here  $x+y = 180^\circ$ ) then that quadrilateral is called cyclic quadrilateral.

- In a cyclic quadrilateral the opposite angles are supplementary.
- Area of a cyclic quadrilateral is  $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$  where  $s = (a+b+c+d)/2$
- Exterior angle is equal to its remote interior opposite angle. (here  $\angle CBX = \angle ADC$ )



# Tip 21

- If  $x$  is the side of an equilateral triangle then the

$$\text{Altitude (h)} = \frac{\sqrt{3}}{2} x$$

$$\text{Area} = \frac{\sqrt{3}}{4} x^2$$

$$\text{Inradius} = \frac{1}{3} * h$$

$$\text{Circumradius} = \frac{2}{3} * h$$

- Area of an isosceles triangle =  $\frac{a}{4} \sqrt{4c^2 - a^2}$

(where  $a$ ,  $b$  and  $c$  are the length of the sides of BC, AC and AB respectively and  $b = c$ )

## Tip 22

### Similar triangles :

If two triangles are similar then their corresponding angles are equal and the corresponding sides will be in proportion.

For any two similar triangles :

- Ratio of sides = Ratio of medians = Ratio of heights = Ratio of circumradii = Ratio of Angular bisectors
- Ratio of areas = Ratio of the square of the sides.

Tests of similarity : (AA / SSS / SAS)

# Tip 23

## **Congruent triangles**

If two triangles are congruent then their corresponding angles and their corresponding sides are equal.

Tests of congruence : (SSS / SAS / AAS / ASA)

# Tip 24

## Area of a triangle, A :

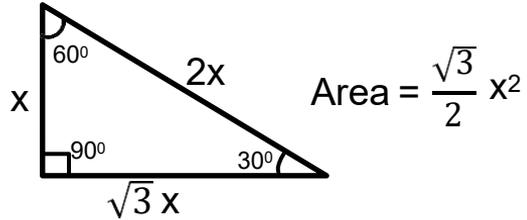
- $A = \sqrt{s(s-a)(s-b)(s-c)}$  where  $s = (a+b+c)/2$ .
- $A = 1/2 * \text{base} * \text{altitude}$
- $A = 1/2 * ab * \sin C$  (C is the angle formed between sides a and b)
- $A = \frac{abc}{4R}$  where R is the circumradius
- $A = r * s$  where r is the inradius and s is the semi perimeter.

(where a, b and c are the lengths of the sides BC, AC and AB)

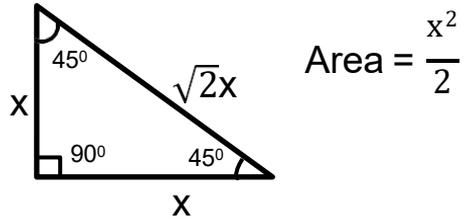
# Tip 25

Special triangles :

30°, 60° and 90°

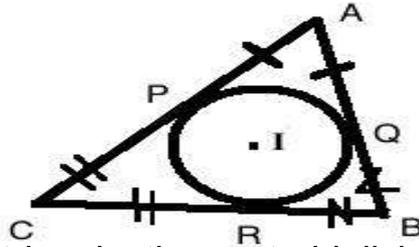


45°, 45°, 90°



## Tip 26

- Consider the triangle ABC with incentre I, and the incircle touching the triangle at P,Q,R as shown in the diagram. As tangents drawn from a point are equal,  $AP=AQ$ ,  $CP=CR$  and  $BQ=BR$ .

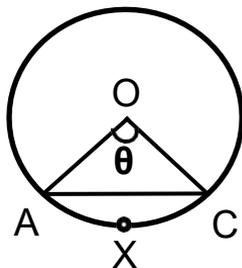


- In an equilateral triangle, the centroid divides the median in the ratio 2:1. As the median is also the perpendicular bisector, angle bisector, G is also the circumcentre and incentre.
- If  $a$  is the side of an equilateral triangle, circumradius =  $a/\sqrt{3}$  and inradius =  $a/(2\sqrt{3})$

## Tip 27

- The angle subtended by a diameter of circle on the circle =  $90^\circ$
- Angles subtended by a equal chord are equal. Also, angles subtended in the major segment are half the angle formed by the chord at the centre
- Equal chords of a circle or equidistant from the centre
- The radius from the centre to the point where a tangent touches a circle is perpendicular to the tangent
- Tangents drawn from the same point to a circle are equal in length
- A perpendicular drawn from the centre to any chord, bisects the chord

## Tip 28

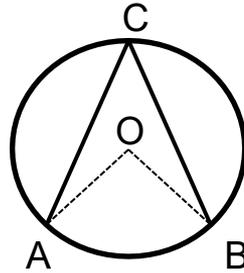


$$\text{Area of sector OAXC} = \frac{\theta}{360} * \pi r^2$$

$$\text{Area of minor segment AXC} = \frac{\theta}{360} \pi r^2 - \frac{1}{2} r^2 \sin \theta$$

## Tip 29

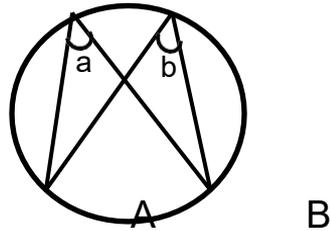
Inscribed angle Theorem :



$$2\angle ACB = \angle AOB$$

The angle inscribed by the two points lying on the circle, at the centre of the circle is twice the angle inscribed at any point on the circle by the same points.

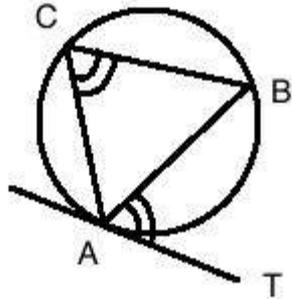
## Tip 30



Angles subtended by the same segment on the circle will be equal. So here angles a and b will be equal

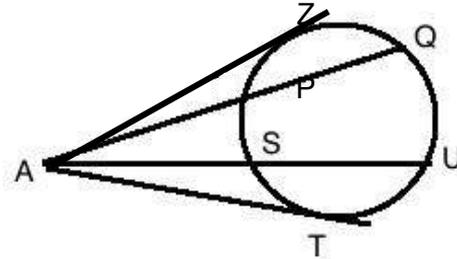
## Tip 31

The angle made by a chord with a tangent to one of the ends of the chord is equal to the angle subtended by the chord in the other segment. As shown in the figure,  $\angle ACB = \angle BAT$ .



## Tip 32

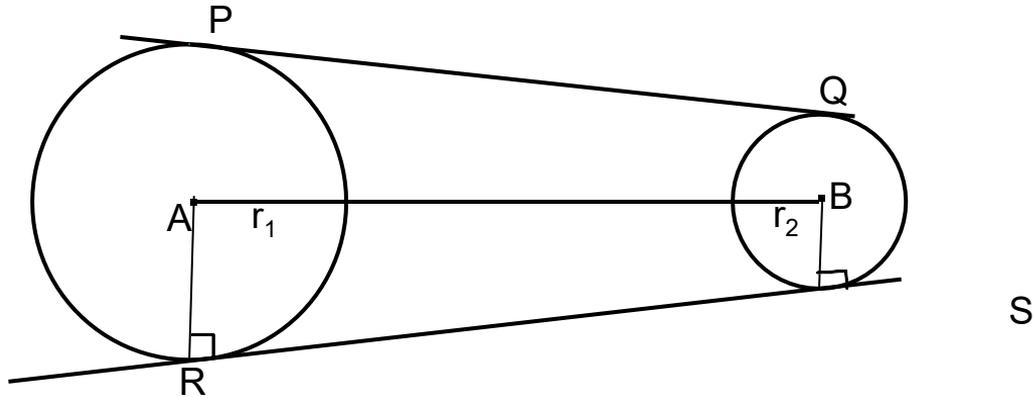
Consider a circle as shown in the image. Here,  $AP \cdot AQ = AS \cdot AU = AT^2$



Two tangents drawn to a circle from an external common point will be equal in length. So here  $AZ = AT$

## Tip 33

Direct common tangent :

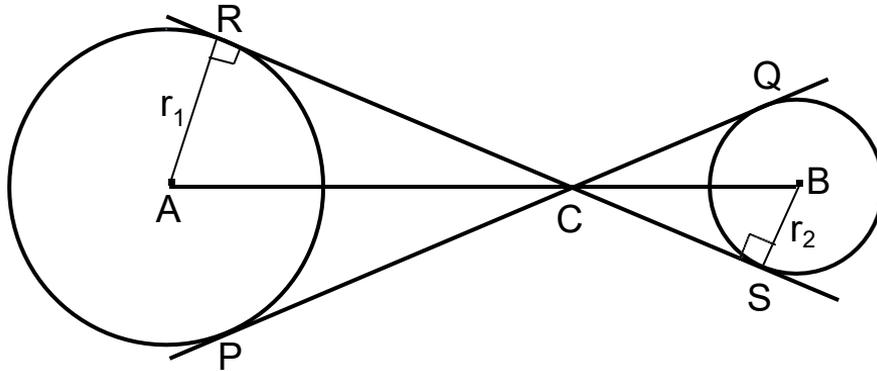


In this figure PQ and RS are the direct common tangents and let AB  
(Distance between the two centres) = D

$$PQ^2 = RS^2 = D^2 - (r_1 - r_2)^2$$

# Tip 34

Transverse common tangent :



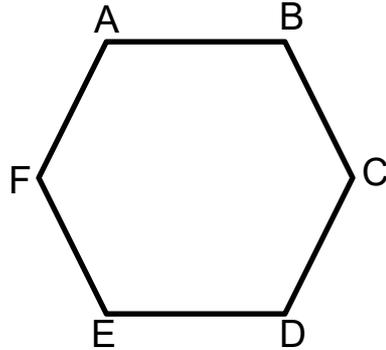
In this figure PQ and RS are the transverse common tangents and let AB (Distance between the two centres) = D

$$PQ^2 = RS^2 = D^2 - (r_1 + r_2)^2$$

## Tip 35

- If all sides and all angles are equal, then the polygon is a regular polygon
- A regular polygon of  $n$  sides has  $\frac{n(n-3)}{2}$  diagonals
- In a regular polygon of  $n$  sides, each exterior angle is  $\frac{360}{n}$  degrees.
- Sum of measure of all the interior angles of a regular polygon is  $180(n-2)$  degrees (where  $n$  is the number of sides of the polygon)
- Sum of measure of all the exterior angles of regular polygon is 360 degrees

## Tip 36



ABCDEF is a regular hexagon with each side equal to 'x' then

- Each interior angle =  $120^\circ$
- Each exterior angle =  $60^\circ$
- Sum of all the exterior angles =  $360^\circ$
- Sum of all the interior angles =  $720^\circ$
- Area =  $\frac{3\sqrt{3}}{2}a^2$

# Tip 37

## Areas of different geometrical figures

Triangle	$\frac{1}{2} * \text{base} * \text{height}$
Rectangle	length * width
Trapezoid	$\frac{1}{2} * \text{sum of bases} * \text{height}$
Parallelogram	base * height
Circle	$\pi * \text{radius}^2$
Rhombus	$\frac{1}{2} * \text{product of diagonals}$
square	side <sup>2</sup> or $\frac{1}{2}$ diagonal <sup>2</sup>
Kite	$\frac{1}{2} * \text{product of the diagonals}$

# Tip 38

## Volume of different solids

Cube	$\text{length}^3$
Cuboid	$\text{Length} * \text{base} * \text{height}$
Prism	$\text{Area of base} * \text{height}$
Cylinder	$\pi r^2 h$
Pyramid	$1/3 * \text{Area of base} * \text{Height}$
Cone	$1/3 * \pi r^2 * h$
Cone Frustum (If R is the base radius, r is the upper surface radius and h is the height of the frustum)	$1/3 \pi h(R^2 + Rr + r^2)$
Sphere	$4/3 * \pi * r^3$
Hemi-sphere	$2/3 \pi r^3$

## Tip 39

### Total Surface area of different solids

Prism	$2 * \text{base area} + \text{base perimeter} * \text{height}$
Cube	$6 * \text{length}^2$
Cuboid	$2(lh+bh+lb)$
Cylinder	$2\pi rh + 2\pi r^2$
Pyramid	$1/2 * \text{Perimeter of base} * \text{slant height} + \text{Area of base}$
Cone (l is the slant height)	$\pi r(l+r)$
Cone Frustum (where R and r are the radii of the base faces and l is the slant height)	$\pi(R^2+r^2+Rl+rl)$
Sphere	$4\pi r^2$
Hemi-sphere	$3\pi r^2$

# Tip 40

## Lateral/Curved surface area

Prism	base perimeter*height
Cube	$4 * \text{length}^2$
Cuboid	$2\text{length} * \text{height} + 2 * \text{breadth} * \text{height}$
Cylinder	$2\pi r h$
Pyramid	$1/2 * \text{Perimeter of base} * \text{slant height}$
Cone (l is the slant height)	$\pi r l$
Cone Frustum (where R and r are the radii of the base faces and l is the slant height)	$\pi(R+r)L$

## Tip 41

- The angle subtended by a diameter of circle on the circle = 90 degrees
- Angles subtended by a equal chords are equal. Also, angles subtended in the major segment are half the angle formed by the chord at the centre
- The radius from the centre to the point where a tangent touches a circle is perpendicular to the tangent
- Tangents drawn from the same point to a circle are equal in length
- A perpendicular drawn from the centre to any chord, bisects the chord
- In a regular polygon of  $n$  sides, each exterior angle is  $360/n$  degrees.