CBSE Test Paper 05

Chapter 8 Introduction to Trigonometry

---

1. The value of cos 48^o - sin 42^o is **(1)**
   a. 0
   b. 1
   c. $\sqrt{2}$
   d. $\frac{1}{2}$
2. If $2sin2\theta = \sqrt{3}$, then the value of θ **(1)**
   a. 60^o
   b. 45^o
   c. 0^o
   d. 30^o
3. Choose the correct option and justify your choice: $\frac{2\tan 30^0}{1-\tan^2 30^0}$ **(1)**
   a. cos60^o
   b. sin 30^o
   c. sin 60^o
   d. tan 60^o
4. If A, B and C are interior angles of a triangle ABC, then the value of $\tan\left(\frac{B+C}{2}\right)$ is **(1)**
   a. $\cot\frac{A}{2}$
   b. None of these
   c. $\tan\frac{A}{2}$
   d. $\sin\frac{A}{2}$
5. If ΔPQR is right angled at Q, then the value of sin (P + R) is **(1)**
   a. $\frac{1}{2}$
   b. 1
   c. $\sqrt{2}$
   d. 0
6. If secθsinθ= 0, then find the value of θ. **(1)**

7. Solve $2\sin^2\theta = \frac{1}{2}$ when $0^\circ < \theta < 90^\circ$. **(1)**

8. Prove the trignometric identity: (1 - cos$^2\theta$)cosec$^2\theta$ = 1 **(1)**

9. Prove the trigonometric identity: **(1)**
   $\cos^2\theta + \frac{1}{1+\cot^2\theta} = 1$

10. If tan A = cot B, prove that $A + B = 90^\circ$. **(1)**

11. Prove that: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$ **(2)**

12. Prove the trigonometric identity: **(2)**
    If $x = a\cos^3\theta, y = b\sin^3\theta,$prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

13. Evaluate $2\left(\frac{\cos 58^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left(\frac{\cos 38^\circ cosec 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ}\right)$. **(2)**

14. If $\sin\theta = \frac{3}{4}$, show that $\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3}$. **(3)**

15. In $\triangle PQR$, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sinP, cosP and tanP. **(3)**

16. If $cosec\ A = \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$. **(3)**

17. Prove the trigonometric identity: **(3)**
    $(\sec A + \tan A - 1)(\sec A - \tan A + 1) = 2\tan A$

18. Prove the trigonometric identity: **(4)**
    $\frac{\cot^2 A(\sec A - 1)}{1 + \sin A} = \sec^2 A\left(\frac{1 - \sin A}{1 + \sec A}\right)$

19. If $\sec\theta = \frac{13}{5}$, show that $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$. **(4)**

20. Evaluate without using trigonometric tables: **(4)**
    $\frac{\sec^2(90^\circ - \theta) - \cot^2\theta}{2(\sin^2 25^\circ + \sin^2 65^\circ)} + \frac{2\cos^2 60^\circ \tan^2 28^\circ \tan^2 62^\circ}{3(\sec^2 43^\circ - \cot^2 47^\circ)}$

CBSE Test Paper 05

Chapter 8 Introduction to Trigonometry

Solution

1. a. 0

   Explanation: Here, $\cos 48^\circ - \sin 42^\circ$

   = $\cos 48^\circ - \sin(90^\circ - 48^\circ)$

   = $\cos 48^\circ - \cos 48^\circ$ = 0

2. d. 30^0

   Explanation: Given: $2\sin 2\theta = \sqrt{3}$

   $\sin 2\theta = \frac{\sqrt{3}}{2}$

   but we know that sin 60 = $\frac{\sqrt{3}}{2}$

   $\sin 2\theta = \sin 60^\circ$

   on comparing on both sides

   $2\theta = 60^\circ$

   $\theta = 30^\circ$

3. d. tan 60^0

   Explanation: $\frac{2tan\,30^0}{1-tan^2\,30^0}$

   $= \frac{2\times\frac{1}{\sqrt{3}}}{1-(\frac{1}{\sqrt{3}})^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$

   $= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3} = tan\,60^0$

4. a. $\cot\frac{A}{2}$

   Explanation: According to question,

   A + B + C = 180°

   $\Rightarrow \frac{\mathrm{A+B+C}}{2} = 90^\circ$

   $\Rightarrow \frac{\mathrm{A}}{2} + \frac{\mathrm{B+C}}{2} = 90^\circ$

   $\Rightarrow \frac{\mathrm{B+C}}{2} = 90^\circ - \frac{\mathrm{A}}{2}$

   $\Rightarrow \tan\left(\frac{\mathrm{B+C}}{2}\right) = \tan\left(90^\circ - \frac{\mathrm{A}}{2}\right)$

$\Rightarrow \tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$

5. b. 1

Explanation: If in right angled triangle PQR, right angled at Q, then P and R are acute angles.

Let $\angle$P = θ, then $\angle$R
= $90^\circ - \theta$
Now, $\sin(P + R) = \sin(\theta + 90^\circ - \theta)$= $\sin 90^\circ$ = 1

6. Given secθ.sinθ = 0
$\frac{1}{cos\theta} \times sin\theta = 0$
or, $\frac{\sin\theta}{\cos\theta}$ =0
tanθ = 0
$tan\theta = tan 0^\circ$
θ = 0°

7. Here it is given $2\sin^2\theta = \frac{1}{2}$ when $0^\circ < \theta < 90^\circ$.
We have,
$2\sin^2\theta = \frac{1}{2}$
$\Rightarrow \quad \sin^2\theta = \frac{1}{4}$
$\Rightarrow \quad \sin\theta = \frac{1}{2}\, [\because \sin\theta > 0 \text{ for } 0^\circ < \theta < 90^\circ]$
$\Rightarrow \quad \theta = 30^\circ$

8. LHS = (1 - cos$^2\theta$)cosec$^2\theta$
= sin$^2\theta$(cosec$^2\theta$) [(1 - cos$^2\theta$) = sin$^2\theta$]
= sin$^2\theta$ $\times \frac{1}{\sin^2\theta}$ = 1 = RHS

9. We have,
LHS = cos$^2\theta$ + $\frac{1}{1+cot^2\theta}$
$\Rightarrow$ LHS = cos$^2\theta$ + $\frac{1}{cosec^2\theta}$ [$\because$ 1 + cot$^2\theta$ = cosec$^2\theta$]
$\Rightarrow$ LHS = cos$^2\theta$ + sin$^2\theta$ = 1 = RHS $\left[\because \frac{1}{cosec\theta} = \sin\theta\right]$

10. Given: tan A = cot B

$\Rightarrow \cot(90^\circ - A) = \cot B$

$\Rightarrow 90^\circ - A = B$

$\Rightarrow 90^\circ = \mathrm{A} + \mathrm{B}$

$\Rightarrow A + B = 90^\circ$

Hence proved

11. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$

$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)} \left[\because a^3 - b^3 = (a-b)(a^2+ab+b^2)\right.$

$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \tan\theta + \cot\theta + 1 = 1 + \tan\theta + \cot\theta = R.HS$ proved

Since, $\tan A = \frac{\sin A}{\cos A}$

$\cot A = \frac{\cos A}{\sin A}$

12. We have,

$x = a\cos^3\theta, y = b\sin^3\theta$

$\frac{x}{a} = \cos^3\theta \; and \; \frac{y}{b} = \sin^3\theta$

$\text{L.H.S} = \left[\frac{x}{a}\right]^{2/3} + \left[\frac{y}{b}\right]^{2/3}$

$= \left[\cos^3\theta\right]^{2/3} + \left[\sin^3\theta\right]^{2/3} \left[\because (a^m)^n = a^{m\times n}\right]$

$= \cos^2\theta + \sin^2\theta$

= 1 $\left[\because \cos^2\theta + \sin^2\theta = 1\right]$

= R.H.S.

Hence proved.

13. We have,

$2\left(\frac{\cos 58^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left(\frac{\cos 38^\circ cosec 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ}\right)$

$= 2\left\{\frac{\cos(90^\circ-32^\circ)}{\sin 32^\circ}\right\} - \sqrt{3}\left\{\frac{\cos 38^\circ cosec(90^\circ-38^\circ)}{\tan 15^\circ \tan 60^\circ \tan(90^\circ-15^\circ)}\right\}$

$= 2\left(\frac{\sin 32^\circ}{\sin 32^\circ}\right) - \sqrt{3}\left\{\frac{\cos 38^\circ \sec 38^\circ}{\tan 15^\circ\times\sqrt{3}\times\cot 15^\circ}\right\}$

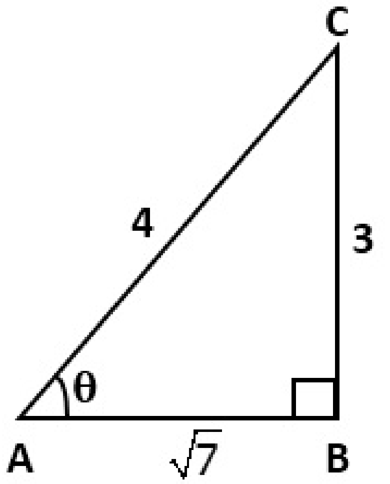
$[\because\ \cos(90-\theta)=\sin\theta\ ,\ \cos ec\,(90-\theta)=sec\theta,\ \tan(90-\theta)=cot\theta\ ]$

$= 2-\sqrt{3}\left\{\frac{\cos 38^{\circ}\times\frac{1}{\cos 38^{\circ}}}{\tan 15^{\circ}\times\sqrt{3}\times\frac{1}{\tan 15^{\circ}}}\right\}=2-\frac{\sqrt{3}}{\sqrt{3}}=2-1=1$

$\left[sec\theta=\frac{1}{\cos\theta},\ cot\theta=\frac{1}{\tan\theta}\right]$

therefore, $2\left(\frac{\cos 58^{\circ}}{\sin 32^{\circ}}\right)-\sqrt{3}\left(\frac{\cos 38^{\circ} cosec 52^{\circ}}{\tan 15^{\circ}\tan 60^{\circ}\tan 75^{\circ}}\right)=1$

14. Let us draw a triangle ABC in which ∠B = 90°.



Let ∠A = θ°.

Given,

$\sin\theta=\frac{3}{4}$

$\Rightarrow$ cosec$\theta=\frac{4}{3}$ ......(1)

But ,$\sin\theta=\frac{\text{Perpendicular}}{\text{Hypotenuse}}=\frac{BC}{AC}=\frac{3}{4}$

Let BC = 3 and AC = 5,

By Pythagoras' theorem in Δ ABC , we have :-

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2$

$= 4^2 - 3^2 = 16 - 9 = 7$

$\Rightarrow AB=\sqrt{7}$

Now,

$\cot\theta=\frac{\text{Base}}{\text{Perpendicular}}=\frac{AB}{BC}=\frac{\sqrt{7}}{3}$ ....(2)

$\sec\theta=\frac{\text{Hypotenuse}}{\text{Base}}=\frac{AC}{AB}=\frac{4}{\sqrt{7}}$ ....(3)

LHS

$= \sqrt{\frac{\cos ec^2\theta - \cot^2\theta}{\sec^2\theta - 1}}$

$= \sqrt{\frac{(4/3)^2 - (\sqrt{7}/3)^2}{(4/\sqrt{7})^2 - 1}}$ [from (2) & (3)]

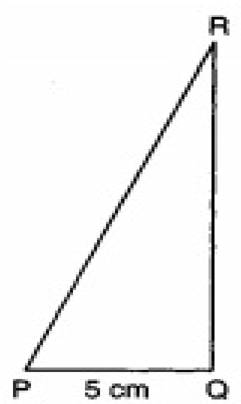
$= \sqrt{\frac{16/9 - 7/9}{12}}$

$= \sqrt{\frac{1}{9/7}}$

$= \sqrt{\frac{7}{9}}$

$= \frac{\sqrt{7}}{3}$

= RHS (Hence Proved).

15. 

In $\triangle PQR$ by Pythagoras theorem

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (25 - QR)^2 = 5^2 + QR^2 [\because PR + QR = 25 \text{ cm} \Rightarrow PR = 25 - \text{QR}]$

$625 - 50QR + QR^2 = 25 + QR^2$

$\Rightarrow 600 - 50QR = 0$

$\Rightarrow QR = \frac{600}{50} = 12 \text{ cm}$

Now, PR + QR = 25 cm

$\Rightarrow$ PR = 25 - Q R = 25 - 12 = 13 cm

Hence, $\sin P = \frac{QR}{PR} = \frac{12}{13}, \cos P = \frac{PQ}{PR} = \frac{5}{13}$ and, $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

16. We have

$\cos ecA = \sqrt{2}$

$\Rightarrow \cos ec^2 A = (\sqrt{2})^2 = 2$

$\cot^2 A = \cos ec^2 A - 1$

$\Rightarrow \cot^2 A = 2 - 1$

$\Rightarrow \cot^2 A = 1$

$\tan^2 A = \frac{1}{\cot^2 A} = \frac{1}{1} = 1$
$\sec^2\theta = 1 + \tan^2\theta$
$\sec^2\theta = 1 + 1 = 2$
$\cos^2\theta = \frac{1}{\sec^2\theta} = \frac{1}{2} \left[\because \cos\theta = \frac{1}{\sec\theta}\right]$
$\sin^2\theta = 1 - \cos^2\theta$
$\Rightarrow \sin^2\theta = 1 - \frac{1}{2}$
$\Rightarrow \sin^2\theta = \frac{1}{2}$
Now, $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$
$= \frac{2\times\frac{1}{2} + 3\times 1}{4\left(1 - \frac{1}{2}\right)}$
$= \frac{1+3}{4\times\frac{1}{2}}$
$= \frac{4}{2}$
= 2
Hence $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)} = 2$

17. We have, (secA + tanA - 1)(secA - tanA + 1)

= (sec A + tan A - (sec^{2}A - tan^{2}A)) (sec A - tan A + (sec^{2}A - tan^{2}A))

= (sec A + tan A - (sec A + tan A)(sec A - tan A)) (sec A - tan A + (sec A + tan A)(sec A - tan A))

= {(sec A + tan A) (1 - (sec A - tan A))} { (sec A - tan A) (1 + (sec A + tan A))}

= {(sec A + tan A)(1 - sec A + tan A)} {(sec A - tan A)(1 + sec A + tan A)}

= (sec A + tan A) (sec A - tan A) (1 - sec A + tan A) (1 + sec A + tan A)

= (sec^{2}A - tan^{2}A) (1 - sec A + tan A) (1 + sec A + tan A)

$= 1\left(1 - \frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)\left(1 + \frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$
$= 1\left(\frac{\cos A - 1 + \sin A}{\cos A}\right)\left(\frac{\cos A + 1 + \sin A}{\cos A}\right)$
$= \left(\frac{\cos A + \sin A - 1}{\cos A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$
$= \frac{(\cos A + \sin A)^2 - 1}{\cos^2 A}$
$= \frac{\cos^2 A + \sin^2 A + 2\cos A\sin A - 1}{\cos^2 A}$
$= \frac{1 + 2\cos A\sin A - 1}{\cos^2 A} \left[\because \sin^2\theta + \cos^2\theta = 1\right]$
$= \frac{2\cos A\sin A}{\cos A\cos A}$

$$= 2\frac{\sin A}{\cos A} = 2\tan A = RHS.$$

Hence proved.

18. LHS $= \frac{\cot^2 A(\sec A - 1)}{1+\sin A}$

$$= \frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1}{\cos A} - 1\right)}{1+\sin A}$$

$$= \frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A}$$

$$= \frac{\frac{\cos A \times \cos A}{1-\cos^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{\frac{\cos A}{(1)^2 - \cos^2 A}(1-\cos A)}{1+\sin A}$$

$$= \frac{\frac{\cos A}{(1+\cos A)(1-\cos A)}(1-\cos A)}{(1+\sin A)}$$

$$= \frac{\cos A}{(1+\cos A)(1+\sin A)}$$

RHS $= \sec^2 A\left[\frac{1-\sin A}{1+\sec A}\right]$

$$= \frac{1}{\cos^2 A}\left[\frac{1-\sin A}{1+\frac{1}{\cos A}}\right]$$

$$= \frac{1}{\cos^2 A}\left[\frac{1-\sin A}{\frac{\cos A+1}{\cos A}}\right]$$

$$= \frac{1}{\cos^2 A}\left[\frac{(1-\sin A)\cos A}{(1+\cos A)}\right]$$

$$= \frac{1}{\cos A \times \cos A}\left[\frac{(1-\sin A)\cos A}{(1+\cos A)}\right]$$

$$= \frac{1-\sin A}{\cos A(1+\cos A)}$$

Multiplying numerator and denominator by (1 + sinA)

$$= \frac{(1-\sin A)}{\cos A(1+\cos A)} \times \frac{(1+\sin A)}{1+\sin A}$$

$$= \frac{(1)^2 - \sin^2 A}{\cos A(1+\cos A)(1+\sin A)}$$

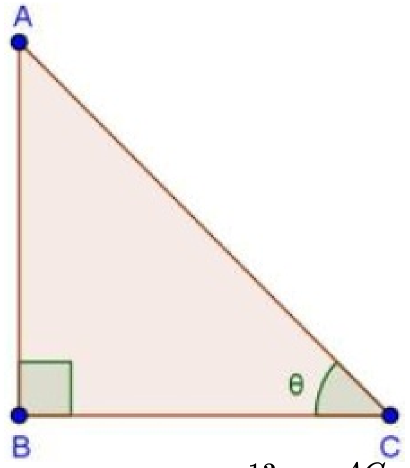
$$= \frac{\cos^2 A}{\cos A(1+\cos A)(1+\sin A)}$$

$$= \frac{\cos A \times \cos A}{\cos A(1+\cos A)(1+\sin A)}$$

$$= \frac{\cos A}{(1+\cos A)(1+\sin A)}$$

$\therefore$ LHS = RHS

19.



Given, $\sec\theta = \frac{13}{5} = \frac{AC}{BC}$

Let AC = 13K

and BC = 5K

In $\triangle$ABC, By Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$AB^2 + (5K)^2 = (13K)^2$

$AB^2 + 25K^2 = 169K^2$

$AB^2 = 169K - 25K^2 = 144K^2$

$AB = \sqrt{1}44K^2 = 12K$

$\therefore \sin\theta = \frac{AB}{AC} = \frac{12K}{13K} = \frac{12}{13}$

$\cos\theta = \frac{BC}{AC} = \frac{5K}{13K} = \frac{5}{13}$

LHS = $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

$= \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} + 9\times\frac{5}{13}}$

$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} + \frac{45}{13}}$

$= \frac{\frac{9}{13}}{\frac{3}{13}}$

$= \frac{9}{13} \times \frac{13}{3}$

= 3 = R.H.S.

20. $\frac{\sec^2(90^\circ - \theta) - \cot^2\theta}{2(\sin^2 25^\circ + \sin^2 65^\circ)} + \frac{2\cos^2 60^\circ \tan^2 28^\circ \tan^2 62^\circ}{3(\sec^2 43^\circ - \cot^2 47^\circ)}$

$\frac{cosec^2\theta - \cot^2\theta}{2\{\sin^2 25^\circ + \sin^2(90^\circ - 25^\circ)\}} + \frac{2\left(\frac{1}{2}\right)^2 \tan^2 28^\circ \tan^2(90^\circ - 28^\circ)}{3\{\sec^2 43^\circ - \cot^2(90^\circ - 43^\circ)\}} \because \sec(90^\circ - \theta) = cosec\theta$

$$= \frac{1}{2\left(\sin^2 25^\circ + \cos^2 25^\circ\right)} + \frac{2 \cdot \frac{1}{4} \cdot \tan^2 28^\circ \cdot \cot^2 28^\circ}{3\left(\sec^2 43^\circ - \tan^2 43^\circ\right)}$$

$\because \cos ec^2\theta - \cot^2\theta = 1$

$\sin(90^\circ - \theta) = \cos\theta$

$\tan(90^\circ - \Theta) = \cot\theta$

$\cot(90^\circ - \theta) = \tan\theta$

$$= \frac{1}{2(1)} + \frac{2 \cdot \frac{1}{4} \cdot \tan^2 28^\circ \cdot \frac{1}{\tan^2 28^\circ}}{3(1)}$$

$\because \sin^2\theta + \cos^2\theta = 1$

$\cot\theta = \frac{1}{\tan\theta}$

$\sec^2\theta - \tan^2\theta = 1$

$$= \frac{1}{2} + \frac{1}{6} = \frac{3+1}{6} = \frac{4}{6} = \frac{2}{3}$$