SURFACE AREAS AND VOLUMES

(A) Objective Type Questions — 1 Mark Each

Stand Alone MCQs (1 Mark Each)

1. A cylindrical pencil sharpened at one edge is the combination of:

(A) a cone and a cylinder
(B) frustum of a cone and a cylinder
(C) a hemisphere and a cylinder
(D) two cylinders U

Ans. Option (A) is correct.

Explanation: The sharpened part of the pencil is cone and unsharpened part is cylinder.

2. A surahi is the combination of:

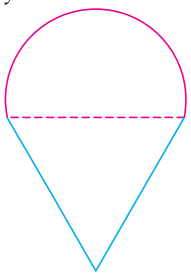
(A) a sphere and a cylinder
(B) a hemisphere and a cylinder
(C) two hemispheres
(D) a cylinder and a cone U

Ans. Option (A) is correct.

Explanation: A surahi is the combination of a sphere and a cylinder.

3. A plumbline (Sahul) is the combination of:

(A) a cone and a cylinder
(B) a hemisphere and a cone
(C) frustum of a cone and a cylinder
(D) sphere and cylinder



U

Ans. Option (B) is correct.

Explanation: Plumbline is an instrument used to check the verticality of an object. It is a combination of a hemisphere and a cone.

4. The shape of a gilli, in the gilli-danda game (see in figure) is a combination of:

(A) two cylinders
(B) a cone and a cylinder
(C) two cones and a cylinder
(D) two cylinders and a cone U

Ans. Option (C) is correct.

Explanation: The shape of gilli, in the gilli-danda game is a combination of two cones and a cylinder.

5. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then the number of marbles that the cube can accommodate is:

(A) 142296 **(B)** 142396
(C) 142496 **(D)** 142596 A + U

Ans. Option (A) is correct.

Explanation: Let the spherical marble has radius r.
Diameter of the marble = 0.5 cm

$\Rightarrow r = \frac{0.5}{2}$ cm = 0.25 cm

Length of side of cube l = 22 cm
Let n marbles can fill the cube.

$\therefore$ Volume of n marbles = $(1 - \frac{1}{8})$ part of volume of cube

$$\Rightarrow n.\frac{4}{3}\pi r^3 = \frac{7}{8} \times l^3$$

$$n = \frac{7l^3}{8} \times \frac{3}{4\pi r^3}$$

$$\Rightarrow \quad n = \frac{7 \times 3 \times 22 \times 22 \times 22 \times 7}{8 \times 4 \times 22 \times 0.25 \times 0.25 \times 0.25}$$

$\Rightarrow \quad n = 7 \times 3 \times 22 \times 22 \times 2 \times 7$

$= 42 \times 484 \times 7$

$n = 142296$

So, cube can accommodate 142296 marbles.

6. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each to its ends. The length of entire capsule is 2 cm. The capacity of the capsule is :

(A) 0.36 cm^3 **(B)** 0.35 cm^3

(C) 0.34 cm^3 **(D)** 0.33 cm^3 A

Ans. Option (A) is correct.

Explanation: Capsule consists of 2 hemispheres and a cylinder.

$r = \frac{0.5}{2}$ cm = 0.25 cm

$\Rightarrow r = 0.25$ cm

Total length of capsule = $r + h + r$

$\Rightarrow \quad 2 = 2r + h$

$\Rightarrow \quad 2 = 2 \times 0.25 + h$

$\Rightarrow \quad h = 2 - 0.5 = 1.5$ cm

Volume of capsule = Volume of two hemispheres + Volume of cylinder

$$= 2\times\left(\frac{4}{3}\pi r^3 \times \frac{1}{2}\right) + \pi r^2 h$$

$$= \frac{4}{3}\pi r^3 + \pi r^2 h$$

$$= \pi r^2\left(\frac{4}{3}r + h\right)$$

$$= \frac{22}{7}\times 0.25 \times 0.25\left(\frac{4}{3}\times 0.25 + \frac{15}{10}\right)$$

$$= \frac{22}{7}\times 0.25 \times 0.25\left(\frac{1}{3} + \frac{3}{2}\right)$$

$$= \frac{22}{7}\times\frac{25}{100}\times\frac{25}{100}\times\frac{11}{6} = \frac{121}{336} = 0.3601$$

$\therefore$ Volume of capsule = 0.3601 cm^3 = 0.36 cm^3.

7. If two solid hemispheres of same base radius 'r' are joined together along their bases, then curved surface area of this new solid is :

(A) $4\pi r^2$ **(B)** $6\pi r^2$

(C) $3\pi r^2$ **(D)** $8\pi r^2$ A

Ans. Option (A) is correct.

Explanation: When two hemispheres of equal radii are joined base to base, new solid becomes sphere and curved surface area of sphere is $4\pi r^2$.

8. A right circular cylinder of radius r cm and height h cm (where $h > 2r$) just encloses of sphere of diameter :

(A) r cm **(B)** $2r$ cm

(C) h cm **(D)** $2h$ cm A

Ans. Option (B) is correct.

Explanation: As the cylinder just encloses the sphere so the cylinder and diameter of sphere are equal, *i.e.*, $2r$ and height $h > 2r$.

9. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively, is melted and recast into the form of a cone of base diameter 8 cm. The height of the cone is:

(A) 12 cm **(B)** 14 cm

(C) 15 cm **(D)** 18 cm A

Ans. Option (B) is correct.

Explanation: During recasting a shape into another shape it's volume does not change.

For spherical shell

$r_1 = \frac{4}{2} = 2$ cm

$r_2 = \frac{8}{2} = 4$ cm

For cone $r = \frac{8}{2} = 4$ cm

$h = ?$

During recasting volume remains same so,

Volume of cone = Volume of hollow spherical shell

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi {r_2}^3 - \frac{4}{3}\pi {r_1}^3$$

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi({r_2}^3 - {r_1}^3)$$

$$\Rightarrow \quad r^2 h = 4({r_2}^3 - {r_1}^3)$$

$$\Rightarrow \quad (4 \times 4)h = 4[(4)^3 - (2)^3]$$

$$\Rightarrow \quad 4h = 64 - 8$$

$$\Rightarrow \quad h = \frac{56}{4}$$

$$\Rightarrow \quad h = 14 \text{ cm}$$

10. A solid piece of iron in the form of a cuboid of dimensions 49 cm × 33 cm × 24 cm, is moulded to form a solid sphere. The radius of the sphere is:

(A) 21 cm **(B)** 23 cm

(C) 25 cm **(D)** 19 cm A

Ans. Option (A) is correct.

Explanation: Solid cuboid of iron is moulded into solid sphere.

Hence, volume of cuboid and sphere are equal.

For sphere $r = ?$

For cuboid $l = 49$ cm

$b = 33$ cm

$h = 24$ cm

$\therefore$ Volume of sphere (solid) = Volume of cuboid

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = l \times b \times h$$

$$\Rightarrow \quad r^3 = \frac{(l \times b \times h \times 3)}{(4 \times \pi)}$$

$$= \frac{(49 \times 33 \times 24 \times 3 \times 7)}{(4 \times 22)}$$

$$\Rightarrow \quad r^3 = 7 \times 7 \times 7 \times 3 \times 3 \times 3$$

$$\Rightarrow \quad r = 21 \text{ cm}$$

11. A mason constructs a wall of dimensions 270 cm × 300 cm × 350 cm with the bricks each of size 22.5 cm × 11.25 cm × 8.75 cm and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Then the number of bricks used to construct the wall is:

(A) 11100 **(B)** 11200

(C) 11000 **(D)** 11300 A

Ans. Option (B) is correct.

Explanation: The volume of the wall covered by mortar = $\frac{1}{8}$ part.

So, the volume of wall covered by bricks

$$= \left(1-\frac{1}{8}\right) \text{ volume of wall}$$

$$= \frac{7}{8} \text{ volume of wall}$$

Bricks (Cuboid)	Wall (Cuboid)
l_1 = 22.5 cm	l = 270 cm
b_1 = 11.25 cm	b = 300 cm
h_1 = 8.75 cm	h = 350 cm

Let n be the number of bricks.

According to the question, we have

Volume of n bricks = $\frac{7}{8}$ Volume of wall

$$\Rightarrow n \times l_1 \times b_1 \times h_1 = \frac{7}{8} \times l \times b \times h$$

$$\Rightarrow n = \frac{(7 \times l \times b \times h)}{(8 \times l_1 \times b_1 \times h_1)}$$

$$= \frac{(7 \times 270 \times 300 \times 350)}{(8 \times 22.5 \times 11.25 \times 8.75)}$$

$$\Rightarrow n = \frac{(7 \times 270 \times 300 \times 350 \times 100 \times 10 \times 100)}{(8 \times 225 \times 1125 \times 875)}$$

$$\Rightarrow n = 2 \times 4 \times 350 \times 4$$
$$= 32 \times 350$$
$$= 11{,}200 \text{ bricks}$$

12. Twelve solid sphere of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is:

(A) 4 cm **(B)** 3 cm

(C) 2 cm **(D)** 6 cm A

Ans. Option (C) is correct.

Explanation: Solid cylinder is recasted into 12 spheres.

So, the volume of 12 spheres will be equal to the volume of the cylinder.

For spheres

$$R = ?$$

For cylinder

$$r = \frac{2}{2} = 1 \text{ cm}$$

$$h = 16 \text{ cm}$$

$\therefore$ Volume of 12 spheres = Volume of cylinder

$$\Rightarrow 12 \frac{4}{3}\pi R^3 = \pi r^2 h$$

$$\Rightarrow R^3 = \frac{(3r^2h)}{(4 \times 12)}$$

$$= \frac{(3 \times 1 \times 1 \times 16)}{(4 \times 12)} = 1$$

$$\Rightarrow R = 1 \text{ cm}$$

Hence, diameter = $2R = 2 \times 1 = 2$ cm

13. During conversion of a solid from one shape to another, the volume of new shape will:

(A) increase **(B)** decrease

(C) remains unaltered **(D)** be doubled A

Ans. Option (C) is correct.

Explanation: During reshaping a solid, the volume of new solid will be equal to old one or remains unaltered.

14. A rectangular sheet of paper 40 cm × 22 cm, is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is:

(A) 3.5 **(B)** 7

(C) $\frac{80}{7}$ **(D)** 5 A

Ans. Option (A) is correct.

Explanation: Circumference = 22 cm

$$2\pi r = 22$$

$$2 \times \frac{22}{7} \times r = 22$$

$$r = 3.5 \text{ cm}$$

15. The number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm, is:

(A) 3 **(B)** 5

(C) 4 **(D)** 6 A

Ans. Option (B) is correct.

Explanation:

$$\text{No. of solid spheres} = \frac{\text{Volume of cylinder}}{\text{Volume of sphere}}$$

$$= \frac{\pi R^2 h}{\frac{4}{3}\pi r^3}$$

$$= \frac{\pi(2)^2 \times 45 \times 3}{4 \times \pi \times (3)^3}$$

$$= 5$$

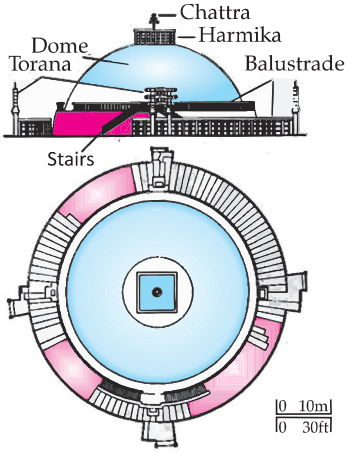
Case-based MCQs (1 Mark Each)

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the questions that follows, on the basis of the same: U

The Great Stupa at Sanchi is one of the oldest stone structures in India, and an important monument of Indian Architecture.

It was originally commissioned by the emperor Ashoka in the 3rd century BCE. Its nucleus was a simple hemispherical brick structure built over the relics of the Buddha. It is a perfect example of combination of solid figures. A big hemispherical dome with a cuboidal structure mounted on it. $\left(\text{Take } \pi = \frac{22}{7}\right)$ [CBSE QB, 2021]



1. Calculate the volume of the hemispherical dome if the height of the dome is 21 m:

(A) 19404 cu. m **(B)** 2000 cu. m
(C) 15000 cu. m **(D)** 19000 cu. m

Ans. Option (A) is correct.

Explanation: Height of hemispherical dome = Radius of hemispherical dome = 21 m.

$$\text{Volume of dome} = \frac{2}{3}\pi r^3$$
$$= \frac{2}{3}\times\frac{22}{7}\times 21\times 21\times 21$$
$$= 19{,}404 \text{ m}^3$$

2. The formula to find the volume of sphere is:

(A) $\frac{2}{3}\pi r^3$ **(B)** $\frac{4}{3}\pi r^3$
(C) $4\pi r^2$ **(D)** $2\pi r^2$

Ans. Option (B) is correct.

3. The cloth require to cover the hemispherical dome if the radius of its base is 14 m is:

(A) 1222 sq.m **(B)** 1232 sq.m
(C) 1200 sq.m **(D)** 1400 sq.m

Ans. Option (B) is correct.

Explanation: The cloth required to cover the hemispherical dome it the radius of its base is 14 cm is

$$2\pi R^2 = 2\times\frac{22}{7}(14)^2 \text{ m}^2$$
$$= 1232 \text{ sq. m}$$

4. The total surface area of the combined figure *i.e.* hemispherical dome with radius 14 m and cuboidal shaped top with dimensions 8 m × 6 m × 4 m is:

(A) 1200 sq. m **(B)** 1232 sq. m
(C) 1392 sq.m **(D)** 1932 sq. m

Ans. Option (C) is correct.

Explanation: Total surface Area of Combined figure

$$= 2\pi r^2 + 2(lb + bh + hl) - lb$$
$$= 2\times\frac{22}{7}\times 14\times 14 + 2(8\times 6 + 6\times 4 + 4\times 8) - 8\times 6] \text{ m}^2$$
$$= [1232 + 208 - 48] \text{ m}^2$$
$$= 1392 \text{ m}^2$$

5. The volume of the cuboidal shaped top is with dimensions mentioned in question 4 is:

(A) 182.45 m^3 **(B)** 282.45 m^3
(C) 292 m^3 **(D)** 192 m^3

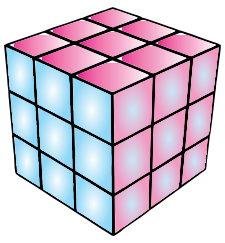
Ans. Option (D) is correct.

Explanation: Volume of the cuboidal shaped top

$$= l \times b \times h$$
$$= 8 \text{ m} \times 6 \text{ m} \times 4 \text{ m}$$
$$= 192 \text{ m}^3$$

AI II. Read the following text and answer the questions that follows, on the basis of the same:

On a Sunday, your Parents took you to a fair. You could see lot of toys displayed, and you wanted them to buy a RUBIK's cube and strawberry ice-cream for you. [CBSE QB, 2021]



1. The length of the diagonal if each edge measures 6 cm is:

(A) $3\sqrt{3}$ cm **(B)** $3\sqrt{6}$ cm
(C) $\sqrt{12}$ cm **(D)** $6\sqrt{3}$ cm

Ans. Option (D) is correct.

Explanation: We know that,
Length of the diagonal = $\sqrt{3} \times$ side
$\therefore$ Length of the diagonal of cube with edge 6 cm
$= \sqrt{3} \times 6$
$= 6\sqrt{3}$ cm

2. Volume of the solid figure if the length of the edge is 7 cm is:

(A) 256 cm^3 **(B)** 196 cm^3
(C) 343 cm^3 **(D)** 434 cm^3

Ans. Option (C) is correct.

Explanation:
Volume of cube $= (side)^3$
$= (7)^3 = 343\ cm^3$

3. What is the curved surface area of hemisphere (ice cream) if the base radius is 7 cm ?

(A) 309 cm^2 **(B)** 308 cm^2
(C) 803 cm^2 **(D)** 903 cm^2

Ans. Option (B) is correct.

Explanation: We know that,
CSA(curved surface area) of hemisphere
$= 2\pi r^2$
Given, $r = 7$ cm
So, $CSA = 2 \times \frac{22}{7} \times 7 \times 7$
$= 308\ cm^2$

4. Slant height of a cone if the radius is 7 cm and the height is 24 cm___.

(A) 26 cm **(B)** 25 cm
(C) 52 cm **(D)** 62 cm

Ans. Option (B) is correct.

Explanation: Slant height of cone,
$l = \sqrt{r^2 + h^2}$
Hence, $r = 7$ cm and $h = 24$ cm
$\therefore \quad l = \sqrt{(7)^2 + (24)^2}$
$= \sqrt{625} = 25$ cm

5. The total surface area of cone with hemispherical ice cream is:

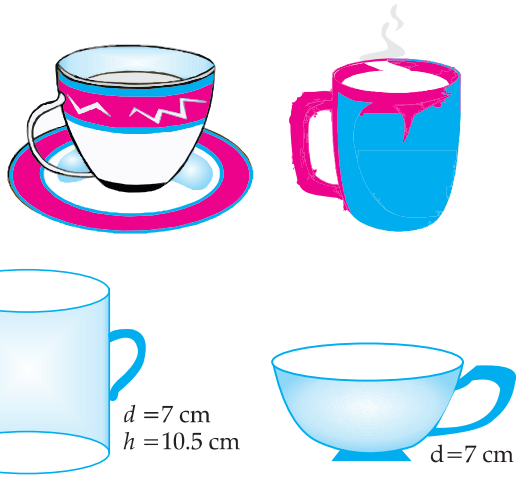
(A) 858 cm^2 **(B)** 885 cm^2
(C) 588 cm^2 **(D)** 855 cm^2

Ans. Option (A) is correct.

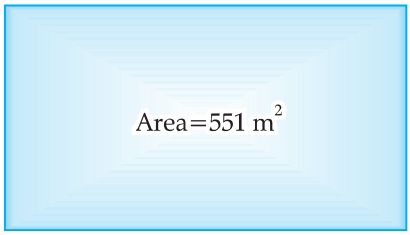
Explanation: Total surface area of the cone with hemispherical ice-cream
= curved surface area of the cone
+ curved surface area of the hemisphere
= 550 + 308
= 858 cm^2

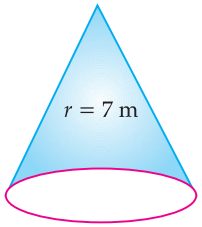
III. Read the following text and answer the following questions that follows, on the basis of the same.

Adventure camps are the perfect place for the children to practice decision making for themselves without parents and teachers guiding their every move. Some students of a school reached for adventure at Sakleshpur. At the camp, the waiters served some students with a welcome drink in a cylindrical glass and some students in a hemispherical cup whose dimensions are shown below.



After that they went for a jungle trek. The jungle trek was enjoyable but tiring. As dusk fell, it was time to take shelter. Each group of four students was given a canvas of area 551 m^2. Each group had to make a conical tent to accommodate all the four students. Assuming that all the stitching and wasting incurred while cutting, would amount to 1 m^2, the students put the tents. The radius of the tent is 7 m.





1. The volume of cylindrical cup is:

(A) 295.75 cm^3 **(B)** 7415.5 cm^3

(C) 384.88 cm^3 **(D)** 404.25 cm^3

Ans. Option (D) is correct.

Explanation:

$$\text{diameter, } d = 7 \text{ cm}$$

$$\text{radius, } r = \frac{d}{2} = 3.5 \text{ cm}$$

$$\text{height, } h = 10.5 \text{ cm}$$

Volume of cylindrical cup

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 10.5$$

$$= 404.25 \text{ cm}^2$$

2. The volume of hemispherical cup is:

(A) 179.67 cm^3 **(B)** 89.83 cm^3

(C) 172.25 cm^3 **(D)** 210.60 cm^3

Ans. Option (B) is correct.

Explanation: For hemispherical cup,

$$d = 7$$

$$\Rightarrow \quad r = \frac{d}{2} = \frac{7}{2} \text{ cm}$$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3 \text{ cm}^3$$

$$= 89.83 \text{ cm}^3$$

3. Which container had more juice and by how much?

(A) Hemispherical cup, 195 cm^3

(B) Cylindrical glass, 207 cm^3

(C) Hemispherical cup, 280.85 cm^3

(D) Cylindrical glass, 314.42 cm^3

Ans. Option (D) is correct.

Explanation: Volume of cylindrical cup

$$= \pi r^2 h$$

$$\left(\text{Given, } r = \frac{7}{2} \text{ cm and } h = 10.5 \text{ cm}\right)$$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 10.5$$

$$= 404.25 \text{ cm}^3$$

Also, volume of hemispherical cup

$$= 89.83 \text{ cm}^3$$

[As calculated in Q.2.]

Thus, cylindrical cup has more volume.

So, difference in volumes

$$= (404.25 - 89.83) \text{ cm}^3$$

$$= 314.42 \text{ cm}^3$$

4. The height of the conical tent prepared to accommodate four students is:

(A) 18 m **(B)** 10 m

(C) 24 m **(D)** 14 m

Ans. Option (C) is correct.

Explanation:

$$\text{Radius} = 7 \text{ m}$$

$$\text{Area of conical tent} = 551 \text{ m}^2 - 1 \text{ m}^2$$

$$= 550 \text{ m}^2$$

$$\pi r l = 550$$

$$\frac{22}{7} \times 7\sqrt{r^2 + h^2} = 550$$

$$\frac{22}{7} \times 7\sqrt{7^2 + h^2} = 550$$

$$\sqrt{7^2 + h^2} = \frac{550}{22}$$

$$\sqrt{7^2 + h^2} = \frac{50}{2}$$

$$\sqrt{7^2 + h^2} = 25$$

$$7^2 + h^2 = (25)^2$$

$$h^2 = 625 - 49$$

$$h^2 = 576$$

$$h = \sqrt{576}$$

$$= 24 \text{ m}$$

5. How much space on the ground is occupied by each student in the conical tent?

(A) 54 m^2 **(B)** 38.5 m^2

(C) 86 m^2 **(D)** 24 m^2

Ans. Option (B) is correct.

Explanation: Area of Base of conical tent $= \pi r^2$

$$= \frac{22}{7} \times 7 \times 7$$

$$= 154 \text{ m}^2$$

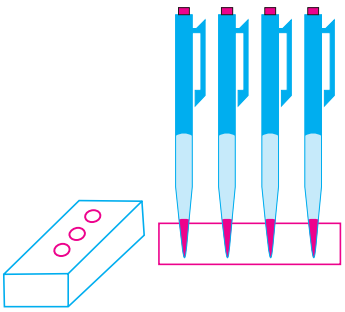
Area of occupied by

$$\text{each student} = \frac{1}{4} \times 154 \text{ m}^2$$

$$= 38.5 \text{ m}^2$$

AI IV. Read the following text and answer the questions that follows, on the basis of the same:

A carpenter made a wooden pen stand. It is in the shape of cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. (See Figure). C + AE



1. What is the volume of cuboid?

(A) 525 cm^3 **(B)** 225 cm^3

(C) 552 cm^3 **(C)** 255 cm^3

Ans. Option (A) is correct.

Explanation: For cuboid

$l = 15$ cm, $b = 10$ cm and $h = 3.5$ cm

Volume of the cuboid $= l \times b \times h$

$= 15 \times 10 \times 3.5$

$= 525\ cm^3$

2. What is the volume of a conical depression ?

(A) $\frac{11}{3}\ cm^3$ **(B)** $\frac{11}{30}\ cm^3$

(C) $\frac{3}{11}\ cm^3$ **(D)** $\frac{30}{11}\ cm^3$

Ans. Option (B) is correct.

Explanation: For conical depression:

$r = 0.5$ cm,

$h = 1.4$ cm

Volume of conical depression

$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

$= \frac{11}{30}\ cm^3$

3. What is the total volume of conical depressions?

(A) 1.74 cm^3 **(B)** 1.44 cm^3

(C) 1.47 cm^3 **(D)** 1.77 cm^3

Ans. Option (C) is correct.

Explanation: Volume of four conical depressions

$= 4 \times \frac{11}{30} = 1.47\ cm^3$

4. What is the volume of wood in the entire stand?

(A) 522.35 cm^3 **(B)** 532.53 cm^3

(C) 523.35 cm^3 **(D)** 523.53 cm^3

Ans. Option (D) is correct.

Explanation: Volume of the wood in the entire stand = Volume of cuboid

– Volume of 4 conical depressions

$= 525 - 1.47$

$= 523.53\ cm^3$

5. The given problem is based on which mathematical concept?

(A) Triangle

(B) Surface Areas and Volumes

(C) Height and Distances

(D) None of these

Ans. Option (B) is correct.

(B) Subjective Questions

Very Short Answer Type Questions

(1 Mark Each)

AI **1. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes ?** A **[CBSE Delhi Set-I, 2020]**

Sol. Let h_1 and h_2 be height and r_1, r_2 be radii of two cones, then ratio of their volumes

$= \frac{\frac{1}{3}\pi {r_1}^2 h_1}{\frac{1}{3}\pi {r_2}^2 h_2}$

Given, $\frac{h_1}{h_2} = \frac{1}{3}$ and $\frac{r_1}{r_2} = \frac{3}{1}$

$= \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right)$

$= \left(\frac{3}{1}\right)^2 \left(\frac{1}{3}\right)$

$= \frac{3}{1}$

Hence, ratio of their volumes is 3 : 1.

2. Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.

R **[CBSE OD Comptt. Set-I, II, III, 2018]**

Sol. Let the sides of two cubes be a and A, then

$\frac{a^3}{A^3} = \frac{1}{27}$ [Given] ½

$\Rightarrow \quad \frac{a}{A} = \frac{1}{3}$

We know that the surface area of a cube is $6(\text{side})^2$.

$\therefore$ Ratio of surface areas $= \frac{6a^2}{6A^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$ ½

[CBSE Marking Scheme, 2018]

AI 3. If the volume of a cube is 8 cm^3, then what is the surface area of a cube. A [O.E.B.]

Sol. Given, volume of a cube = 8 cm^3

Let the side of a cube be a cm, then

$$a^3 = 8$$

$$\Rightarrow \quad a = \sqrt[3]{8}$$

$$= \sqrt[3]{2\times2\times2}$$

$$= 2 \text{ cm}$$

Then, surface area of a cube

$$= 6a^2$$

$$= 6 \times 2 \times 2$$

$$= 24 \text{ cm}^2$$

4. Find the number of solid spheres of diameter 6 cm can be made by melting a solid metallic cylinder of height 45 cm and diameter 4 cm. A [O.E.B.]

Sol. Let the number of spheres be n.

Radius of sphere, $r_1 = 3$ cm

Radius of cylinder, $r_2 = 2$ cm

Volume of spheres = Volume of cylinder

$$n \times \frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

or, $$n \times \frac{4}{3} \times \frac{22}{7} \times (3)^3 = \frac{22}{7} \times (2)^2 \times 45$$

or, $$36n = 180$$

or, $$n = \frac{180}{36} = 5$$

Thus, the number of solid spheres = 5

5. Three solid metallic spherical balls of radii 3 cm, 4 cm and 5 cm are melted into a single spherical ball, find its radius. A [O.E.B.]

Sol. Let the radius of spherical ball be R.

Volume of spherical ball= Volume of three balls

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi [(3)^3 + (4)^3 + (5)^3]$$

or, $$R^3 = 27 + 64 + 125$$

or, $$R^3 = 216$$

or, $$R = 6 \text{ cm}$$

6. 12 solid spheres of the same size are made by melting a solid metallic cone of base radius 1 cm and height of 48 cm. Find the radius of each sphere. A [O.E.B.]

Sol. No. of spheres = 12

Radius of cone, $r = 1$ cm

Height of the cone = 48 cm

$\therefore$ Volume of 12 spheres = Volume of cone

Let the radius of sphere be R cm,

$$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

or, $$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (1)^2 \times 48$$

$$16R^3 = 16$$

or, $$R^3 = 1$$

or, $$R = 1 \text{ cm}$$ **1**

[CBSE Marking Scheme, 2014]

7. The radii of the top and bottom of a bucket of slant height 45 cm are 28 cm and 7 cm respectively. Find the curved surface area of the bucket. A [O.E.B.]

Sol. Here, $r_1 = 7$ cm, $r_2 = 28$ cm, $l = 45$ cm

Curved surface area of bucket = $\pi l(r_1 + r_2)$

$$= \frac{22}{7} \times 45(7 + 28)$$

$$= \frac{22}{7} \times 45 \times 35$$

$\Rightarrow$ Curved surface area of bucket = $22 \times 45 \times 5$ cm^2

= 4,950 cm^2

Short Answer Type Questions-I

(2 Marks Each)

AI 1. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. $\left(\text{Use } \pi = \frac{22}{7}\right)$

A [CBSE OD Set-I, 2020]

Sol. Given,

Volume of a right circular cylinder = $25\frac{1}{7}$ cm^3

i.e., $$\pi r^2 h = \frac{176}{7}$$

where h is height and r is radius then

$$\frac{22}{7} \times h^2 \times h = \frac{176}{7}$$

$$\Rightarrow \quad h^3 = \frac{176}{22} = 8 = 2^3$$

Hence, height of the cylinder = 2 cm

AI 2. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

A [CBSE OD Set-II, 2020]

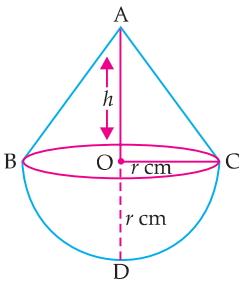
Sol. Let ABC be a cone, which is mounted on a hemi-sphere.

Given: $OC = OD = r$ cm

Curved surface area of the hemispherical part

$$= \frac{1}{2}(4\pi r^2)$$

$$= 2\pi r^2$$



Slant height of a cone,

$$l = \sqrt{r^2 + h^2}$$

So, curved surface area of cone

$$= \pi r l$$

$$= \pi r \sqrt{h^2 + r^2}$$

i.e., $2\pi r^2 = \pi r \sqrt{h^2 + r^2}$ [Given]

$\Rightarrow$ $2r = \sqrt{h^2 + r^2}$

Squaring both the sides, we get

$$4r^2 = h^2 + r^2$$

$\Rightarrow$ $4r^2 - r^2 = h^2$

$\Rightarrow$ $3r^2 = h^2$

$\Rightarrow$ $\frac{r^2}{h^2} = \frac{1}{3}$

$\Rightarrow$ $\frac{r}{h} = \frac{1}{\sqrt{3}}$

Hence, the ratio of the radius and the height

$$= 1 : \sqrt{3}$$

AI 3. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

A [CBSE OD Set-III, 2020]

Sol. Given, Height (h) = 14 cm

and Base radius (r) = 6 cm

Volume of the remaining solid

= Volume of a right circular cylinder – Volume of a right circular cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h$$

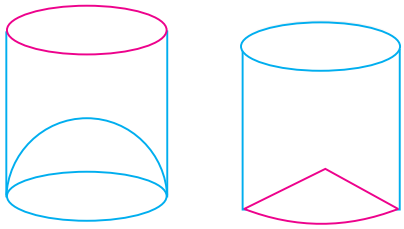
$$= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 14$$

$$= 1056 \text{ cm}^3$$

AI 4. Isha is 10 years old girl. On the result day, Isha and her father Suresh were very happy as she got first position in the class. While coming back to their home, Isha asked for a treat from her father as a reward for her success. They went to a juice shop and asked for two glasses of juice.

Aisha, a juice seller, was serving juice to her customers in two type of glasses. Both the glasses had inner radius 3 cm. The height of both the glasses was 10 cm.

First type: A Glass with hemispherical raised bottom.



Second type: A glass with conical raised bottom of height 1.5 cm.

Isha insisted to have the juice in first type of glass and her father decided to have the juice in second type of glass. Out of the two, Isha or her father Suresh, who got more quantity of juice to drink and by how much ? U [CBSE SQP, 2020]

Sol. Capacity of first glass $= \pi r^2 H - \frac{2}{3}\pi r^3$

$$= \pi \times 9(10 - 2)$$

$$= 72\pi \text{ cm}^3$$ **1**

Capacity of second glass $= \pi r^2 H - \frac{1}{3}\pi r^2 h$

$$= \pi \times 3 \times 3(10 - 0.5)$$

$$= 85.5\pi \text{ cm}^3$$ **1**

$\because$ Suresh got 42.39 cm^3 more quantity of juice.

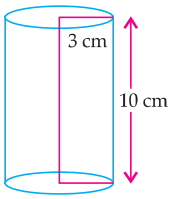
[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

In case of first type glass, we have

radius (r) = 3 cm

Height (H) = 10 cm



Capacity of Juice in cylindrical part

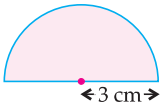
$$= \pi r^2 H$$

$$= \pi \times (3)^3 \times 10$$

Capacity of hemispherical part

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\pi \times (3)^3$$

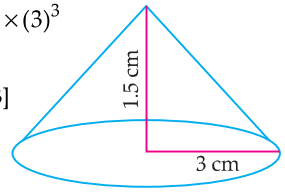


$\therefore$ Capacity of Juice in first glass

$$= \pi \times (3)^2 \times 10 - \frac{2}{3}\pi \times (3)^3$$

$$= \pi \times (3)^2 [10 - \frac{2}{3} \times 3]$$

$$= 9\pi \times 8$$



$= 72\pi \text{ cm}^3$

$= 72 \times 3.14 \text{ cm}^3$

$= 226.08 \text{ cm}^3$

In case of second glass, we have

radius of base (r) = 3 am

height of bottom part (h) = 1.5 cm

$\therefore$ Capacity of conical part

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times (3)^2 \times 1.5$$

$$= 4.5\pi \text{ cm}^3$$

Now, capacity of Juice in second glass

$$= \pi r^2 H - \frac{1}{3}\pi r^2 h$$

$$= \pi(3)^2 \times 10 - 4.5\pi$$

$$= 85.5 \times 3.14$$

$$= 268.47 \text{ cm}^3$$

Since, Suresh used second glass for drinking juice, so he got more quantity of juice.

i.e., Suresh got 268.47 – 226.08 = 42.39 cm^3 more juice than Isha.

5. A right circular cylinder and a cone have equal bases and equal heights. If their curved surface areas are in the ratio 8 : 5, show that the ratio between radius of their bases to their height is 3 : 4.

U **[CBSE Comptt. OD Set-I, II, III, 2018]**

Sol. Let r be the radii of bases of cylinder and cone and h be the height

Slant height of cone = $\sqrt{r^2 + h^2}$ ½

$$\therefore \quad \frac{2\pi rh}{\pi r\sqrt{r^2 + h^2}} = \frac{8}{5} \quad ½$$

$$\frac{h}{\sqrt{r^2 + h^2}} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{h^2}{r^2 + h^2} = \frac{16}{25}$$

$$\Rightarrow \quad 25h^2 = 16r^2 + 16h^2$$

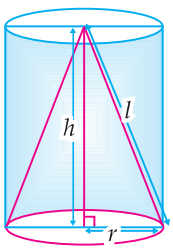
$$\Rightarrow \quad 9h^2 = 16r^2 \quad ½$$

$$\Rightarrow \quad \frac{r^2}{h^2} = \frac{9}{16}$$

$$\Rightarrow \quad \frac{r}{h} = \frac{3}{4} \quad ½$$

[CBSE Marking Scheme, 2018]

Detailed Solution:



Let r be the radii of bases of cylinder and cone and h be the height, then Curved surface area of cylinder

$$= 2\pi rh.$$

and curved surface area of a cone

$$= \pi rl$$

$$= \pi r \times \sqrt{h^2 + r^2}$$

where, $l = \sqrt{h^2 + r^2}$

Given,

$$\frac{2\pi rh}{\pi r \times \sqrt{h^2 + r^2}} = \frac{8}{5}$$

$$\Rightarrow \quad \frac{h}{\sqrt{h^2 + r^2}} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{h^2}{r^2 + h^2} = \frac{16}{25}$$

$$\Rightarrow \quad 25h^2 = 16r^2 + 16h^2$$

$$\Rightarrow \quad 25h^2 - 16h^2 = 16r^2$$

$$\Rightarrow \quad 9h^2 = 16r^2$$

$$\Rightarrow \quad \frac{r^2}{h^2} = \frac{9}{16}$$

$$\Rightarrow \quad \frac{r}{h} = \frac{3}{4}$$

Hence, the ratio between radius of their bases to their heights is 3 : 4. **Proved.**

COMMONLY MADE ERRORS

- In such type of problems, mostly students write incorrect formulae of surface area of cylinder and cone and also they do errors in calculation.
- Students write the formula of cylinder in place of cone and vice-versa.

ANSWERING TIP

- Adequate practice and remembering of formulae is necessary.

6. A cylindrical glass tube with radius 10 cm has water upto a height of 9 cm. A metal cube of 8 cm edge is immersed completely. By how much the water level will rise in the glass tube ?

A **[CBSE Term-II, 2015]**

Sol. Let the height of water raised measured be h cm. ½

$\therefore$ Volume of water displaced in cylinder $= \pi(10)^2 h$ ½

Volume of cube $= 8 \times 8 \times 8\ \text{cm}^3$ ½

$\therefore \quad \pi(10)^2 h = 8 \times 8 \times 8$

$\therefore \quad h = \dfrac{8\times8\times8\times7}{22\times10\times10}$ ½

$= 1.629$ cm

[CBSE Marking Scheme, 2015]

7. A solid metallic cuboid of dimensions 9 m × 8 m × 2 m is melted and recast into solid cubes of edge 2 m. Find the number of cubes so formed.

A [CBSE Foreign Set-I, II, 2017]

Sol. Volume of cuboid $= 9 \times 8 \times 2\ \text{m}^3$ ½

Volume of cube $= 2 \times 2 \times 2\ \text{m}^3$ ½

Let number of recast cubes be n.

$\therefore$ Volume of n cubes = Volume of cuboid

$n \times 2 \times 2 \times 2 = 9 \times 8 \times 2$

$n = \dfrac{9\times8\times2}{2\times2\times2}$ **1**

$= 18$

Hence, number of cubes recast = 18.

[CBSE Marking Scheme, 2017]

8. A solid metallic cylinder of radius 3.5 cm and height 14 cm is melted and recast into a number of small solid metallic balls, each of radius $\frac{7}{12}$ cm. Find the number of balls so formed.

A [CBSE Term-II, 2016]

Sol. Let the number of recast balls be n

radius of cylinder, $R = 3.5$ cm

height of cylinder, $h = 14$ cm

radius of recast balls, $r = \dfrac{7}{12}$ cm

$\therefore$ Volume of n balls = Volume of cylinder

$\Rightarrow \quad n\dfrac{4}{3}\pi r^3 = \pi R^2 h$ **1**

$\Rightarrow \quad n \times \dfrac{4}{3} \times \dfrac{7}{12} \times \dfrac{7}{12} \times \dfrac{7}{12} = 3.5 \times 3.5 \times 14$

$\Rightarrow \quad n = \dfrac{3.5\times3.5\times14\times3\times12\times12\times12}{4\times7\times7\times7}$

$= 0.5 \times 0.5 \times 2 \times 3 \times 3 \times 12 \times 12$

$= 648$

Hence, number of recast balls = 648 **1**

[CBSE Marking Scheme, 2016]

9. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ?

A [CBSE SQP, 2016]

Sol. Diameter of sphere = 6 cm

Diameter of cylindrical vessel = 12 cm

Volume of sphere $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3} \times \pi \times 3 \times 3 \times 3$

$= 36\pi\ \text{cm}^3$ **1**

$\therefore$ Volume of sphere = Increase in volume of cylinder

$36\pi = \pi(6)^2 \times h$

$h = 1$ cm

$\therefore$ Level of water rise in vessel = 1 cm **1**

[CBSE Marking Scheme, 2016]

10. Find the number of coins of 1.5 cm diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

A [CBSE SQP, 2016]

Sol. Volume of coin $= \pi r^2 h$

$= \dfrac{22}{7} \times (0.75)^2 \times 0.2\ \text{cm}^3$ ½

Volume of cylinder $= \dfrac{22}{7} \times (2.25)^2 \times 10\ \text{cm}^3$ ½

No. of coins $= \dfrac{\text{Volume of cylinder}}{\text{Volume of coin}}$ ½

$= \dfrac{\left(\frac{22}{7} \times (2.25)^2 \times 10\right)}{\left(\frac{22}{7} \times (0.75)^2 \times 0.2\right)}$

$= 450$ ½

[CBSE Marking Scheme, 2016]

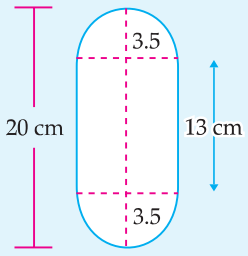
Short Answer Type Questions-II

(3 Marks Each)

AI **1. A solid is in the form of a cylinder with hemispherical end. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid.** $\left(\text{Use } \pi = \frac{22}{7}\right)$

A [CBSE OD Set-I, 2019]

Sol. Height of cylinder $= 20 - 7 = 13$ cm. **1**



Total volume $= \pi\left(\dfrac{7}{2}\right)^2 \times 13 + \dfrac{4}{3}\pi\left(\dfrac{7}{2}\right)^3\ \text{cm}^3$ **1**

$$= \frac{22}{7} \times \frac{49}{4}\left(13 + \frac{4}{3} \cdot \frac{7}{2}\right) \text{cm}^3$$

$$= \frac{77 \times 53}{6}$$

$= 680.17 \text{ cm}^3$ (Approx) **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Height of the cylinder $(h) = (20 - 7)$ cm $= 13$ cm

Radius of circular part $(r) = \frac{7}{2}$ cm

Volume of solid = Volume of cylinder + 2 × Volume of hemisphere

$$V = \pi r^2 h + 2 \times \left(\frac{2\pi}{3} r^3\right)$$

$$= \pi r^2 \left(h + \frac{4}{3} r\right)$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left[13 + \frac{4}{3} \times \frac{7}{2}\right]$$

$$= \frac{77}{2}\left(\frac{53}{3}\right) \text{cm}^3$$

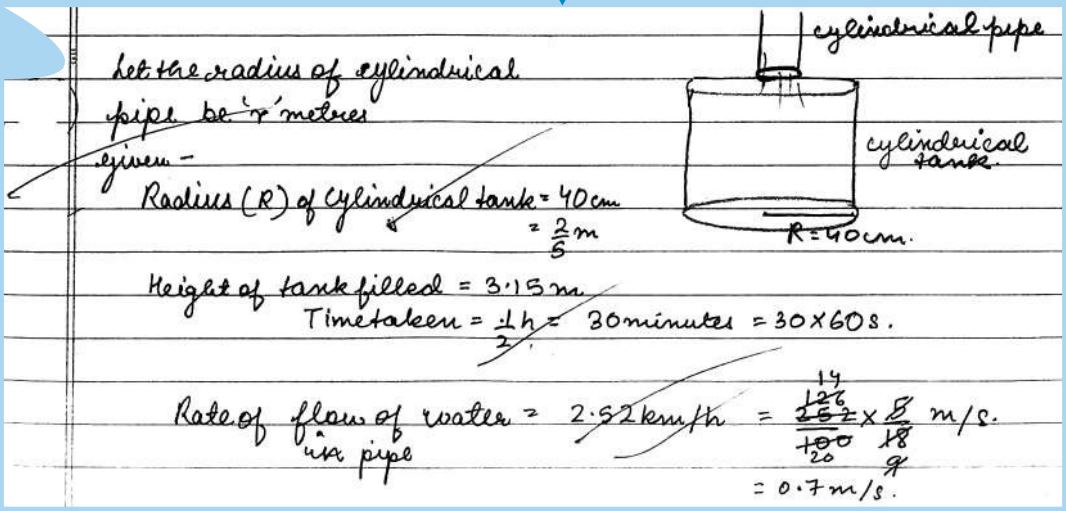
$= 680.17 \text{ cm}^3$ (Approx)

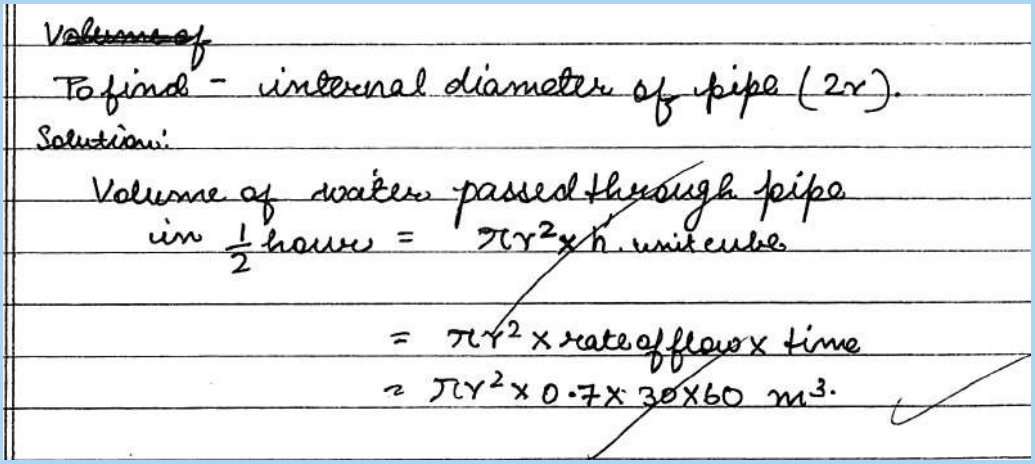
2. A cylindrical tank of radius 40 cm is filled upto height 3.15 m by an other cylindrical pipe with the rate of 2.52 km/h in $\frac{1}{2}$ hour. Calculate the diameter of cylindrical pipe ?

C + A [CBSE Delhi Region, 2019]

[CBSE Delhi Set-I, II, III, 2015]

Topper's Answer, 2019





Volume of water in tank in $\frac{1}{2}$ hour = $\pi R^2 \times h$

$= \pi\left(\frac{2}{5}\right)^2 \times 3.15\ m^3$

But, volume of water passed through pipe = Volume of water collected in tank

$\therefore \pi r^2 \times \frac{7}{10} \times 30 \times 60 = \pi\left(\frac{2}{5}\right)^2 \times \frac{315}{100}$

$\Rightarrow r^2 = \frac{4}{5\times 8} \times \frac{315}{8100} \times \frac{1}{7} \times \frac{1}{3} \times \frac{1}{60}$

$\Rightarrow r^2 = \frac{1}{2500} \Rightarrow r = \sqrt{\frac{1}{50^2}} \Rightarrow r = \pm\frac{1}{50}$

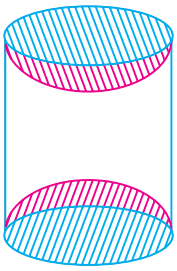
Radius is always positive, so $r = -\frac{1}{50}$ can be ignored.

$\Rightarrow r = \frac{1}{50}$ m.

$\Rightarrow r = \frac{1}{50} \times 100$ cm $\Rightarrow r = 2$ cm

Internal diameter of pipe = $d = 2r = 4$ cm or 0.04 m.

3. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in fig. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.



C + A

[CBSE Delhi/OD 2018]

Sol. Total surface area of article = CSA of cylinder + CSA of two hemispheres

CSA of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 3.5 \times 10$

$= 220\ cm^2$ **1**

Surface area of two hemispherical scoops

$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 154\ cm^2$ **1**

$\therefore$ Total surface area of article $= 220 + 154$

$= 374\ cm^2$ **1**

[CBSE Marking Scheme, 2018]

COMMONLY MADE ERROR

- Mostly students are unable to find the radius of hemisphere also they use the value of , 3.14 in place of $\frac{22}{7}$ and they do errors in calculation, they subtract the area of hemisphere from T.S.A. of cylinder in place of adding these.

ANSWERING TIP

- They should read the question clearly and use right formula and correct calculation for which good practice is necessary.

4. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap ?

A **[CBSE Delhi/OD 2018]**

Sol. Radius of conical heap = 12 m ½

Volume of rice $= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5\ m^3$

$= 528\ m^3$ **1**

Area of canvas cloth required $= \pi rl$

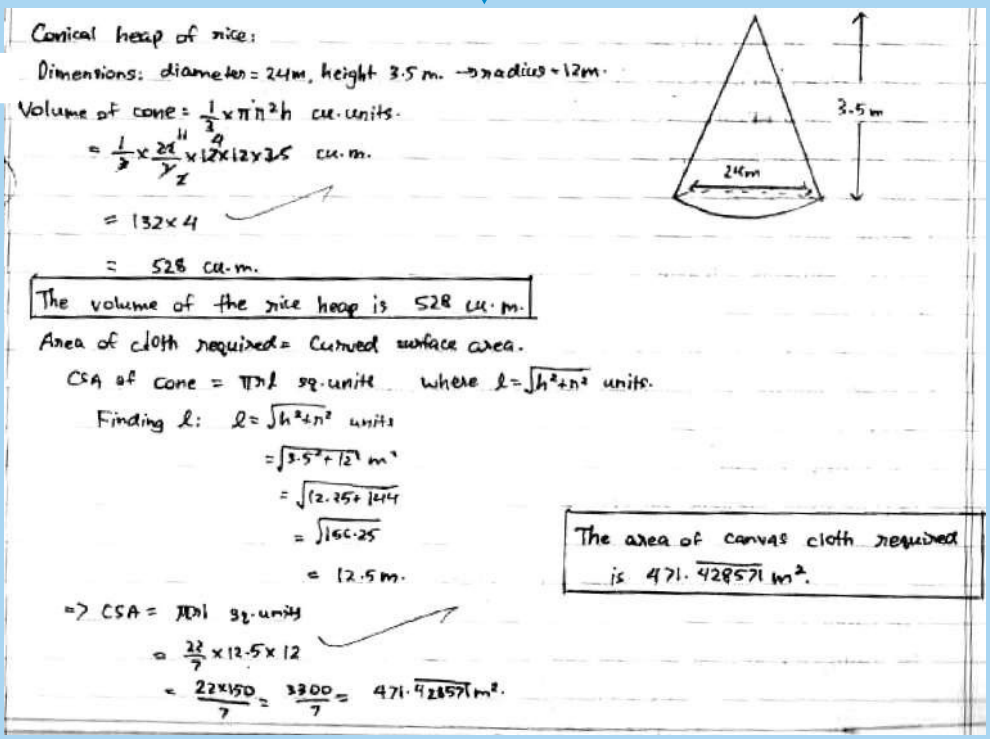
$l = \sqrt{12^2 + (3.5)^2} = 12.5$ m ½

$\therefore$ Area of canvas required $= \frac{22}{7} \times 12 \times 12.5$

$= 471.42$ m^2 [CBSE Marking Scheme, 2018] 1

Detailed Solution:

Topper's Answer, 2018



COMMONLY MADE ERROR

- Sometimes the students find TSA of the canvas in place of CSA.

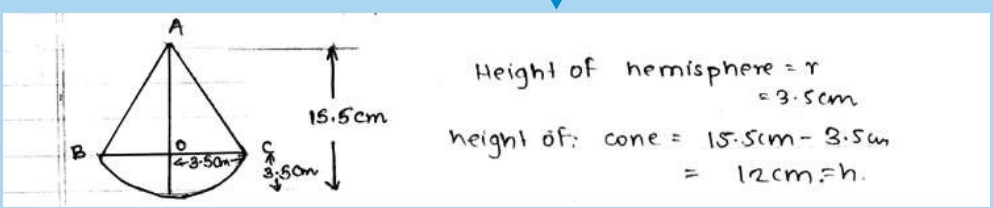
ANSWERING TIP

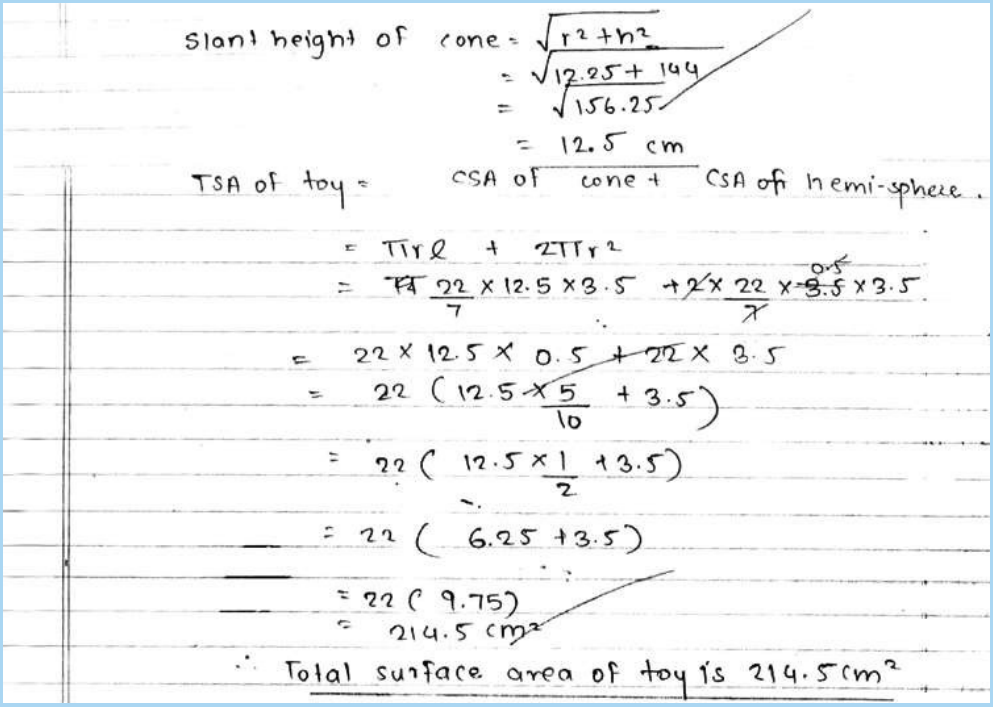
- They should have clear idea about C.S.A and T.S.A and volume.

5. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy. A [CBSE Delhi/OD 2017]

Topper's Answer, 2017

Sol.





6. The $\frac{3}{4}$ th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel. A **[CBSE Delhi Set-I, 2017]**
[CBSE OD Set-II, 2016]

Sol. Radius of conical vessel = 5 cm
and its height = 24 cm

Volume of this vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 5 \times 5 \times 24$

$= 200\pi\ cm^3$. **1**

Internal radius of cylindrical vessel = 10 cm
Let the height of emptied water be h.
∴ Volume of water in cylinder

$= \frac{3}{4} \times$ Volume of cone

$\Rightarrow \quad \pi r^2 h = \frac{3}{4} \times$ Volume of cone **1**

$\Rightarrow \quad \pi \times 10 \times 10 \times h = 150\pi$

$\Rightarrow \quad h = 1.5$ cm

Hence the height of water = 1.5 cm **1**

[CBSE Marking Scheme, 2017]

7. Rampal decided to donate canvas for 10 tents conical in shape with base diameter 14 m and height 24 m to a centre for handicapped person's welfare. If the cost of 2 m wide canvas is ₹ 40 per metre, find the amount by which Rampal helped the centre. C + A **[CBSE Comptt. OD Set-I, II, III, 2017]**
[CBSE OD Set-I, II, III, 2015, 2016]

Sol. Diameter of tent = 14 m and height = 24 m

∴ radius of tent = 7 m

Slant height $= \sqrt{h^2+r^2} = \sqrt{24^2+7^2}$

$= \sqrt{576+49} = 25$ m **1**

Surface area of the tent $= \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 550\ m^2$ **1**

Surface area of 10 tents $= 550 \times 10$

$= 5500\ m^2$

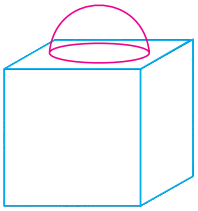
Total cost $= 5500 \times \frac{40}{2}$

$= ₹\ 110000$

Hence, the amount by which Rampal helped the centre = ₹ 110000 **1**

[CBSE Marking Scheme, 2017]

8. The given figure is a decorative block, made up of two solids – a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. $\left[\text{Use } \pi = \frac{22}{7}\right]$.



A [CBSE Delhi Set-I, II, III, 2016]

Sol. Surface area of block

$= 216 - \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} + 2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$ 1+½+½

$= 225.625 \text{ cm}^2$. [CBSE Marking Scheme, 2016] 1

Detailed Solution:

Given, side of cube = 6 cm

diameter of hemisphere = 3.5 cm

radius of hemisphere = $\frac{3.5}{2}$

Total surface area of cube = $6a^2$

$= 6 \times (6)^2 = 216 \text{ cm}^2$

Total surface area of solid = TSA of cube

– Area of circle

+ CSA of hemisphere.

$= 216 \text{ cm}^2 - \pi . \left(\frac{3.5}{2}\right)^2 \text{ cm}^2 + 2.\pi . \left(\frac{3.5}{2}\right)^2 \text{ cm}^2$

$= \left(216 - \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} + 2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right) \text{cm}^2$

$= \left(216 - \frac{77}{8} + \frac{77}{4}\right) \text{cm}^2$

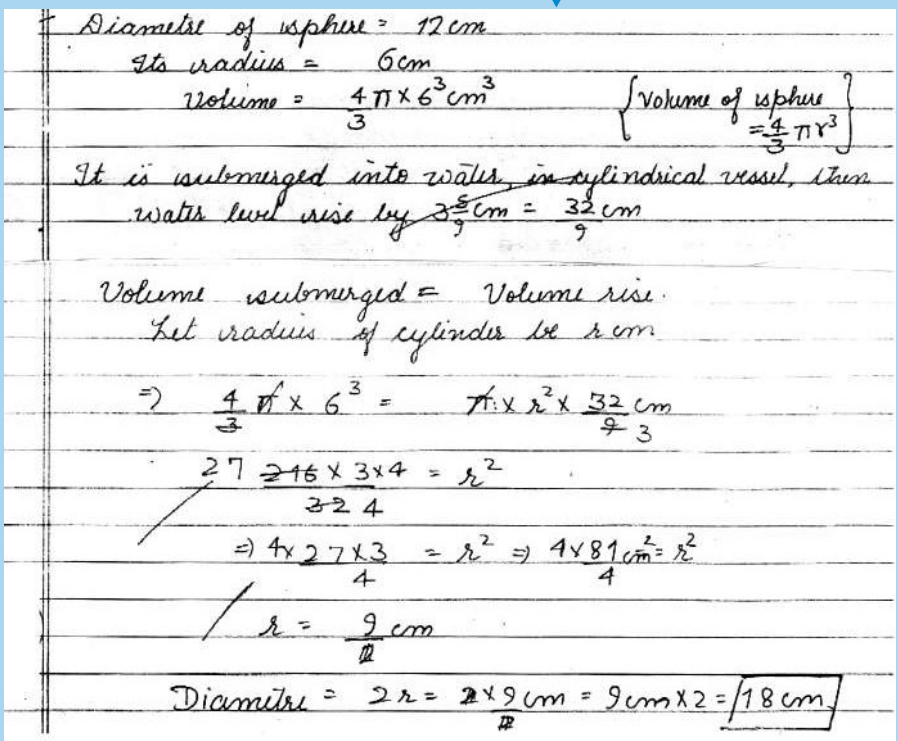
$= \left(\frac{1728 - 77 + 154}{8}\right) \text{cm}^2$

$= \frac{1805}{8} = 225.625 \text{ cm}^2$

9. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level into the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

A [CBSE OD Set-II, 2016]

Topper's Answer, 2016



Diameter of sphere = 12 cm

Its radius = 6 cm

Volume = $\frac{4}{3}\pi \times 6^3 \text{ cm}^3$ {Volume of sphere $= \frac{4}{3}\pi r^3$}

It is submerged into water, in cylindrical vessel, then water level rise by $3\frac{5}{9}$ cm = $\frac{32}{9}$ cm

Volume submerged = Volume rise

Let radius of cylinder be r cm

$\Rightarrow \frac{4}{3}\pi \times 6^3 = \pi \times r^2 \times \frac{32}{9} \text{ cm}$

$\frac{27}{32} \quad \frac{216 \times 3 \times 4}{4} = r^2$

$\Rightarrow \frac{4 \times 27 \times 3}{4} = r^2 \Rightarrow \frac{4 \times 81 \text{ cm}^2}{4} = r^2$

$r = \frac{9}{\pi} \text{ cm}$

Diameter = $2r = 2 \times \frac{9}{\pi} \text{ cm} = 9 \text{ cm} \times 2 = 18 \text{ cm}$

10. A well of diameter 4 m dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment. U [CBSE Delhi Set-I, II, III, 2016, 2015]

Sol. Diameter of earth dug out = 4 m

Radius of earth dug out, r = 2 m

Depth of the earth, h = 21 m,

Volume of earth dug out = $\pi r^2 h$

$$= \frac{22}{7} \times 2 \times 2 \times 21$$

$$= 264 \text{ m}^3 \quad \mathbf{1}$$

Width of embankment = 3 m

Outer radius of ring = 2 + 3 = 5 m

Let the height of embankment be h.

$\therefore$ Volume of embankment = Volume of earth dug out

$$\pi(R^2 - r^2)h = 264$$

$$\frac{22}{7} \times (25 - 4) \times h = 264 \quad \mathbf{1}$$

$$h = \frac{264 \times 7}{22 \times 21} = 4$$

$\therefore$ Height of embankment = 4 m **1**

[CBSE Marking Scheme, 2016]

11. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$

U **[CBSE Delhi Set-I, 2016]**

Sol. Here $r + h = 37$ and $2\pi r(r + h) = 1628$ ½

or, $2\pi r \times 37 = 1628$ ½

or, $2\pi r = \frac{1628}{37}$

or, $r = 7$ cm ½

and $h = 30$ cm ½

Hence, volume of cylinder $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30$

$= 4620 \text{ cm}^3$ **1**

[CBSE Marking Scheme, 2016]

12. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer. A **[CBSE OD Set-I, II, III, 2015]**

Sol. Volume of bowl $= \frac{2}{3}\pi R^3$

Volume of liquid in bowl $= \frac{2}{3}\pi \times (18)^3 \text{ cm}^3$ ½

Volume of liquid after wastage

$= \frac{2}{3}\pi \times (18)^3 \times \frac{90}{100} \text{ cm}^3$ ½

Volume of one bottle $= \pi r^2 h$

Volume of liquid in 72 bottles

$= \pi \times (3)^2 \times h \times 72 \text{ cm}^3$ ½

Volume of bottles = Volume of liquid after wastage

$$\pi \times (3)^2 \times h \times 72 = \frac{2}{3}\pi \times (18)^3 \times \frac{90}{100}$$

or, $$h = \frac{\frac{2}{3}\pi \times (18)^3 \times \frac{90}{100}}{\pi \times (3)^2 \times 72}$$

Hence, the height of bottle = 5.4 cm ½ + **1**

[CBSE Marking Scheme, 2015]

13. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weights, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$ cm and its depth $\frac{8}{9}$ cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

A **[CBSE Foreign Set-I, II, III, 2015]**

Sol. Volume of cylinder $= \pi r^2 h = \pi(3)^2 \times 5$

$= 45\pi \text{ cm}^3$ ½

Volume of conical hole $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times \frac{8}{9}$

$= \frac{2}{3}\pi \text{ cm}^3$ **1**

Metal left in cylinder $= 45\pi - \frac{2}{3}\pi = \frac{133\pi}{3} \text{ cm}^3$

1

Again, the required ratio

$$= \frac{\text{Volume of metal left}}{\text{Volume of metal taken out}}$$

$$= \frac{\frac{133}{3}\pi}{\frac{2}{3}\pi} = 133 : 2 \quad ½$$

Hence, Volume of metal left : Volume of metal taken out = 133 : 2.

[CBSE Marking Scheme, 2015]

14. A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder in cubic metre. $\left[\text{Use } \pi = \frac{22}{7}\right]$.

A **[Foreign Set-I, II, III, 2015]**

Sol. Volume of water in cylinder = Volume of cylinder

$= \pi r^2 h$

$= \pi \times (60)^2 \times 180$

$= 648000\pi \text{ cm}^3$ **1**

Water displaced on dropping cone

= Volume of solid cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (30)^2 \times 60$

$= 18000\pi \text{ cm}^3$ **1**

Volume of water left in cylinder

= Volume of cylinder – Volume of cone

$= 648000\pi - 18000\pi$

$= 630000\pi \text{ cm}^3$

$= \frac{630000 \times 22}{1000000 \times 7}$ m^3

$= 1.98$ m^3 **1**

[CBSE Marking Scheme, 2015]

15. The rain water from 22 m × 20 m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m. If the rain water collected from the roof fills $\frac{4}{5}$th of cylindrical vessel then find the rainfall in cm. A **[CBSE Foreign Set-I, II, III, 2015]**

Sol. Volume of water collected in cylindrical vessel

$= \frac{4}{5} \times \pi \times (1)^2 \times \left(\frac{7}{2}\right)$ m^3 **1**

$= \frac{44}{5}$ m^3 **1**

Let the rainfall is h m.

Volume of rain water from roof $= 22 \times 20 \times h$ m^3

or, $22 \times 20 \times h = \frac{44}{5}$

or, $h = \frac{44}{5} \times \frac{1}{22 \times 20} = \frac{1}{50}$ m

$= \frac{1}{50} \times 100 = 2$ cm **1**

[CBSE Marking Scheme, 2015]

16. A hollow cylindrical pipe is made up of copper. It is 21 dm long. The outer and inner diameters of the pipe are 10 cm and 6 cm respectively. Find the volume of copper used in making the pipe.

A **[CBSE Term-II, 2015]**

Sol. Height of cylindrical pipe, $h = 21$ dm

$= 210$ cm

External radius, $R = \frac{10}{2} = 5$ cm

Internal radius, $r = \frac{6}{2} = 3$ cm **1**

Volume of copper used in making the pipe

= (Volume of external cylinder)

– (Volume of internal cylinder)

$= \pi R^2 h - \pi r^2 h$ **1**

$= \pi h (R^2 - h^2)$

$= \frac{22}{7} \times 210 \times (5^2 - 3^2)$

$= \frac{22}{7} \times 16 \times 210$

$= 10560$ cm^3 **1**

[CBSE Marking Scheme, 2015]

17. A glass is in the shape of a cylinder of radius 7 cm and height 10 cm. Find the volume of juice in litre required to fill 6 such glasses. $\left[\text{Use } \pi = \frac{22}{7}\right]$

A **[CBSE Term-II, 2015]**

Sol. Radius of the glass $= 7$ cm

Height of the glass $= 10$ cm

Volume of 1 glass $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 10$

$= 1540$ cm^3 **1**

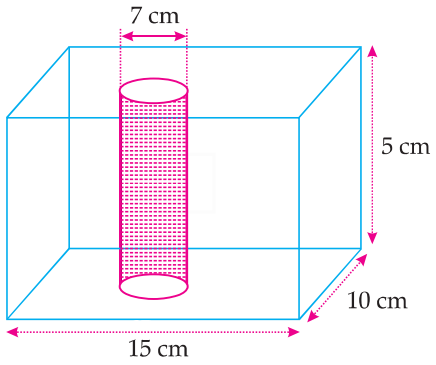
$\therefore$ Volume of juice to fill 6 glasses

$= 6 \times 1540 = 9240$ cm^3 **1**

$\therefore$ Volume in litre $= \frac{9240}{1000} = 9.240$ litre **1**

[CBSE Marking Scheme, 2015]

18. In fig., from a cuboidal solid metallic block of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. $\left[\text{Use } \pi = \frac{22}{7}\right]$



A **[CBSE Delhi Set-I, II, III, 2015]**

Sol. Total surface area $= 2(lb + bh + hl) + 2\pi rh - 2\pi r^2$

Here, $l = 15$ cm, $b = 10$ cm, $h = 5$ cm, $r = \frac{7}{2}$ cm

TSA of cuboidal block $= 2(15 \times 10 + 10 \times 5 + 5 \times 15)$

$= 550$ cm^2 **1**

CSA of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 5$

$= 110$ cm^2 **1**

Area of two circular bases $= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 77$ cm^2 **½**

Required area $= 550 + 110 - 77 = 583$ cm^2 **½**

[CBSE Marking Scheme, 2015]

19. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also find the cost of painting the hemisphere part of the toy at the rate of ₹ 10 per cm^2. $\left[\text{Use } \pi = \frac{22}{7}\right]$

A **[CBSE Delhi, Set-I, II, III, 2015]**

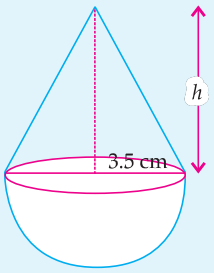
Sol. Given, radius of cone = radius of hemisphere = r

$r = 3.5$ cm.

Total volume, $V = 166\frac{5}{6}\text{ cm}^3 = \frac{1001}{6}\text{ cm}^3$ ½

Let the height of cone be h.

Total volume = Volume of cone + Volume of hemisphere



$$\frac{1001}{6} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

or, $$\frac{1001}{6} = \frac{1}{3}\pi(3.5)^2 h + \frac{2}{3}\pi(3.5)^3$$

or, $$\frac{1001}{6} = \frac{1}{3}\pi[12.25h + 2\times 42.875]$$

or, $$\frac{1001\times 3\times 7}{6\times 22} = 12.25h + 85.75$$

or, $$\frac{21021}{132} = 12.25h + 85.75$$ ½

or, $12.25h = 159.25 - 85.75$

or, $h = \frac{73.5}{12.25} = 6$ cm ½

Height of the toy = 6 + 3.5 = 9.5 cm. ½

Curved surface area of hemisphere = $2\pi r^2$

$= 2 \times \frac{22}{7}\times 3.5\times 3.5$

$= 77\text{ cm}^2$ ½

Cost of painting = ₹ 10 × 77

= ₹ 770 ½

[CBSE Marking Scheme, 2015]

20. Two spheres of same metal weighs 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere.

A **[CBSE OD Set-III, 2019]**

Sol. Radius of first sphere = 3 cm

$\therefore$ Mass $= \frac{4}{3}\pi(3)^3.d = 1$

[d= density]

Let radius of 2nd sphere be r cm

$\therefore$ Mass $= \frac{4}{3}\pi(r)^3.d = 7$

$\Rightarrow r^3 = 7(3)^3$ ½

$\Rightarrow \frac{4}{3}\pi(3)^3 + \frac{4}{3}\pi.(3)^3.7 = \frac{4}{3}\pi R^3$ **1**

$\Rightarrow R^3 = (3)^3(1+7)$

$\Rightarrow R = 3(2) = 6$ ½

$\therefore$ Diameter = 12 cm **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Weight of smaller sphere = 1 kg

Weight of larger sphere = 7 kg

Radius of smaller sphere, r = 3 cm

$\therefore$ Volume of smaller sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(3)^3$

$= \frac{4}{3}\pi(27)$

$= 36\pi\text{ cm}^3$

Since, 1 kg metal sphere occupies 36π cm^3 space.

Now, weight of recasted metal sphere = (1 + 7) kg

= 8 kg

$\therefore$ 8 kg sphere occupies $\frac{4}{3}\pi R^3$

[R is the radius of new sphere.]

$\Rightarrow 8 \times 36\pi = \frac{4}{3}\pi R^3$

$\Rightarrow R^3 = \frac{8\times 36\times 3}{4}$

$\Rightarrow R^3 = 216$

$\Rightarrow R^3 = 6\times 6\times 6$

$\Rightarrow R = 6$ cm

Hence, diameter of new sphere, $D = 2R = 2 \times 6$

= 12 cm.

21. A metallic solid sphere of radius 10.5 cm is melted and recasted into smaller solid cones each of radius 3.5 cm and height 3 cm. How many cones will be made ? A **[CBSE Delhi Set-II, 2017]**

Sol. Radius of given sphere = 10.5 cm

$\therefore$ Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\times 10.5\times 10.5\times 10.5$

$= 4\pi \times 3.5 \times 10.5 \times 10.5\text{ cm}^3$ **1**

Radius of one recasted cone = 3.5 cm

and height = 3 cm

$\therefore$ Volume $= \frac{1}{3}\pi\times 3.5\times 3.5\times 3$

$= \pi \times 3.5 \times 3.5\text{ cm}^3$ **1**

Let the number of recasted cones be n.

$\therefore n \times \pi \times 3.5 \times 3.5\text{ cm}^3 = 4 \times \pi \times 3.5 \times 10.5 \times 10.5\text{ cm}^3$

$\Rightarrow n = \frac{4\times 3.5\times 10.5\times 10.5}{3.5\times 3.5}$

$= 126$

Hence, number of recasted cones = 126 **1**

[CBSE Marking Scheme, 2017]

22. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of same height and same base radius is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)

U [CBSE Comptt. OD Set-I, II, III, 2017]

Sol. Height of cylinder = height of cone = 8 cm

radius of cylinder = radius of cone = 6 cm

$\therefore$ Slant height of cone $= \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36}$

$= 10$ cm ½

Total surface area of remaining solid

= Curved surface area of cylinder + Surface area of cone + Area of top cylinder

$= 2\pi rh + \pi rl + \pi r^2$

$= \pi r(2h + l + r)$ **1**

$= \frac{22}{7} \times 6(2 \times 8 + 10 + 6)$

$= \frac{22}{7} \times 6 \times 32$

$= 603.43$ **1**

Hence total surface area of remaining solid $= 603.43$ cm^2 [CBSE Marking Scheme, 2017] ½

23. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recasted into a metallic sphere. Find the diameter of the sphere and hence find its surface area. $\left[\text{Use } \pi = \frac{22}{7}\right]$

U [CBSE OD Set-I, II, III, 2015]

Sol. Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of metal in 504 cones

$= 504 \times \frac{1}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3$ ½

Volume of Sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times r^3$ **1**

Since, Volume of sphere = Volume of 504 cones

$\frac{4}{3} \times \frac{22}{7} \times r^3 = 504 \times \frac{1}{3} \times \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20} \times 3$ ½

or, $r^3 = \left(\frac{21}{2}\right)^3$

or, $r = 10.5$ cm

$\therefore$ Diameter $= 21$ cm ½

and surface area $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$

$= 1386$ cm^2 ½

[CBSE Marking Scheme, 2015]

Long Answer Type Questions

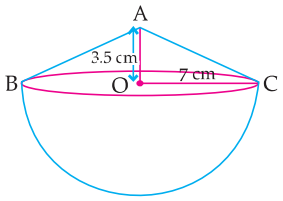
(5 Marks Each)

AI **1. A solid is in the shape of a hemisphere surmounted by a cone. If the radius of hemisphere and base radius of cone is 7 cm and height of cone is 3.5 cm, find the volume of the solid. $\left(\text{Take } \pi = \frac{22}{7}\right)$**

A [CBSE OD Set-III, 2020]

Sol. Here, radius $(r) = 7$ cm

and height of cone $(h) = 3.5$ cm



$\therefore$ Volume of the soild

= Volume of hemisphere + volume of a cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{2}{3} \times \frac{22}{7} \times (7)^3 + \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 3.5$

$= \frac{1}{3}(2156 + 539)$

$= \frac{1}{3} \times 2695$

$= 898.33$ cm^3

AI **2. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed ?**

A [CBSE Delhi Set-I, II, III, 2019, 2018]
[CBSE OD Set-III, 2020, 2017]

Sol. Length of canal covered in 30 min = 5000 m **1½**

$\therefore$ Volume of water flown in 30 min

$= 6 \times 1.5 \times 5000$ m^3 **2**

If 8 cm standing water is needed

then area irrigated $= \frac{6 \times 1.5 \times 5000}{0.08} = 562500$ m^2 **1½**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Canal is of the shape of cuboid, where

Breadth = 6 m

Depth = 1.5 m

and Speed of water = 10 km/hr

Length of water moved in 60 minutes = 10 km

Length of water moved in 1 minute $= \frac{1}{60} \times 10$ km

Length of water moved in 30 minutes

$= \frac{30}{60} \times 10 = 5$ km $= 5000$ m

Now, volume of water in canal
= Length × Breadth × Depth
= $5000 \times 6 \times 1.5\ m^3$

Now, volume of water in canal
= volume of water in area irrigated

$5000 \times 6 \times 1.5\ m^3$ = Area irrigated × 8 cm

$5000 \times 6 \times 1.5\ m^3$ = Area irrigated × $\frac{8}{100}$ m

$\therefore$ Area irrigated = $\frac{5000 \times 6 \times 1.5 \times 100}{8}\ m^2$

= $5.625 \times 10^5\ m^2$

3. Water is flowing at the rate of 15 km/h through a cylindrical pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time the level of water in pond rise by 21 cm ?

A [CBSE SQP 2020, 2018]

Sol. Quantity of water flowing through pipe in 1 hour

$= \pi \times \frac{7}{100} \times \frac{7}{100} \times 15000\ m^3$ 2½

Required time = $\left(50 \times 44 \times \frac{21}{100}\right) \div \left(\pi \times \frac{7}{100} \times \frac{7}{100} \times 15000\right)$

= 2 hours 2½

[CBSE Marking Scheme, 2020]

Detailed Solution:

Speed of water flowing through the pipe
= 15 km/hr = 15000 m/hr

Volume of water flowing in 1hr = $\pi R^2 H$

$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000\ m^3$

$= 231\ m^3$

Volume of water in the tank when the depth is 21 cm
= lbh

$= 50 \times 44 \times \frac{21}{100}\ m^3$

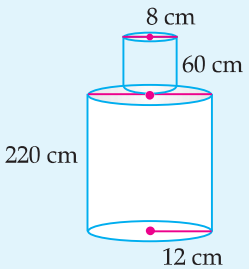
$= 462\ m^3$

Time taken to fill $462\ m^3$ = $\frac{462}{231}$ = 2 hrs.

AI **4. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 gm mass. (Use $\pi = 3.14$)**

A [CBSE OD Set-I, 2019] [CBSE Term-II, 2012]

Sol.



1

Total volume = $3.14\ (12)^2\ (220) + 3.14(8)^2(60)\ cm^3$ 1

= 99475.2 + 12057.6

= $111532.8\ cm^3$ 1

Mass = $\frac{111532.8 \times 8}{1000}$ kg 1

= 892.262 kg 1

[CBSE Marking Scheme, 2019]

Detailed Solution:

To find mass of pole, we need to find volume of pole.

Volume of pole = Volume of small cylinder + Volume of large cylinder

Now, for small cylinder,

Radius, r = 8 cm

Height, h_1 = 60 cm

Volume of small cylinder

$= \pi r^2 h_1$

$= 3.14 \times (8)^2 \times 60$

$= 3.14 \times 64 \times 60$

$= 12057.6\ cm^3$

Volume of large cylinder,

Height = h_2 = 220 cm

Radius = R = $\frac{\text{diameter}}{2}$

= $\frac{24}{2}$ = 12 cm

Volume of large cylinder = $\pi R^2 h_2$

$= 3.14 \times (12)^2 \times 220$

$= 3.14 \times 144 \times 220$

$= 99475.2\ cm^3$

Now, Volume of pole = Volume of small cylinder + Volume of large cylinder

= 12057.6 + 99475.2

= $111532.8\ cm^3$

Since, 1 cm^3 of iron has 8 gm mass

So, given volume of iron has mass

= 111532.8×8 gm

= 892262.4 gm

= $\frac{892262.4}{1000}$ kg

= 892.2624 kg

= 892.262 kg

COMMONLY MADE ERROR

- Many candidates use formula for finding volume of cylinder, $2\pi r^2 h$ instead of $\pi r^2 h$.

ANSWERING TIP

- Remember the correct formula for finding volume of cylinder.

5. Water is flowing at the rate of 5 km/hour through a pipe of diameter 14 cm into a rectangular tank of dimensions 50 m × 44 m. Find the time in which the level of water in the tank will rise by 7 cm.

A [CBSE Comptt. Delhi Set-I, II, III, 2017]

Sol. Try yourself similar to Q. 3. L.A.T.Q.

[CBSE Marking Scheme, 2017]

6. A vessel full of water is in the form of an inverted cone of height 8 cm and the radius of its top, which is open, is 5 cm. 100 spherical lead balls are dropped into vessel. One-fourth of the water flows out of the vessel. Find the radius of a spherical ball. [Foreign Set-I, II, III, 2015]

Sol. Volume of water in cone

$$= \frac{1}{3}\pi r^2 h \qquad ½$$

$$= \frac{1}{3}\pi \times (5)^2 \times 8 \qquad ½$$

$$= \frac{200}{3}\pi \text{ cm}^3 \qquad ½$$

Volume of water flown out

$$= \frac{1}{4} \times \frac{200}{3}\pi = \frac{50}{3}\pi \text{ cm}^3 \qquad \mathbf{1}$$

Let the radius of one spherical ball be r cm **1½**

$$\therefore \quad \frac{4}{3}\pi r^3 \times 100 = \frac{50}{3}\pi$$

$$r^3 = \frac{50}{4 \times 100} = \frac{1}{8}$$

or, $$r = \frac{1}{2} = 0.5 \text{ cm} \qquad \mathbf{1}$$

[CBSE Marking Scheme, 2015]

7. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of a cone of radius 3 cm and height 9 cm. Find the number of toys so formed.

A [CBSE OD Comptt. Set-II, III, 2017]

Sol. Given, height of cylinder = 15 cm

and its diameter = 12 cm

$\therefore$ radius = 6 cm

radius of cone = 3 cm **1**

and height = 9 cm

Let the number of toys recast be n. **1**

$\therefore$ Volume of n conical toys = Volume of cylinder **1**

$$n \times \frac{1}{3}\pi \times 3 \times 3 \times 9 = \pi \times 6 \times 6 \times 15$$

$$n = \frac{6 \times 6 \times 15}{3 \times 9} \qquad \mathbf{1}$$

$$n = 20$$

Hence the number of toys = 20. **1**

[CBSE Marking Scheme, 2017]

Q. 8. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire.

$\left[\text{Use } \pi = \frac{22}{7}\right]$ **A [CBSE OD Set-I, II, III, 2015]**

Sol. Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times 10 \qquad \mathbf{1}$$

$$= 554.40 \text{ cm}^3 \qquad ½$$

Volume of metal scooped out = 2 × volume of hemisphere

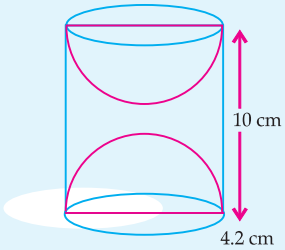
$$= 2 \times \frac{2}{3} \times \pi r^3$$

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{42}{10}\right)^3 \qquad \mathbf{1}$$

$$= 310.46 \text{ cm}^3$$

Volume of rest of cylinder = 554.40 – 310.46



½

$$= 243.94 \text{ m}^3 \qquad ½$$

Let the length of wire be l cm

$\therefore$ Volume of wire $= \pi r^2 l$

and $\pi r^2 l = 243.94 \text{ cm}^3$ ½

$$\frac{22}{7} \times \frac{7}{10} \times \frac{7}{10} \times l = 243.94 \text{ cm}^3 \qquad ½$$

or, $$l = \frac{243.94 \times 10 \times 10}{22 \times 7}$$

$$l = 158.4 \text{ cm} \qquad ½$$

[CBSE Marking Scheme, 2015]