Long Answer Type Questions

[4 marks]

Que 1. From the pair of linear equations in this problem, and find its solution graphically: 10 students of class X took part in a Mathematics quiz. If the girls is 4 more than the number of boys, find the number of boy and girls who took part in the quiz.

Sol. Let x be the number of girls and y be the number of boys.

According to questions, we have

$$x = y + 4$$

$$\Rightarrow \qquad x - y = 4$$

Again, total number of student = 10

Therefore, $\quad x + y = 10$

Hence, we have following system of equations

$$x - y = 4$$

And $\quad x + y = 10$

From equation (i), we have the following system of equations

x	0	4	7
y	-4	0	3

From equation (ii), we have the following table:

x	0	10	7
y	10	0	3

Plotting this, we have

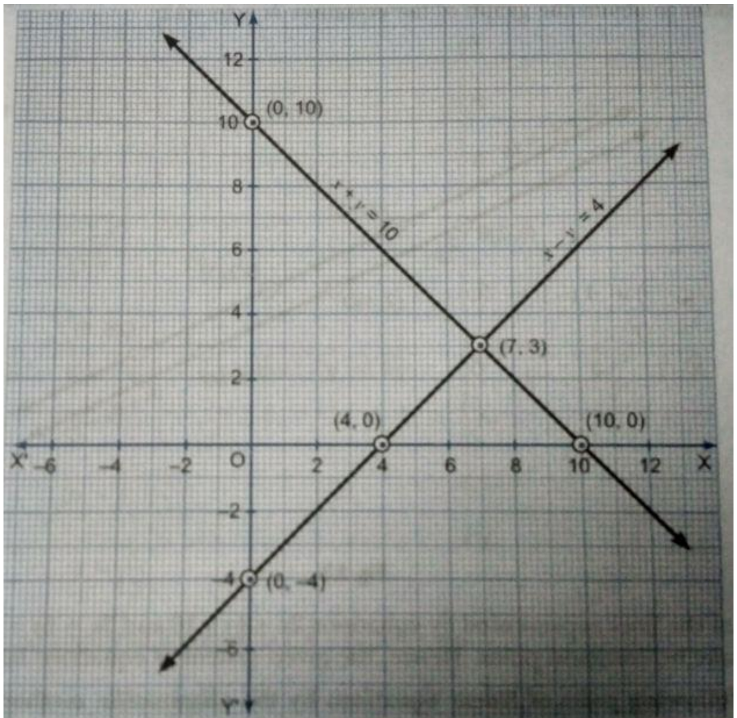

Fig. 3.2

Here, the two lines intersect at point (7,3) $i.e, x = 7, y = 3$.

So, the number of girls = 7

And number of boys = 3

Que 2. Show graphically the given system of equations

$\mathbf{2x + 4y = 10}$ **And** $\mathbf{3x + 6y = 12}$ **Has no solution.**

Sol. We have, $2x + 4y = 10$

$\Rightarrow$ $4y = 10 - 2x$ $\Rightarrow$ $y = \frac{5-x}{2}$

Thus, we have he following table:

x	1	3	5
y	2	1	0

Plot the points A (1, 2), B (3, 1) and C (5, 0) on the graph paper. Join A, B and C and extend it on both sides to obtain the equation $2x + 4y = 10$.

We have, $3x + 6y = 12$

$\Rightarrow$ $6y = 12 - 3x$ $\Rightarrow$ $y = \frac{5-x}{2}$

Thus, we have the following table:

x	2	0	4
y	1	2	0

Plot the points $D(2, 1)$, $E(0, 2)$ and $F(4, 0)$ on the same graph paper. Join D, E and F and extend on both sides to obtain the graph of the equation $3x + 6y = 12$.

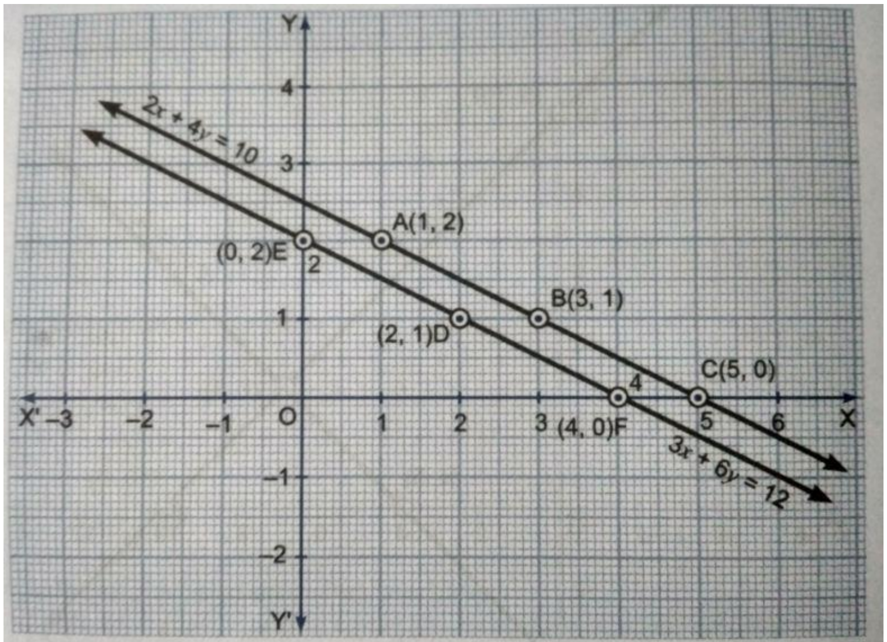

Fig. 3.3

We find that the lines represented by equations $2x + 4y = 10$ and $3x + 6y = 12$ are parallel. So the two lines have no common point. Hence, the given system of equations has no solution.

Que 3. Solve the following pairs of linear equations by the elimination method and the substitution method:

$(\boldsymbol{i})$ $\boldsymbol{3x - 5y - 4 = 0}$ **And** $\boldsymbol{9x = 2y + 7}$

$(\boldsymbol{ii})$ $\boldsymbol{\frac{x}{2} + \frac{2y}{3} = -1}$ **And** $\boldsymbol{x - \frac{y}{3} = 3}$

Sol. (i) we have, $3x - 5y - 4 = 0$

$\Rightarrow$ $3x - 5y = 4$ $\quad\ldots(i)$

Again, $9x = 2y + 7$

$\Rightarrow$ $9x - 2y = 7$ $\quad\ldots(ii)$

By Elimination Method:

Multiplying equation (i) by 3, we get

$$9x - 15y = 12 \quad\ldots(iii)$$

Subtracting (ii) from (iii), we get

$$\begin{array}{rcrcr} 9x & - & 15y & = & 12 \\ 9x & - & 2y & = & 7 \\ - & & + & & - \\ \hline & & -13y & = & 5 \end{array}$$

$\Rightarrow \qquad y = -\frac{5}{13}$

Putting the value of y in equation (ii), we have

$$9x - 2\left(-\frac{5}{13}\right) = 7 \quad \Rightarrow \quad 9x + \frac{10}{13} = 7 \quad \Rightarrow \quad 9x = 7 - \frac{10}{13}$$

$$\Rightarrow \quad 9x = \frac{91-10}{13} \quad \Rightarrow \quad 9x = \frac{81}{13} \quad \Rightarrow \quad x = \frac{9}{13}$$

Hence, the required solution is $x = \frac{9}{13}, y = -\frac{5}{13}$.

By Substitution Method:

Expressing x in terms of y from equation (i), we have

$$x = \frac{4+5y}{3}$$

Substituting the value of x in equation (ii), we have

$$9 \times \left(\frac{4+5y}{3}\right) - 2y = 7$$

$$\Rightarrow \quad 3 \times (4 + 5y) - 2y = 7$$

$$\Rightarrow \quad 12 + 15y - 2y = 7 \quad \Rightarrow \quad 13y = 7 - 12$$

$$\therefore \quad y = -\frac{5}{13}$$

Putting the value of y in equation (i), we have

$$3x - 5 \times \left(-\frac{5}{13}\right) = 4 \quad \Rightarrow \quad 3x + \frac{25}{13} = 4$$

$$\Rightarrow \quad 3x = 4 - \frac{25}{13} \quad \Rightarrow \quad 3x = \frac{27}{13}$$

$$\therefore \quad x = \frac{9}{13}$$

Hence, the required solution is $x = \frac{9}{13}, y = -\frac{5}{13}$.

(ii) We have, $\frac{x}{2} + \frac{2y}{3} = -1 \quad \Rightarrow \quad \frac{3x+4y}{6} = -1$

$\therefore \quad 3x + 4y = -6 \qquad \ldots(i)$

And $\quad x - \frac{y}{3} = 3 \qquad \Rightarrow \qquad \frac{3x-y}{3} = 3$

$\therefore \quad 3x - y = 9 \qquad \ldots(ii)$

By Elimination Method:

Subtracting (ii) from (i), we have

$$5y = -15 \quad \text{or } y = -\frac{15}{5} = -3$$

Putting the value of y in equation (i), we have

$$3x + 4 \times (-3) = -6 \qquad \Rightarrow \qquad 3x - 12 = -6$$

$\therefore \quad 3x = -6 + 12 \qquad \Rightarrow \qquad 3x = 6$

Hence, Solution is $x = 2, y = -3$.

By Elimination Method:

Expressing x in terms of y from equation (i), we have

$$x = \frac{-6-4y}{3}$$

Substituting the value of x in from equation (i), we have

$$3 \times \left(\frac{-6-4y}{3}\right) - y = 9 \quad \Rightarrow \quad -6 - 4y - y = 9 \quad \Rightarrow \quad -6 - 5y = 9$$

$\therefore \quad -5y = 9 + 6 = 15$

$\therefore \quad y = \frac{15}{-5} = -3$

Putting the value of y in equation (i), we have

$$3x + 4 \times (-3) = -6 \qquad \Rightarrow \qquad 3x - 12 = -6$$

$\therefore \quad 3x = 12 - 6 = 6 \qquad \therefore \quad x = \frac{6}{3} = 2$

Hence, the required solution is $x = 2, y = -3$.

Que 4. Draw the graph of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the tringle formed by these lines and the x-axis, and shade the triangular region.

Sol. We have, $\quad x - y + 1 = 0$ and $\quad 3x + 2y - 12 = 0$

Thus, $\quad x - y = -1 \quad \Rightarrow \quad x = y - 1 \qquad \ldots(i)$

$3x + 2y = 12 \quad \Rightarrow \quad x = \frac{12-12y}{3} \qquad \ldots(ii)$

From equation (i), we have

x	-1	0	2
y	0	1	3

From equation (ii), we have

x	0	4	2
y	6	0	3

Plotting this, we have

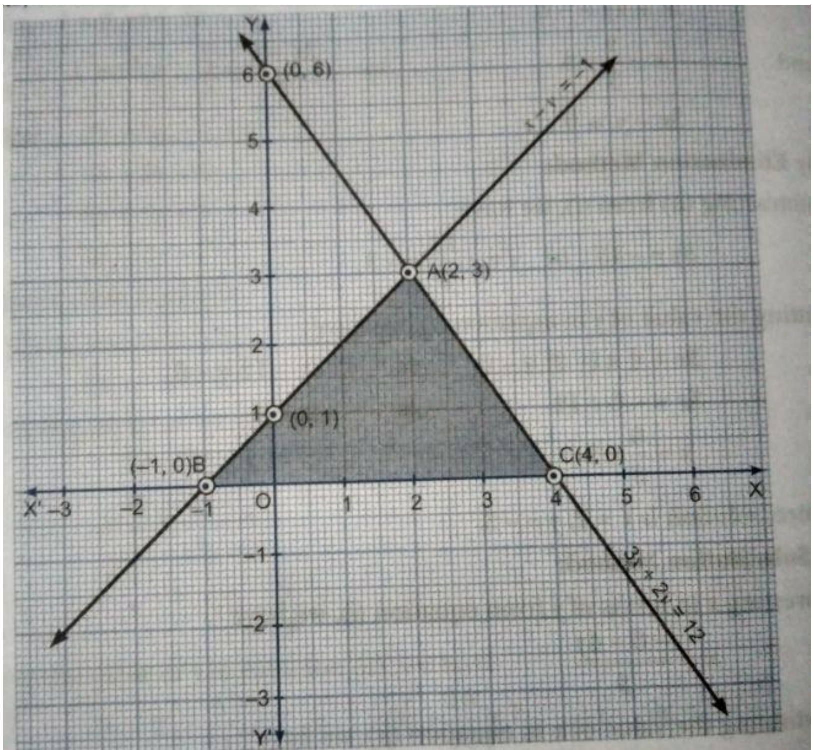

Fig. 3.4

ABC is the required (shaded) region and point of intersection is (2,3).

∴ The vertices of the tringle are (-1, 0), (4, 0), (2, 3).

From the pair of linear equations in the following problem and find their solutions (if they exist by any algebraic method (Q.5 to 8) :

Que 5. A Part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay 1000 as hostel charges whereas a student B, who takes food for 26 days, pays `1180 as hostel charges. Find the fixed charges and the cost of food per day.

Sol. Let the fixed charge be ₹ x and the cost of food per day be ₹ y.

Therefore, according to question,

$$x + 20y = 1000 \quad \ldots(i)$$
$$x + 26y = 1180 \quad \ldots(ii)$$

Now, subtracting equation (ii) from (i), we have

$$x + 20y = 1000$$
$$\underline{-x + -26y = -1180}$$
$$-6y = -180$$
$$y = \frac{-180}{-6} = 30$$

Putting the value of y in equation (i), we have

$$x + 20 \times 30 = 1000 \quad \Rightarrow \quad x + 600 = 1000 \quad \Rightarrow x = 1000 - 600 = 400$$

Hence, fixed charge is `400 and cost of food per day is ₹30.

Que 6. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deduced for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Sol. Let x be the number of questions of right answer and y be the number of questions of wrong answer.

∴ According to question,

$$3x - y = 40 \quad \ldots(i)$$

and $4x - 2y = 50$

or $2x - y = 25 \quad \ldots(ii)$

Subtracting (ii) from (i), we have

$$3x - y = 40$$
$$\underline{-2x \mathop{-}_{+} y = -25}$$
$$x = 15$$

Putting the value of x in equation (i), we have

$$3x\ 15 - y = 40 \quad \Rightarrow \quad 45 - y = 40$$

∴ $y = 45 - 40 = 5$

Hence, total number of question is x + y i. e., 5 + 15 = 20.

Que 7. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Sol. Let the speed of two cars be x km/h and y km/h respectively.

Case I: When two cars move in the same direction, they will meet each other at P after 5 hours.

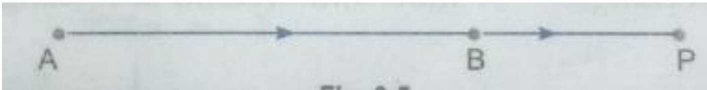

Fig. 3.5

The distance covered by car from A = 5x (Distance = Speed × Time)
and distance covered by the car from B = 5y

$\therefore \quad 5x - 5y = AB = 100 \quad \Rightarrow \quad x - y = \frac{100}{5}$

$\therefore \quad x - y = 20 \qquad \ldots (i)$

Case II: When two cars move in opposite direction, they will meet each other at Q after one hour.

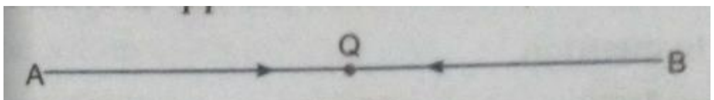

Fig. 3.6

The distance covered by the car from A = x
The distance covered by the car from B = y
$\therefore \quad x + y = AB = 100 \quad \Rightarrow \quad x + y = 100 \qquad \ldots (ii)$
Now, adding equations (i) and (ii), we have
$2x = 120 \quad \Rightarrow \quad x = \frac{120}{2} = 60$

Putting the value of x in equation (i), we get
$60 - y = 20 \quad \Rightarrow \quad -y = -40 \quad \therefore \quad y = 40$
Hence, the speeds of two cars are 60 km/h and 40 km/h respectively.

Que 8. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Sol. Let the length and breadth of a rectangle be x and y respectively.
Then area of the rectangle = xy
According to question, we have
$(x - 5)(y + 3) = xy - 9 \quad \Rightarrow \quad xy + 3x - 5y - 15 = xy - 9$

$\Rightarrow \quad 3x - 5y = 15 - 9 = 6 \quad \Rightarrow \quad 3x - 5y = 6 \qquad \ldots (i)$

Again, we have
$(x - 3)(y + 2) = xy + 67 \quad \Rightarrow \quad xy + 2x + 3y + 6 = xy + 67$

$\Rightarrow \quad 2x - 3y = 67 - 6 = 61 \quad \Rightarrow \quad 2x - 3y = 61 \qquad \ldots (ii)$

Now, from equation (i), we express the value of x in terms of y.

$$x = \frac{6+5y}{3}$$

Substituting the value of x in equation (ii), we have

$$2 \times \left(\frac{6+5y}{3}\right) + 3y = 61 \quad \Rightarrow \quad \frac{12+10y+9y}{3} = 61$$

$$\Rightarrow 19y = 183 - 12 = 171 \quad \Rightarrow \quad y = \frac{171}{19} = 9$$

Putting the value of y in equation (i), we have

$$3x - 5 \times 9 = 6 \quad \Rightarrow \quad 3x = 6 + 45 = 51$$

$$\therefore \quad x = \frac{51}{3} = 17$$

Hence, the length of rectangle = 17 units and breadth of rectangle = 9 units.

Que 9. Formulate the following problems as a pair of equations, and hence find their solutions:
(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find he speed of rowing in still water and the speed of the current.
(ii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by bus and the remaining by train. If she travels 100 km by bus and the remaining by train, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Sol. (i) Let her speed of rowing in still water be x km/h and the speed of the current be y km/h

Case I: When Ritu rows downstream

Her speed (downstream) = $(x + y)$ km/h

Now, We have speed = $\frac{\text{distance}}{\text{time}}$

$$\Rightarrow \quad (x + y) = \frac{20}{2} = 10$$

$$\therefore \quad x + y = 10 \quad \ldots (i)$$

Case II: When Ritu rows upstream

Her speed (upstream) = $(x - y)$ km/h

Again, Speed = $\frac{\text{distance}}{\text{time}}$

$$\Rightarrow \quad x - y = \frac{4}{2} = 2$$

$$\therefore \quad x - y = 2 \quad \ldots (ii)$$

Now, adding (i) and (ii), we have

$$2x = 12 \qquad \Rightarrow \qquad x = \frac{12}{2} = 6$$

Putting the value of x in equation (i), we have

$$6 + y = 10 \qquad \Rightarrow \qquad y = 10 - 6 = 4$$

Hence, speed of Ritu in still water = 6 km/h. and speed of current = 4 km/h.

(ii) Let the speed of the bus be x km/h and speed of the train be y km/h.

According to question, we have

$$\frac{60}{x} + \frac{240}{y} = 4$$

And $\frac{100}{x} + \frac{200}{y} = 4 + \frac{10}{60} = 4 + \frac{1}{6} = \frac{25}{6} \Rightarrow \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$

Now, let $\frac{1}{x} = u$ and $\frac{1}{y} = u$,

$\therefore \quad 60u + 240v = 4 \qquad \ldots (i)$

$100u + 200v = \frac{25}{6} \qquad \ldots (ii)$

Multiplying equation (i) by 5 and (ii) by 6 and subtracting, we have

$$\begin{array}{rcrcr} 300u & + & 1200v & = & 20 \\ \underset{-}{}600u & \overset{+}{-} & 1200v & = & \underset{-}{}25 \\ \hline & & -300u & = & -5 \end{array}$$

$\therefore \quad u = \frac{-5}{-300} = \frac{1}{60}$

Putting the value of u in equation (i), we have

$$60 \text{ x } \frac{1}{60} + 240v = 4 \qquad \Rightarrow \qquad 240v = 4 - 1 = 3$$

$\therefore \quad v = \frac{3}{240} = \frac{1}{80}$

Now, $\quad u = \frac{1}{60} \qquad \Rightarrow \qquad \frac{1}{x} = \frac{1}{60} \qquad \therefore \qquad x = 60$

And $\quad v = \frac{1}{80} \qquad \Rightarrow \qquad \frac{1}{y} = \frac{1}{80} \qquad \therefore \qquad y = 80$

Hence, speed of the bus is 60 km/h and speed of the train is 80 km/h.

Que 10. The sum of a two digit number and the number formed by interchanging its digits is 110.

If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of the digits in the first number. Find the first number.

Sol. Let the digits at unit and tens places be x and y respectively.

Then, number = 10y + x $\quad\ldots(i)$

Number formed by interchanging the digits = $10x + y$

According to the given condition, we have

$(10y + x) + (10x+y) = 110 \quad\Rightarrow\quad 11x + 11y = 110 \quad\Rightarrow\quad x + y - 10 = 0$

Again, according to question, we have

$(10y + x) - 10 = 5(x + y) + 4 \quad\Rightarrow\quad 10y + x - 10 = 5x + 5y + 4$

$\Rightarrow 10y + x - 5x - 5y = 4 + 10$

$5y - 4x = 14$ Or $4x - 5y + 14 = 0$

By using cross-multiplication, we have

$$\frac{x}{1 \times 14 - (-5) \times (-10)} = \frac{-y}{1 \times 14 - 4 \times (-10)} = \frac{1}{1 \times (-5) - 1 \times 4}$$

$$\Rightarrow \frac{x}{14-50} = \frac{-y}{14+40} = \frac{1}{-5-4} \quad\Rightarrow\quad \frac{x}{-36} = \frac{-y}{54} = \frac{1}{-9}$$

$$\Rightarrow x = \frac{-36}{-9} \text{ and } y = \frac{-54}{-9} \quad\Rightarrow\quad x = 4 \text{ and } y = 6$$

Putting the values of x and y in equation (i), we get Number = 10 x 6 + 4 = 64.

Que 11. Jamila sold a table and a chair for ₹1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair she would have got ₹1065. Find cost price of each.

Sol. Let cost price of table be ₹x and the cost price of the chair be ₹y.

The selling price of the table, when it is sold at profit of 10% = ₹ $\left(x + \frac{10x}{100}\right) = \frac{110x}{100}$

The selling price of the chair when it is sold at a profit of 25% = ₹ $\left(y + \frac{25y}{100}\right) = \frac{125y}{100}$

So, $\frac{110x}{100} + \frac{125y}{100} = 1050 \quad\ldots(i)$

When the table is sold at a profit of 25%

Its selling price = ₹$\left(x + \frac{25}{100}x\right)$ = ₹ $\frac{125}{100}x$

When the chair is sold at a profit of 10%

its selling price = ₹ $\left(y + \frac{10y}{100}\right)$ = ₹$\frac{110y}{100}$

So, $\frac{125}{100}x + \frac{110y}{100} = 1065$ … (ii)

Form equation (i) and (ii) we get

$110x + 125y = 105000$

and $125x + 110y = 106500$

On adding and subtracting these equations we get

$235x + 235y = 211500$

and $15x - 15y = 1500$

$i.e.,$ $x + y = 900$ … (iii)

$x - y = 100$ … (iv)

Solving equation (iii) and (iv) we get

$x = 500$, $y = 400$

So, the cost price of the table is ₹ 500 and the cost price of the chair is ₹ 400.