CBSE Test Paper 01

Chapter 7 Coordinate Geometry

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1. The distance between the points $A(p\ sin\ 25^\circ,\ 0)$ and $B(0,\ p\ sin\ 65^\circ)$ is **(1)**
   a. 0 units
   b. p units
   c. p2 units
   d. 1 units
2. If the points (x, y), (1, 2) and (7, 0) are collinear, then the relation between ‘x’ and ‘y’ is given by **(1)**
   a. 3x – y – 7 = 0
   b. 3x + y + 7 = 0
   c. x + 3y – 7 = 0
   d. x – 3y + 7 = 0
3. If the distance between the points (p, – 5) and (2, 7) is 13 units, then the value of ‘p’ is **(1)**
   a. -3, -7
   b. 3, -7
   c. 3, 7
   d. -3, 7
4. If the vertices of a triangle are (1, 1), (– 2, 7) and (3, – 3), then its area is **(1)**
   a. 0 sq. units
   b. 2 sq. units
   c. 24 sq. units
   d. 12 sq. units
5. The distance between the points (x_1, y_1) and (x_2, y_2) is given by **(1)**
   a. $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$ units
   b. $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$ units
   c. $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ units
   d. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ units
6. If the points A(x, 2), B(- 3, - 4), C(7, - 5) are collinear, then find the value of x. **(1)**

7. Find the distance between the points A and B in the following : A(a, 0), B(0, a) **(1)**

8. Find the perpendicular distance of A(5,12) from the y-axis. **(1)**
9. Find the distance of the point (- 4, - 7) from the y-axis. **(1)**
10. Find the coordinates of the centroid of a triangle whose vertices are (0,6), (8,12) and (8,0). **(1)**
11. Find the distance between the points: A(-6, -4) and B(9, -12) **(2)**
12. Find the condition that the point (x, y) may lie on the line joining (3, 4) and (-5, - 6). **(2)**
13. If P (x, y) is any point on the line joining the points A(a,0) and B(0, b), then show that $\frac{x}{a}+\frac{y}{b}=1$. **(2)**
14. The area of triangle formed by the points (p, 2 - 2p), (1, p, 2 p) and (-4 -p, 6 - 2p) is 70 sq. units. How many integral values of p are possible. **(3)**
15. Point A is on x-axis, point B is on y-axis and the point P lies on line segment AB, such that P (4, - 5) and AP : PB = 5 : 3. Find the coordinates of point A and B. **(3)**
16. Show that four points (0, -1), (6, 7), (-2, 3) and (8, 3) are the vertices of a rectangle. Also, find its area. **(3)**
17. Find the co-ordinates of the points of trisection of the line segment joining the points A(1, - 2) and B(- 3,4). **(3)**
18. Show that the points A(3, 5), B(6, 0), C(1, -3) and D (-2, 2) are the vertices of a square ABCD. **(4)**
19. A (4, 2), B (6, 5) and C (1, 4) are the vertices of $\triangle$ABC.
    i. The median from A meets BC in D. Find the coordinates of the point D.
    ii. Find the coordinates of point P on AD such that AP : PD = 2:1.
    iii. Find the coordinates of the points Q and R on medians BE and CP respectively such that BQ : QE = 2 :1 and CR: RF =2: 1.
    iv. What do you observe? **(4)**
20. Find the lengths of the medians of a ΔABC whose vertices are A(0, -1) B(2, 1) and C(0, 3). **(4)**

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Chapter 07 Coordinate Geometry

Solution

1. b. p units

   Explanation: The distance between point A and point B=

   AB = $\sqrt{(0-p\sin 25^\circ)^2+(p\sin 65^\circ-0)^2}$

   = $\sqrt{p^2\sin^2 25^\circ+p^2\sin^2 65^\circ}$

   = $p\sqrt{\sin^2 25^\circ+\sin^2(90^\circ-25^\circ)}$

   = $p\sqrt{\sin^2 25^\circ+\cos^2 25^\circ}[\because \sin(90^\circ-\theta)=\cos\theta]$

   = p units

   $[\because \cos^2\theta+\sin^2\theta=1]$

2. c. x + 3y – 7 = 0

   Explanation: >$\therefore \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|=0$

   $\Rightarrow \frac{1}{2}|x(2-0)+1(0-y)+7(y-2)|=0$

   $\Rightarrow \frac{1}{2}|2x-y+7y-14|=0$

   $\Rightarrow 2x+6y-14=0 \Rightarrow x+3y-7=0$

3. d. -3, 7

   Explanation: Let point A be (p, -5) and point B (2, 7) and distance between A and B = 13 units

   $\therefore\ 13=\sqrt{(2-p)^2+(7+5)^2}$

   $\Rightarrow 13=\sqrt{4+p^2-4p+144}$

   $\Rightarrow 13=\sqrt{p^2-4p+148}$

   $\Rightarrow 169=p^2-4p+148$

   $\Rightarrow p^2-4p-21=0$

   = p^2 - 7p + 3p - 21= 0

   = p(p - 7) + 3(p - 7) = 0

   $\Rightarrow (p-7)(p+3)=0$

   $\Rightarrow p=7, p=-3$

4. a. 0 sq. units

Explanation: Given: $(x_1, y_1) = (1, 1), (x_2, y_2) = (-2, 7)$ and $(x_3, y_3) = (3, -3)$, then the Area of triangle

$\therefore \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

= $\frac{1}{2}|1(7+3) + (-2)(-3-1) + 3(1-7)|$

= $\frac{1}{2}|10 + 8 - 18|$

= $\frac{1}{2}|0|$ = 0 sq. units

Also therefore the three given points(vertices) are collinear.

5. d. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$units

Explanation: The distance between the points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ units. This is known as distance formula.

6. Since the points are collinear, then,

Area of triangle = 0

$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\frac{1}{2}[x(-4+5) + (-3)(-5-2) + 7(2+4)] = 0$

x + 21 + 42 = 0

x = -63

7. A(a, 0), B(0, a)

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0-a)^2 + (a-0)^2}$

$= \sqrt{(a^2 + a^2)} = \sqrt{2a^2} = \sqrt{2}a\ units$

8. The point on the y-axis is (0,12)

∴Distance between (5,12) and (0,12)

d = $\sqrt{(0-5)^2 + (12-12)^2}$

= $\sqrt{25+0}$

= 5 units

9. Points are (- 4, - 7) and (0, - 7)

Distance $= \sqrt{(0+4)^2 + (-7+7)^2}$

= $\sqrt{4^2 + 0} = \sqrt{16}$ = 4 units

10. Coordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3) are

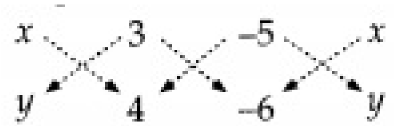
$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$
$$= \left(\frac{0+8+8}{3}, \frac{6+12+0}{3}\right) = \left(\frac{16}{3}, \frac{18}{3}\right) = \left(\frac{16}{3}, 6\right).$$

11. The given points are A(-6, -4) and B(9, -12)

    Then, (x_1 = -6, y_1 = -4) and (x_2 = 9, y_2 = -12)

    $$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
    $$= \sqrt{(6+9)^2 + (-12+4)^2} = \sqrt{(15)^2 + (-8)^2}$$
    $$= \sqrt{225+64} = \sqrt{289} = 17\ units$$

12. Since the point P (x, y) lies on the line joining A (3, 4) and B (-5, -6). Therefore, P, A and B are collinear points.

    $$\therefore \{4x + 3 \times -6 + (-5) \times y\} - \{3y + (-5) \times 4 + x \times (-6)\} = 0$$

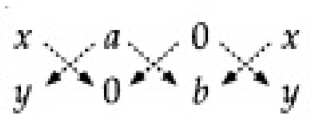
    ⇒ {4x - 18 - 5y) - (3y - 6x - 20) = 0

    ⇒ 10x -8 y + 2 = 0 ⇒ 5x - 4 y + 1 = 0

    Hence, the point (x, y) lies on the line joining (3,4) and (-5, -6), if 5x - 4y+1 = 0

13. It is given that the point P (x, y) lies on the line segment joining points A (a, 0) and B (0, b).

    Therefore, points P (x, y), A (a, 0) and B (0, b) are collinear points.

    $$\therefore (x \times 0 + a \times b + 0 \times y) - (a \times y + 0 \times 0 + x \times b) = 0$$

    ⇒ ab - (ay + bx) = 0

    ⇒ ab = ay + bx

    $\Rightarrow \frac{ab}{ab} = \frac{ay}{ab} + \frac{bx}{ab}$ [Dividing throughout by ab]

    $\Rightarrow 1 = \frac{y}{b} + \frac{x}{a}$ or $\frac{x}{a} + \frac{y}{b} = 1$

14. Area= $\frac{1}{2}$ [p(2p - 6 + 2p) + (1 - p) (6 - 2p - 2 + 2p) + (- 4 - p)(2 - 2p - 2p)]

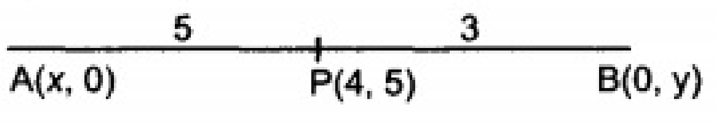
    ⇒ $\frac{1}{2}$ [p(4p - 6) + (1 - p)4 + (- 4 - p) (2 - 4p)] = 70

    ⇒ $4p^2$ - 6p + 4 - 4p - 8 + 16p - 2p + $4p^2$ = 140

    ⇒ $\frac{1}{2}$ [-13k - 9] = 15

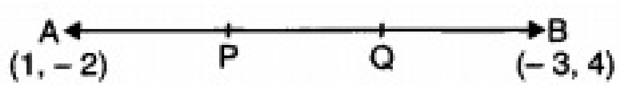
$\Rightarrow$ [-13k - 9] = 30 - 13k - 9 = 30 or - 13k - 9 = - 30
k = - 3 or k $= \frac{21}{13}$
When k = - 3, coordinates = 15 sq. units
$\Rightarrow \frac{1}{2} \times \mathrm{AB} \times$ Altitude = 15
$\Rightarrow \frac{1}{2} \times 3 \times$ Altitude = 15
$\Rightarrow$ Altitude = 10 units

15. Let coordinates of A are (x, 0) and coordinates of B are (0, y)



Using section formula, we get
4 = $\frac{5\times 0+3\times x}{5+3}$
$\Rightarrow$ 32 = 3x
$\Rightarrow$ x = $\frac{32}{3}$
Similarly, 5 = $\frac{5\times y+3\times 0}{5+3}$
$\Rightarrow$ 40 = 5y
$\Rightarrow$ y = 8
$\therefore$ Coordinate of A are $\left(\frac{32}{3}, 0\right)$ and coordinates of B are (0, 8).

16. Let A (0 - 1), B (6, 7), C (-2, 3) and D (8, 3) be the given points. Then,
AD = $\sqrt{(8-0)^2+(3+1)^2} = \sqrt{64+16} = 4\sqrt{5}$
BC = $\sqrt{(6+2)^2+(7-3)^2} = \sqrt{64+16} = 4\sqrt{5}$
AC = $\sqrt{(-2-0)^2+(3+1)^2} = \sqrt{4+16} = 2\sqrt{5}$
and, BD = $\sqrt{(8-6)^2+(3-7)^2} = \sqrt{4+16} = 2\sqrt{5}$
Therefore, AD = BC and AC = BD
So, ADBC is a parallelogram
Now, AB = $\sqrt{(6-0)^2+(7+1)^2} = \sqrt{36+64}$ = 10
and, CD = $\sqrt{(8+2)^2+(3-3)^2}$ = 10
Clearly, $AB^2 = AD^2 + DB^2$ and $CD^2 = CB^2 + BD^2$
Hence, ADBC is a rectangle.
Area of rectangle ADBC = $AD \times DB = (4\sqrt{5} \times 2\sqrt{5})$sq. units = 40 sq. units.

17. 

Let P(x_1, y_1), Q(x_2, y_2) divides AB into 3 equal parts.

$\therefore$ P divides AB in the ratio of 1: 2

$\therefore x_1 = \frac{1\times -3+2\times 1}{1+3}$ and $y_1 = \frac{1\times 4+2\times -2}{1+2}$

$\Rightarrow\ x_1 = \frac{2-3}{3} = \frac{-1}{3}\ \ y_1 = \frac{-4+4}{3} = 0$

$\therefore$Co-ordinates of P$\left(-\frac{1}{3}, 0\right)$.

Q is the mid-point of PB.

$\therefore x_2 = \frac{\frac{-1}{3}+(-3)}{2}$

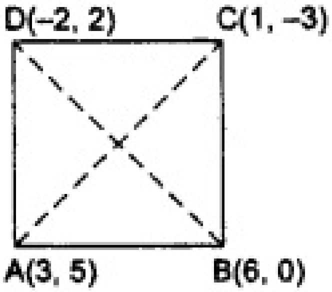
$= \frac{-10}{6} = \frac{-5}{3}$

$y_2 = \frac{0+4}{2} = 2$

$\therefore$Co-ordinates of Q$\left(-\frac{5}{3}, 2\right)$.

18. Let A(3,5), B(6, 0), C(1, -3) and D(-2, 2) be the angular points of a quadrilateral ABCD.

Join AC and BD



Now $AB = \sqrt{(6-3)^2+(0-5)^2}$

$= \sqrt{3^2+(-5)^2}$

$= \sqrt{9+25} = \sqrt{34}\ units,$

$BC = \sqrt{(1-6)^2+(-3-0)^2} = \sqrt{(-5)^2+(-3)^2}$

$= \sqrt{25+9} = \sqrt{34}\ units,$

$CD = \sqrt{(-2-1)^2+(2+3)^2} = \sqrt{(-3)^2+5^2}$

$= \sqrt{9+25} = \sqrt{34}\ units,$

and $DA = \sqrt{(3+2)^2+(5-2)^2} = \sqrt{5^2+3^2}$

$= \sqrt{25+9} = \sqrt{34}\ units,$

Thus, AB = BC =CD= DA.

Diagonal $AC = \sqrt{(1-3)^2+(-3-5)^2} = \sqrt{(-2)^2+(-8)^2}$

$= \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}\ units$

Diagonal $BD = \sqrt{(-2-6)^2+(2-0)^2}$

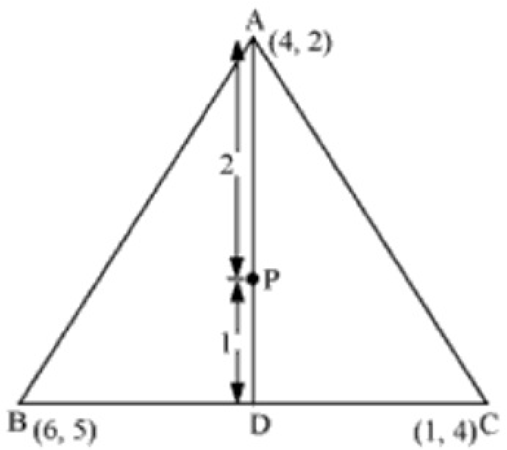
$= \sqrt{(-8)^2 + 2^2} = \sqrt{64 + 4}$

$= \sqrt{68} = 2\sqrt{17}\ units$

$\therefore \quad \text{diag} \cdot AC = \text{diag}.\ BD$

Thus, ABCD is a quadrilateral in which all sides are equal and the diagonals are equal. Hence, quad. ABCD is a square.

19.



i. Median AD of the triangle will divide the side BC in two equal parts. So D is the midpoint of side BC.

   Coordinates of $D = \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$

ii. Point P divides the side AD in a ratio 2 : 1.

   Coordinates of $P = \left(\frac{2 \times \frac{7}{2} + 1 \times 4}{2+1}, \frac{2 \times \frac{9}{2} + 1 \times 2}{2+1}\right)$

   $= \left(\frac{11}{3}, \frac{11}{3}\right)$

iii. Median BE of the triangle will divide the side AC in two equal parts. So E is the midpoint of side AC.

   Coordinates of $E = \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

   Point Q divides the side BE in a ratio 2:1

   Coordinates of $Q = \left(\frac{2 \times \frac{5}{2} + 1 \times 6}{2+1}, \frac{2 \times 3 + 1 \times 5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

   Median CF of the triangle will divide the side AB in two equal parts. So F is the midpoint of side AB.

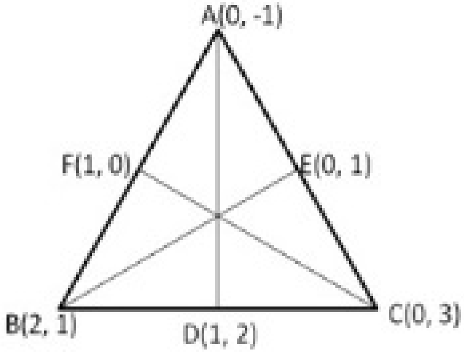
   Coordinates of $F = \left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

   Point R divides the side CF in a ratio 2:1.

Coordinates of $R = \left(\frac{2\times5+1\times1}{2+1}, \frac{2\times\frac{7}{2}+1\times4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

iv. Now we may observe that coordinates of point P, Q are same. So, all these are representing same point on the plane i.e. centroid of the triangle.

20. Let D, E, F be the midpoint of the side BC, CA and AB respectively in ΔABC



Then, by the midpoint formula, we have

$D\left(\frac{2+0}{2}, \frac{1+3}{2}\right), E\left(\frac{0+0}{2}, \frac{3-1}{2}\right) F\left(\frac{0+2}{2}, \frac{-1+1}{2}\right)$

i.e., D(1, 2), E(0, 1), F(1, 0)

Hence the lengths of medians AD, BE and CF are given by

$AD = \sqrt{(1-0)^2+(2+1)^2} = \sqrt{1+9} = \sqrt{10}\ units$

$BE = \sqrt{(0-2)^2+(1-1)^2} = \sqrt{4+0} = \sqrt{4} = 2\ units$

$CF = \sqrt{(1-0)^2+(0-3)^2} = \sqrt{1+9} = \sqrt{10}\ units$

Hence, AD = $\sqrt{10}$, BE= 2, CF = $\sqrt{10}$