Long Answer Type Questions

[4 MARKS]

Que 1. Find the value of:

$$\frac{\mathbf{4}}{(\mathbf{216})^{\frac{-2}{3}}} + \frac{\mathbf{1}}{(\mathbf{256})^{-\frac{3}{4}}} + \frac{\mathbf{2}}{(\mathbf{243})^{-\frac{1}{5}}}.$$

Sol.

$$\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}.$$

$$= 4(216)^{\frac{2}{3}} + (256)^{\frac{3}{4}} + 2(243)^{\frac{1}{5}} = 4(6^3)^{\frac{2}{3}} + (4^4)^{\frac{3}{4}} + 2(3^5)^{\frac{1}{5}}$$

$$= 4 \times 6^2 + 4^3 + 2 \times 3 = 4 \times 36 + 64 + 6$$

$$= 144 + 64 + 6 = 214$$

Que 2. Find the values of a and b:

$$\frac{\mathbf{7+\sqrt{5}}}{\mathbf{7-\sqrt{5}}} - \frac{\mathbf{7-\sqrt{5}}}{\mathbf{7+\sqrt{5}}} = \boldsymbol{a} + \frac{\mathbf{7}}{\mathbf{11}}\sqrt{\mathbf{5}}\boldsymbol{b}.$$

Sol. LHS

$$\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}}$$

Rationalising the denominator, we get

$$\frac{7+\sqrt{5}}{7-\sqrt{5}} \times \frac{7+\sqrt{5}}{7+\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} \times \frac{7-\sqrt{5}}{7-\sqrt{5}}$$

$$= \frac{\left(7+\sqrt{5}\right)^2}{7^2 - \left(\sqrt{5}\right)^2} - \frac{\left(7-\sqrt{5}\right)^2}{7^2 - \left(\sqrt{5}\right)^2}$$

$$= \frac{7^2 + \left(\sqrt{5}\right)^2 + 2 \times 7 \times \sqrt{5}}{49-5} - \frac{7^2 + \left(\sqrt{5}\right)^2 - 2 \times 7\sqrt{5}}{49-5}$$

$$= \frac{49+5+14\sqrt{5}}{44} - \frac{49+5-14\sqrt{5}}{44} = \frac{54+14\sqrt{5}}{44} - \frac{54-14\sqrt{5}}{44}$$

$$= \frac{54+14\sqrt{5}-54+14\sqrt{5}}{44} = \frac{28\sqrt{5}}{44} = \frac{7\sqrt{5}}{11} = 0 + \frac{7\sqrt{5}}{11}$$

Hence,

$$0+\frac{7\sqrt{5}}{11} = a+\frac{7\sqrt{5}b}{11}$$

$$\Rightarrow a = 0, b = 1$$

Que 3. Simplify: $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$.

Sol.

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} = \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}\times\frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}} = \frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{(\sqrt{10})^2-(\sqrt{3})^2} = \frac{7(\sqrt{30}-3)}{10-3}$$

$$\therefore \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} = \frac{7(\sqrt{30}-3)}{7} = \sqrt{30}-3$$

$$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{2\sqrt{30}-2\times 5}{(\sqrt{6})^2-(\sqrt{5})^2}$$

$$\therefore \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{(2\sqrt{30}-10)}{6-5} = 2\sqrt{30}-10$$

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}\times\frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}} = \frac{3\sqrt{30}-18}{15-18} = \frac{3\sqrt{30}-18}{-3}$$

$$\therefore \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3(\sqrt{30}-6)}{-3} = -(\sqrt{30}-6) = 6-\sqrt{30}$$

Therefore,

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$= \sqrt{30}-3-(2\sqrt{30}-10)-(6-\sqrt{30})$$

$$= \sqrt{30}-3-2\sqrt{30}+10-6+\sqrt{30}$$

$$= 10-9+2\sqrt{30}-2\sqrt{30} = 1$$

Que 4. If $a = \frac{2+\sqrt{5}}{2-\sqrt{5}}$ and $b = \frac{2-\sqrt{5}}{2+\sqrt{5}}$, then find the value of $a^2 - b^2$.

Sol.

$$a = \frac{2+\sqrt{5}}{2-\sqrt{5}}$$

$$a = \frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}, \qquad (by\ rationalising\ the\ denominator)$$

$$= \frac{(2+\sqrt{5})^2}{2^2-\left(\sqrt{5}\right)^2} = \frac{4+5+4.\sqrt{5}}{4-5} = \left(9+4\sqrt{5}\right)$$

$$Also, b = \frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{\left(2-\sqrt{5}\right)^2}{2^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{2^2+\left(\sqrt{5}\right)^2-2.2\sqrt{5}}{4-5} = \frac{4+5-4\sqrt{5}}{-1}.$$

$$\therefore\ b = -\left(9-4\sqrt{5}\right) \Rightarrow b = 4\sqrt{5}-9$$

$$We\ know, \qquad a^2-b^2 = (a+b)(a-b)$$

$$Here,\ a+b = -9-4\sqrt{5}+4\sqrt{5}-9 = -18$$

$$a-b = -9-4\sqrt{5}-\left(4\sqrt{5}-9\right) = -8\sqrt{5}$$

$$Hence, a^2-b^2 = (a+b)(a-b) = -18\left(-8\sqrt{5}\right)$$

$$\Rightarrow a^2-b^2 = 144\sqrt{5}$$

Que 5. If $a = \frac{1}{7-4\sqrt{3}}$ *and* $b = \frac{1}{7+4\sqrt{3}}$, then find the value of:

(i) **$A^2 + b^2$** **(ii) $a^3 + b^3$**

Sol.

$$a = \frac{1}{7-4\sqrt{3}} = \frac{1}{7+4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{7+4\sqrt{3}}{7^2-\left(4\sqrt{3}\right)^2}$$

$$= \frac{7+4\sqrt{3}}{49-16\times 3} = \frac{7+4\sqrt{3}}{49-48}$$

$$\therefore\ a = \frac{1}{7-4\sqrt{3}} = 7+4\sqrt{3}$$

$$b = \frac{1}{7+4\sqrt{3}} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{7^2-\left(4\sqrt{3}\right)^2}$$

$$= \frac{7-4\sqrt{3}}{49-16\times 3} = \frac{7-4\sqrt{3}}{49-48}$$

$$\therefore\ b = \frac{1}{7+4\sqrt{3}} = 7-4\sqrt{3}$$

$$\therefore\ a+b = 7+4\sqrt{3}+7-4\sqrt{3} = 14\ and\ ab = \left(7+4\sqrt{3}\right)\left(7-4\sqrt{3}\right)$$

$$= 7^2 - \left(4\sqrt{3}\right)^2 = 49-16\times 3 = 49-48$$

$$\Rightarrow\ ab = 1$$

$$Now, a^2 + b^2 = (a+b)^2 - 2ab = (14)^2 - 2\times 1 = 196-2$$

$$\therefore\ a^2 + b^2 = 194$$

$$Also, a^3 + b^3 = (a+b)^3 - 3ab(a+b) = (14)^3 - 3\times 1\,(14)$$

$$a^3 + b^3 = 2744 - 42 = 2702$$