Long Answer Questions-I-C (PYQ)

[4 Marks]

Q.1. If $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$, then show that
$$y^2\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0.$$

Ans.

Given, $x = a \cos \theta + b \sin \theta$

$$\Rightarrow \quad \frac{dx}{d\theta} = -a\sin\theta + b\cos\theta \qquad \ldots(i)$$

Also, $y = a \sin \theta - b \cos \theta$

$$\frac{dy}{d\theta} = a\cos\theta + b\sin\theta \qquad \ldots(ii)$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\cos\theta + b\sin\theta}{-a\sin\theta + b\cos\theta} \qquad [\text{From } (i) \text{ and } (ii)]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{a\cos\theta + b\sin\theta}{b\cos\theta - a\sin\theta} \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y} \qquad \ldots(iii)$$

Differentiating again with respect to x, we get

$$\frac{d^2y}{dx^2} = -\frac{y - x.\frac{dy}{dx}}{y^2}$$

$$\Rightarrow \quad y^2\frac{d^2y}{dx^2} = -y + x\frac{dy}{dx}$$

$$\Rightarrow \quad y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$

Q.2. If $y = Pe^{ax} + Qe^{bx}$, then show that $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0$

Ans.

Given, $y = Pe^{ax} + Qe^{bx}$

On differentiating with respect to x, we have

$$\frac{dy}{dx} = Pae^{ax} + Qbe^{bx}$$

Again, differentiating with respect to x, we have

$$\frac{d^2y}{dx^2} = Pa^2 e^{ax} + Qb^2 e^{bx}$$

Now, LHS $= \frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby$

$$= Pa^2e^{ax} + Qb^2e^{bx} - (a+b)\left(Pae^{ax} + Qbe^{bx}\right) + ab\left(Pe^{ax} + Qe^{bx}\right)$$

$$= Pa^2e^{ax} + Qb^2e^{bx} - Pa^2e^{ax} - Pabe^{ax} - Qabe^{bx} - Qb^2e^{bx} + Pabe^{ax} + Qabe^{bx}$$

$$= 0 = \text{RHS}$$

Q.3. If $y = \sin(\log x)$, then prove that $x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0.$

Ans.

Given, $y = \sin(\log x)$

$$\Rightarrow \frac{dy}{dx} = \cos(\log x) \times \frac{1}{x} = \frac{\cos(\log x)}{x}$$

Again, $\frac{d^2y}{dx^2} = \frac{x\left[-\sin(\log x)\times\frac{1}{x}\right] - \cos(\log x)}{x^2} = \frac{-\cos(\log x) - \sin(\log x)}{x^2}$

Now, LHS $= x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y$

$$= \frac{x^2\{-\cos(\log x) - \sin(\log x)\}}{x^2} + \frac{x\cos(\log x)}{x} + \sin(\log x)$$

$$= -\cos(\log x) - \sin(\log x) + \cos(\log x) + \sin(\log x) = 0 = \text{RHS}$$

Q.4. If $y = \log[x + \sqrt{x^2+1}]$, then prove that $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

Ans.

Given, $y=\log[x+\sqrt{x^2+1}]$

Differentating with respect to x, we get

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{x+\sqrt{x^2+1}}\times\left[1+\frac{2x}{2\sqrt{x^2+1}}\right]=\frac{(x+\sqrt{x^2+1})}{(x+\sqrt{x^2+1})\times\sqrt{x^2+1}}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{\sqrt{x^2+1}}$$

Differentiating again with respect to x, we get

$$\frac{d^2y}{dx^2}=-\ \frac{1}{2}(x^2+1)^{-\ 3/2}.2x=\frac{-x}{(x^2+1)^{3/2}}$$

$$\Rightarrow \quad (x^2+1)\frac{d^2y}{dx^2}=\frac{-x}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad (x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$$

Q.5. If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, then show that $(1-x^2)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0.$

Ans.

Given, $y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{\sqrt{1 - x^2} . \frac{1}{\sqrt{1 - x^2}} - \sin^{-1} x . \frac{-2x}{2\sqrt{1 - x^2}}}{(\sqrt{1 - x^2})^2} = \frac{1 + xy}{1 - x^2} \qquad \ldots(i)$$

Again differentiating with respect to x, we get

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(1 - x^2) . \left(x . \frac{dy}{dx} + y\right) + (1 + xy) . 2x}{(1 - x^2)^2}$$

$$\Rightarrow \quad (1 - x^2) . \frac{d^2y}{dx^2} = x . \frac{dy}{dx} + y + \frac{(1 + xy) . 2x}{1 - x^2}$$

$$\Rightarrow \quad (1 - x^2) . \frac{d^2y}{dx^2} = x \frac{dy}{dx} + y + 2x \frac{dy}{dx} \qquad [\text{using } (i)]$$

$$\Rightarrow \quad (1 - x^2) \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - y = 0$$

Q.6. If $y = e^x (\sin x + \cos x)$, then show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Ans.

Given, $y = e^x (\sin x + \cos x)$

$$\therefore \quad \frac{dy}{dx} = e^x (\cos x - \sin x) + (\sin x + \cos x) . e^x$$

$$\Rightarrow \quad \frac{dy}{dx} = 2e^x \cos x$$

$$\Rightarrow \quad \frac{dy}{dx} = 2e^x \cos x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2 (-e^x \sin x + \cos x . e^x$$

$$= -2e^x \sin x + 2e^x \cos x = -2e^x \sin x - 2e^x \cos x + 4e^x \cos x$$

Now, $= -2e^x (\sin x + \cos x) + 2 . (2e^x \cos x) = -2y + 2\frac{dy}{dx}$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Q.7. If $y = \operatorname{cosec}^{-1} x$, $x > 1$, then show that $x(x^2-1)\frac{d^2y}{dx^2} + (2x^2-1)\frac{dy}{dx} = 0$

Ans.

$\because \quad y = \operatorname{cosec}^{-1} x$

Differentiating with respect to x, we get

$\therefore \quad \frac{dy}{dx} = \frac{-1}{x\sqrt{x^2-1}}$

Again differentiating with respect to x, we get

$$\frac{d^2y}{dx^2} = \frac{x\sqrt{x^2-1}\,.\,0 + 1.\left\{x\,.\,\frac{2x}{2\sqrt{x^2-1}} + \sqrt{x^2-1}\right\}}{x^2(x^2-1)}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{x^2+x^2-1}{x^2(x^2-1)\,.\,\sqrt{x^2-1}} = \frac{2x^2-1}{\sqrt{x^2-1}\,.\,x^2(x^2-1)}$$

$$\Rightarrow \quad x(x^2-1)\frac{d^2y}{dx^2} = \frac{2x^2-1}{x\sqrt{x^2-1}} = (2x^2-1)\left(-\frac{dy}{dx}\right)$$

$$\Rightarrow \quad x(x^2-1)\frac{d^2y}{dx^2} + (2x^2-1)\frac{dy}{dx} = 0$$

Q.8. If $y = \sin^{-1} x$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$

Ans.

$\because \quad y = \sin^{-1} x$

Differentiating with respect to x, we get

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \sqrt{1-x^2}\frac{dy}{dx} = 1$$

Again differentiating with respect to x, we get

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{1\times(-2x)}{2\sqrt{1-x^2}} = 0$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - \frac{xdy}{dx} = 0$$

Q.9. **If $y = 3\cos(\log x) + 4\sin(\log x)$, show that**

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$$

Ans.

Given, $y = 3\cos(\log x) + 4\sin(\log x)$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -\frac{3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x}$$

$$\Rightarrow \quad y_1 = \frac{1}{x}[-3\sin(\log x) + 4\cos(\log x)]$$

Again differentiating with respect to x, we get

$$\frac{d^2y}{dx^2} = \frac{x\left[\frac{-3\cos(\log x)}{x} - \frac{4\sin(\log x)}{x}\right] - [-3\sin(\log x) + 4\cos(\log x)]}{x^2}$$

$$= \frac{-3\cos(\log x) - 4\sin(\log x) + 3\sin(\log x) + 4\cos(\log x)}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{-\sin(\log x) - 7\cos(\log x)}{x^2}$$

$$\Rightarrow \quad y_2 = \frac{-\sin(\log x) - 7\cos(\log x)}{x^2}$$

Now, LHS $= x^2 y_2 + x y_1 + y$

$$= x^2\left(\frac{-\sin(\log x) - 7\cos(\log x)}{x^2}\right) + x \times \frac{1}{x}[-3\sin(\log x) + 4\cos(\log x)] + 3\cos(\log x) + 4\sin(\log x)$$

$= -\sin(\log x) - 7\cos(\log x) - 3\sin(\log x) + 4\cos(\log x) + 3\cos(\log x) + 4\sin(\log x)$

$= 0 =$ RHS

Q.10.

If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, $0 < t < \frac{\pi}{2}$, find $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

Ans.

Given, $x = a(\cos t + t\sin t)$

Differentiating both sides with respect to t, we get

$$\frac{dx}{dt} = a(-\sin t + t\cos t + \sin t)$$

$$\Rightarrow \quad \frac{dx}{dt} = at\cos t \qquad \ldots(i)$$

Differentiating again with respect to t, we get

$$\frac{d^2x}{dt^2} = a(-t\sin t + \cos t) = a(\cos t - t\sin t)$$

Again, $y = a(\sin t - t\cos t)$

Differentiating with respect to t, we get

$$\frac{dy}{dt} = a(\cos t + t\sin t - \cos t) \qquad \ldots(ii)$$

$\Rightarrow \quad \frac{dy}{dt} = at \ \sin \ t$

Differentiating again with respect to *t* we get

$$\frac{d^2y}{dt^2} = a(t\cos t + \sin t)$$

Now, $\frac{dy}{dx} = \frac{dt/dy}{dt/dx}$ [from (*i*) and (*ii*)]

$\Rightarrow \quad \frac{dy}{dx} = \frac{at \ \sin \ t}{at \ \cos \ t}$

$\Rightarrow \quad \frac{dy}{dx} = \tan t$

Differentiating again with respect to *x*, we get

$$\frac{d^2y}{dx^2} = \sec^2 t.\frac{dt}{dx} = \sec^2 \ t.\frac{1}{dt/dx} = \frac{\sec^2 t}{at \ \cos \ t} \qquad \text{[from } (i)\text{]}$$

$$= \frac{\sec^3 t}{at}$$

Hence $\frac{d^2x}{dt^2} = a(\cos t - t\sin t)$

and $\frac{d^2y}{dt^2} = a\,(t\cos t + \sin t)$ and $\frac{d^2y}{dx^2} = \frac{\sec^3 t}{at}$

Q.11. If $x = a\left(\cos t + \log \tan \frac{t}{2}\right), y = a \sin t,$ then find $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

Ans.

Given, $x = a\left(\cos t + \log\ \tan\frac{t}{2}\right)$

Differentiating with respect to t, we get

$$\frac{dx}{dt} = a\left(-\sin t + \frac{1}{\tan\frac{t}{2}}.\sec^2\frac{t}{2}.\frac{1}{2}\right)$$

$$= a\left\{-\sin t + \frac{1}{2\sin\frac{t}{2}.\cos\frac{t}{2}}\right\} = a\left\{-\sin t + \frac{1}{\sin t}\right\}$$

$$\frac{dx}{dt} = a\left(\frac{1-\sin^2 t}{\sin t}\right) = a\,\frac{\cos^2 t}{\sin t}$$

$\because\quad y = a\sin t$

Differentiating with respect to t, we get

$$\frac{dy}{dt} = a.\cos t \quad\Rightarrow\quad \frac{d^2y}{dt^2} = -a\sin t$$

$$\therefore\quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a\ \cos\ t.\sin\ t}{a\ \cos^2\ t} = \tan t$$

$$\therefore\quad \frac{d^2y}{dx^2} = \sec^2 t.\frac{dt}{dx} = \sec^2 t.\frac{1\times\sin t}{a\ \cos^2 t} = \frac{1}{a}\sec^4 t.\sin t$$

Hence, $\frac{d^2y}{dx^2} = -\ a\ \sin\ t$ and $\frac{d^2y}{dx^2} = \frac{\sec^4 t\ \sin\ t}{a}$

Q.12. If $x = a\sin t$ and $y = a\left(\cos t + \log\ \tan\frac{t}{2}\right)$, then find $\frac{d^2y}{dx^2}$.

Ans.

Given, $x = a \sin t$

Differentiating both sides with respect to t, we get

$$\frac{dx}{dt} = a \cos t \qquad \ldots(i)$$

Again, $\because y = a\left[\cos t + \log\left(\tan\frac{t}{2}\right)\right]$

Differentiating both sides with respect to t, we get

$$\frac{dy}{dt} = a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}.\sec^2\frac{t}{2}.\frac{1}{2}\right]$$

$$\Rightarrow \quad \frac{dy}{dt} = a\left[-\sin t + \frac{1}{\sin t}\right]$$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{a\left(1-\sin^2 t\right)}{\sin t}$$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{a\cos^2 t}{\sin t} \qquad \ldots(ii)$$

$$\because \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{a\cos^2 t}{\sin t} \times \frac{1}{a\cos t} \qquad [\text{From } (i) \text{ and } (ii)]$$

$$\frac{dy}{dx} = \cot t$$

Differentiating again with respect to x, we get

$$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2 t.\frac{dt}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 t.\frac{1}{a\cos t} = \frac{-\operatorname{cosec}^2 t}{a\cos t}$$

Q.13. If $= \log\left[x + \sqrt{x^2 + a^2}\right]$, show that $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$

Ans.

Given $y = \log [x + \sqrt{x^2 + a^2}]$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left[1 + \frac{2x}{2\sqrt{x^2+a^2}}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x+\sqrt{x^2+a^2}}{(x+\sqrt{x^2+a^2})(\sqrt{x^2+a^2})}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\sqrt{x^2+a^2}} \qquad \ldots(i)$$

Differentiating again with respect to x, we get

$$\frac{d^2y}{dx^2} = -\frac{1}{2}(x^2 + a^2)^{-\frac{3}{2}}.2x = \frac{-x}{(x^2+a^2)^{\frac{3}{2}}}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-x}{(x^2+a^2).\sqrt{x^2+a^2}}$$

$$\Rightarrow \quad (x^2 + a^2)\frac{d^2y}{dx^2} = -\frac{x}{\sqrt{x^2+a^2}}$$

$$\Rightarrow \quad (x^2 + a^2)\frac{d^2y}{dx^2} + x.\frac{dy}{dx} = 0 \qquad [\text{from } (i)]$$

Q.14. **If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.**

Ans.

Given, $x = a\cos^3\theta$

Differentiating both sides with respect to θ, we get

$$\frac{dx}{d\theta} = -3a\cos^2\theta.\sin\theta \qquad ...(i)$$

Also, $y = a\sin^3\theta$

Differentiating both sides with respect to θ, we get

$$\frac{dy}{d\theta} = 3a\sin^2\theta.\cos\theta \qquad ...(ii)$$

Now $\frac{dy}{dx} = \frac{d\theta/dy}{d\theta/dx} = \frac{3a\sin^2\theta.\cos\theta}{-3a\cos^2\theta.\sin\theta}$

$\Rightarrow \quad \frac{dy}{dx} = -\tan\theta$

$\Rightarrow \quad \frac{d^2y}{dx^2} = -\sec^2\theta.\frac{d\theta}{dx}$

$$= \frac{-\sec^2\theta}{-3a\cos^2\theta.\sin\theta}$$

$$= \frac{1}{3a}\sec^4\theta.\ \text{cosec}\ \theta$$

$\therefore \quad \left.\frac{d^2y}{dx^2}\right]_{x=\frac{\pi}{6}} = \frac{1}{3a}\sec^4\frac{\pi}{6}.\ \text{cosec}\ \frac{\pi}{6}$

$$= \frac{1}{3a}.\left(\frac{2}{\sqrt{3}}\right)^4 \times 2 = \frac{32}{27a}$$

Q.15. If $y = x^x$, then prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$

Ans.

Given, $y = x^x$

Taking logarithm on both sides, we get

$$\log y = x \cdot \log x$$

Differentiating both sides, we get

$$\Rightarrow \quad \frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x$$

$$\Rightarrow \quad \frac{dy}{dx} = y(1 + \log x) \qquad \ldots(i)$$

Again differentiating both sides, we get

$$\frac{d^2y}{dx^2} = y \cdot \frac{1}{x} + (1 + \log x) \cdot \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{y}{x} + \frac{1}{y} \cdot \frac{dy}{dx} \cdot \frac{dy}{dx} \qquad [\text{From } (i)]$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{y}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$

Q.16. **If $y = x^3 \log\left(\frac{1}{x}\right)$, then prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$.**

Ans.

The given differential equation is $y = x^3 \log\left(\frac{1}{x}\right)$

$$\Rightarrow \quad \frac{dy}{dx} = x^3 . x\left(\frac{-1}{x^2}\right) + \log\ \frac{1}{x} . 3x^2 = -\ x^2 + 3x^2 . log\left(\frac{1}{x}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = -\ x^2 + \frac{3}{x} . x^3 \log\ \left(\frac{1}{x}\right)$$

$$\Rightarrow \quad x\frac{dy}{dx} = -\ x^3 + 3y$$

Again differentiating with respect to x

$$\Rightarrow \quad \frac{dy}{dx} + x^2\frac{d^2y}{dx^2} = -3x^2 + 3\frac{dy}{dx}$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$$

Hence proved.

Q.17. **If** $y = \left(x + \sqrt{1 + x^2}\right)^n$**, then show that**

$$(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$$

Ans.

Given $y=\left(x+\sqrt{1+x^2}\right)^n$

Differentiating with respect to *x*, we get

$$\Rightarrow \quad \frac{\mathrm{dy}}{\mathrm{dx}}=n\left(x+\sqrt{1+x^2}\right)^{n-1}.\left\{1+\frac{2x}{2\sqrt{1+x^2}}\right\}$$

$$\Rightarrow \quad \frac{\mathrm{dy}}{\mathrm{dx}}=n\left(x+\sqrt{1+x^2}\right)^{n-1}.\left(\frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad \frac{\mathrm{dy}}{\mathrm{dx}}=\frac{n\left(x+\sqrt{1+x^2}\right)^n}{\sqrt{1+x^2}}$$

$$\Rightarrow \quad \frac{\mathrm{dy}}{\mathrm{dx}}=\frac{\mathrm{ny}}{\sqrt{1+x^2}}$$

$$\Rightarrow \quad \sqrt{1+x^2}.\frac{\mathrm{dy}}{\mathrm{dx}}=\mathrm{ny}$$

Again differentiating with respect to *x*, we get

$$\sqrt{1+x^2}.\frac{d^2y}{\mathrm{dx}^2}+\frac{2x}{2\sqrt{1+x^2}}.\frac{\mathrm{dy}}{\mathrm{dx}}=n\frac{\mathrm{dy}}{\mathrm{dx}}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{\mathrm{dy}}{\mathrm{dx}}+x.\frac{\mathrm{dy}}{\mathrm{dx}}=n\ .\sqrt{1+x^2}.\frac{\mathrm{dy}}{\mathrm{dx}}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{\mathrm{dy}}{\mathrm{dx}}+x\frac{\mathrm{dy}}{\mathrm{dx}}=n.\sqrt{1+x^2}.\frac{\mathrm{ny}}{\sqrt{1+x^2}}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{\mathrm{dx}^2}+x\frac{\mathrm{dy}}{\mathrm{dx}}=n^2y$$