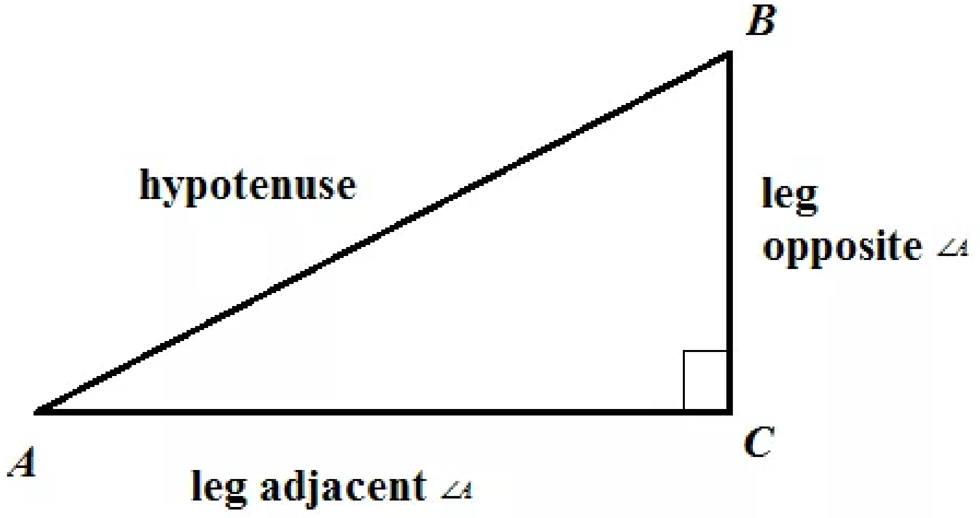
Chapter 11. Radical Expressions and Triangles

Ex. 11.7

Answer 1CU.

The trigonometric ratios are ratios of the measure of two sides of a right triangle and these ratios are sine, cosine and tangent. A model of the right triangle is drawn below:



In word trigonometric ratios

$$\text{Sine of } \angle A = \frac{\text{measure of leg opposite} \angle A}{\text{measure of hypotenuse}}$$

$$\text{Coine of } \angle A = \frac{\text{measure of leg adjacent } \angle A}{\text{measure of hypotenuse}}$$

$$\text{Tangent of } \angle A = \frac{\text{measure of leg opposite } \angle A}{\text{measure of leg adjacent to } \angle A}$$

In symbolic format

$$\sin A = \frac{BC}{AB}$$

$$\cos A = \frac{AC}{AB}$$

$$\tan A = \frac{BC}{AC}$$

Answer 1RM.

Factor:

In mathematical meaning:

A number that divides into another number exactly.

In everyday meaning:

A circumstances contributing to a result when making a decision factor comes.

Leg:

In mathematical meaning:

A section of race or line length of a figure.

In everyday meaning:

A part that supports a chair, table etc.

Rationalize:

In mathematical meaning:

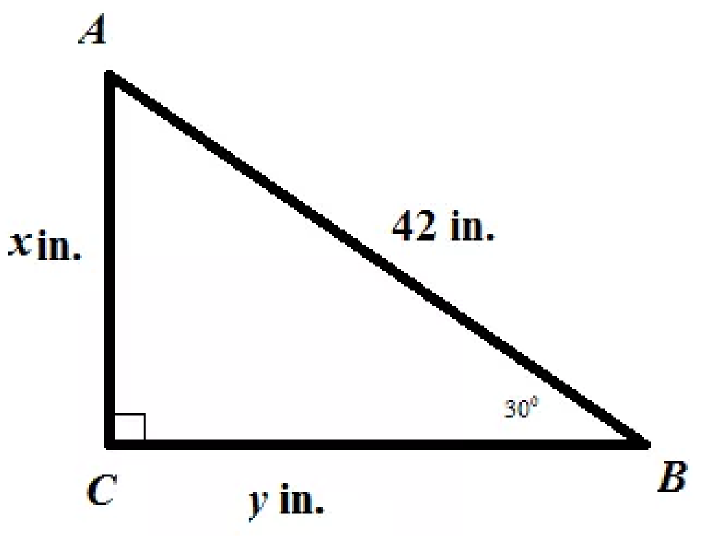
Try to find a logical reason for.

In everyday meaning:

Reorganize so as to become more efficient.

Answer 2CU.

Consider a right angled triangle is drawn taking hypotenuse 42 in. and acute angle *B* is 30^0 as shown below:



To find the measure of $\angle A, \overline{AC}, \overline{BC}$ follow the steps

Step 1: Find the measure of $\angle A$. The sum of the measures of the angles in a triangle is 180^0.

$180^0 - 90^0 - 30^0 = 60^0$

The measure of $\angle A$ is 60^0.

Step 3: Find the value of *y*, which is the measure of the sides adjacent to $\angle B$. Use the cosine ratio.

$$\cos 30^0 = \frac{y}{42} \quad [\text{Definition of cosine}]$$

$$0.8660 \approx \frac{y}{42} \quad [\text{Evaluate } \cos 30^0]$$

$$36.4 \approx y \quad [\text{Multiplied by } 42]$$

$\overline{BC}$ is about 36.4 inches long.

Therefore, the missing measures are $\boxed{60^0, 21 \text{ in.}, 36.4 \text{ in.}}$.

Answer 2RM.

Degree:

A unit for measuring angles

A stage in a scale for measurement

Example: temperature in degree.

Range:

The limit between which something varies

Travel over or cover a wide area

Example: An area for testing military equipment.

Round:

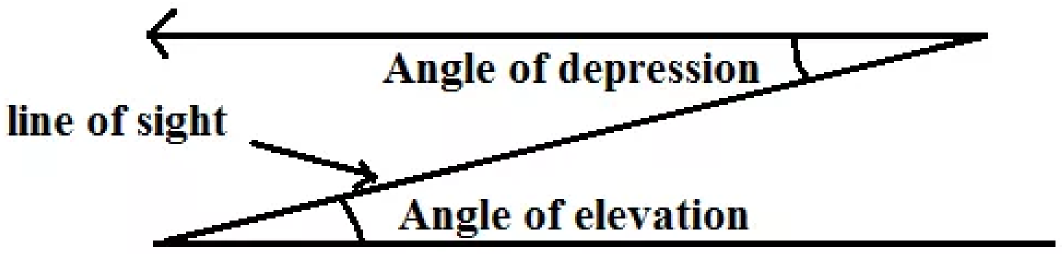
Shape of figure is curved, circular or cylindrical.

It is expressed in convenient units rather than exactly.

Example: A route by which a number of people or places are visited in turn.

Answer 3CU.

An angle of elevation is formed by a horizontal line of sight above it and an angle of depression is formed by a horizontal line of sight below it.



The relationship of angle of measure is

$\tan(\text{angle of elevation}) = \dfrac{\text{height of object} - x}{\text{distance of object}}$, *x* represents distance from the ground to our eye level, to find the height of the object.

Answer 3RM.

The three additional words that come from the same root are

(a) Altitude, from the root word Altus, which means high.

Altitude is used in geometrical figure for the perpendicular distance form one base of geometric figure to vertex of opposite.

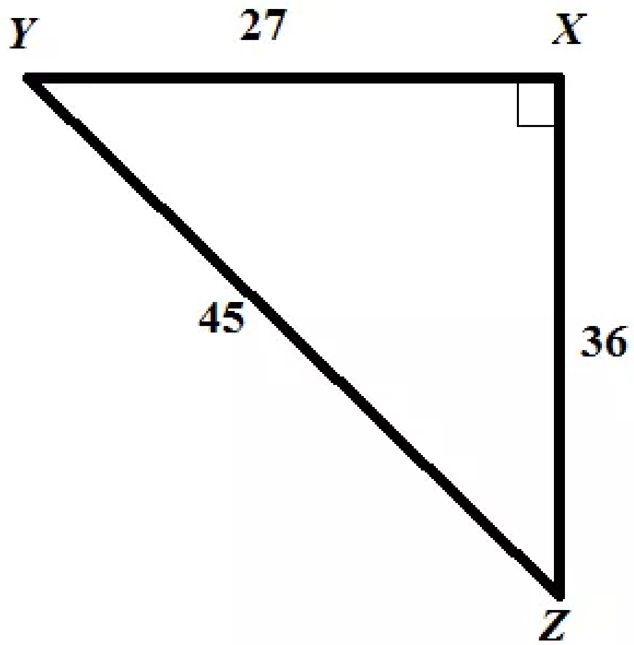
(b) Calculus, from the root word Calyx, which means pebble or small stone.

To determine small part and to do arithmetic operations gives the meaning of calculate and calculus.

(c) Gross, from the root word Grosses, which means thick or large.

The gross is used for large quantities or grocer sold wholesale.

Answer 4CU.

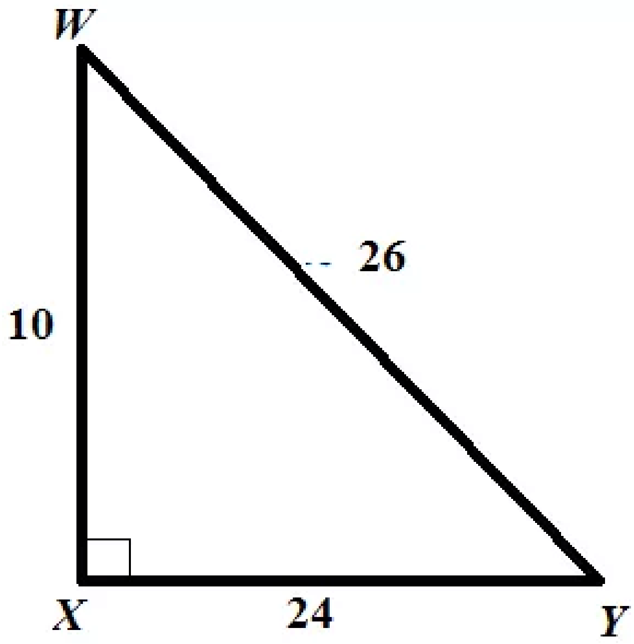


Write the trigonometric ratio for the following function

$$\sin Y = \frac{ZX}{YZ} = \frac{36}{45} = \boxed{0.8000} \qquad \cos Y = \frac{XY}{YZ} = \frac{27}{45} = \boxed{0.6000}$$

$$\tan Y = \frac{ZX}{XY} = \frac{36}{27} = \boxed{1.3333}$$

Answer 5CU.



Write the trigonometric ratio for the following function

$$\sin Y = \frac{WX}{WY} = \frac{10}{26} = \boxed{0.3846} \qquad \cos Y = \frac{XY}{WY} = \frac{24}{26} = \boxed{0.9230}$$

$$\tan Y = \frac{WX}{XY} = \frac{10}{24} = \boxed{0.4167}$$

Answer 6CU.

Consider the trigonometric ratio is

$\sin 60^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 60 [SIN] 0.866025403

Rounded to the nearest ten thousand, $\sin 60^0 \approx \boxed{0.8660}$

Answer 7CU.

Consider the trigonometric ratio is

$\cos 75^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 75 [COS] 0.258819045

Rounded to the nearest ten thousand, $\cos 75^0 \approx \boxed{0.2588}$

Answer 8CU.

Consider the trigonometric ratio is

$\tan 10^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 10 [TAN] 0.17632698084

Rounded to the nearest ten thousand, $\tan 10^0 \approx \boxed{0.1763}$

Answer 9CU.

Consider the equation is

$\sin W = 0.9848$

Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\sin^{-1}}$ 0.9848 $\boxed{=}$ 79.99

Therefore, the measure of the angle nearest to degree is $\boxed{W = 80^0}$.

Answer 10CU.

Consider the equation is

$\cos X = 0.6157$

Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\cos^{-1}}$ 0.6157 $\boxed{=}$ 51.99

Therefore, the measure of the angle nearest to degree is $\boxed{X = 52^0}$.

Answer 11CU.

Consider the equation is

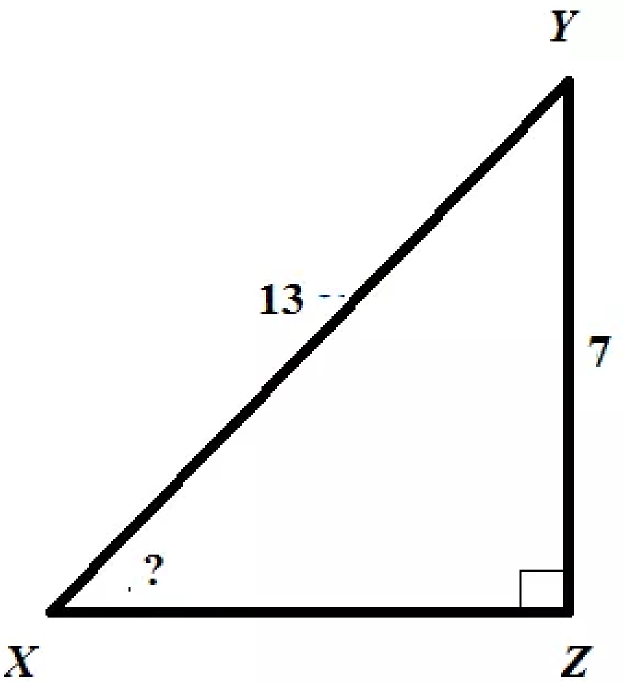
$\tan C = 0.3249$

Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\tan^{-1}}$ 0.3249 $\boxed{=}$ 17.99

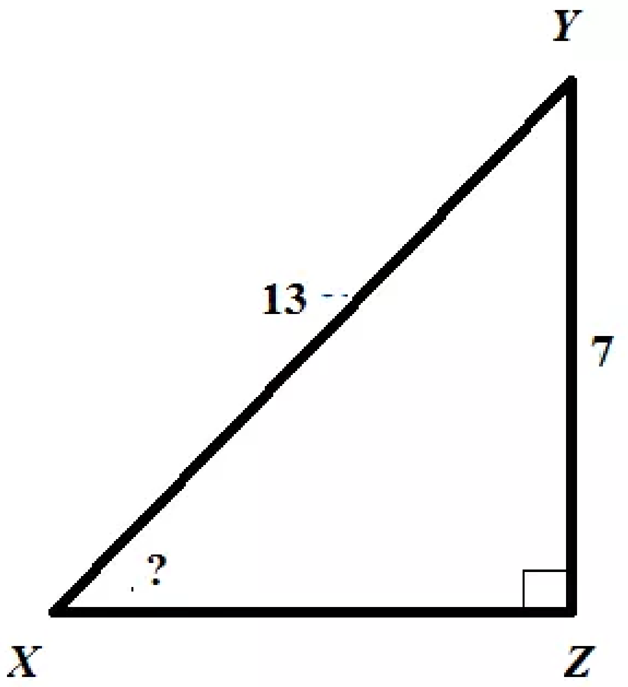
Therefore, the measure of the angle nearest to degree is $\boxed{C = 18^0}$.

Answer 12CU.



Since the lengths of the opposite leg by hypotenuse are known, use the sine ratio.

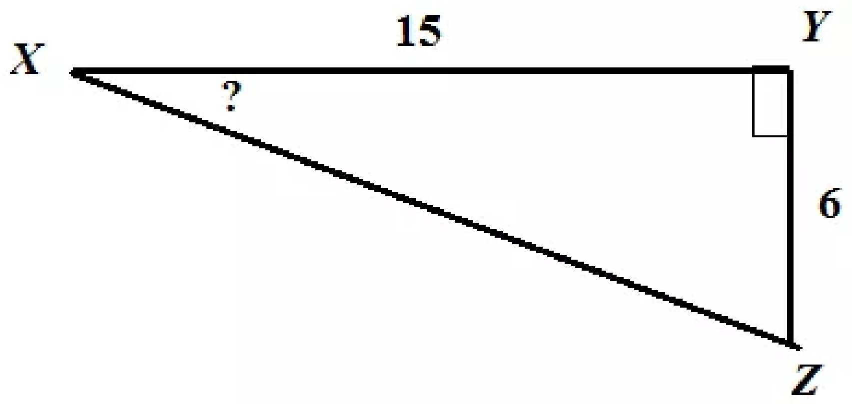
$$\begin{aligned}\sin X &= \frac{\text{opposite leg}}{\text{hypotenuse}} && [\text{Definition of sine}] \\ &= \frac{ZY}{XY} \\ &= \frac{7}{13} && [ZY = 7, XY = 13]\end{aligned}$$



Since the lengths of the opposite leg by hypotenuse are known, use the sine ratio.

$$\sin X = \frac{\text{opposite leg}}{\text{hypotenuse}} \quad \text{[Definition of sine]}$$
$$= \frac{ZY}{XY}$$
$$= \frac{7}{13} \quad [ZY = 7, XY = 13]$$

Answer 13CU.



Since the lengths of the opposite leg and adjacent sides are known, use the tangent ratio.

$$\tan X = \frac{\text{opposite leg}}{\text{adjacent leg}} \quad \text{[Definition of tangent]}$$
$$= \frac{ZY}{XY}$$
$$= \frac{6}{15} \quad [ZY = 6, XY = 15]$$
$$= \frac{2}{5}$$

Now, use the $\left[\tan^{-1}\right]$ on a calculator to find the measure of the angle whose tangent ratio is $\frac{2}{5}$

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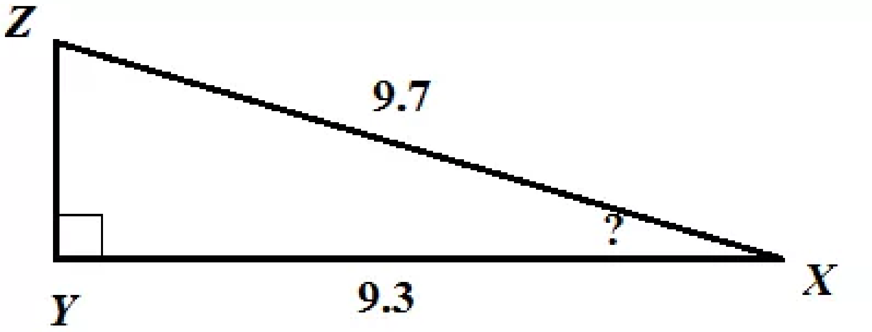
KEYSTROKES: $\boxed{\tan^{-1}}$ (2 $\boxed{\div}$ 5) 21.80

To the nearest degree, the measure of unknown angle *X* is $\boxed{29^0}$.

Answer 14CU.



Since the lengths of the hypotenuse and adjacent sides are known, use the cosine ratio.

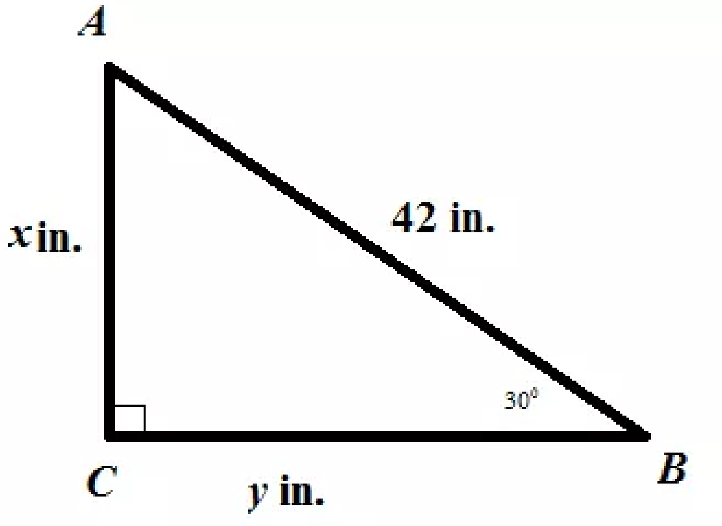
$$\begin{aligned}\cos X &= \frac{\text{adjacent leg}}{\text{hypotenuse}} && [\text{Definition of tangent}]\\ &= \frac{XY}{XZ}\\ &= \frac{9.3}{9.7} && [XY = 9.3, XZ = 9.7]\end{aligned}$$

Now, use the $\left[\cos^{-1}\right]$ on a calculator to find the measure of the angle whose cosine ratio is $\frac{9.3}{9.7}$.

KEYSTROKES: $\boxed{\cos^{-1}}$ (9.3 $\boxed{\div}$ 9.7) 16.51

To the nearest degree, the measure of unknown angle *X* is $\boxed{17^0}$.

Answer 15CU.



To find the measure of $\angle A, \overline{AC}, \overline{BC}$ follow the steps

Step 1: Find the measure of $\angle A$. The sum of the measures of the angles in a triangle is 180^0.

$180^0 - 90^0 - 30^0 = 60^0$

The measure of $\angle A$ is 60^0.

Step 2: Find the value of *x*, which is the measure of the side opposite $\angle B$. Use the sine ratio. Use the sine ratio.

$$\sin 30^0 = \frac{x}{42} \quad [\text{Definition of sine}]$$
$$0.5000 = \frac{x}{42} \quad [\text{Evaluate } \sin 30^0]$$
$$21 = x \quad [\text{Multiply by } 42]$$

$\overline{AC}$ is about 21in. long.

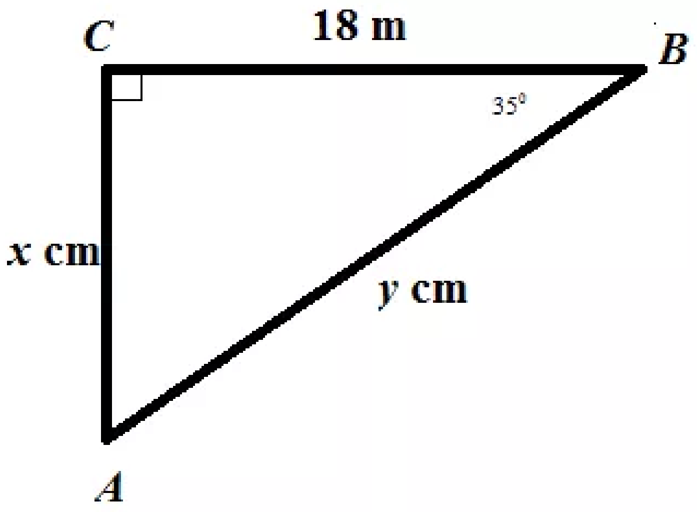
Step 3: Find the value of *y*, which is the measure of the sides adjacent to $\angle B$. Use the cosine ratio.

$$\cos 30^0 = \frac{y}{42} \quad [\text{Definition of cosine}]$$
$$0.8660 \approx \frac{y}{42} \quad [\text{Evaluate } \cos 30^0]$$
$$36.4 \approx y \quad [\text{Multiplied by } 42]$$

$\overline{BC}$ is about 36.4 inches long.

Therefore, the missing measures are $\boxed{60^0, 21 \text{ in.}, 36.4 \text{ in.}}$.

Answer 16CU.



To find the measure of $\angle A, \overline{AC}, \overline{AB}$ follow the steps

Step 1: Find the measure of $\angle A$. The sum of the measures of the angles in a triangle is 180^0.

$180^0 - 90^0 - 35^0 = 55^0$

The measure of $\angle A$ is 55^0.

Step 2: Find the value of *x*, which is the measure of the side opposite $\angle B$. Use the tangent ratio. Use the tangent ratio.

$$\tan 35^0 = \frac{x}{18} \quad [\text{Definition of tangent}]$$

$$0.7002 \approx \frac{x}{18} \quad [\text{Evaluate } \tan 35^0]$$

$$12.6 \approx x \quad [\text{Multiply by } 18]$$

$\overline{AC}$ is about 12.6 cm long.

Step 3: Find the value of *y*, which is the measure of the sides adjacent to $\angle B$. Use the cosine ratio.

$$\cos 35^0 = \frac{18}{y} \quad [\text{Definition of cosine}]$$

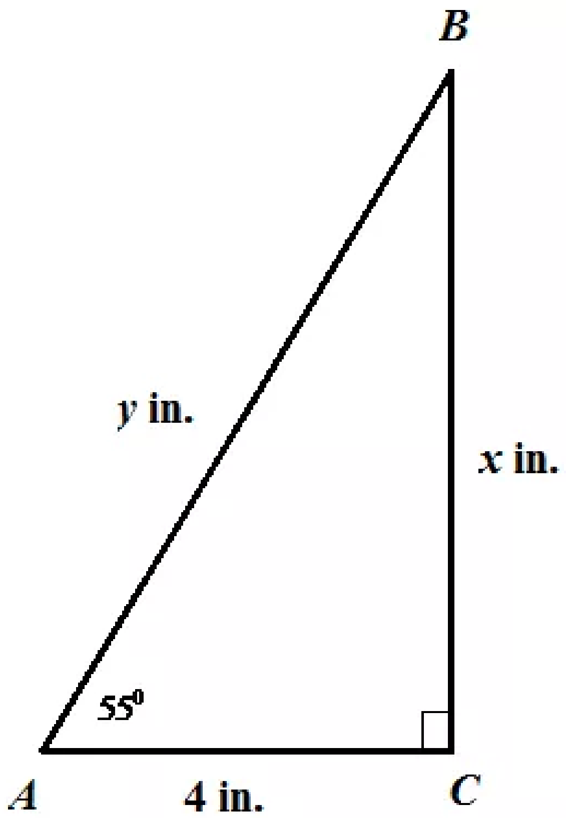
$$y = \frac{18}{\cos 35^0} \quad [\text{Evaluate } \cos 35^0]$$

$$y \approx 21.9$$

$\overline{AB}$ is about 21.9 cm long.

Therefore, the missing measures are $\boxed{55^0, 12.6 \text{ cm}, 21.9 \text{ cm}}$.

Answer 17CU.



To find the measure of $\angle B, \overline{BC}, \overline{AB}$ follow the steps

Step 1: Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180^0.

$$180^0 - 90^0 - 55^0 = 35^0$$

The measure of $\angle B$ is 35^0.

Step 2: Find the value of x, which is the measure of the side opposite $\angle A$. Use the tangent ratio. Use the tangent ratio.

$$\tan 55^0 = \frac{x}{4} \quad [\text{Definition of tangent}]$$

$$1.4281 \approx \frac{x}{4} \quad [\text{Evaluate } \tan 55^0]$$

$$5.7 \approx x \quad [\text{Multiply by } 4]$$

$\overline{BC}$ is about 5.7 in. long.

Step 3: Find the value of *y*, which is the measure of the sides adjacent to $\angle A$. Use the cosine ratio.

$$\cos 55^0 = \frac{4}{y} \quad \text{[Definition of cosine]}$$
$$y = \frac{4}{\cos 55^0} \quad \text{[Evaluate } \cos 55^0\text{]}$$
$$y \approx 6.9$$

$\overline{AB}$ is about 6.97 in. long.

Therefore, the missing measures are $\boxed{35^0, 6.9 \text{ in.}, 5.7 \text{ in.}}$.

Answer 18CU.

Consider *x* be the angle of elevation the road makes with the horizontal.

It is given that

$$\text{Percent grade of road} = \frac{\text{road rises or falls}}{\text{horizontal distance}}$$

Since the lengths of the opposite leg and adjacent sides are known, use the tangent ratio.

$$\tan X = \frac{\text{opposite leg}}{\text{adjacent leg}} \quad \text{[Definition of tangent]}$$
$$= \frac{AB}{BC}$$
$$= \frac{40}{1000} \quad [AB = 40, BC = 1000]$$

Now, use the $\left[\tan^{-1}\right]$ on a calculator to find the measure of the angle whose tangent ratio is $\frac{40}{1000}$.

KEYSTROKES: $\boxed{\tan^{-1}}$ (40 $\boxed{\div}$ 1000) 2.29

To the nearest degree, the measure of unknown angle *x* is 2^0.

Therefore, the angle of elevation is $\boxed{2^0}$.

Now, find *AC*

$$\sin 2^0 = \frac{AB}{AC}$$

$$0.034 = \frac{40}{AC}$$

$$AC = \frac{40}{0.034}$$

$$AC \approx 1176$$

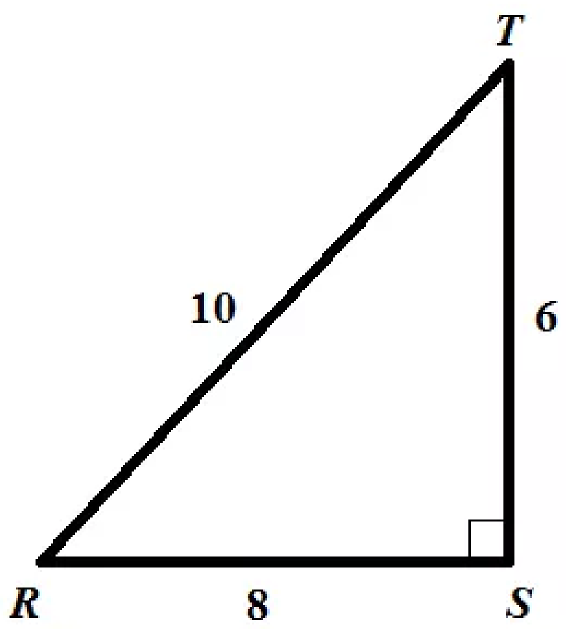
Percent grade of road $= \frac{\text{road rises or falls}}{\text{horizontal distance}} \times 100$

$$= \frac{1176}{1000} \times 100$$

$$= \boxed{117.6\%}$$

Answer 19PA.

Consider the figure below:



Write each ratio and substitute the measures. Use a calculator to find each value.

$$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} \qquad \cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

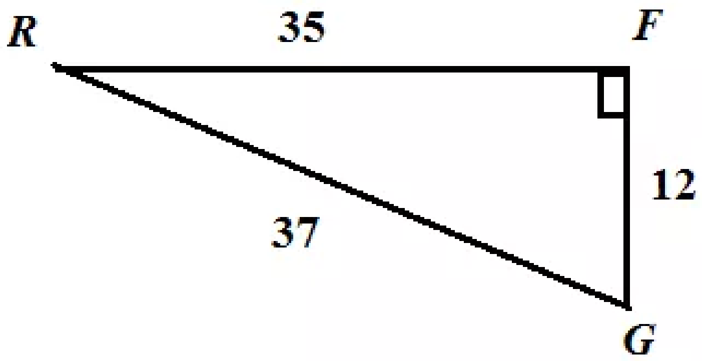
$$= \frac{6}{10} \qquad = \frac{8}{10}$$

$$= \boxed{0.6000} \qquad = \boxed{0.8000}$$

$$\tan R = \frac{\text{opposite leg}}{\text{adjacent leg}}$$
$$= \frac{6}{8}$$
$$= \boxed{0.7500}$$

Answer 20PA.

Consider the figure below:



Write each ratio and substitute the measures. Use a calculator to find each value.

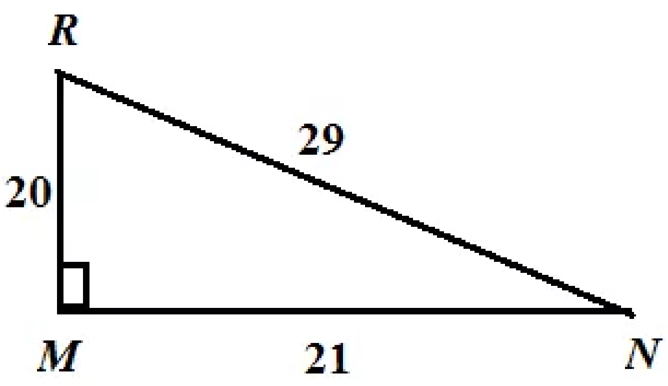
$$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{12}{37} = \boxed{0.3243}$$

$$\cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{35}{37} = \boxed{0.9459}$$

$$\tan R = \frac{\text{opposite leg}}{\text{adjacent leg}}$$
$$= \frac{12}{35}$$
$$= \boxed{0.3429}$$

Answer 21PA.

Consider the figure below:



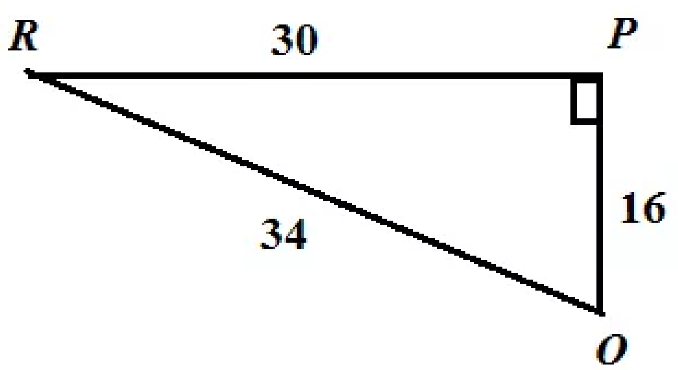
Write each ratio and substitute the measures. Use a calculator to find each value.

$$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{21}{29} = \boxed{0.7241} \qquad \cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{20}{29} = \boxed{0.6896}$$

$$\tan R = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{21}{20} = \boxed{1.05}$$

Answer 22PA.

Consider the figure below:



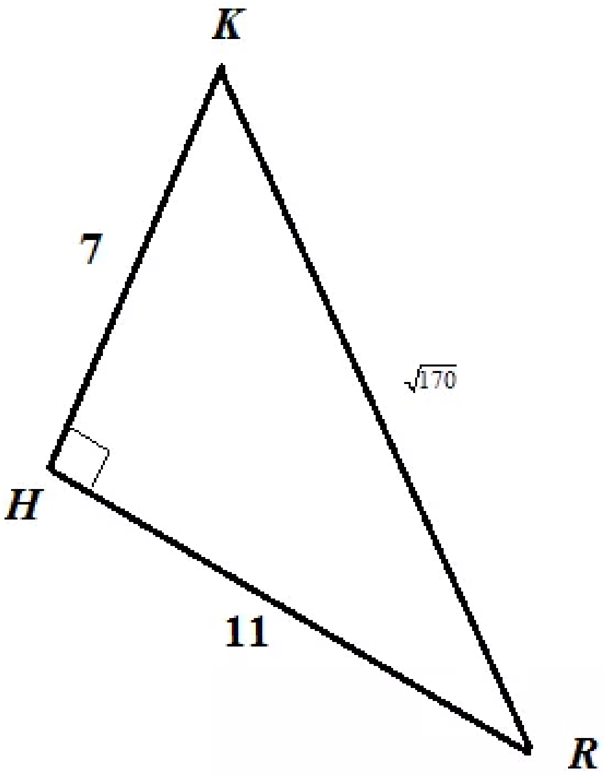
Write each ratio and substitute the measures. Use a calculator to find each value.

$$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{16}{34} = \boxed{0.4706} \qquad \cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{30}{34} = \boxed{0.8824}$$

$$\tan R = \frac{\text{opposite leg}}{\text{adjacent leg}}$$
$$= \frac{16}{30}$$
$$= \boxed{0.5333}$$

Answer 23PA.

Consider the figure below:



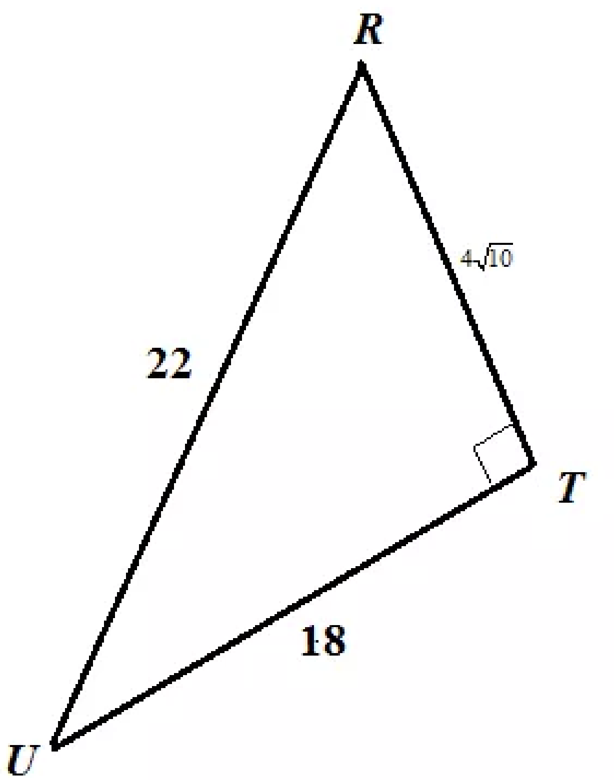
Write each ratio and substitute the measures. Use a calculator to find each value.

$$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} \qquad \cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$
$$= \frac{7}{\sqrt{170}} \qquad = \frac{11}{\sqrt{170}}$$
$$= \boxed{0.5369} \qquad = \boxed{0.8437}$$

$$\tan R = \frac{\text{opposite leg}}{\text{adjacent leg}}$$
$$= \frac{7}{11}$$
$$= \boxed{0.6364}$$

Answer 24PA.

Consider the figure below:



Write each ratio and substitute the measures. Use a calculator to find each value.

$$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{18}{22} = \boxed{0.8182}$$

$$\cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{4\sqrt{10}}{22} = \boxed{0.5749}$$

$$\tan R = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{18}{4\sqrt{10}} = \boxed{1.4230}$$

Answer 25PA.

Consider the trigonometric ratio is

$\sin 30^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 30 SIN 0.5

Rounded to the nearest ten thousand, $\sin 30^0 \approx \boxed{0.5000}$

Answer 26PA.

Consider the trigonometric ratio is

$\sin 80^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 80 SIN 0.984807753

Rounded to the nearest ten thousand, $\sin 80^0 \approx \boxed{0.9848}$

Answer 27PA.

Consider the trigonometric ratio is

$\cos 45^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 45 COS 0.70710678

Rounded to the nearest ten thousand, $\cos 45^0 \approx \boxed{0.7071}$

Answer 28PA.

Consider the trigonometric ratio is

$\cos 48^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 48 COS 0.669130606

Rounded to the nearest ten thousand, $\cos 48^0 \approx \boxed{0.6691}$

Answer 29PA.

Consider the trigonometric ratio is

$\tan 32^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 32 [TAN] 0.624869351909

Rounded to the nearest ten thousand, $\tan 32^0 \approx \boxed{0.6249}$

Answer 30PA.

Consider the trigonometric ratio is

$\tan 15^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 15 [TAN] 0.2679491924

Rounded to the nearest ten thousand, $\tan 15^0 \approx \boxed{0.2679}$

Answer 31PA.

Consider the trigonometric ratio is

$\tan 67^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 67 [TAN] 2.3558523658

Rounded to the nearest ten thousand, $\tan 67^0 \approx \boxed{2.3559}$

Answer 32PA.

Consider the trigonometric ratio is

$\sin 53^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 53 [SIN] 0.798635510047

Rounded to the nearest ten thousand, $\sin 53^0 \approx \boxed{0.7986}$

Answer 33PA.

Consider the trigonometric ratio is

$\cos 12^0$

Using Non graphing Scientific Calculator

KEYSTROKE: 12 $\boxed{\text{COS}}$ 0.978147600733

Rounded to the nearest ten thousand, $\cos 12^0 \approx \boxed{0.9781}$

Answer 34PA.

Consider the equation is

$\cos V = 0.5000$

Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\cos^{-1}}$ 0.5000 $\boxed{=}$ 60

Therefore, the measure of the angle nearest to degree is $\boxed{V = 60^0}$.

Answer 35PA.

Consider the equation is

$\cos Q = 0.7658$

Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\cos^{-1}}$ 0.7658 $\boxed{=}$ 40

Therefore, the measure of the angle nearest to degree is $\boxed{Q = 40^0}$.

Answer 36PA.

Consider the equation is

$\sin K = 0.9781$

Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\sin^{-1}}$ 0.9781 $\boxed{=}$ 77.98

Therefore, the measure of the angle nearest to degree is $\boxed{K = 78^0}$.

Answer 37PA.

Consider the equation is

$$\sin A = 0.8827$$

Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\sin^{-1}}$ 0.8827 $\boxed{=}$ 61.96

Therefore, the measure of the angle nearest to degree is $\boxed{A = 62^0}$.

Answer 38PA.

Consider the equation is

$$\tan S = 1.2401$$

Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\tan^{-1}}$ 1.2401 $\boxed{=}$ 51.11

Therefore, the measure of the angle nearest to degree is $\boxed{S = 51^0}$.

Answer 39PA.

Consider the equation is

$$\tan H = 0.6473$$

Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\tan^{-1}}$ 0.6473 $\boxed{=}$ 32.91

Therefore, the measure of the angle nearest to degree is $\boxed{H = 33^0}$.

Answer 40PA.

Consider the equation is

$$\sin V = 0.3832$$

Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\sin^{-1}}$ 0.3832 $\boxed{=}$ 22.53

Therefore, the measure of the angle nearest to degree is $\boxed{V = 23^0}$.

Answer 42PA.

Consider the equation is

$\tan L = 3.6541$

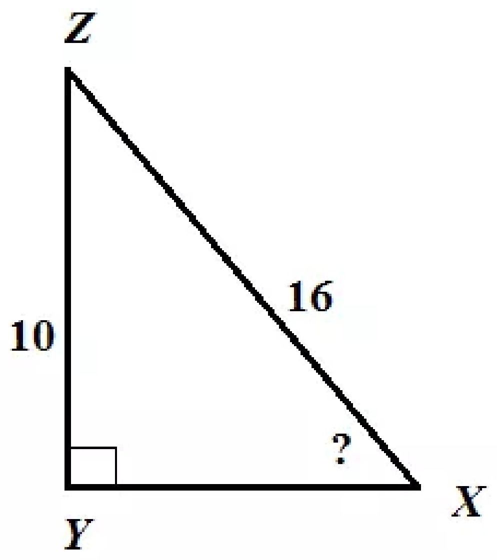
Using Non graphing Scientific Calculator

KEYSTROKE: $\boxed{\tan^{-1}}$ 3.6541 $\boxed{=}$ 74.69

Therefore, the measure of the angle nearest to degree is $\boxed{L = 75^0}$.

Answer 43PA.

Consider the figure below:



Since the lengths of the opposite leg and hypotenuse are known, use the sine ratio.

$$\sin X = \frac{\text{opposite leg}}{\text{hypotenuse}} \qquad [\text{Definition of sine}]$$
$$= \frac{ZY}{XZ}$$
$$= \frac{10}{16} \qquad [ZY = 10, XZ = 16]$$

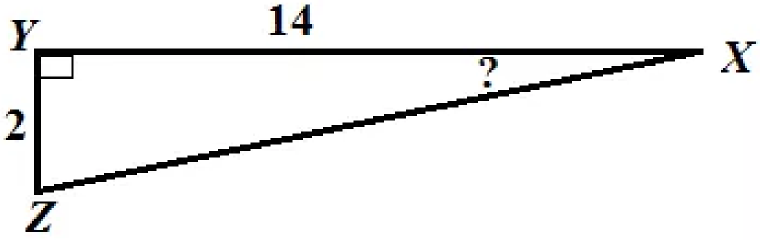
Now, use the $[\sin^{-1}]$ on a calculator to find the measure of the angle whose sine ratio is $\frac{10}{16}$.

KEYSTROKES: $\boxed{\sin^{-1}}$ (10 $\boxed{\div}$ 16) 38.68

To the nearest degree, the measure of unknown angle X is $\boxed{39^0}$.

Answer 44PA.

Consider the figure below:



Since the lengths of the opposite leg and adjacent are known, use the tangent ratio.

$$\begin{aligned} \tan X &= \frac{\text{opposite leg}}{\text{adjacent leg}} && [\text{Definition of tangent}] \\ &= \frac{ZY}{XY} \\ &= \frac{2}{14} && [ZY = 2, XY = 14] \end{aligned}$$

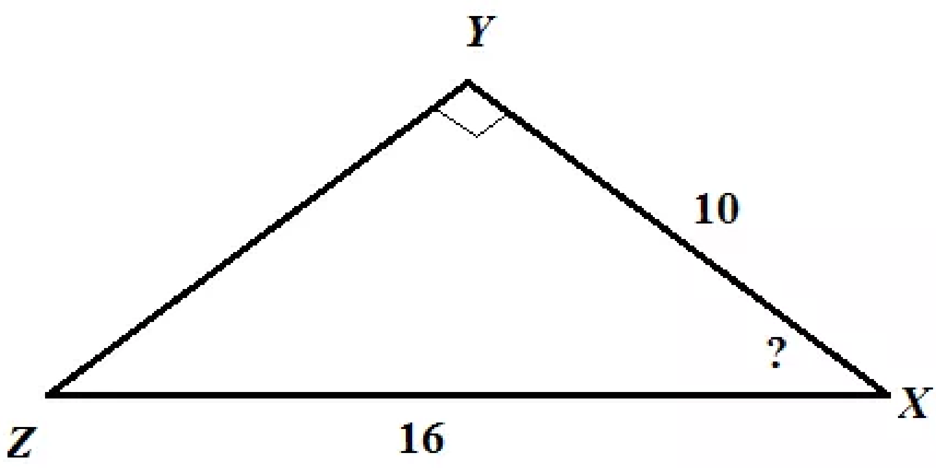
Now, use the $\left[\tan^{-1}\right]$ on a calculator to find the measure of the angle whose tangent ratio is $\frac{2}{14}$.

KEYSTROKES: $\boxed{\tan^{-1}}$ (2 $\boxed{\div}$ 14) 8.13

To the nearest degree, the measure of unknown angle X is $\boxed{8^0}$.

Answer 45PA.

Consider the figure below:



Since the lengths of the hypotenuse and adjacent are known, use the cosine ratio.

$$\cos X = \frac{\text{adjacent leg}}{\text{hypotenuse}} \quad [\text{Definition of cosine}]$$
$$= \frac{XY}{ZX}$$
$$= \frac{10}{16} \quad [XY = 10, ZX = 16]$$

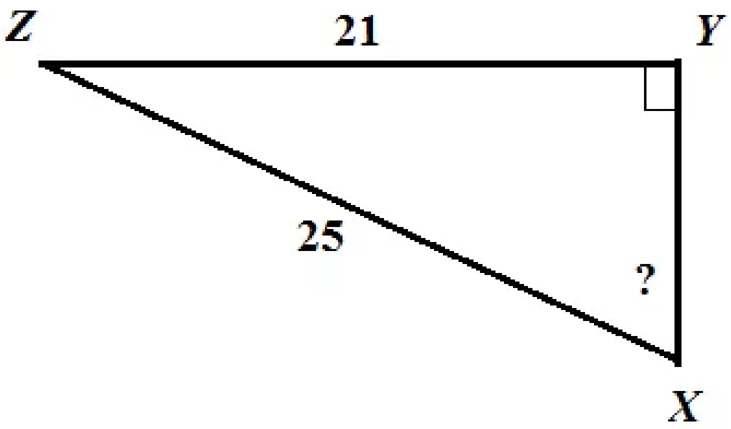
Now, use the $[\cos^{-1}]$ on a calculator to find the measure of the angle whose cosine ratio is $\frac{10}{16}$.

KEYSTROKES: $\boxed{\cos^{-1}}$ (10 $\boxed{\div}$ 16) 51.31

To the nearest degree, the measure of unknown angle X is $\boxed{51^0}$.

Answer 46PA.

Consider the figure below:



Since the lengths of the opposite leg and hypotenuse are known, use the sine ratio.

$$\sin X = \frac{\text{opposite leg}}{\text{hypotenuse}} \quad [\text{Definition of sine}]$$
$$= \frac{ZY}{XZ}$$
$$= \frac{21}{25} \quad [ZY = 21, XZ = 25]$$

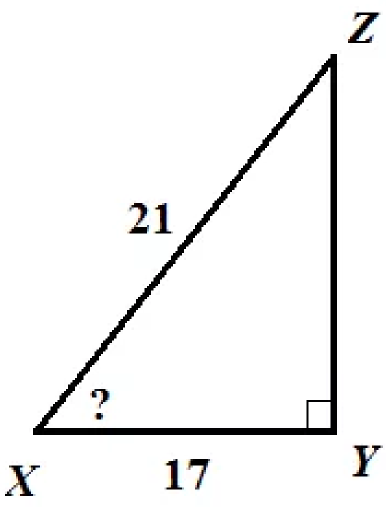
Now, use the $[\sin^{-1}]$ on a calculator to find the measure of the angle whose sine ratio is $\frac{10}{16}$.

KEYSTROKES: $\boxed{\sin^{-1}}$ (21 $\boxed{\div}$ 25) 57.14

To the nearest degree, the measure of unknown angle X is $\boxed{57^0}$.

Answer 47PA.

Consider the figure below:



Since the lengths of the hypotenuse and adjacent are known, use the cosine ratio.

$$\cos X = \frac{\text{adjacent leg}}{\text{hypotenuse}} \quad \text{[Definition of cosine]}$$
$$= \frac{XY}{ZX}$$
$$= \frac{17}{21} \quad [XY = 17, ZX = 21]$$

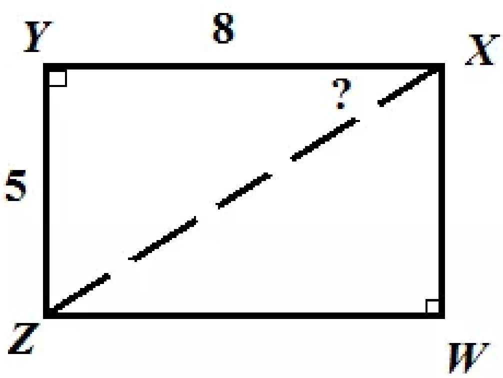
Now, use the $[\cos^{-1}]$ on a calculator to find the measure of the angle whose cosine ratio is $\frac{17}{21}$.

KEYSTROKES: $\boxed{\cos^{-1}}$ (17 $\boxed{\div}$ 21) 35.95

To the nearest degree, the measure of unknown angle X is $\boxed{36^0}$.

Answer 48PA.

Consider the figure below:



Consider the ΔXYZ and the unknown angle is $\angle YXZ$.

Since the lengths of the opposite leg and adjacent are known, use the tangent ratio.

$$\begin{aligned}\tan(YXZ) &= \frac{\text{opposite leg}}{\text{adjacent leg}} && [\text{Definition of tangent}] \\ &= \frac{ZY}{XY} \\ &= \frac{5}{8} && [ZY = 5, XY = 8]\end{aligned}$$

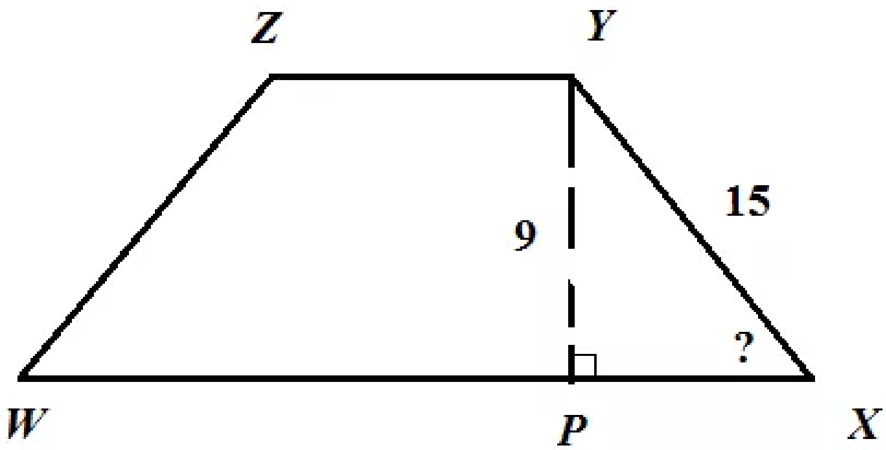
Now, use the $[\tan^{-1}]$ on a calculator to find the measure of the angle whose tangent ratio is $\frac{5}{8}$.

KEYSTROKES: $\boxed{\tan^{-1}}$ (5 $\boxed{\div}$ 8) 32.00

To the nearest degree, the measure of unknown angle *X* is $\boxed{32^0}$.

Answer 49PA.

Consider the figure below:



Consider the triangle is ΔPXY and the unknown angle is $\angle X$.

Since the lengths of the opposite leg and hypotenuse are known, use the sine ratio.

$$\sin X = \frac{\text{opposite leg}}{\text{hypotenuse}} \quad [\text{Definition of sine}]$$
$$= \frac{PY}{XY}$$
$$= \frac{9}{15} \quad [PY = 9, XY = 15]$$

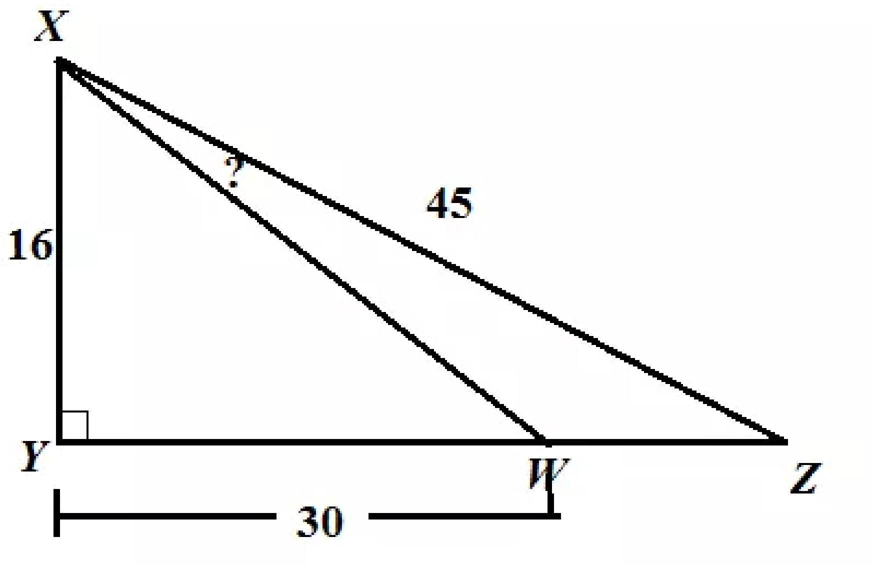
Now, use the $[\sin^{-1}]$ on a calculator to find the measure of the angle whose sine ratio is $\frac{9}{15}$.

KEYSTROKES: $\boxed{\sin^{-1}}$ (9 $\boxed{\div}$ 15) 36.86

To the nearest degree, the measure of unknown angle X is $\boxed{37^0}$.

Answer 50PA.

Consider the figure below:



Consider two triangles are ΔXWY and ΔWXZ. The unknown angle is $\angle X$.

First determine $\angle YXW$.

Since the lengths of the opposite leg and adjacent are known, use the tangent ratio.

$$\tan(YXW) = \frac{\text{opposite leg}}{\text{adjacent leg}} \quad [\text{Definition of tangent}]$$
$$= \frac{YW}{XY}$$
$$= \frac{30}{16} \quad [YW = 30, XY = 16]$$

Now, use the $\left[\tan^{-1}\right]$ on a calculator to find the measure of the angle whose tangent ratio is $\frac{30}{16}$.

KEYSTROKES: $\boxed{\tan^{-1}}$ (30 $\boxed{\div}$ 16) 61.92

To the nearest degree, the measure of angle $\angle YXW$ is 62^0.

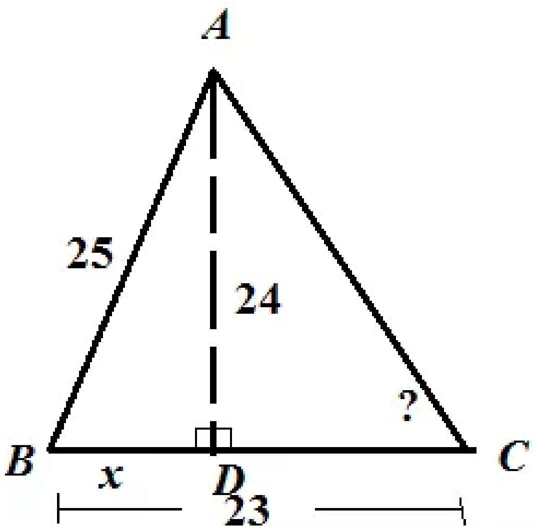
Now, determine $\angle X$ in the triangle *YXZ*.

Since the lengths of the hypotenuse and adjacent are known, use the cosine ratio.

$$\begin{aligned}\cos X &= \frac{\text{adjacent leg}}{\text{hypotenuse}} && [\text{Definition of cosine}]\\ &= \frac{XY}{ZX}\\ &= \frac{16}{45} && [XY = 16, ZX = 45]\end{aligned}$$

Answer 51PA.

Consider the figure below:



Consider two triangles are ΔABD and ΔCDB. The unknown angle is $\angle C$.

First determine *x*.

Using Pythagoras theorem

$$x = \sqrt{25^2 - 24^2}$$
$$= \sqrt{49}$$
$$x = 7$$

The length is $\overline{BD} = 7$. So, *CD* is in length is $(23 - x) = (23 - 7) = 16$.

Since the lengths of the opposite leg and adjacent are known, use the tangent ratio.

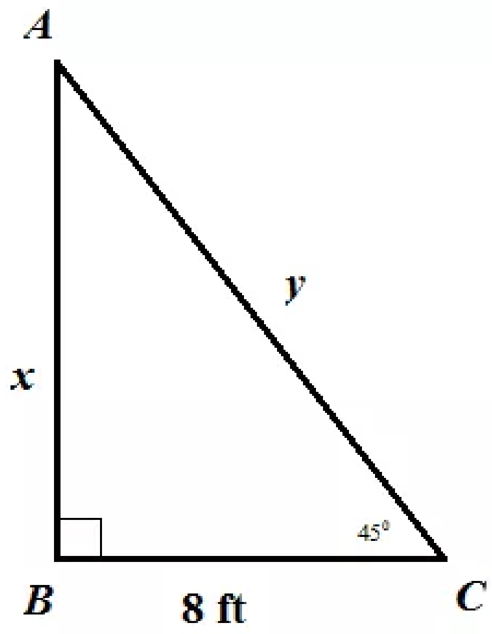
$$\tan C = \frac{\text{opposite leg}}{\text{adjacent leg}} \quad [\text{Definition of tangent}]$$
$$= \frac{AD}{CD}$$
$$= \frac{24}{16} \quad [AD = 24, CD = 16]$$

Now, use the $[\tan^{-1}]$ on a calculator to find the measure of the angle whose tangent ratio is $\frac{24}{16}$.

KEYSTROKES: $\boxed{\tan^{-1}}$ (24 $\boxed{\div}$ 16) 56.30

To the nearest degree, the measure of unknown angle $\angle C$ is $\boxed{56^0}$.

Answer 52PA.



To find the measure of $\angle A, \overline{AC}, \overline{AB}$ follow the steps

Step 1: Find the measure of $\angle A$. The sum of the measures of the angles in a triangle is 180^0.

$$180^0 - 90^0 - 45^0 = 45^0$$

The measure of $\angle A$ is 45^0.

Step 2: Find the value of x, which is the measure of the side opposite $\angle C$. Use the tangent ratio. Use the tangent ratio.

$$\tan 45^0 = \frac{x}{8} \quad [\text{Definition of tangent}]$$
$$1 = \frac{x}{8} \quad [\text{Evaluate } \tan 45^0]$$
$$8 = x \quad [\text{Multiply by } 8]$$

$\overline{AB}$ is about 8.0 ft long.

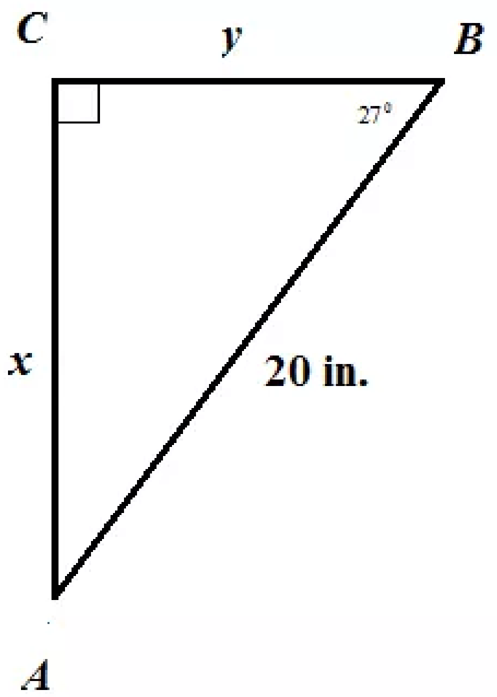
Step 3: Find the value of y, which is the measure of the sides adjacent to $\angle C$. Use the cosine ratio.

$$\cos 45^0 = \frac{8}{y} \quad [\text{Definition of cosine}]$$
$$y = \frac{4}{\cos 45^0} \quad [\text{Evaluate } \cos 45^0]$$
$$y \approx 5.7$$

$\overline{AC}$ is about 5.7 ft. long.

Therefore, the missing measures are $\boxed{45^0, 8.0 \text{ ft}, 5.7 \text{ ft}}$.

Answer 53PA.



To find the measure of $\angle A, \overline{AC}, \overline{BC}$ follow the steps

Step 1: Find the measure of $\angle A$. The sum of the measures of the angles in a triangle is 180^0.

$$180^0 - 90^0 - 27^0 = 63^0$$

The measure of $\angle A$ is 63^0.

Step 2: Find the value of x, which is the measure of the side opposite $\angle B$. Use the sine ratio. Use the sine ratio.

$$\sin 27^0 = \frac{x}{20} \quad \text{[Definition of sine]}$$
$$0.4539 = \frac{x}{20} \quad \text{[Evaluate } \sin 27^0\text{]}$$
$$9.0 = x \quad \text{[Multiply by 20]}$$

$\overline{AC}$ is about 9.0 in. long.

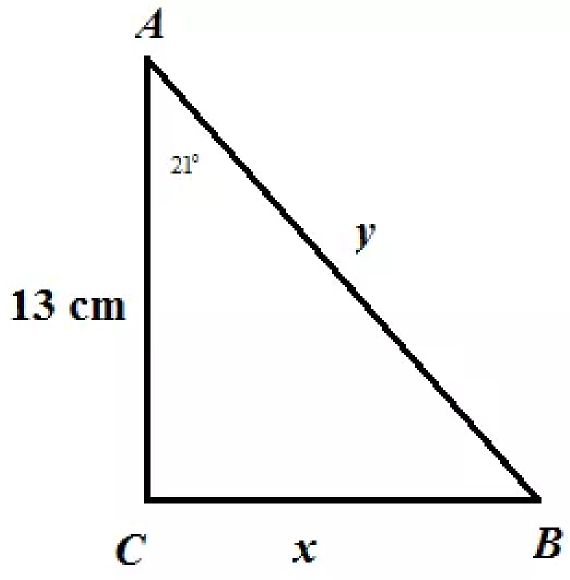
Step 3: Find the value of y, which is the measure of the sides adjacent to $\angle B$. Use the cosine ratio.

$$\cos 27^0 = \frac{y}{20} \quad \text{[Definition of cosine]}$$
$$0.9810 = \frac{y}{20} \quad \text{[Evaluate } \cos 27^0\text{]}$$
$$17.8 \approx y \quad \text{[Multiply by 20 both sides]}$$

$\overline{BC}$ is about 17.8 in. long.

Therefore, the missing measures are $\boxed{63^0, 9.0 \text{ in}, 17.8 \text{ in.}}$.

Answer 54PA.



To find the measure of $\angle B, \overline{AB}, \overline{BC}$ follow the steps

Step 1: Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180^0.

$$180^0 - 90^0 - 21^0 = 69^0$$

The measure of $\angle B$ is 69^0.

Step 2: Find the value of *x*, which is the measure of the side opposite $\angle A$. Use the tangent ratio. Use the tangent ratio.

$$\tan 21^0 = \frac{x}{13} \quad [\text{Definition of tangent}]$$

$$0.3838 = \frac{x}{13} \quad [\text{Evaluate } \tan 21^0]$$

$$4.9 = x \quad [\text{Multiply by } 20]$$

$\overline{BC}$ is about 4.9 cm long.

Step 3: Find the value of *y*, which is the measure of the sides adjacent to $\angle A$. Use the cosine ratio.

$$\cos 21^0 = \frac{13}{y} \quad [\text{Definition of cosine}]$$

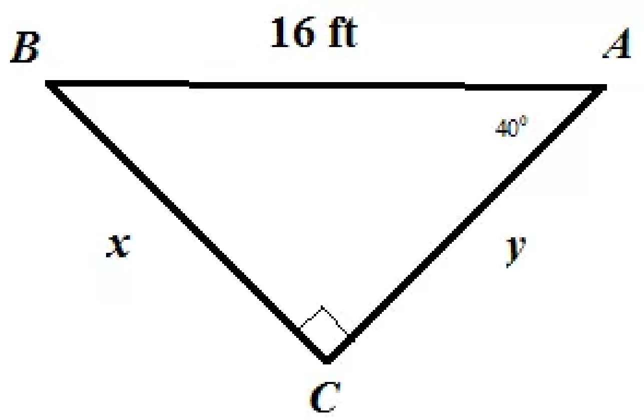
$$y = \frac{13}{\cos 21^0} \quad [\text{Evaluate } \cos 21^0]$$

$$y \approx 13.9$$

$\overline{AB}$ is about 13.9 cm long.

Therefore, the missing measures are $\boxed{69^0, 4.9\text{ cm}, 13.9\text{ cm}}$.

Answer 55PA.



To find the measure of $\angle B, \overline{AB}, \overline{AC}$ follow the steps

Step 1: Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180^0.

$$180^0 - 90^0 - 40^0 = 50^0$$

The measure of $\angle B$ is 50^0.

Step 2: Find the value of x, which is the measure of the side opposite $\angle A$. Use the sine ratio. Use the sine ratio.

$$\sin 40^0 = \frac{x}{16} \quad [\text{Definition of sine}]$$

$$0.6427 = \frac{x}{16} \quad [\text{Evaluate } \sin 40^0]$$

$$10.2 \approx x \quad [\text{Multiply by } 16]$$

$\overline{BC}$ is about 10.2 ft long.

Step 3: Find the value of y, which is the measure of the sides adjacent to $\angle A$. Use the cosine ratio.

$$\cos 40^0 = \frac{y}{16} \quad [\text{Definition of cosine}]$$

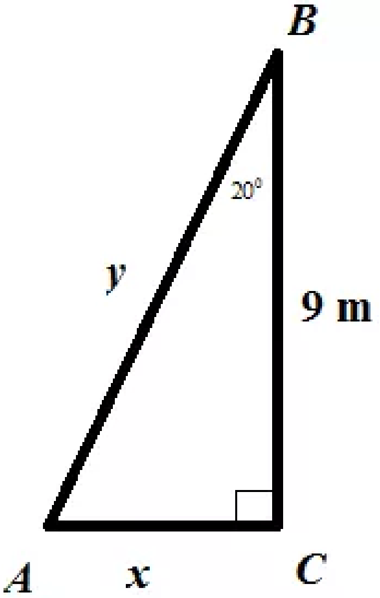
$$0.7660 = \frac{y}{16} \quad [\text{Evaluate } \cos 40^0]$$

$$y \approx 12.3$$

$\overline{AC}$ is about 12.3 ft long.

Therefore, the missing measures are $\boxed{50^0, 10.2 \text{ ft}, 12.3 \text{ ft}}$.

Answer 56PA.



To find the measure of $\angle A, \overline{AB}, \overline{AC}$ follow the steps

Step 1: Find the measure of $\angle A$. The sum of the measures of the angles in a triangle is 180^0.

$$180^0 - 90^0 - 20^0 = 70^0$$

The measure of $\angle A$ is 70^0.

Step 2: Find the value of *x*, which is the measure of the side opposite $\angle B$. Use the tangent ratio. Use the tangent ratio.

$$\tan 20^0 = \frac{x}{9} \quad \text{[Definition of tangent]}$$

$$0.3639 = \frac{x}{9} \quad \left[\text{Evaluate } \tan 20^0\right]$$

$$3.3 \approx x \quad \text{[Multiply by 9]}$$

$\overline{AC}$ is about 3.3 m long.

Step 3: Find the value of *y*, which is the measure of the sides adjacent to $\angle B$. Use the cosine ratio.

$$\cos 20^0 = \frac{9}{y} \quad \text{[Definition of cosine]}$$

$$0.9396 = \frac{9}{y} \quad \left[\text{Evaluate } \cos 20^0\right]$$

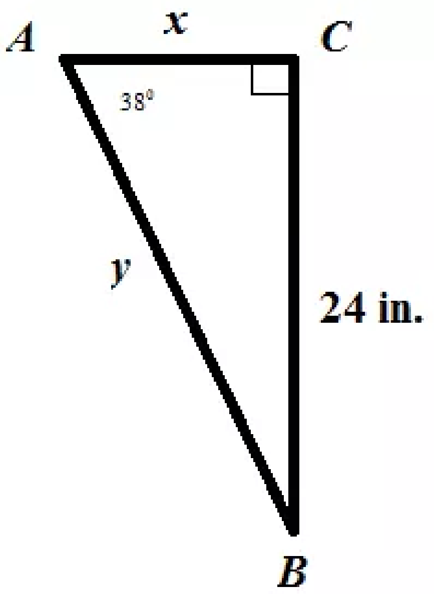
$$y \approx \frac{9}{0.9396}$$

$$y \approx 9.6$$

$\overline{AB}$ is about 9.6 m long.

Therefore, the missing measures are $\boxed{70^0, 3.3\,\text{m}, 9.6\text{ m}}$.

Answer 57PA.

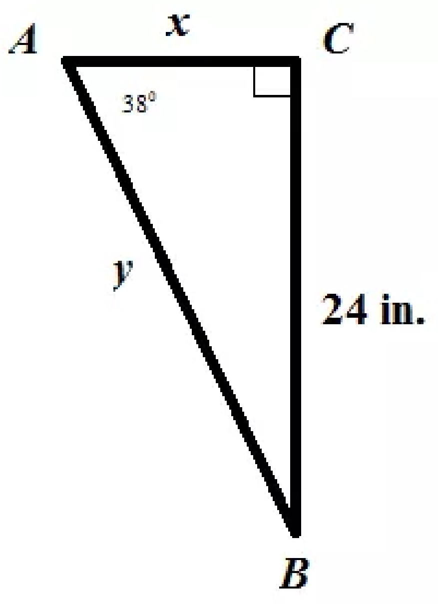


To find the measure of $\angle B, \overline{AB}, \overline{AC}$ follow the steps

Step 1: Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180^0.

$$180^0 - 90^0 - 38^0 = 52^0$$

The measure of $\angle B$ is 52^0.



To find the measure of $\angle B, \overline{AB}, \overline{AC}$ follow the steps

Step 1: Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180^0.

$$180^0 - 90^0 - 38^0 = 52^0$$

The measure of $\angle B$ is 52^0.

Step 3: Find the value of *y*, which is the measure of the sides adjacent to $\angle A$. Use the cosine ratio.

$$\cos 38^0 = \frac{x}{y} \qquad [\text{Definition of cosine}]$$

$$0.7880 = \frac{30.7}{y} \qquad [\text{Evaluate } \cos 38^0, x = 30.7]$$

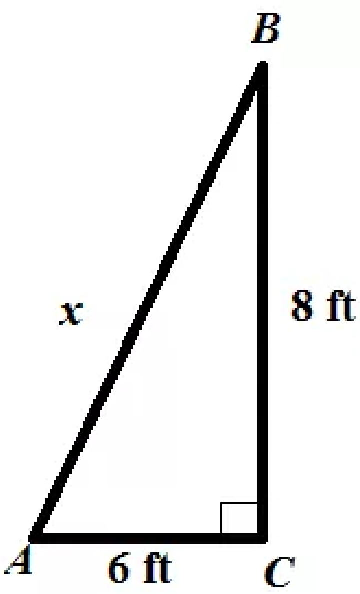
$$y \approx \frac{30.7}{0.7880}$$

$$y \approx 38.1$$

$\overline{AB}$ is about 38.1 in. long.

Therefore, the missing measures are $\boxed{52^0, 30.7\text{ in.}, 38.1\text{ in.}}$.

Answer 58PA.



To find the measure of $\angle A, \angle B, \overline{AB}$ follow the steps

Step 1: Since the lengths of the opposite leg and adjacent sides are known, use the tangent ratio.

$$\begin{aligned} \tan A &= \frac{\text{opposite leg}}{\text{adjacent leg}} && [\text{Definition of tangent}] \\ &= \frac{BC}{AC} \\ &= \frac{8}{6} && [BC = 8, AC = 6] \end{aligned}$$

Now, use the $[\tan^{-1}]$ on a calculator to find the measure of the angle whose tangent ratio is $\frac{8}{6}$

.

KEYSTROKES: $\boxed{\tan^{-1}}$ (8 $\boxed{\div}$ 6) 53.13

To the nearest degree, the measure of unknown angle *A* is $\boxed{53^0}$.

Step 2: Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180^0.

$180^0 - 90^0 - 53^0 = 37^0$

The measure of $\angle B$ is 37^0.

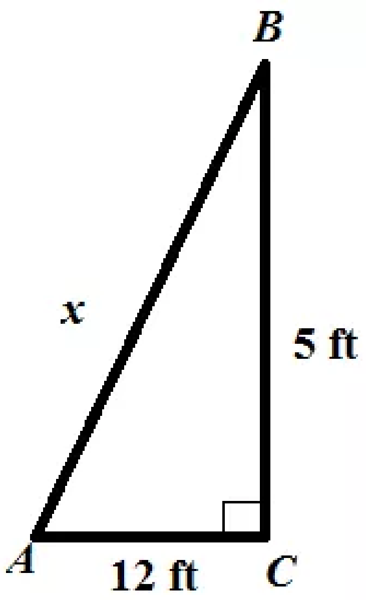
Step 3: Find the value of *x*, which is the measure of the sides adjacent to $\angle A$. Use the cosine ratio.

$$\cos 53^0 = \frac{6}{x} \quad [\text{Definition of cosine}]$$
$$0.6018 = \frac{6}{x} \quad [\text{Evaluate } \cos 53^0]$$
$$x \approx \frac{6}{0.6018}$$
$$x \approx 9.9$$

$\overline{AB}$ is about 9.9 ft long.

Therefore, the missing measures are $\boxed{53^0, 37^0, 9.9 \text{ ft}}$.

Answer 59PA.



To find the measure of $\angle A, \angle B, \overline{AB}$ follow the steps

Step 1: Since the lengths of the opposite leg and adjacent sides are known, use the tangent ratio.

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} \quad [\text{Definition of tangent}]$$
$$= \frac{BC}{AC}$$
$$= \frac{5}{12} \quad [BC = 5, AC = 12]$$

Now, use the $\left[\tan^{-1}\right]$ on a calculator to find the measure of the angle whose tangent ratio is $\frac{5}{12}$.

KEYSTROKES: $\boxed{\tan^{-1}}$ (5 $\boxed{\div}$ 12) 22.61

To the nearest degree, the measure of unknown angle *A* is $\boxed{23^0}$.

Step 2: Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180^0.

$$180^0 - 90^0 - 23^0 = 67^0$$

The measure of $\angle B$ is 67^0.

Step 3: Find the value of *x*, which is the measure of the sides adjacent to $\angle A$. Use the cosine ratio.

$$\cos 23^0 = \frac{12}{x} \quad \left[\text{Definition of cosine}\right]$$

$$0.9205 = \frac{12}{x} \quad \left[\text{Evaluate } \cos 23^0\right]$$

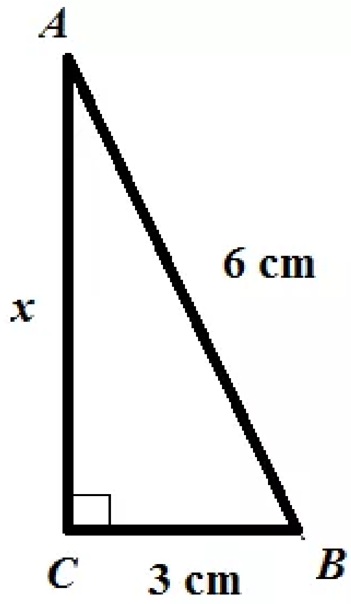
$$x \approx \frac{12}{0.9205}$$

$$x \approx 13.0$$

$\overline{AB}$ is about 13.0 ft long.

Therefore, the missing measures are $\boxed{23^0, 67^0, 13.0 \text{ ft}}$.

Answer 60PA.



To find the measure of $\angle A, \angle B, \overline{AC}$ follow the steps

Step 1: Since the lengths of the hypotenuse and adjacent sides are known, use the cosine ratio.

$$\cos B = \frac{\text{adjacent leg}}{\text{hypotenuse}} \quad [\text{Definition of tangent}]$$
$$= \frac{BC}{AB}$$
$$= \frac{3}{6} \quad [BC = 3, AB = 6]$$

Now, use the $[\cos^{-1}]$ on a calculator to find the measure of the angle whose cosine ratio is $\frac{3}{6}$.

KEYSTROKES: $\boxed{\cos^{-1}}$ (3 $\boxed{\div}$ 6) 60

To the nearest degree, the measure of unknown angle *A* is $\boxed{60^0}$.

Step 2: Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180^0.

$$180^0 - 90^0 - 60^0 = 30^0$$

The measure of $\angle B$ is 30^0.

Step 3: Find the value of *x*, which is the measure of the sides adjacent to $\angle A$. Use the cosine ratio.

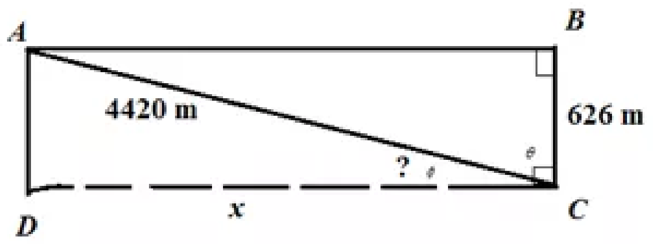
$$\cos 60^0 = \frac{x}{6} \quad [\text{Definition of cosine}]$$
$$0.5 = \frac{x}{6} \quad [\text{Evaluate } \cos 60^0]$$
$$x = 3.0$$

$\overline{AC}$ is about 3.0 cm long.

Therefore, the missing measures are $\boxed{30^0, 60^0, 3.0\text{ cm}}$.

Answer 61PA.

Consider the triangle ΔABC, *AC* is the hypotenuse, *AB* is the perpendicular and *BC* is the base.



Since the adjacent and hypotenuse are known, so use the cosine ratio to find $\angle\theta$.

Write the trigonometric ration of cosine as

$$\cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$
$$= \frac{BC}{AC}$$
$$= \frac{626}{4420} \qquad [BC = 626, AC = 4420]$$

Now, use the $\left[\cos^{-1}\right]$ on a calculator to find the measure of the angle whose cosine ratio is $\frac{626}{4420}$.

KEYSTROKES: $\boxed{\cos^{-1}}$ (626 $\boxed{\div}$ 4420) 81.85

To the nearest degree, the measure of unknown angle θ is 82^0.

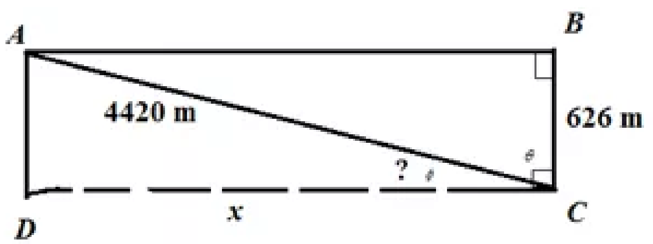
Since *ABCD* from a rectangle so each angle is 90^0, so we can evaluate

$$\angle\phi = 90^0 - \theta$$
$$= 90^0 - 82^0$$
$$= 8^0$$

Therefore, the measure of unknown angle is $\boxed{8^0}$.

Answer 62PA.

Consider the triangle ΔABC, *AC* is the hypotenuse, *AB* is the perpendicular and *BC* is the base. To find horizontal distance *x* follow the process.



Since the adjacent angle and hypotenuse are known, so use the cosine ratio to find *x*.

Write the trigonometric ration of cosine as

$$\cos\phi = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$\cos\phi = \frac{CD}{AC}$$

$$\cos 8^0 = \frac{x}{4420} \qquad [\phi = 8, CD = x, AC = 4420]$$

$$0.99 = \frac{x}{4420} \qquad [\cos 8^0 = 0.99]$$

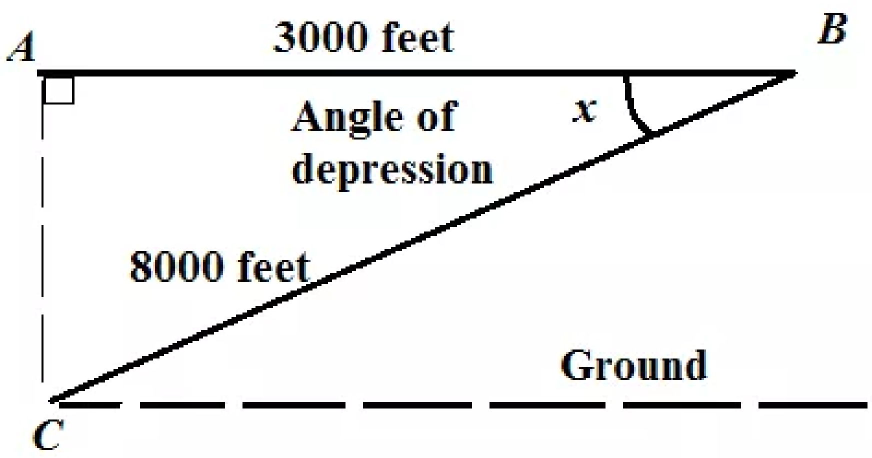
$$x \approx 4376.98$$

$$x \approx 4377\,\text{m}$$

Therefore, the horizontal distance is $\boxed{4377\,\text{m}}$.

Answer 63PA.

Consider the figure below:



Consider the triangle ΔABC, *BC* is the hypotenuse, *AC* is the perpendicular and *AB* is the base. To find depression angle *x* follow the process.

Since the adjacent angle and hypotenuse are known, so use the cosine ratio to find *x*.

Write the trigonometric ration of cosine as

$$\cos x = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$\cos x = \frac{AB}{BC}$$

$$\cos x = \frac{3000}{8000} \qquad [AB = 3000, BC = 8000]$$

$$\cos x = \frac{3}{8}$$

Now, use the $\left[\cos^{-1}\right]$ on a calculator to find the measure of the angle whose cosine ratio is $\frac{3}{8}$.

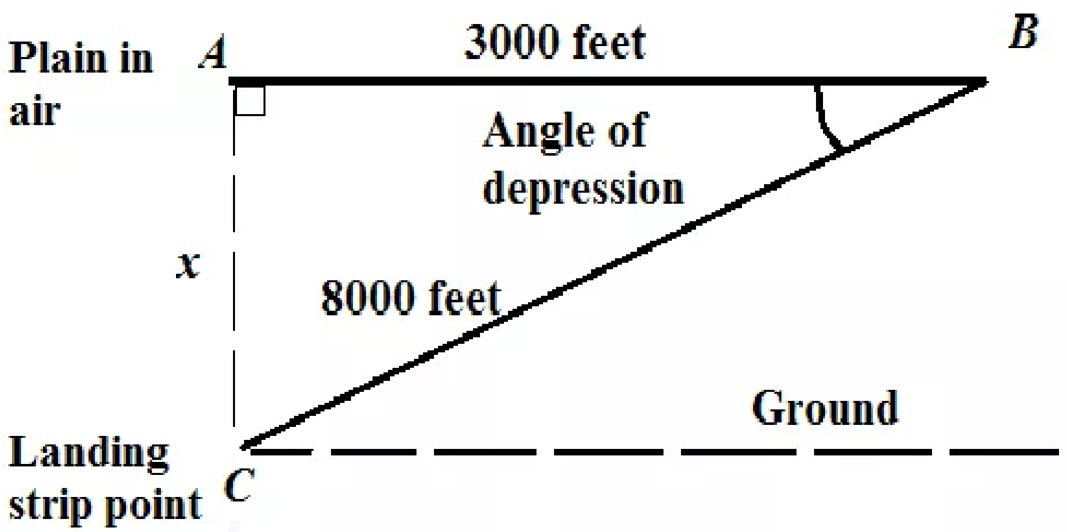
KEYSTROKES: $\boxed{\cos^{-1}}$ (3 $\boxed{\div}$ 8) 67.97

To the nearest degree, the measure of unknown angle *x* is 68^0.

Therefore, the measure of depression angle is $\boxed{68^0}$.

Answer 64PA.

Consider the figure below:



Consider the triangle ΔABC, *BC* is the hypotenuse, *AC* is the perpendicular and *AB* is the base. To find distance *x* follow the process. The depression angle is $\angle B = 68^0$.

Since the opposite angle and hypotenuse are known, so use the sine ratio to find *x*.

Write the trigonometric ration of sine as

$$\sin B = \frac{\text{opposite leg}}{\text{hypotenuse}} \qquad [\text{Definition of sine}]$$

$$\sin 68^0 = \frac{AC}{BC}$$

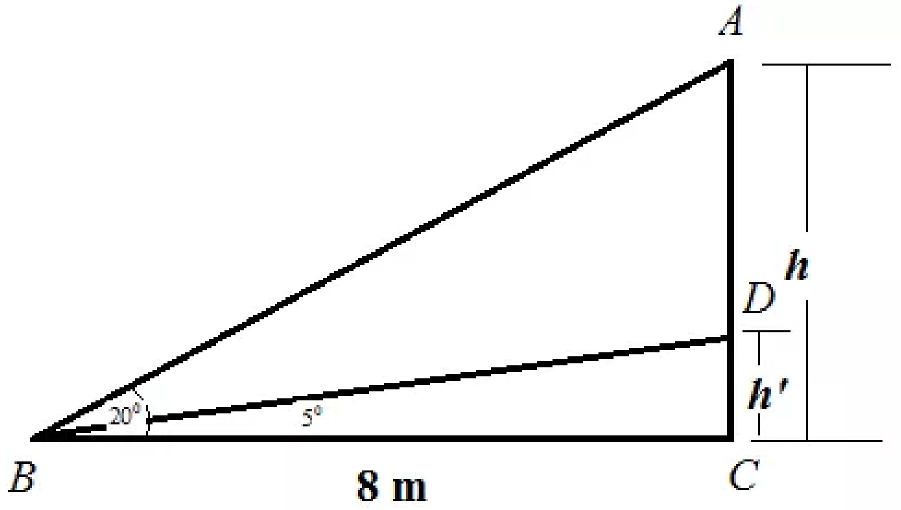
$$0.9271 \approx \frac{x}{8000} \qquad \left[\begin{array}{l} B = 68^0, AC = x, \\ BC = 8000 \end{array}\right]$$

$$7416.8 \approx x \qquad \left[\begin{array}{l} \text{Mulltiply both} \\ \text{sides by } 8000 \end{array}\right]$$

$$7417 \text{ feet} \approx x$$

Therefore, the distance between the plane and the landing strip is $\boxed{7417 \text{ feet}}$.

Answer 65PA.



Consider the triangle ΔABC, *AB* is the hypotenuse, *AC* is the perpendicular and *BC* is the base. To find distance *x* follow the process. The depression angle is $\angle B = 20^0, \angle DBC = 5^0$

Since the opposite angle and adjacent are known, so use the tangent ratio to find *x*.

Write the trigonometric ration of tangent as

$$\tan B = \frac{\text{opposite leg}}{\text{adjacent leg}} \quad [\text{Definition of tangent}]$$

$$\tan 20^0 = \frac{AC}{BC}$$

$$0.3639 \approx \frac{h}{8} \quad \begin{bmatrix} B = 20^0, AC = h, \\ BC = 8 \end{bmatrix}$$

$$h \approx 2.9 \quad \begin{bmatrix} \text{Mulltiply both} \\ \text{sides by 8} \end{bmatrix}$$

$$\tan B = \frac{\text{opposite leg}}{\text{adjacent leg}} \quad [\text{Definition of tangent}]$$

$$\tan 5^0 = \frac{DC}{BC}$$

$$0.0874 \approx \frac{h'}{8} \quad \begin{bmatrix} B = 5^0, AC = h', \\ BC = 8 \end{bmatrix}$$

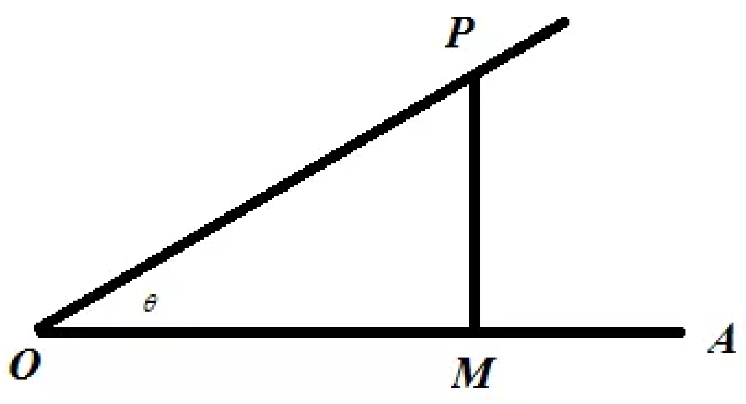
$$h' \approx 0.69 \quad \begin{bmatrix} \text{Mulltiply both} \\ \text{sides by 8} \end{bmatrix}$$

$$h' \approx 0.7$$

Therefore, the heights are ranges from $\boxed{2.9 \text{ feet to } 0.7 \text{ feet}}$.

Answer 66PA.

Consider the figure below:



Consider the triangle ΔMOP, *OP* is the hypotenuse, *PM* is the perpendicular and *OM* is the base.

In trigonometric ratios of the angle *AOP* are defined as follows:

Let the angle *AOP* be denoted by θ.

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin\theta = \frac{MP}{OP}$$

$$\cos\theta = \frac{\text{base}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{OM}{OP}$$

In the right angled triangle *MOP* we have

$$MP^2 + OM^2 = OP^2 \quad [\text{Use Pythagorean theorem}]$$

$$\frac{MP^2 + OM^2}{OP^2} = \frac{OP^2}{OP^2} \quad \left[\text{Divide both side by } OP^2\right]$$

$$\left(\frac{MP}{OP}\right)^2 + \left(\frac{OM}{OP}\right)^2 = 1$$

$$(\sin\theta)^2 + (\cos\theta)^2 = 1 \quad \left[\sin\theta = \frac{MP}{OP}, \cos\theta = \frac{OM}{OP}\right]$$

Therefore, the trigonometric identity $\boxed{(\sin\theta)^2 + (\cos\theta)^2 = 1}$ is proved.

Answer 67PA.

To write the trigonometric ratios, we have to know the distance between two points and the angles from these two points to a third point. We can evaluate the distance to the third point by forming a triangle and using trigonometric ratios.

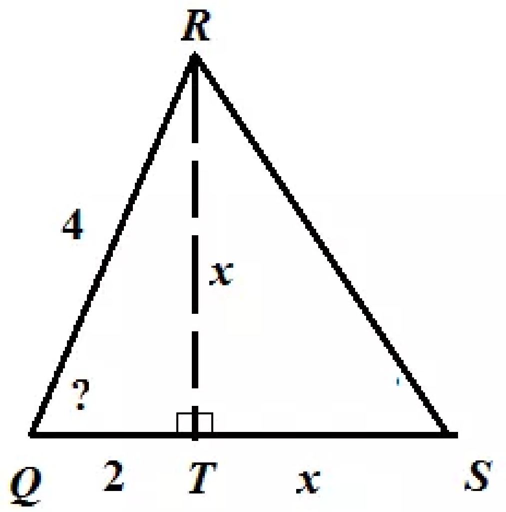
Therefore, the determination process should follow below two main tests:

If we measure our distance from the mountain and the angle of elevation to the peakof the mountain from two different points, it can be written an equation using trigonometric ratios to determine its height.

We have to know the altitude of the two points we are measuring.

Answer 68PA.

Consider the figure below:



Consider two triangles are ΔRQT and ΔRTS, *x* be the side in length of *RT*.

It is given that *RT* is equal to *TS*, so using the Pythagoras theorem length *x* can be evaluated as

First determine *x*.

Using Pythagoras theorem

$$\begin{aligned} x &= \sqrt{4^2 - 2^2} \\ &= \sqrt{16-4} \\ &= \sqrt{12} \\ x &= 2\sqrt{3} \end{aligned}$$

Now, find the length of *RS*.

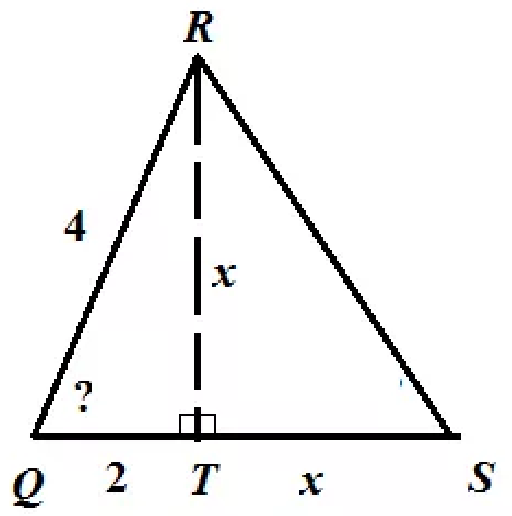
Using Pythagoras theorem

$$\begin{aligned} RS &= \sqrt{RT^2 + TS^2} \\ &= \sqrt{x^2 + x^2} \qquad [\text{Since } RT = TS = x] \\ &= \sqrt{2x^2} \\ &= x\sqrt{2} \\ &= \left(2\sqrt{3}\right)\left(\sqrt{2}\right) \left[\text{Substitute } x = 2\sqrt{3}\right] \\ RS &= 2\sqrt{6} \qquad [\text{Simpliy}] \end{aligned}$$

Therefore, (A) the length of *RS* is $\boxed{2\sqrt{6}}$.

Answer 69PA.

Consider the figure below:



Consider two triangles are ΔRQT.

Since the adjacent and hypotenuse are known, so use the cosine ratio to find $\angle Q$.

Write the trigonometric ration of cosine as

$$\begin{aligned} \cos Q &= \frac{\text{adjacent leg}}{\text{hypotenuse}} \\ &= \frac{QT}{RQ} \\ &= \frac{2}{4} \qquad [QT = 2, RQ = 4] \\ &= \frac{1}{2} \end{aligned}$$

Now, use the $\left[\cos^{-1}\right]$ on a calculator to find the measure of the angle whose cosine ratio is $\frac{1}{2}$.

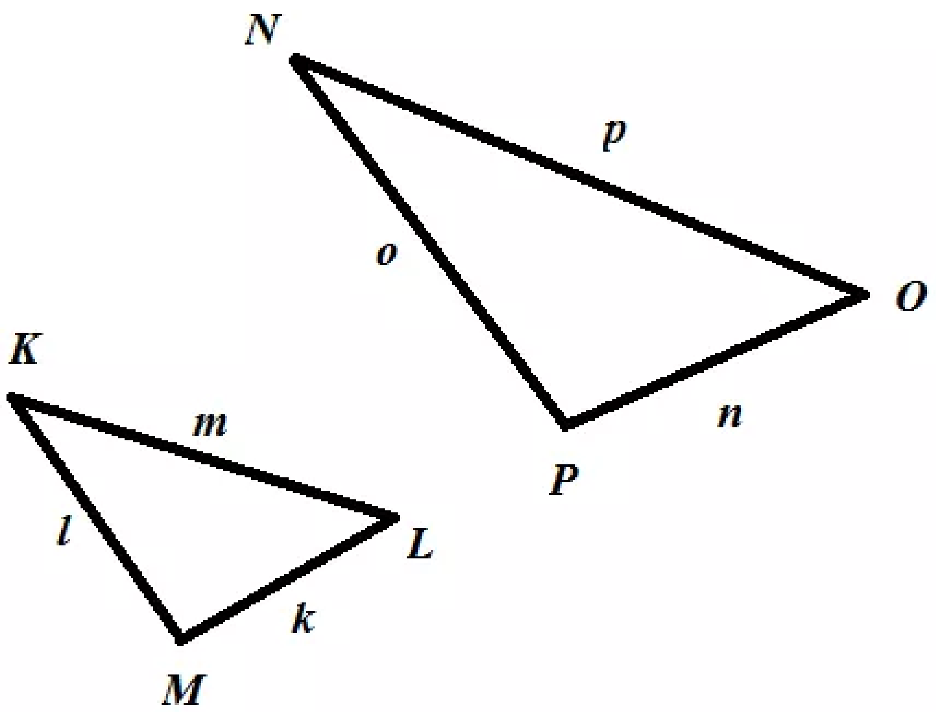
KEYSTROKES: $\boxed{\cos^{-1}}$ (1 $\boxed{\div}$ 2) 60

To the nearest degree, the measure of unknown angle Q is 60^0.

Therefore, (D) the measure of $\angle Q$ is $\boxed{60^0}$.

Answer 70MYS.

Consider the figure below:



It is given that

$\Delta KLM \sim \Delta NOP$ and $k = 5, l = 3, m = 6, n = 10$

Since $\Delta KLM \sim \Delta NOP$, so

$$\frac{KM}{NP} = \frac{ML}{PO} = \frac{KL}{NO}$$

$$\frac{l}{o} = \frac{k}{n} = \frac{m}{p}$$

$$\frac{3}{o} = \frac{5}{10} = \frac{6}{p} \qquad [k = 5, l = 3, m = 6, n = 10]$$

Now,

$$\frac{5}{10}=\frac{6}{p} \quad \left[\begin{array}{l}\text{Corresponding sides of similar triangles} \\ \text{are proportional}\end{array}\right]$$
$$5p=60 \quad [\text{Find the cross product}]$$
$$p=12 \quad [\text{Divide both sides by } 5]$$

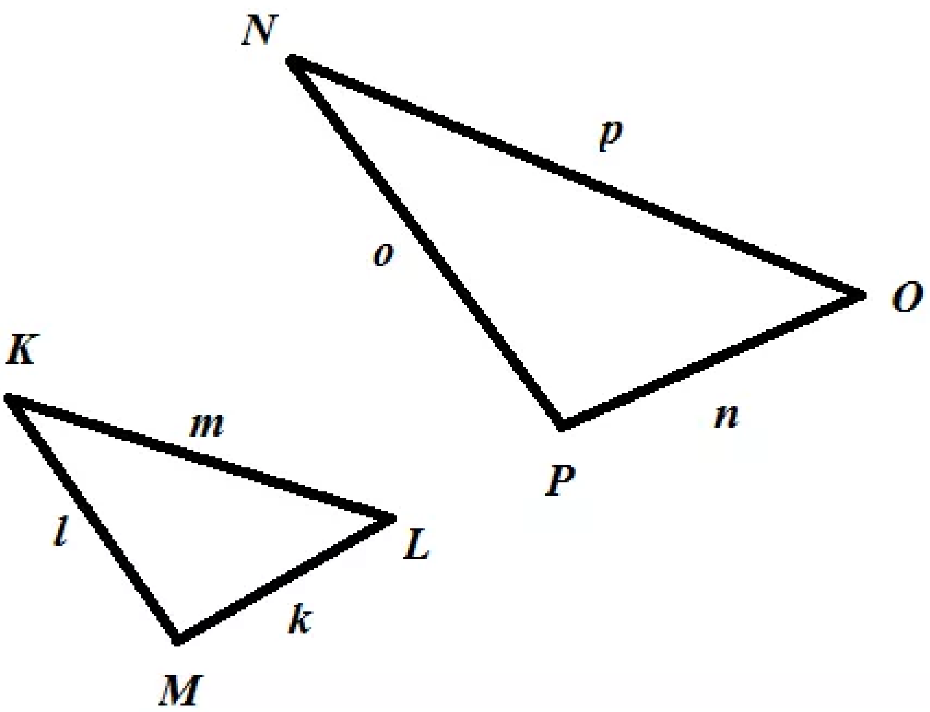
Again,

$$\frac{3}{o}=\frac{5}{10} \quad \left[\begin{array}{l}\text{Corresponding sides of similar triangles} \\ \text{are proportional}\end{array}\right]$$
$$5o=30 \quad [\text{Find the cross product}]$$
$$o=6 \quad [\text{Divide both sides by } 5]$$

Therefore, the measures of unknown sides are $\boxed{p=12, o=6}$.

Answer 71MYS.

Consider the figure below:



It is given that

$\Delta KLM \sim \Delta NOP$ and $l=9, m=3, n=12, p=4.5$

Since $\Delta KLM \sim \Delta NOP$, so

$$\frac{KM}{NP}=\frac{ML}{PO}=\frac{KL}{NO}$$
$$\frac{l}{o}=\frac{k}{n}=\frac{m}{p}$$
$$\frac{9}{o}=\frac{k}{12}=\frac{3}{4.5} \quad [l=9, m=3, n=12, p=4.5]$$

Now,

$$\frac{9}{o}=\frac{3}{4.5} \quad \left[\begin{array}{l}\text{Corresponding sides of similar triangles} \\ \text{are proportional}\end{array}\right]$$

$$3o=40.5 \quad [\text{Find the cross product}]$$

$$o=13.5 \quad [\text{Divide both sides by } 3]$$

Again,

$$\frac{k}{12}=\frac{3}{4.5} \quad \left[\begin{array}{l}\text{Corresponding sides of similar triangles} \\ \text{are proportional}\end{array}\right]$$

$$4.5k=36 \quad [\text{Find the cross product}]$$

$$k=8 \quad [\text{Divide both sides by } 4.5]$$

Therefore, the measures of unknown sides are $\boxed{k=8, o=13.5}$.

Answer 72MYS.

It is given that

Point coordinates are $(x_1, y_1)=(9,28),(x_2, y_2)=(a,-8)$, distance $d=39$.

The distance between the two points will be:

$$\begin{aligned}
\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} &= d \\
\sqrt{(a-9)^2+((-8)-(28))^2} &= 39 \\
(a-9)^2+36^2 &= 1521 \\
a^2-18a+81+1296 &= 1521 \\
a^2-18a+1377-1521 &= 0 \\
a^2-18a-144 &= 0 \\
(a-24)(a+6) &= 0 \\
a &= 24,-6
\end{aligned}$$

Therefore the values of *a* are $\boxed{24,-6}$.

Answer 73MYS.

It is given that

Point coordinates are $(x_1, y_1) = (3, a), (x_2, y_2) = (10, -1)$, distance $d = \sqrt{65}$.

The distance between the two points will be:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$$
$$\sqrt{(10-3)^2 + ((-1) - a)^2} = \sqrt{65}$$
$$7^2 + (a+1)^2 = 65$$
$$a^2 + 2a + 1 + 49 = 65$$
$$a^2 + 2a + 50 - 65 = 0$$
$$a^2 + 2a - 15 = 0$$
$$(a+5)(a-3) = 0$$
$$a = -5, 3$$

Therefore the values of *a* are $\boxed{-5, 3}$.

Answer 74MYS.

Consider the expression

$$c^2(c^2 + 3c)$$

To find the product follow the steps

$$c^2(c^2 + 3c) = c^2 \cdot c^2 + c^2 \cdot 3c \quad [\text{Use distributive property}]$$
$$= c^4 + 3c^3 \quad [\text{Multiply the product}]$$

Therefore, the product is $\boxed{c^4 + 3c^3}$.

Answer 45MYS.

Consider the expression

$$s(4s^2 - 9s + 12)$$

To find the product follow the steps

$$s(4s^2 - 9s + 12) = s \cdot 4s^2 - s \cdot 9s + s \cdot 12 \quad [\text{Use distributive property}]$$
$$= 4s^3 - 9s^2 + 12s \quad [\text{Multiply the product}]$$

Therefore, the product is $\boxed{4s^3 - 9s^2 + 12s}$.

Answer 76MYS.

Consider the expression

$$xy^2\left(2x^2+5xy-7y^2\right)$$

To find the product follow the steps

$$\begin{aligned} xy^2\left(2x^2+5xy-7y^2\right) &= xy^2\cdot 2x^2+xy^2\cdot 5xy \\ &\quad -xy^2\cdot 7y^2 && \left[\begin{array}{l}\text{Use distributive}\\ \text{property}\end{array}\right] \\ &= 2x^3y^2+5x^2y^3-7xy^4 && \left[\begin{array}{l}\text{Multiply the}\\ \text{product}\end{array}\right] \end{aligned}$$

Therefore, the product is $\boxed{2x^3y^2+5x^2y^3-7xy^4}$.

Answer 77MYS.

Consider the system

$$a=3b+2 \quad ......(1)$$
$$4a-7b=23 \quad ......(2)$$

To solve system substitute $a=3b+2$ into the equation (2) follow the steps

$$\begin{aligned} 4a-7b&=23 \\ 4(3b+2)-7b&=23 \\ 12b+8-7b&=23 \\ 5b+8&=23 \\ 5b&=15 \\ \frac{5b}{5}&=\frac{15}{5} \\ b&=3 \end{aligned}$$

Substitute $b=3$ in to the equation (1)

$$\begin{aligned} a&=3b+2 \\ &=3(3)+2 \\ &=9+2 \\ &=11 \end{aligned}$$

Therefore, solution is $\boxed{(a,b)=(11,3)}$.

Answer 78MYS.

Consider the system

$$p+q=10 \qquad \text{......(1)}$$
$$3p-2q=-5 \qquad \text{......(2)}$$

Rewrite the equation (1) as

$$p=10-q$$

To solve system substitute $p=10-q$ into the equation (2) follow the steps

$$\begin{aligned} 3p-2q&=-5 \\ 3(10-q)-2q&=-5 \\ 30-3q-2q&=-5 \\ 30-5q+5&=0 \\ 35&=5q \\ 7&=q \end{aligned}$$

Substitute $q=7$ in to the equation (1)

$$\begin{aligned} p&=10-q \\ &=10-7 \\ &=3 \end{aligned}$$

Therefore, solution is $\boxed{(p,q)=(3,7)}$.

Answer 79MYS.

Consider the system

$$3r+6s=0 \qquad \text{......(1)}$$
$$-4r-10s=-2 \qquad \text{......(2)}$$

Rewrite the equation (1) as

$$3r=-6s$$
$$r=-2s \qquad [\text{Divide both sides by } 3]$$

To solve system substitute $r=-2s$ into the equation (2) follow the steps

$$\begin{aligned} -4r-10s&=-2 \\ -4(-2s)-10s&=-2 \\ 8s-10s&=-2 \\ -2s&=-2 \\ \frac{-2s}{-2}&=\frac{-2}{-2} \\ s&=1 \end{aligned}$$

Substitute $s=1$ in to the equation $r=-2s$ as

$$\begin{aligned} r&=-2s \\ &=-2(1) \\ &=-2 \end{aligned}$$

Therefore, solution is $\boxed{(r,s)=(-2,1)}$.