JCECE
ENGINEERING ENTRANCE EXAM.
SOLVED PAPER 2004

Physics

1. The direction of vector $\vec{A}$ is reversed. What are the values of $\Delta\vec{A}$ and $\Delta|\vec{A}|$?

(a) $+2\vec{A}, 0$ (b) $+\vec{A}, 0$

(c) $-\vec{A}, 0$ (d) $-2\vec{A}, 0$

2. The velocity of a body moving with uniform acceleration at a given instant of time t is 10 m/s. After 5 s its velocity is 20 m/s. Distance travelled in that time is :

(a) 300 m (b) 400 m

(c) 150 m (d) 75 m

3. A stone falls freely such that the distance covered by it in the last second of its motion is equal to the distance covered by it in the first 5 s. It is in air for :

(a) 26 s (b) 25 s

(c) 13 s (d) 12 s

4. A child is swinging a swing. Minimum and maximum heights of swing from earth's surface are 0.75 m and 2m respectively. The maximum velocity of this swing is :

(a) 5 m/s (b) 10 m/s

(c) 15 m/s (d) 20 m/s

5. An object is thrown along a direction making an angle 45° with the horizontal direction. The horizontal range of the object is equal to :

(a) twice the vertical height

(b) vertical height

(c) four times the vertical height

(d) three times the vertical height

6. A fireman wants to slide down a rope. The breaking load for the rope is $\frac{3}{4}$th of the weight of the man. With what acceleration should the fireman slide down ? (Acceleration due to gravity is g)

(a) $\frac{g}{2}$ (b) $\frac{g}{4}$ (c) $\frac{3g}{4}$ (d) zero

7. A body of mass 1 kg is rotating in a vertical circle of radius 1 m. What will be the difference in kinetic energy at the top and at the bottom of the circle ? (Take $g = 10\ m/s^2$)

(a) 50 J (b) 30 J (c) 20 J (d) 10 J

8. A circular ring of mass m and radius r is rolling on a smooth horizontal surface with speed v. Its kinetic energy is :

(a) $\frac{1}{8}mv^2$ (b) $\frac{1}{4}mv^2$

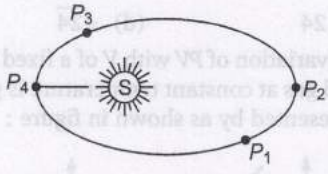
(c) $\frac{1}{3}mv^2$ (d) mv^2

9. A satellite is revolving around the earth in a circular orbit of radius 4 times that of the parking orbit. The time period of the satellite is :

(a) 16 days (b) 2 days

(c) 4 days (d) 8 days

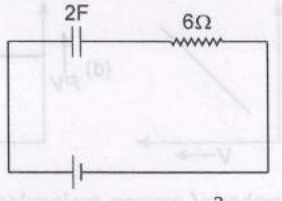
10. As shown in figure, a planet revolves in elliptical orbit around the sun. Where is KE of the planet maximum ?



(a) At P_4 (b) At P_1

(c) At P_2 (d) At P_3

11. If the condenser shown in the circuit is charged to 5 V and left in the circuit, in 12 s the charge on the condenser will become :



(a) $\frac{10}{e}$ coulomb (b) $\frac{e^2}{10}$ coulomb

(c) $\frac{10}{e^2}$ coulomb (d) $\frac{e}{10}$ coulomb

12. A sings with a frequency (n) and B sings with a frequency 1/8 that of A. If the energy remains the same and the amplitude of A is a, the amplitude of B will be :
(a) $2a$ (b) $8a$
(c) $4a$ (d) a

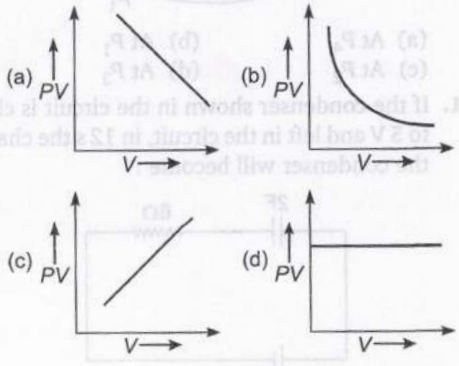
13. Reactance of a capacitor of capacitance C μF for AC frequency $\frac{400}{\pi}$ Hz is 25Ω, the value of C is :
(a) 75μF (b) 100μF
(c) 25μF (d) 50μF

14. An electron and a proton have same de-Broglie wavelength, then kinetic energy of the electron is :
(a) greater than KE of proton
(b) zero
(c) equal to KE of proton
(d) infinite

15. Relative permeability of iron is 5500. Its magnetic susceptibility is :
(a) 5499 (b) 5500×10^{7}
(c) 5500×10^{-7} (d) 5501

16. A force of $7\hat{\mathbf{i}} + 6\hat{\mathbf{k}}$ makes a body to move on a plane with the velocity $3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. The power developed is :
(a) $\sqrt{45}$ (b) 45
(c) 24 (d) $\sqrt{24}$

17. The variation of PV with V of a fixed mass of an ideal gas at constant temperature is graphically represented by as shown in figure :



18. The number of oxygen molecules in a cylinder of volume 1m^3 at a temperature of 27°C and pressure of 13.8 Pa is : (Boltzmann's constant $k = 1.38 \times 10^{-23}\,\text{J K}^{-1}$)
(a) 6.23×10^{26} (b) 0.33×10^{28}
(c) 3.3×10^{21} (d) none of these

19. The minimum magnifying power of a telescope is M. If the focal length of the eye lens is halved, the magnifying power will become :
(a) $\frac{M}{4}$ (b) $3M$
(c) $2M$ (d) $4M$

20. The surface temperature is maximum for :
(a) blue star (b) yellow star
(c) green star (d) red star

21. A convergent doublet of separated lens, corrected for spherical aberration, are separated by 2 cm and has an equivalent focal length of 10 cm. The focal length of its component lenses are :
(a) $f_1 = 18$ cm, $f_2 = 10$ cm
(b) $f_1 = 20$ cm, $f_2 = 28$ cm
(c) $f_1 = 20$ cm, $f_2 = 18$ cm
(d) $f_1 = 24$ cm, $f_2 = 18$ cm

22. In Young's experiment using light of wavelength 6000Å, fringe width obtained at distance 2.5 m is 0.8 mm. If the entire apparatus is immersed in a liquid of refractive index 1.6, the fringe width will be :
(a) 0.2 mm (b) 0.4 mm
(c) 0.5 mm (d) 0.6 mm

23. In double slit experiment, the angular width of interference fringes with sodium light ($\lambda = 5890$ Å) is 0.20°. The change in wavelength required to increase the angular width by 10% will be :
(a) zero
(b) increased by 6479 Å
(c) increased by 589 Å
(d) decreased by 589 Å

24. SHM is executed by a particle of mass m. The displacement of the particle is $\left(\frac{1}{\sqrt{2}}\right)$ times the amplitude. What fraction of the total energy is kinetic at this displacement ?
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{3}{4}$ (d) $\frac{1}{2}$

25. The period of SHM of a particle is 12 s. The phase difference between the positions at $t = 3$ s and $t = 4$ s will be :
(a) $\pi/4$ (b) $3\pi/5$
(c) $\pi/6$ (d) $\pi/2$

26. The volume of a metal sphere increases by 0.15% when its temperature is raised by 24°C. The coefficient of linear expansion of metal is :

(a) $2.5 \times 10^{-5}/°C$

(b) $2.0 \times 10^{-5}/°C$

(c) $1.5 \times 10^{-5}/°C$

(d) $1.2 \times 10^{-5}/°C$

27. The radii of two soap bubbles are r_1 and r_2 ($r_2 > r_1$). When they come into contact, the radius of their common interface is :

(a) $r_2 - r_1$ (b) $\sqrt{r_1^2 - r_2^2}$

(c) $\frac{r_1 + r_2}{2}$ (d) $\frac{r_1 r_2}{r_1 - r_2}$

28. A ray of light is incident on the surface of a glass slab at 60°. The refractive index of glass, if the reflected and refracted rays are mutually perpendicular to each other, will be :

(a) 1.5 (b) 4.16 (c) 2.25 (d) 1.73

29. A stretched string is 1 m long. Its mass per unit length is 0.5 g/m. It is stretched with a force of 20 N. It is plucked at a distance of 25 cm from one end. The frequency of note emitted by it will be :

(a) 400 Hz (b) 300 Hz

(c) 200 Hz (d) 100 Hz

30. A transverse wave is given by $y = A \sin 2\pi (ft - x/\lambda)$. The maximum particle velocity is 4 times the wave velocity when :

(a) $\lambda = 2\pi A$ (b) $\lambda = \pi A$

(c) $\lambda = \pi A/2$ (d) $\lambda = \pi A/4$

31. The energy in eV of red light of wavelength $\lambda = 6560$ Å is :

(a) 1.89 eV (b) 2.89 eV

(c) 3.89 eV (d) 4.89 eV

32. The wavelength of first member of Lyman series is 1215 Å. The wavelength of H_α line is :

(a) 6561 Å (b) 5464 Å

(c) 800 Å (d) 4840 Å

33. When silver is irradiated by ultraviolet light of 1000 Å, potential of 7.7 V is required to stop the photo electrons. The work function of silver will be :

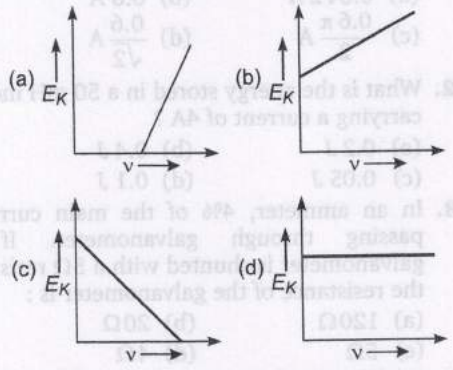
(a) 3.72 eV

(b) 6.72 eV

(c) 5.72 eV

(d) 4.67 eV

34. Maximum kinetic energy E_k of a photoelectron varies with the frequency ν of the incident radiation as :



35. A radioactive element has half-life period 1600 yr. After 6400 yr, what part of element will remain ?

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{2}$

36. For a transistor $I_E = 25$ mA and $I_B = 1$ mA, the value of current gain α will be :

(a) $\frac{25}{24}$ (b) $\frac{24}{25}$ (c) $\frac{25}{26}$ (d) $\frac{26}{25}$

37. When two semiconductors of p and n type are brought into contact, they form a p- n junction which acts like a :

(a) rectifier (b) amplifier

(c) oscillator (d) conductor

38. The de-Broglie wavelength of 1 kg mass moving with a velocity of 10 m/s, will be :

(a) 6.626×10^{-35} m (b) 6.626×10^{-34} m

(c) 6.626×10^{-33} m (d) none of these

39. Radioactive nuclei that are injected into a patient collected at certain sites within its body, undergoing radioactive decay and emitting electromagnetic radiation. These radiations can then be recorded by a detector. This procedure provides an important diagnostic tools called :

(a) gamma camera

(b) CAT scan

(c) radiotracer technique

(d) gamma ray spectroscopy

40. If R, C and L denote resistance, capacitance and inductance. Which of the following will not have the dimensions of frequency ?

(a) RL^{-1} (b) $R^{-1}C^{-1}$

(c) $L^{-1/2}C^{-1/2}$ (d) RCL

41. A 10 μF capacitor is connected across a 200 V, 50 Hz AC supply. The peak current through the circuit is :
(a) $0.6\sqrt{2}$ A (b) 0.6 A
(c) $\frac{0.6\pi}{2}$ A (d) $\frac{0.6}{\sqrt{2}}$ A

42. What is the energy stored in a 50 mH inductor carrying a current of 4A ?
(a) 0.2 J (b) 0.4 J
(c) 0.05 J (d) 0.1 J

43. In an ammeter, 4% of the main current is passing through galvanometer. If the galvanometer is shunted with a 5Ω resistance, the resistance of the galvanometer is :
(a) 120Ω (b) 20Ω
(c) 5Ω (d) 4Ω

44. If 100 kWh of energy is consumed at 33 V in a copper voltameter, what is the mass of copper liberated ? (Electrochemical equivalent of copper is 0.33×10^{-6} kg/C)
(a) 1 mg (b) 3.6 kg
(c) 3.3 kg (d) 1 kg

45. Four identical resistors when connected in series dissipate 5 W power. If they are connected in parallel, the power dissipated will be :
(a) 80 W (b) 60 W
(c) 40 W (d) 20 W

46. What is the magnitude of the point charge due to which the electric field 30 cm away has the magnitude of 2N/C ?
$\left(\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{Nm}^2/\text{C}^2\right)$
(a) 4×10^{-11}C (b) 2×10^{-11}C
(c) 5.4×10^{-11}C (d) 7.5×10^{-11}C

47. A parallel plate capacitor is immersed in an oil of dielectric constant 2. The field between the plates is :
(a) decreased, by a factor of $\frac{1}{\sqrt{2}}$
(b) increased, by a factor of $\sqrt{2}$
(c) increased, by a factor of $\frac{1}{2}$
(d) decreased, by a factor of 2

48. A triode valve has an amplification factor of 20 and its plate is given a potential of 300 V. The grid voltage to reduce the plate current to zero, is :
(a) 25 V (b) 15 V
(c) 12 V (d) 10 V

49. In an ideal parallel *LC* circuit, the capacitor is charged by connecting it to a DC source which is then disconnected. The current in the circuit :
(a) becomes zero instantaneously
(b) grows monotonically
(c) decays monotonically
(d) oscillates instantaneously

50. For a satellite moving in an orbit around the earth, the ratio of kinetic to potential energy is :
(a) 1 : 2 (b) 2 : 1
(c) 1 : 1 (d) 4 : 1

Chemistry

1. Which of the following statements, about zero order reaction is not true?
(a) Its rate constant unit is s^{-1}
(b) The graph between time and rate of reaction is a straight line
(c) Rate of reaction is independent of concentration of reactants
(d) None of the above

2. The amount of H_2S, required to precipitate 1.69 g BaS from $BaCl_2$ solution is :
(a) 3.4 g (b) 0.034 g
(c) 0.34 g (d) 0.17 g

3. Which of the following is not a protein?
(a) Testosterone (b) Lipase
(c) Keratin (d) Haemoglobin

4. The reaction,
$$CH_3COCH_3 \xrightarrow[\text{(ii) KOH, glycol}]{\text{(i) } NH_2NH_2} CH_3CH_2CH_3$$
is known as :
(a) Clemmensen's reduction
(b) Blanc's reduction
(c) Birch reduction
(d) Wolff-Kishner reduction

5. pH-scale was proposed by:
(a) Arrhenius
(b) Sorenson
(c) Pierre de Coubertin
(d) none of the above

6. Thomas slag is:
(a) $Ca_3(PO_4)_2 \cdot 2H_2O$
(b) $Ca_3(PO_4)_2 \cdot CaSiO_3$
(c) $MgSiO_3$
(d) $CaSiO_3$

7. Mohr's salt is:
(a) $Na_2SO_4 \cdot Al_2(SO_4) \cdot 24H_2O$
(b) $CuSO_4 \cdot Al_2(SO_4)_3 \cdot 24H_2O$
(c) $FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$
(d) $K_2SO_4 \cdot Fe_2(SO_4)_3 \cdot 24H_2O$

8. Which of the following has maximum ionisation energy?
(a) Caesium (b) Fluorine
(c) Xenon (d) Nitrogen

9. Which of the following give ammonia, on heating with NaOH?
(a) Ethyl amine (b) Ethane nitrile
(c) Nitroethane (d) Ethanamide

10. Which of the following is poisonous?
(a) Methanol (b) Ethanol
(c) Glycerol (d) Castor oil

11. Which of the following fraction of coal tar distillation is obtained at 270°–360°C?
(a) Light oil (b) Middle oil
(c) Green oil (d) Heavy oil

12. The amount of oxalic acid, required to prepare 300 mL 2.5 M solution, is:
(a) 67.5 g (b) 9.45 g
(c) 6.75 g (d) 94.5 g

13. The carbon di-oxide gas does not follow gaseous laws at all ranges of pressure and temperature because:
(a) it is triatomic gas
(b) its internal energy is quite high
(c) there is attraction between its, molecules
(d) it solidify at low temperature

14. The bond-angle in AsH_3 is greater than that in:
(a) NH_3 (b) H_2O
(c) BCl_3 (d) none of these

15. Which of the following gas, diffuse most slowly?
(a) C_2H_6
(b) C_2H_4
(c) CH_4
(d) All of these diffuse at equal rate

16. Oxidation number of fluorine is +1 in:
(a) F_2O (b) NF_3
(c) both (a) and (b) (d) none of these

17. Which radioisotope is used in the treatment of Leukeamia?
(a) P–32 (b) Na–24
(c) I–128 (d) Co–59

18. A metal having higher value of E°, can:
(a) replace hydrogen from dilute H_2SO_4
(b) replace hydrogen from concentrate H_2SO_4
(c) act as oxidising agent
(d) all of the above

19. The method, which is used to purify colloidal solution is:
(a) electrophoresis (b) dialysis
(c) chromatography (d) peptisation

20. Four gases ethane, ethene, ethyne and propene are passed into a wolf bottle containing ammoniacal $AgNO_3$ solution. Which gas will come out of wolf bottle?
(a) Only ethyne
(b) Ethane and ethyne
(c) Ethane and ethene
(d) Ethane, ethene and propene

21. How many conformers are known of ethane?
(a) Two
(b) Three
(c) > 3
(d) Ethane cannot show conformation

22. Interfering radicals interfere in the usual test of:
(a) acid radicals (b) basic radicals
(c) both (a) and (b) (d) none of these

23. Flux in the smelting process is added to:
(a) decrease the solubility of impurities
(b) increase the fusion temperature of roasted ore
(c) convert impurities into slag
(d) all of the above

24. The heat of formation of MgO, Al_2O_3 and SiO_2 are –692, –1676, –911, kJ mol^{-1}. Most stable oxide is:
(a) MgO (b) Al_2O_3
(c) SiO_2 (d) cannot be predicted

25. Which of the following reaction, will be favoured by low pressure?
(a) $PCl_5 \rightleftharpoons PCl_3 + Cl_2$
(b) $N_2 + 3H_2 \rightleftharpoons 2NH_3$
(c) $H_2 + \frac{1}{2}O_2 \rightleftharpoons H_2O$
(d) $2NO + O_2 \rightleftharpoons 2NO_2$

26. In III group of basic radical analysis, NH_4Cl is added before group reagent NH_4OH. This is done to:
(a) increase the solubility product of III group salts
(b) decrease the solubility product of III group salts

(c) increase the ionic product of OH^- and III group radicals
(d) decrease the degree of ionisation of group reagent

27. Representative elements are:
(a) *s*-block elements
(b) *p*-block elements
(c) both (a) and (b)
(d) none of these

28. Which of the following equation is used in Pauling's scale of electronegativity?
(a) $x_A = 0.187(IE + EA) + 0.17$
(b) $x_A - x_B = 0.182 [E_{A-B} - \sqrt{(E_{A-A} \times E_{B-B})}]^{1/2}$
(c) $x_A = \frac{0.359\, Z_{eff}}{r_A^2} + 0.744$
(d) None of the above

29. Which hybridisation has sulphur in SO_2?
(a) sp^2 (b) sp^3d^2
(c) sp^3 (d) sp

30. The energy of last electron of Li will be:
(a) –30.6 eV (b) –13.6 eV
(c) –24.6 eV (d) –28.6 eV

31. Benzaldehyde, in presence of dil. NaOH, changes into benzyl alcohol and sodium benzoate. This reaction is an example of :
(a) rearrangement
(b) oxidation
(c) disproportionation
(d) none of the above

32. Which of the following is the strongest acid?
(a) *o*-nitro benzoic acid
(b) *m*-nitro benzoic acid
(c) *m*-chloro benzoic acid
(d) *m*-cyano benzoic acid

33. What is the product '*X*' in the reaction, $CH_3COCl \xrightarrow{CH_3COONa} X$?
(a) CH_3COOH
(b) $(CH_3CO)_2O$
(c) $CH_3COCH_2COOC_2H_5$
(d) CH_3COCH_2COOH

34. A 0.5 g/L solution of glucose is found to be isotonic with a 2.5 g/L solution of an organic compound. What will be the molecular weight of that organic compound?
(a) 300 (b) 600
(c) 925 (d) 1200

35. Which of the following ion does not follow EAN rule?
(a) $[Fe(CN)_6]^{4-}$ (b) $[Ag(CN)_2]^-$
(c) $[PtCl_6]^{2-}$ (d) None of these

36. Potassium, on burning in oxygen atmosphere, gives:
(a) K_2O (b) K_2O_3 (c) KO_2 (d) K_2O_2

37. Which method is employed to prepare copper sol?
(a) Peptisation
(b) Bredig's arc method
(c) Double decomposition
(d) Reduction of $CuCl_2$

38. How much copper will be deposited at cathode, on passing 4 coulomb charge in 15 min?
(a) 0.0195 g (b) 0.0013 g
(c) 0.020 g (d) 0.0020 g

39. If 50 % of a radioactive substance dissociates in 15 min, then the time taken by substance to dissociate 99% will be :
(a) 50 min (b) 100 min
(c) 99 min (d) 150 min

40. Which of the following alcohol give positive iodoform test?
(a) Methanol (b) Ethyl carbinol
(c) Pentanol-3 (d) Methyl carbinol

41. Human body is an example of:
(a) open system (b) closed system
(c) isolated system (d) none of these

42. The atomic size of cerium and promethium is quite close, due to :
(a) they are in same period in periodic table
(b) their electronic configuration is same
(c) *f*-electrons have poor shielding effect
(d) nuclear charge is higher on cerium than promethium

43. Brown glass and cement have, which element common in them?
(a) Fe (b) Al
(c) Na (d) All of these

44. The most common isomerism found in ethers, is :
(a) chain isomerism (b) tautomerism
(c) metamerism (d) optical isomerism

45. *t*-butyl chloride preferrably undergo hydrolysis by :
(a) S_N1 mechanism
(b) S_N2 mechanism
(c) any of (a) and (b)
(d) none of the above

46. Which of the following has lowest boiling point?
(a) NaCl (b) CuCl
(c) $CuCl_2$ (d) CsCl

47. Which of the following is true for the reaction
$CO(g)+\frac{1}{2}O_2(g) \rightleftharpoons CO_2(g)$
(a) $K_p > K_c$
(b) $K_p < K_c$
(c) $K_p = K_c$
(d) $K_p \geq K_c$

48. Which of the following is not a buffer solution?
(a) $CH_3COOH + CH_3COONa$
(b) $H_3BO_3 + Na_3BO_3$
(c) $HClO_4 + NaClO_4$
(d) $NH_4OH + (NH_4)_2SO_4$

49. Which of the following ore is concentrated by both, magnetic and gravimetric separation?
(a) Dolomite (b) Tin stone
(c) Galena (d) Bauxite

50. Which of the following is not a polymer?
(a) RNA (b) Oxytocin
(c) Cholesterol (d) Amylose

Mathematics

1. The minimum value of $x^2 + \frac{1}{1+x^2}$ is at :
(a) $x = 0$ (b) $x = 4$
(c) $x = 1$ (d) $x = 3$

2. If $\vec{a}, \vec{b}, \vec{c}$ are the non-coplanar vectors, then the value of $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}} + \frac{\vec{b}\cdot(\vec{a}\times\vec{c})}{\vec{c}\cdot(\vec{a}\times\vec{b})}$ is :
(a) 1 (b) 2
(c) 0 (d) none of these

3. If $x - 2y = 4$, the minimum value of xy is :
(a) -2 (b) 0 (c) 0 (d) -3

4. If $z = x + iy$ and $\left|\frac{1-iz}{z-i}\right| = 1$, the locus of z is :
(a) x-axis
(b) y-axis
(c) circle with unity radius
(d) none of the above

5. The vertex of an equilateral triangle is $(2, -1)$ and the equation of its base is $x + 2y = 1$, the length of its sides is :
(a) $2/\sqrt{15}$ (b) $4/3\sqrt{3}$
(c) $1/\sqrt{5}$ (d) $4/\sqrt{15}$

6. The resultant of two forces P and Q is R. If the direction of P is reversed keeping the direction Q same, the resultant remains unaltered. The angle between P and Q is :
(a) 90° (b) 60°
(c) 45° (d) 30°

7. The distance s (in cm) travelled by a particle in t seconds is given by, $s = t^3 + 2t^2 + t$. The speed of the particle after 1 s will be :
(a) 2 cm/s (b) 8 cm/s
(c) 6 cm/s (d) none of these

8. The roots of $|x-2|^2 + |x-2| - 6 = 0$ are :
(a) 4, 2 (b) 0, 4
(c) -1, 3 (d) 5, 1

9. The height of a tower is 7848 cm. A particle is thrown from the top of the tower with the horizontal velocity of 1784 cm/s. The time taken by the particle to reach the ground is ($g = 981$ cm/s^2) :
(a) $\sqrt{8}$ s (b) 2 s (c) 4 s (d) 8 s

10. The directrix of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is :
(a) $y = 6/\sqrt{13}$ (b) $x = 6/\sqrt{13}$
(c) $y = 9/\sqrt{13}$ (d) $x = \pm 9/\sqrt{13}$

11. The value of
$\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\cos\frac{5\pi}{3}\right)$ is :
(a) $10\pi/3$ (b) 0
(c) $\pi/2$ (d) $5\pi/3$

12. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ will be equal to :
(a) $2f(x^2)$ (b) $f(x^2)$
(c) $2f(2x)$ (d) $2f(x)$

13. If $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots + a_{12}x^{12}$ then the value of $a_2 + a_4 + \ldots + a_{12}$ is :
(a) 31 (b) 32
(c) 64 (d) 1024

14. $2x^3 - 6x + 5$ is an increasing function, if :
(a) $0 < x < 1$ (b) $-1 < x < 1$
(c) $x < -1$ or $x > 1$ (d) $-1 < x < -\frac{1}{2}$

15. Two trains are 2 km apart their lengths are 200 m and 300 m. They are approaching towards each other with speed of 20 m/s and 30 m/s respectively. After how much time will they cross each other ?

(a) 150 s (b) 100 s
(c) 50 s (d) $\frac{25}{3}$ s

16. $\frac{d^3y}{dx^3} + 2\left[1 + \frac{d^2y}{dx^2}\right] = 1$, has degree and order as :

(a) 3, 1 (b) 3, 2
(c) 1, 3 (d) 2, 3

17. The value of $I = \int_0^1 x \left|x - \frac{1}{2}\right| dx$ is :

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{8}$ (d) none of these

18. If $A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$, $|\text{adj } A|$ is equal to :

(a) 6 (b) 16
(c) 10 (d) none of these

19. $\vec{a} \cdot (\vec{b} + \vec{c}) \times (\vec{a} + \vec{b} + \vec{c})$ is equal to :

(a) $[\vec{a}\ \vec{b}\ \vec{c}]$ (b) $3[\vec{a}\ \vec{b}\ \vec{c}]$
(c) 0 (d) $2[\vec{a}\ \vec{b}\ \vec{c}]$

20. A block weighing w, is supported on an inclined surface with the help of a horizontal force P. The same block can be supported with the help of another force Q acting parallel to the inclined surface, then $\frac{1}{P^2} + \frac{1}{w^2}$ is equal to :

(a) $w \sin \alpha$ (b) 1
(c) $1/Q$ (d) $1/Q^2$

21. $\int_0^2 |x - 1|\, dx$ is equal to :

(a) 0 (b) 1/2
(c) 1 (d) 2

22. Two cards are drawn one by one from a pack of cards. The probability of getting first card an ace and second a coloured one is (before drawing second card first card is not placed again in the pack) :

(a) 4/13
(b) 1/26
(c) 5/52
(d) 5/221

23. A particle is displaced from the point $A(5, -5, -7)$ to the point $B(6, 2, -2)$ under the influence of the forces $P_1 = 10\hat{i} - \hat{j} + 11\hat{k}$, $P_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$, $P_3 = -2\hat{i} + \hat{j} - 9\hat{k}$, the work done is :

(a) 87 unit (b) 85 unit
(c) 81 unit (d) none of these

24. If $\sin x + \cos x = \frac{1}{5}$, then $\tan 2x$ is :

(a) $\frac{25}{17}$ (b) $\frac{24}{7}$ (c) $\frac{7}{25}$ (d) $\frac{25}{7}$

25. In a ΔABC, $\angle B = \pi/3$ and $\angle C = \pi/4$. If D divides BC internally in ratio 1 : 3, then $\frac{\sin \angle BAD}{\sin \angle CAD}$ is equal to :

(a) $1/\sqrt{3}$ (b) $1/\sqrt{6}$
(c) $\sqrt{2/3}$ (d) 1/3

26. If $|\vec{a} \times \vec{b}| = |\vec{a} \cdot \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(a) π (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

27. Let A, B and C are the angles of a plain triangle and $\tan\left(\frac{A}{2}\right) = \frac{1}{3}$, $\tan\left(\frac{B}{2}\right) = \frac{2}{3}$. Then $\tan\left(\frac{C}{2}\right)$ is equal to :

(a) 1/3 (b) 2/3
(c) 2/9 (d) 7/9

28. The value of $\lim_{x \to 1} (1 - x) \tan\left(\frac{\pi}{2} x\right)$:

(a) $3\pi/4$ (b) $2\pi/3$
(c) $2/\pi$ (d) $\pi/4$

29. If $f(x) = \left(\frac{1}{x}\right)^x$, then the maximum value of $f(x)$ is :

(a) e (b) $(e)^{1/e}$
(c) $\left(\frac{1}{e}\right)^e$ (d) none of these

30. The volume of the solid formed by rotating the area enclosed between the curve $y = x^2$ and the line $y = 1$ about $y = 1$ is (in cubic unit) :

(a) $9\pi/5$ (b) $2\pi/5$
(c) $8\pi/3$ (d) $7\pi/5$

31. $\int_8^{15} \frac{dx}{(x - 3)\sqrt{x + 1}}$ is equal to :

(a) $\frac{1}{2} \log \frac{5}{3}$ (b) $\frac{1}{3} \log \frac{5}{3}$
(c) $\frac{1}{5} \log \frac{3}{5}$ (d) $\frac{1}{2} \log \frac{3}{5}$

32. Area of the square formed by $|x| + |y| = 1$ is :
(a) 0 sq unit (b) 1 sq unit
(c) 2 sq unit (d) 4 sq unit

33. If $x = 3 + i$, then $x^3 - 3x^2 - 8x + 15$ is equal to :
(a) 45 (b) -15
(c) 10 (d) 6

34. The function $f(x) = \log(x + \sqrt{x^2 + 1})$ is :
(a) even function
(b) odd function
(c) neither even nor odd
(d) periodic function

35. The locus of the intersection point of $x\cos\alpha + y\sin\alpha = a$ and $x\sin\alpha - y\cos\alpha = b$ is :
(a) parabola (b) ellipse
(c) hyperbola (d) none of these

36. If $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is :
(a) 0 (b) -1
(c) $-\frac{3}{2}$ (d) 3

37. If $x = \log_b a$, $y = \log_c b$, $z = \log_a c$, then xyz is :
(a) 0 (b) 1
(c) 3 (d) none of these

38. The value of the determinant
$$\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$$ is :
(a) 0 (b) 1
(c) $\alpha^2 - \beta^2$ (d) $\alpha^2 + \beta^2$

39. If $P(A) = P(B) = x$ and $P(A \cap B) = P(A' \cap B') = \frac{1}{3}$, then x is equal to :
(a) 1/2 (b) 1/4
(c) 1/3 (d) 1/6

40. If p and q are the roots of the equation $x^2 + px + q = 0$, thus :
(a) $p = 1$ or 0 (b) $p = -2$ or 0
(c) $p = -2$ (d) $p = 1$

41. If a dice is thrown twice, the probability of occurrence of 4 atleast once, is :
(a) 11/36 (b) 35/36
(c) 7/12 (d) none of these

42. The value of $\int_0^8 |x - 5|\, dx$ is :
(a) 9 (b) 12
(c) 17 (d) 18

43. The value of $\int_0^{\pi} |\sin^3\theta|\, d\theta$ is :
(a) 0 (b) π
(c) 4/3 (d) 3/8

44. A ball weighing 2 kg and speed 6 m/s collides with another ball of 4 kg moving in opposite direction with speed of 3 m/s. They combine after the collision. The speed of this combined mass in m/s is :
(a) 4 (b) 2
(c) 0 (d) 3

45. If α, β, γ are the roots of the equation $x^3 + 4x + 1 = 0$, then $(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1}$ is equal to :
(a) 2 (b) 3
(c) 4 (d) 5

46. If $\cos\theta + \cos 2\theta + \cos 3\theta = 0$, the general value of θ is :
(a) $\theta = 2m\pi \pm \pi/4$
(b) $\theta = m\pi + (-1)^n 2\pi/3$
(c) $\theta = m\pi + (-1)^n \pi/3$
(d) $\theta = 2m\pi \pm 2\pi/3$

47. Forces P and Q acting at a point O make an angle 150° between them. Their resultant acts at O, has magnitude 2 unit and is perpendicular to P. Then in the same unit, the magnitudes of P and Q are :
(a) $2\sqrt{3}$, 4 (b) $\sqrt{\frac{3}{2}}$, 2
(c) 3, 4 (d) 4, 5

48. A person observes the angle of depression of a building as 30°. The person proceeds towards the building with a speed of $25(\sqrt{3} - 1)$ m/h. After two hours, he observes the angle of elevation as 45°. The height of the building (in m) is :
(a) $50(\sqrt{3} - 1)$ (b) $50(\sqrt{3} + 1)$
(c) 50 (d) 100

49. The value of $\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+2}$ is :
(a) 0 (b) 1
(c) e^2 (d) e^4

50. If $A + B + C = 270°$, then $\cos 2A + \cos 2B + \cos 2C + 4\sin A \sin B \sin C$ is equal to :
(a) 0 (b) 1
(c) 2 (d) 3

ANSWERS

PHYSICS

1. (d)	2. (d)	3. (c)	4. (a)	5. (c)	6. (b)	7. (c)	8. (b)	9. (d)	10. (a)
11. (a)	12. (b)	13. (d)	14. (a)	15. (a)	16. (c)	17. (d)	18. (c)	19. (c)	20. (a)
21. (c)	22. (c)	23. (c)	24. (d)	25. (c)	26. (b)	27. (d)	28. (d)	29. (c)	30. (c)
31. (a)	32. (a)	33. (d)	34. (a)	35. (c)	36. (b)	37. (a)	38. (a)	39. (c)	40. (d)
41. (a)	42. (b)	43. (a)	44. (b)	45. (a)	46. (b)	47. (d)	48. (b)	49. (d)	50. (a)

CHEMISTRY

1. (a)	2. (c)	3. (a)	4. (d)	5. (b)	6. (b)	7. (c)	8. (b)	9. (d)	10. (a)
11. (c)	12. (a)	13. (c)	14. (d)	15. (a)	16. (d)	17. (a)	18. (c)	19. (b)	20. (d)
21. (c)	22. (b)	23. (c)	24. (a)	25. (a)	26. (d)	27. (c)	28. (b)	29. (a)	30. (a)
31. (c)	32. (a)	33. (b)	34. (c)	35. (b)	36. (c)	37. (b)	38. (b)	39. (c)	40. (d)
41. (a)	42. (c)	43. (a)	44. (c)	45. (a)	46. (c)	47. (b)	48. (c)	49. (b)	50. (c)

MATHEMATICS

1. (a)	2. (c)	3. (a)	4. (a)	5. (a)	6. (a)	7. (b)	8. (b)	9. (c)	10. (d)
11. (c)	12. (d)	13. (a)	14. (c)	15. (c)	16. (c)	17. (c)	18. (c)	19. (c)	20. (d)
21. (c)	22. (d)	23. (a)	24. (b)	25. (b)	26. (c)	27. (d)	28. (c)	29. (b)	30. (b)
31. (a)	32. (c)	33. (b)	34. (b)	35. (d)	36. (c)	37. (b)	38. (a)	39. (a)	40. (a)
41. (a)	42. (c)	43. (c)	44. (c)	45. (c)	46. (d)	47. (a)	48. (c)	49. (c)	50. (b)

HINTS & SOLUTIONS

Physics

1. Key Idea : *When direction of a vector is reversed it has same magnitude but with minus sign.*

The value of

$$\Delta\vec{A} = \vec{A}_2 - \vec{A}_1$$

Given, $\vec{A}_1 = A, \vec{A}_2 = -A$

$$\therefore \quad \Delta\vec{A} = -A - A = -2\vec{A}$$

Also, $|\vec{A}| = |-\vec{A}|$

$$\therefore \quad \Delta|\vec{A}| = 0$$

2. From equation of motion

$$v = u + at$$

Given, $v = 20$ m/s, $u = 10$ m/s, $t = 5$ s

$$\therefore \quad a = \frac{10}{5} = 2\ \text{m/s}^2$$

Also, $v^2 = u^2 + 2as$

$$(20)^2 = (10)^2 + 2 \times 2s$$

$$\Rightarrow \quad s = \frac{300}{4} = 75\ \text{m}$$

3. Key Idea : *In case of freely falling body initial velocity is zero.*

From equation of motion the distance travelled in the nth second of motion is

$$s_n = u + \frac{1}{2}g(2n-1) \qquad \ldots(\text{i})$$

Also, $s = ut + \frac{1}{2}gt^2$ ...(ii)

Putting $u = 0$ in Eqs. (i) and (ii) and $t = 5$ s in Eq. (ii), we get

$$s = \frac{1}{2}g(5)^2 = \frac{25}{2}g,\ s_n = \frac{1}{2}g(2n-1)$$

Given, $s_n = s$

$$\therefore \quad \frac{1}{2}g(2n-1) = \frac{25}{2} \times g$$

$$\Rightarrow \quad 2n - 1 = 25$$

$$\Rightarrow \quad n = 13\ \text{s}$$

4. Key Idea : *Maximum kinetic energy of swing should be equal to difference in potential energies to conserve energy.*

From energy conservation

$$\frac{1}{2}mv_{\max}^2 = mg\,(H_2 - H_1)$$

Here, H_1 = maximum height of swing from earth's surface

$$= 0.75\ \text{m}$$

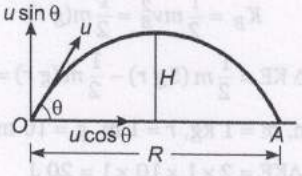
H_2 = maximum height of swing from earth's surface

$$= 2\ \text{m}$$

$$\therefore \quad \frac{1}{2}mv_{\max}^2 = mg\,(2 - 0.75)$$

or $v_{\max} = \sqrt{2 \times 10 \times 1.25} = \sqrt{25} = 5$ m/s

5. Let u be the velocity of projection and θ is angle of projection, then



$$R = \frac{u^2 \sin 2\theta}{g}$$

and $$H = \frac{u^2 \sin^2\theta}{g}$$

Given $\theta = 45°$

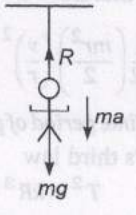
$$\therefore \quad R = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} \qquad \ldots(\text{i})$$

$$H = \frac{u^2 \sin^2 45°}{g} = \frac{u^2}{4g} \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$R = 4H$$

6. Let the resultant acceleration of fireman be a then



$$R = mg - ma$$

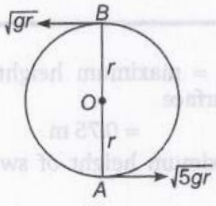
Given, $R = \frac{3}{4}mg$

$$\therefore \quad \frac{3}{4}mg = mg - ma$$

$$\Rightarrow \quad a = \frac{g}{4}$$

7. Key Idea : *Velocity at bottom is $\sqrt{5}$ times the velocity at top.*

The energy possessed by a body due to velocity v is given by



$$K = \frac{1}{2}mv^2$$

Difference in KE $= K_A - K_B$

Given, $K_A = \frac{1}{2}mv_A^2 = \frac{1}{2}m(5gr)$

$$K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(gr)$$

$$\therefore \quad \Delta KE = \frac{1}{2}m(5gr) - \frac{1}{2}m(gr) = 2mgr$$

Given, $m = 1$ kg, $r = 1$ m, $g = 10$ m/s^2

$$\therefore \quad \Delta KE = 2 \times 1 \times 10 \times 1 = 20 \text{ J}$$

8. Kinetic energy of rotation of a body having moment of inertia I and angular velocity ω is given by

$$K = \frac{1}{2}I\omega^2$$

For a circular ring of radius r, and mass m moment of inertia (I) about its diameter is given by

$$I = \frac{Mr^2}{2}$$

$$\therefore \quad K = \frac{1}{2}\left(\frac{Mr^2}{2}\right)\omega^2$$

Also, $v = r\omega$, therefore $\omega = \frac{v}{r}$

$$\therefore \quad K = \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}mv^2$$

9. Key Idea : *Time period of parking orbit is 1 day.*

From Kepler's third law

$$T^2 = kR^3$$

where, T is time period and R is radius of earth.

Given, $T_1 = 1$ day, $R_2 = 4R_1$

$$\therefore \quad \frac{T_2}{1} = \left(\frac{4R_1}{R_1}\right)^{3/2}$$

$$\Rightarrow \quad T_2 = (4)^{3/2} = 8 \text{ days}$$

10. Key Idea : *Angular momentum is conserved.*

From law of conservation of angular momentum, we have

$$J = I\omega = \text{constant}$$

where, I is moment of inertia and ω is angular velocity.

Also, $I = MR^2$ and $\omega = \frac{v}{R}$

$$\therefore \quad v \propto \frac{1}{R}$$

At P_4 the value of R is minimum, hence the velocity is maximum or kinetic energy will be maximum.

11. Key Idea : *Exponential decay of charge takes place.*

In a C-R circuit, discharging takes place.

The q-t equation is

$$q = q_0 e^{-t/RC}$$

where R is resistance, C is capacitance and $q_0 = CV_0$

Hence, $q = CV_0 e^{-t/RC}$

Given, $V_0 = 5$ V, $C = 2$ F, $t = 12$ s, $R = 6\,\Omega$

$$\therefore \quad q = 2 \times 5e^{-12/6 \times 2} = \frac{10}{e} \text{ coulomb}$$

12. The energy (E) of a wave of amplitude a, and angular velocity ω is

$$E = \frac{1}{2}ma^2\omega^2$$

Also, $\omega = 2\pi n$

$$\therefore \quad E = \frac{1}{2}ma^2(2\pi n)^2 = 2ma^2\pi^2n^2$$

$$\therefore \quad \frac{E_A}{E_B} = \frac{(a_A n_A)^2}{(a_B n_B)^2}$$

Given, $E_A = E_B$, $n_A = n$, $n_B = \frac{n}{8}$

$$\therefore \quad 1 = \frac{a_A^2 \times 64\, n^2}{a_B^2 n^2}$$

$$\Rightarrow \quad a_B = 8a_A = 8a$$

13. Capacitive reactance X_C is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

where, $\omega = 2\pi f$

Given, $X_C = 25\Omega$, $f = \frac{400}{\pi}$ Hz

Hence, $25 = \frac{1}{2\pi \times \frac{400}{\pi} \times C}$

$$\Rightarrow \quad C = \frac{1}{25 \times 800} = 50\,\mu\text{F}$$

14. Key Idea : *Relation between kinetic energy K and momentum (p) is* $p = \sqrt{2mK}$

The de-Broglie wavelength (λ) is given by

$$\lambda = \frac{h}{p} \quad ...(i)$$

where, h is Planck's constant, and p is momentum.

Also, $$p = \sqrt{2mK} \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mK}}$$

$$\Rightarrow \quad K = \frac{h^2}{2m\lambda^2} \Rightarrow K \propto \frac{1}{m}$$

Hence, kinetic energy of electron will be greater than that of proton because proton has more mass than electron.

15. The relation for magnetic susceptibility is given by

$$\mu_r = 1 + \chi$$

or $$\chi = \mu_r - 1$$

Given, $\mu_r = 5500$

$$\therefore \quad \chi = 5500 - 1 = 5499$$

16. Key Idea : *Power developed is equal to scalar product of force and velocity vector.*

Power is given by

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

Given, $\vec{\mathbf{F}} = 7\hat{\mathbf{i}} + 6\hat{\mathbf{k}}$, $\vec{\mathbf{v}} = 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

$$\therefore \quad P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

$$P = (7\hat{\mathbf{i}} + 6\hat{\mathbf{k}}) \cdot (3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

$$P = 6 \times 4$$

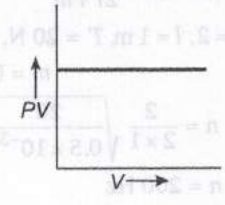
$$P = 24 \text{ unit}$$

Note : Power is a scalar quantity.

17. From ideal gas equation

$$PV = RT = \text{constant}$$

Since, PV remains constant, the correct variation of PV with respect to V is as shown.



18. From ideal gas equation

$$PV = nkT$$

$$\Rightarrow \quad n = \frac{PV}{kT}$$

Putting $P = 13.8$ Pa, $V = 1\ m^3$

$$k = 1.38 \times 10^{-23}\ JK^{-1},\ T = 300\ K$$

$$\therefore \quad n = \frac{13.8 \times 1}{1.38 \times 10^{-23} \times 300}$$

$$n = 3.3 \times 10^{21}$$

19. Key Idea : *Minimum magnifying power is equal to ratio of focal length of objective to that of eyepiece.*

Magnifying power M is given by

$$M = \frac{f_o}{f_e}$$

where, f_o is focal length of objective, and f_e is focal length of eyepiece.

Given, $$f_e' = \frac{f_e}{2}$$

$$\therefore \quad M' = \frac{f_o}{f_e/2} = 2 \cdot \frac{f_o}{f_e} = 2M$$

20. Key Idea : *Apply Wien's displacement law.*

From Wien's displacement law,

$$\lambda_m T = \text{constant}$$

where, λ_m is maximum wavelength and T is absolute temperature.

Since, wavelength of blue colour is minimum, hence, surface temperature of blue star is maximum.

Note : Surface temperature will be minimum for red star.

21. Since, the doublet is corrected for spherical aberration, it satisfies the following condition.

$$f_1 - f_2 = d = 2\ cm$$

$$\Rightarrow \quad f_1 = f_2 + 2\ cm$$

Let the equivalent focal length = F

$$\therefore \quad F = \frac{f_1 f_2}{f_1 + f_2 - d} = 10\ cm$$

Solving it, $f_1 = 20$ cm, $f_2 = 18$ cm.

22. Key Idea : *Refractive index* $\mu = \frac{\lambda_a}{\lambda_w}$.

The fringe width (W) is given by

$$W = \frac{D\lambda}{d}$$

where, λ is wavelength, D is distance between source and screen and d is distance between coherent sources.

$$\therefore \quad \frac{W_a}{W_w} = \frac{\lambda_a}{\lambda_w}$$

Also, $\frac{\lambda_a}{\lambda_w} = \mu$

$\therefore \quad \frac{0.8}{W_w} = \mu = 1.6$

$\Rightarrow \quad W_w = \frac{0.8}{1.6} = \frac{1}{2} = 0.5 \text{ mm}$

23. Angular fringe width $\theta = \frac{\beta}{D} = \frac{\lambda}{2d}$

$\therefore \quad \frac{\theta'}{\theta} = \frac{\lambda'}{\lambda}$

or $\quad \frac{\theta' - \theta}{\theta} = \frac{\lambda' - \lambda}{\lambda}$

$\therefore \quad \Delta\lambda = \frac{\Delta\theta}{\theta}\lambda$

Given, $\frac{\Delta\theta}{\theta} = 10\%, \lambda = 5890\text{Å}$

$\Delta\lambda = \frac{10}{100} \times 5890 = 589\text{Å}$

Hence, fringe width increases by 589Å.

24. For a body executing SHM, the total energy E_T is given by

$E_T = \frac{1}{2}m\omega^2A^2$ and $E_K = \frac{1}{2}m\omega^2(A^2 - y^2)$

Given, $\quad y = \frac{A}{\sqrt{2}}$

$$E_K = \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{2}\right) = \frac{E_T}{2}$$

Hence, $\quad \frac{E_K}{E_T} = \frac{1}{2}$

Note : In SHM kinetic energy is converted to potential energy and *vice versa*, but total energy remains same.

25. The phase of particle at $t = 3s$ is

$$\phi_1 = \omega t = \left(\frac{2\pi}{T}\right)t$$

$$= \frac{2\pi \times 3}{12} = \frac{\pi}{2}$$

At $t = 4s$

$$\phi_2 = \omega t = \left(\frac{2\pi}{T}\right)t$$

$$= \frac{2\pi \times 4}{12} = \frac{2\pi}{3}$$

Phase difference $\Delta\phi = \phi_2 - \phi_1 = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$

26. Linear expansion coefficient

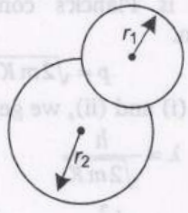
$$\alpha = \frac{\gamma}{3} = \frac{1}{3}\cdot\frac{\Delta V}{V\Delta t}$$

$\Rightarrow \quad \alpha = \frac{1}{3}\cdot\frac{0.15}{100 \times 24}$

$\Rightarrow \quad \alpha = 2 \times 10^{-5}/°\text{C}$

27. **Key Idea :** *Excess pressure inside a bubble of radius R is given by* $P = \frac{4T}{r}$.

Let P_1 and P_2 are pressure differences across the common interface. Let r is radius of curvature of the common surface.



$$P_2 - P_1 = \frac{4T}{r}$$

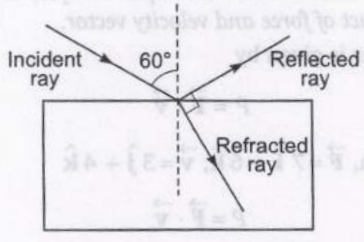
$$\frac{4T}{r} = \frac{4T}{r_2} - \frac{4T}{r_1} \quad (T = \text{surface tension})$$

$$\frac{1}{r} = \frac{1}{r_2} - \frac{1}{r_1}$$

$$r = \frac{r_1 r_2}{r_1 - r_2}$$

28. From Snell's law

$$\mu = \frac{\sin i}{\sin r}$$



Given, $i + r = 90° \Rightarrow \quad r = 90° - i$

$\therefore \quad \mu = \frac{\sin i}{\sin(90° - i)} = \tan i$

Given, $i = 60°$, therefore

$\mu = \tan 60° = \sqrt{3} = 1.73$

29. As stretched string of 1m long is plucked at a distance of 25 cm from end, so there will be 2 loops.

Frequency, $\quad n = \frac{p}{2l}\sqrt{\frac{T}{m}}$

Given, $p = 2, l = 1\text{ m}, T = 20\text{ N}$,

$m = 0.5 \times 10^{-3}$ kg/m

$\therefore \quad n = \frac{2}{2 \times 1}\sqrt{\frac{20}{0.5 \times 10^{-3}}}$

$\Rightarrow \quad n = 200\text{ Hz}$

30. The equation of given wave travelling with amplitude A is given by

$$y = A\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$$

Maximum velocity $v_m = A\omega = A \times 2\pi f$

Also, $c = f\lambda$

$\therefore \quad v_m = \dfrac{A2\pi c}{\lambda}$

Given, $v_m = 4c$, then

$$4c = \frac{A \cdot 2\pi c}{\lambda}$$

$\Rightarrow \quad \lambda = \dfrac{\pi A}{2}$

31. From Planck's law, the energy (E) of a wave of wavelength λ is

$$E = \frac{hc}{\lambda}$$

where, h is Planck's constant, c is speed of light, and λ is wavelength.

Given, $h = 6.625 \times 10^{-34}$ J-s

$c = 3 \times 10^8$ m/s, $\lambda = 6560\text{Å} = 6560 \times 10^{-10}$ m,

$1\text{ eV} = 1.6 \times 10^{-19}$ J

$$\therefore \quad E = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{6560 \times 10^{-10} \times 1.6 \times 10^{-19}}\text{ eV}$$

$\Rightarrow \quad E = 1.89$ eV

32. **Key Idea :** *For H_α line $n_1 = 2$, $n_2 = 3$.*

From the relation

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For Lyman series, $n_1 = 1$, $n_2 = 2$

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1} - \frac{1}{2^2}\right) = \frac{3R}{4} \qquad \ldots(i)$$

For H_α line, $n_1 = 2$, $n_2 = 3$

$$\frac{1}{\lambda_{H_\alpha}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{\lambda_{H_\alpha}}{\lambda_L} = \frac{3}{4} \times \frac{36}{5} = \frac{27}{5}$$

$\Rightarrow \quad \lambda_{H_\alpha} = \dfrac{27}{5} \times 1215 = 6561\text{Å}$

33. From Einstein's theory, maximum kinetic energy (E_k) of emitted photoelectron is given by

$$E_k = h\nu - W$$

where W is work function of silver, ν is frequency.

$\Rightarrow \quad W = \dfrac{hc}{\lambda} - E_k$

Putting $h = 6.6 \times 10^{-34}$ J-s,

$c = 3 \times 10^8$ m/s, $\lambda = 10^{-7}$ m

$E_k = 7.7$ eV

$= 7.7 \times 1.6 \times 10^{-19}$ V

$$\therefore \quad W = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{10^{-7}} - 7.7 \times 1.6 \times 10^{-19}\text{ J}$$

$\Rightarrow \quad W = 19.8 \times 10^{-19} - 12.32 \times 10^{-19}$

$\Rightarrow \quad W = 7.48 \times 10^{-19}$

$$W = \frac{7.48 \times 10^{-19}}{1.6 \times 10^{-19}}\text{ eV}$$

$= 4.67$ eV

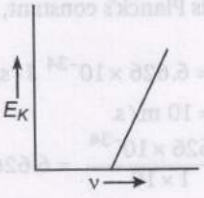
34. From Einstein's theory, the maximum kinetic energy (E_k) of emitted photoelectron is

$$E_k = h\nu - W \qquad \ldots(i)$$

where, ν is frequency and W is work function. Also equation of straight line, with slope m, and intercept c on x-axis given by

$$y = mx + c \qquad \ldots(i)$$

Comparing Eqs. (i) and (ii), we get



35. From Rutherford Soddy law, the number of atoms left after n half-lives is

$$N = N_0\left(\frac{1}{2}\right)^n$$

where, $n = \dfrac{\text{time } (t)}{\text{half-life } (T_{1/2})}$

Given, $t = 6400$ yr, $T = 1600$ yr

$$\therefore \quad n = \frac{6400}{1600} = 4$$

So, $\dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^n = \left(\dfrac{1}{2}\right)^4 = \dfrac{1}{16}$

Hence, $\dfrac{1}{16}$ part of element will remain.

36. Key Idea : *Emitter current is sum of base current and collector current.*

The current gain (α) for a transistor is given by

$$\alpha = \frac{\Delta I_C}{\Delta I_E} \quad ...(i)$$

Also, $\Delta I_E = \Delta I_C + \Delta I_B$ ...(ii)

From Eqs. (i) and (ii), we get

$$\alpha = \frac{\Delta I_E - \Delta I_B}{\Delta I_E}$$

Given, $\Delta I_E = 25$ mA, $\Delta I_B = 1$ mA

$$\therefore \quad \alpha = \frac{25-1}{25} = \frac{24}{25}$$

Note : Current gain (α) is usually less than one, hance, options (a) and (d) cannot be correct.

37. A *p-n* junction diode works as a rectifier. It offers a low resistance for the current to flow when it is forward biased but a very high resistance when reverse-biased, thus it allows current through it only in one direction and acts as a rectifier. The junction diode can be used either as a half wave rectifier or as a full wave rectifier.

Note : A junction diode cannot amplify a signal.

38. From de-Broglie wavelength (λ), we have

$$\lambda = \frac{h}{mv}$$

where, h is Planck's constant, m is mass and v is velocity.

Given, $h = 6.626 \times 10^{-34}$ J-s, $m = 1$ kg,

$v = 10$ m/s.

$$\therefore \quad \lambda = \frac{6.626 \times 10^{-34}}{1 \times 10} = 6.626 \times 10^{-35} \text{ m}$$

39. The radiotracer group applies a variety of techniques involving radioactive atoms as tracers for diffusion studies. In the classical radiotracer technique, radioactive isotopes of the element whose diffusion is to be investigated are deposited on the specimen surface. Then the specimen is diffusion annealed in a furnace at the temperature T at which the diffusion coefficient is to be measured. From the radioactivity of the taken-off layers the penetration profile of the radiotracers which results from diffusion for the time t at the temperature T may be reconstructed. A comparison of this with the radiotracer distribution prior to diffusion annealing yields the diffusion coefficient. This is an important diagnostic tool.

A gamma camera is an imaging device most commonly used as a medical imaging device in nuclear medicine. It produces images of the distribution of gamma ray emitting radionuclides. While CAT (Computed axial tomography) is a medical imaging method employing tomography where digital geometry processing is used to generate a three dimensional image of the internals.

40. Key Idea : *Frequency* $= \frac{1}{\text{time constant}}$

We know that

Dimensions of $R = [ML^2T^{-3}I^{-2}]$

Dimensions of $L = [ML^2T^{-2}I^{-2}]$

Dimensions of $C = [M^{-1}L^{-2}T^4I^2]$

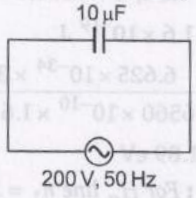
Hence, dimensions of $RCL = [ML^2T^{-1}I^{-2}]$

Which are not equal to dimensions of frequency.

Note : The constant is given by CR and LC.

41. Key Idea : *Peak current* $i_0 = \sqrt{2} i_{rms}$.

In a capacitive circuit, the capacitive reactance



$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

Given, $f = 50$ Hz, $C = 10\ \mu\text{F} = 10 \times 10^{-6}$ F

$$\therefore X_C = \frac{1}{2 \times \pi \times 50 \times 10 \times 10^{-6}} = \frac{1000}{\pi}\ \Omega$$

and $i_{rms} = \frac{V}{X_C} = \frac{200}{1000/\pi} = \frac{3.14}{5} \approx 0.6$ A

Peak value of current $i_0 = \sqrt{2} i_{rms}$

$$i_0 = 0.6\sqrt{2}$$

42. Energy stored in an inductor is given by

$$E = \frac{1}{2} Li^2$$

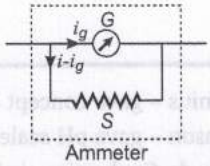
Given, $L = 50$ mH $= 50 \times 10^{-3}$ H, $i = 4$ A

$$\therefore \quad E = \frac{1}{2} \times 50 \times 10^{-3} \times (4)^2$$

$$\Rightarrow \quad E = 0.4 \text{ J}$$

43. Key Idea : *Potential difference across galvanometer resistance and shunt is same.*

Let i_g be the current across galvanometer and $i - i_g$ across shunt, then



Potential difference across G = potential difference across S

i.e., $i_g \times G = (i - i_g) \times S$

Given, $\frac{i_g}{i} = \frac{4}{100} = 0.04$, $S = 5\Omega$

$\therefore \quad \frac{G}{5} = \frac{1}{0.04} - 1 = 24$

$\Rightarrow \quad G = 24 \times 5 = 120\Omega$

44. From Faraday's law of electrolysis

$$m = zq \qquad ...(i)$$

where, z is electrochemical equivalent, and q is charge.

Also, Energy per sec = qV

or $E = qV$

$$\Rightarrow \quad q = \frac{E}{V} = \frac{3.6 \times 10^8}{33} \qquad ...(ii)$$

From Eqs. (i) and (ii), we get

$$m = \frac{0.33 \times 10^{-6} \times 3.6 \times 10^8}{33} = 3.6 \text{ kg}$$

45. **Key Idea :** *Power dissipated is inversely proportional to equivalent resistance.*

Let R be the resistance of individual component. When such resistances are connected in series, the equivalent resistance is

$$R_{eq} = R + R + R + R = 4R$$

When, they are connected in parallel, equivalent resistance is

$$\frac{1}{R'_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{4}{R}$$

$$\Rightarrow \quad R'_{eq} = \frac{R}{4}$$

From Joule's law, the power dissipated is

$$\Rightarrow \quad P = \frac{V^2}{R_{eq}}$$

Hence, $\frac{P_s}{P_p} = \frac{R'_{eq}}{R_{eq}} = \frac{R}{4} \times \frac{1}{4R} = \frac{1}{16}$

$$\Rightarrow \quad P_p = 16 \times P_s$$

Given, $P_s = 5$ W

$$\therefore \quad P_p = 16 \times 5 = 80 \text{ W}$$

46. The electric field E due to point charge q is given by

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

$$\Rightarrow \quad q = E.4\pi\varepsilon_0 r^2$$

Given, $r = 30$ cm $= 30 \times 10^{-2}$ m, $E = 2$ N/C

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

Hence, $q = 2 \times \frac{1}{9 \times 10^9} \times \left(\frac{30}{100}\right)^2 = 2 \times 10^{-11}$ C

47. When a dielectric material is inserted between the plates of a capacitor (keeping the charge to be constant) or capacitor is immersed in dielectric medium, the electric field decreases by a factor k (the dielectric constant of the dielectric)

$$\therefore \quad E = \frac{E_0}{k}$$

Given, $k = 2$

Hence, $E = \frac{E_0}{2}$

Thus, electric field between the plates is decreased by a factor of 2.

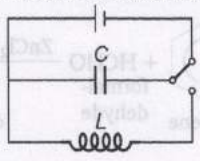
48. The grid potential required to make plate current zero is given by

$$V_g = \frac{V_p}{\mu}$$

Given, $V_p = 300$ V, $\mu = 20$

$$\therefore \quad V_g = \frac{300}{20} = 15 \text{ V}$$

49. When a charged capacitor is discharged through an inductor of negligble ohmic resistance, electrical oscillations takes place in the circuit. These are called L - C oscillations.



50. **Key Idea :** *The potential energy of satellite is twice its kinetic energy but opposite in sign.*

Potential energy,

$$U = -\frac{GM_e m}{R_e} \text{ or } |U| = \frac{GM_e m}{R_e}$$

Kinetic energy,

$$K = \frac{1}{2}\frac{GM_e m}{R_e}$$

Thus, $\frac{K}{|U|} = \frac{1}{2}\frac{GM_e m}{R_e} \times \frac{R_e}{GM_e m} = \frac{1}{2}$

Chemistry

1. Key Idea : *For zero order reaction*

$r = k[\text{conc}]^0$

$\therefore$ Units of k are mol L^{-1} time^{-1}

$\therefore$ The statement that unit of rate constant for zero order reaction is s^{-1} is wrong.

It is infact unit of k for first order reaction. The other statements are correct.

2. Key Idea : *First write balanced chemical reaction and then find the answer.*

$BaCl_2 + H_2S \longrightarrow BaS + 2HCl$

Molecular mass of $H_2S = 1 \times 2 + 32 = 34$

Molecular mass of $BaS = 137 + 32 = 169$

Given mass of $BaS = 1.69$ g

According to reaction.

$\because$ 169 g of BaS is obtained by = 34 g of H_2S

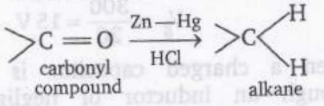
$\therefore$ 1.69 g of BaS is obtained by $= \frac{34}{169} \times 1.69$

$= 0.34$ g of H_2S

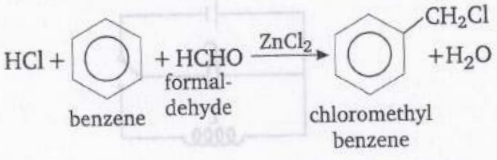
3. Testosterone is a harmone and is chemically a steroid. Lipase, keratin and haemoglobin are proteins.

4. Key Idea : *Define all terms to find correct answer.*

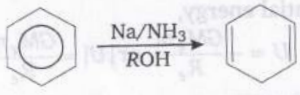
(a) **Clemmensen's reduction**



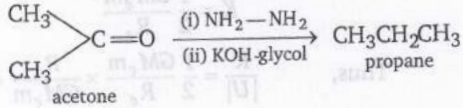
(b) **Blanc reduction :**



(c) **Birch reduction :**



(d) **Wolff-Kishner reduction**



5. (a) Arrhenius – gave concept of acid and base

(b) Sorenson – gave pH scale

(c) Pierre de Coubertin – is father of modern olympic games.

6. Thomas slag is also called as phosphatic slag. It is a mixture of calcium phosphate and calcium silicate. It is used as manure. Chemically it is $Ca_3(PO_4)_2 \cdot CaSiO_3$.

7. (i) $Na_2SO_4 \cdot Al_2(SO_4)_3 \cdot 24H_2O$ and $K_2SO_4 \cdot Fe_2(SO_4)_3 \cdot 24H_2O$ are alum.

(ii) $FeSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$ is Mohr's salt.

8. Key Idea : *Ionisation energy is the amount of energy required to take out most loosely bonded electron from isolated gaseous atom. Its value increases in a period and decreases in a group.*

$\therefore$ Among Cs, F, Xe and N, ionisation energy of F is highest.

9. Key Idea : *Amides give ammonia on heating with* NaOH.

Choice (d) ethanamide is the only amide in given choices.

$CH_3CONH_2 + NaOH \xrightarrow{\Delta} CH_3COONa + NH_3 \uparrow$

10. (i) Methyl alcohol is poisonous.

(ii) Ethyl alcohol is drinking alcohol.

11. (a) Light oil fraction is distilled upto 170°C. It has benzene, toluene and xylene.

(b) Middle oil fraction or carbolic oil fraction distills between 170–230°C. It has phenol and pyridine.

(c) Green oil or anthracene oil fraction distills between 270–360°C. It has anthracene and phenanthrene.

(d) Heavy oil distills between 230–270K. It has methyl and higher alkyl phenols.

12. Key Idea : *Molarity*

$= \frac{\text{moles of solute}}{\text{volume of solution in L}}$

$= \frac{\text{mass/molecular mass}}{V \text{ in L}}$

Given molarity = 2.5 M

Volume = 300 mL $= \frac{300}{1000}$ L

Molecular mass of oxalic acid $(COOH)_2$

$= (12 + 16 \times 2 + 1)_2 = 90$

$$\therefore \quad 2.5 = \frac{\text{mass}/90}{300/1000}$$

or $$2.5 = \frac{\text{mass}}{90} \times \frac{1000}{300}$$

$$\therefore \quad \text{mass} = \frac{90 \times 300 \times 2.5}{1000}$$

$$= 67.5\,\text{g}$$

13. The real gases do not follow gaseous laws completely at all the ranges of temperature and pressures. The reason is that in them, the attractive forces rust between molecules of real gases.

14. Key Idea : *The bond angle of* AsH_3 *is smaller than that of* NH_3 *because electronegativity of* As *is less than* N.

(a) NH_3 (b) H_2O (c) BCl_3

Bond angle = 107° Bond angle = 105° Bond angle = 120°

∴ Bond angle of AsH_3 is greater than all of them.

15. Key Idea : *According to Graham's law*

$$r \propto \frac{1}{\sqrt{M}}$$

∴ Greater the molecular mass, lesser will be rate of diffusion.

(a) C_2H_6 – molecular mass

$$= 2 \times 12 + 1 \times 6 = 30$$

(b) C_2H_4 – molecular mass $= 2 \times 12 + 1 \times 4 = 28$

(c) CH_4 – molecular mass $= 12 + 1 \times 4 = 16$

∵ C_2H_6 has highest molecular mass

∴ Rate of diffusion is least for C_2H_6.

16. Key Idea : *Sum of oxidation number of all elements in a compound is always zero.*

(a) Let oxidation number of F in $F_2O = x$

$$\therefore \quad 2x + (-2) = 0$$

$$x = +1$$

(b) Let oxidation number of F in $NF_3 = x$

$$\therefore \quad +3 + 3x = 0$$

$$\therefore \quad x = -1$$

17. P-32 is used in treatment of Leukeamia.

18. A metal having higher value of $E°$ is reduced easily.

∴ It is good oxidising agent (oxidising agent undergoes reduction).

19. Key Idea : *Define all terms to find correct answer.*

(a) **Electrophoresis** is movement of colloidal particles under the influence of an electric field.

(b) **Dialysis** is method for purification of colloids. In this method particles are separated from those of crystalloids.

(c) Chromatography is method for purification of organic compounds.

(d) Peptisation is process of conversion of colloid into precipitate by addition of electrolyte.

20. Key Idea : *Terminal alkynes are precipitated in form of acetylides when they react with* $AgNO_3$. *Alkane and alkene do not react with* $AgNO_3$.

(i) $\underset{\text{ethane}}{C_2H_6} + AgNO_3 \longrightarrow$ No reaction

(ii) $\underset{\text{ethene}}{C_2H_4} + AgNO_3 \longrightarrow$ No reaction

(iii) $\underset{\text{propene}}{C_3H_6} + AgNO_3 \longrightarrow$ No reaction

(iv) $\underset{\text{ethyne}}{HC \equiv CH} + AgNO_3 \longrightarrow \underset{\text{white ppt. of silver acetylide}}{Ag—C \equiv C—Ag}$

∵ Ethane, ethene and propene do not react with $AgNO_3$.

∴ They come out of wolf bottle.

21. Key Idea : *The conformers are formed by free rotation across carbon-carbon single bond.*

A compound can have infinite number of conformers.

22. Key Idea : *Interfering radicals are found in inorganic salt analysis.*

The interfering amounts are PO_4^{3-}, F^-, BO_3^{3-} and $C_2O_4^{2-}$. They interfere in cation or basic radical analysis after group II.

23. During smelting flux is added to convert impurities into slag. The slag floats over pure metal and can be easily removed.

24. Key Idea : *Greater the amout of given out as heat of formation more will be stability of oxide.*

Given, MgO, $\Delta H_f = -692\,\text{kJ mol}^{-1}$

SiO_2, $\Delta H_f = -911\,\text{kJ mol}^{-1}$

Al_2O_3, $\Delta H_f = -1676\,\text{kJ mol}^{-1}$

∴ MgO is most stable oxide among given choices.

25. Key Idea : *According to Le-Chatelier's principle, the reactions in which number of moles are increasing are favourable at low pressure.*

(a) $PCl_5 \rightleftharpoons PCl_3 + Cl_2$

Moles of reactants = 1

Moles of products = 2

∵ Number of moles are increasing.

∴ The reaction is favoured at low pressure.

(b) $N_2 + 3H_2 \rightleftharpoons 2NH_3$

∵ Number of moles are decreasing.

∴ The reaction is favoured at high pressure.

(c) $H_2 + \frac{1}{2}O_2 \rightleftharpoons H_2O$

∵ Number of moles are decreasing.

∴ The reaction is favoured at high pressure.

(d) $2NO + O_2 \rightleftharpoons 2NO$

∵ Number of moles are decreasing.

∴ The reaction is favoured at high pressure.

26. Key Idea : *The compounds precipitate only when their solubility products are less than their ionic products.*

When NH_4Cl is added before NH_4OH in basic radical analysis, the dissociation of NH_4OH decreases due to common ion effect. It gives only as many OH^- ions which are required for precipitation of group III cations only.

27. Both *s* and *p*-block elements are called representative elements because their properties are predicted by looking at their electronic configuration.

28. According to Pauling's scale of electronegativity.

$$x_A - x_B = 0.182[(E_{A-B} - (E_{A-A} \times E_{B-B})^{1/2}]$$

29. Key Idea :

Hybridisation $= \frac{1}{2}$ [*number of valence* e^- *in central atom + number of monovalent atoms – charge on cation + charge on anion*]

For SO_2

$$\text{Hybridisation} = \frac{1}{2}(6 + 0 - 0 + 0) = 3$$

∴ sp^2-hybridisation.

30. Key Idea : *Energy of electron* $= \frac{-13.6Z^2}{n^2}$ *eV*

where Z = atomic number of element, for Li = 3

n = number of orbit, for last e^- *of* Li, $n = 2$

∴ Energy of last electron of Li

$$= \frac{-13.6 \times (3)^2}{(2)^2} = \frac{-13 \times 9}{4} = -30.6 \text{ eV}$$

31. Key Idea : *Define all terms to find correct answer.*

(a) **Rearrangement reaction** : The atoms within molecule change their position during rearrangement reaction.

(b) **Oxidation** : Oxygen adds during oxidation reaction.

(c) **Disproportionation** : During this reaction half of the molecules of some substance are oxidised and other half are reduced.

$$\underset{\text{benzaldehyde}}{C_6H_5CHO} + NaOH \longrightarrow \underset{\text{sodium benzoate}}{C_6H_5COONa} + \underset{\text{benzyl alcohol}}{C_6H_5CH_2OH}$$

∵ Half of C_6H_5CHO molecules are oxidised to acid and other half of molecules are reduced to alcohol.

∴ It is disproportionation reaction.

32. Key Idea :

(i) *Electron withdrawing groups increase the acidity.*

(ii) *The effect of withdrawing groups is maximum at ortho position due to nearness.*

∴ Among given choices *o*-nitro benzoic acid is most acidic.

33. $\underset{\text{acid chloride}}{CH_3COCl} + \underset{\text{sodium acetate}}{CH_3COONa} \longrightarrow \underset{\text{acetic anhydride}}{(CH_3CO)O} + NaCl$

34. Given, Glucose solution = 0.5 g/L is isotonic with 2.5 g/L solution of organic compound.

$$\text{Molarity of glucose} = \frac{\text{mass of glucose/ molecular mass of glucose}}{V \text{ of solution in L}}$$

$$= \frac{0.5/180}{1} = 0.0027 \text{ M}$$

∵ Glucose solution is isotonic with organic compound solution.

∴ molarity of glucose = molarity of organic compound = 0.0027 M

$$\text{Molarity of organic compound} = \frac{\text{mass of organic compound/ molecular mass of organic compound}}{V \text{ of solution}}$$

$$\text{or} \quad 0.0027 = \frac{2.5/\text{molecular mass}}{1}$$

$$\therefore \quad \text{molecular mass} = \frac{2.5}{0.0027} = 925$$

35. Key Idea :

EAN = Atomic number of central atom
+ number of e^- gained via ligands
– oxidation state of central atom

According to EAN for stable complex value EAN = Atomic number of noble gas.

(a) $[Fe(CN)_6]^{4-}$

$$EAN = 26 + 2 \times 6 - 2 = 36$$

36 is atomic number of Ar (noble gas). So this complex follows EAN rule.

(b) $[Ag(CN)_2]^-$

$$EAN = 47 + 2 \times 2 - 1 = 50$$

$\because$ 50 is not atomic number of any noble gas.

$\therefore$ it does not follow EAN rule.

(c) $[PtCl_6]^{2-}$

$$EAN = 78 + 2 \times 6 - 4 = 86$$

86 is atomic number of noble gas Rn. So it follows EAN rule.

36. $K + O_2 \longrightarrow \underset{\text{potassium superoxide}}{KO_2}$

37. (i) Metal sols are usually prepared by Bredig's arc method.

(ii) Colloidal metal sulphides are formed by double decomposition.

38. Key Idea : *According to Faraday's first law of electrolysis.*

$$w = Zit$$

or $$= \frac{E}{F} it \quad \left(\because Z = \frac{E}{F}\right)$$

$$= \frac{E}{F} \times q \quad (\because it = q)$$

Given $q = 4$ coulomb

E for Cu $= 31.75$

$\therefore$ $$w = \frac{31.75}{96500} \times 4$$

$$= 0.0013\,g$$

39. Key Idea :

$$k = \frac{2.303}{t} \log \frac{a}{a-x}$$

Given, $t = 15$, $a = 100$, $a - x = 50$

$\therefore$ $$k = \frac{2.303}{15} \log \frac{100}{50}$$

$$k = \frac{2.303}{15} \log 2 \qquad \ldots(i)$$

Also $t = ?$, $a = 100$, $a - x = 100 - 99 = 1$

$\therefore$ $$k = \frac{2.303}{t} \log \frac{100}{1}$$

or $$k = \frac{2.303}{t} \log 100 \qquad \ldots(ii)$$

From Eqs. (i) and (ii).

$$\frac{2.303}{15} \log 2 = \frac{2.303}{t} \log 100$$

or $$\frac{1}{15} \log 2 = \frac{1}{t} \log 100$$

or $$\frac{0.3010}{15} = \frac{2}{t}$$

or $$t = \frac{15 \times 2}{0.3010}$$

$$= 99 \text{ min}$$

40. Key Idea : *Iodoform test is given by alcohols which produce carbonyl compounds having CH_3CO group.*

(i) $CH_3OH \xrightarrow{[O]} HCHO$, no CH_3CO group, no iodoform test.

(ii) $\underset{\text{methyl carbinol}}{C_2H_5OH} \xrightarrow{[O]} CH_3CHO$, has CH_3CO group, gives iodoform test

(iii) $\underset{\text{ethyl carbinol}}{C_2H_5CH_2OH} \xrightarrow{[O]} C_2H_5CHO$, no CH_3CO group no iodoform test.

(iv) $CH_3—CH_2—CH(OH)—CH_2—CH_3 \xrightarrow{[O]}$

$$CH_3—CH_2—\overset{\overset{O}{\|}}{C}—CH_2—CH_3$$

No CH_3CO group, no iodoform test.

$\therefore$ Only methyl carbinol gives iodoform test.

41. Key Idea :

(i) ***Open system*** *: Heat and matter both are exchanged between system and surrounding.*

(ii) ***Closed system*** *: Only heat is exchanged between system and surroundings.*

(iii) ***Isolated system*** *: Neither heat nor matter is exchanged between system and surroundings.*

Human body is an example of open system because it can exchange both heat and matter with its surroundings.

42. The size of cerium and promethium is quite close due to poor shielding effect of *f*-electrons.

43. (i) Brown glass contains Fe_2O_3 and sodium and calcium silicate.

(ii) Cement has about 1% Fe_2O_3.

∴ Cement and brown glass both have iron.

44. Key Idea :

(i) ***Chain isomerism*** *: The two isomers have different chain lengths.*

(ii) ***Tautomerism*** *: It is characteristic of compounds having labile hydrogen. It is dynamic isomerism.*

(iii) ***Optical isomerism*** *: It is possible in compounds having at least one asymmetric carbon atom.*

(iv) ***Metamerism*** *: It is shown by compounds which have at two alkyl groups attached to same functional group.*

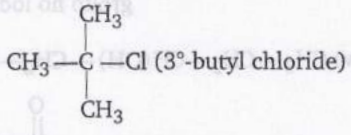
Ethers are *R*—O—*R* OR *R*—O—*R'*. Since they have two alkyl groups in their structure. So they show metamerism.

45. Key Idea :

(i) *Alkyl halides undergo nucleophilic substitution.*

(ii) *3° alkyl halides undergo S_N2 reaction mechanism.*

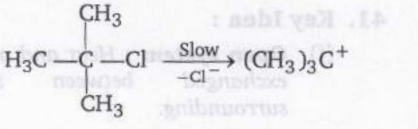
(iii) *1° alkyl halides undergo S_N1 reaction mechanism.*



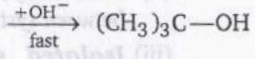
$CH_3—C(CH_3)(CH_3)—Cl$ (3°-butyl chloride)

It is 3° alkyl halide, so it will undergo S_N1 reaction mechanism.

S_N1 mechanism :



$H_3C—C(CH_3)(CH_3)—Cl \xrightarrow[-Cl^-]{Slow} (CH_3)_3C^+$



$\xrightarrow[fast]{+OH^-} (CH_3)_3C—OH$

46. Key Idea :

(i) *More the covalent character, less is boiling point.*

(ii) *Smaller the size of cation, more is polarising power and more is covalent character.*

Among Na^+ (of NaCl), Cu^+ (of CuCl), Cu^{2+} (of $CuCl_2$) and Cs^+ (of CsCl) size of Cu^{2+} is smallest.

∴ $CuCl_2$ has highest covalent character.

47. Key Idea :

$$K_p = K_c(RT)^{\Delta n}$$

$$CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$$

$$\Delta n = 1 - 1\frac{1}{2}$$

$$= -\frac{1}{2}$$

$$\therefore \quad K_p = K_c(RT)^{-1/2}$$

$$\therefore \quad K_c < K_p$$

48. Key Idea : *Buffer are mixture of ether weak acid and its salt with strong base or weak base and its salt with strong acid.*

(a) $CH_3COOH + CH_3COONa$ is buffer because CH_3COOH is weak acid and CH_3COONa is it salt with strong base.

(b) $H_3BO_3 + Na_3BO_3$ is buffer because H_3BO_4 is weak acid and Na_3BO_3 is its salt with strong base.

(c) $HClO_4 + NaClO_4$ is not buffer both acid and salt are strong.

(d) $NH_4OH + (NH_4)_2SO_4$ is buffer because NH_4OH is weak base and $(NH_4)_2SO_4$ is its salt with strong acid.

49. Tin stone is SnO_2. It has magnetic properties and it separated by both magnetic and gravimetric method.

50. Key Idea : *Define all of them to find correct answer.*

(a) RNA — It is polymer of nucleotides

(b) Oxytocin — It is polypeptide

(c) Cholestrol — It is a steroid

(d) Amylose — It is polysaccharide

∴ RNA is correct answer.

Mathematics

1. Let $f(x) = x^2 + \frac{1}{1+x^2}$

On differentiating w.r.t. x, we get

$$f'(x) = 2x - \frac{1}{(1+x^2)^2}(2x)$$

For maxima or minima, put $f'(x) = 0$

$$\therefore \quad 2x - \frac{2x}{(1+x^2)^2} = 0$$

$$\Rightarrow \quad 2x\left[1 - \frac{1}{(1+x^2)^2}\right] = 0$$

$$\Rightarrow \quad x = 0$$

$\therefore$ The minimum value at $x = 0$.

2. **Key Idea :** *If* $\vec{a}, \vec{b}$ *and* $\vec{c}$ *are non-coplanar vectors, then* $[\vec{a}\ \vec{b}\ \vec{c}] \neq 0$.

$$\therefore \quad \frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}} + \frac{\vec{b}\cdot(\vec{a}\times\vec{c})}{\vec{c}\cdot(\vec{a}\times\vec{b})}$$

$$= \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{c}\ \vec{a}\ \vec{b}]} + \frac{[\vec{b}\ \vec{a}\ \vec{c}]}{[\vec{c}\ \vec{a}\ \vec{b}]}$$

$$= \frac{[\vec{a}\ \vec{b}\ \vec{c}] - [\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{c}\ \vec{a}\ \vec{b}]} = 0$$

3. Given that, $x - 2y = 4$

Let $A = xy$

$$\therefore \quad A = x\left(\frac{x-4}{2}\right)$$

$$= \frac{1}{2}(x^2 - 4x)$$

On differentiating w.r.t. x, we get

$$\frac{dA}{dx} = \frac{1}{2}(2x - 4)$$

For maxima or minima, put $2x - 4 = 0$

$\therefore \quad x = 2$

Again differentiating, we get

$$\frac{d^2A}{dx^2} = \frac{1}{2}(2) = 1$$

At $x = 2$, $\frac{d^2A}{dx^2} = 1$, minima

$\therefore$ The minimum value of

$$A = xy = \frac{1}{2}(4 - 8)$$

$$= -2$$

Note : Minimum and maximum values of a curve comes alternatively.

4. We have $\left|\frac{1 - iz}{z - i}\right| = 1$

$$\Rightarrow \quad |1 - iz| = |z - i|$$

$$\Rightarrow \quad |1 - i(x + iy)| = |x + iy - i|$$

$$\Rightarrow \quad |1 - ix + y| = |x + i(y - 1)|$$

$$\Rightarrow \quad |(1 + y) - ix| = |x + i(y - 1)|$$

$$\Rightarrow \quad (1 + y)^2 + x^2 = x^2 + (y - 1)^2$$

$$\Rightarrow 1 + y^2 + 2y + x^2 = x^2 + y^2 + 1 - 2y$$

$$\Rightarrow \quad 4y = 0$$

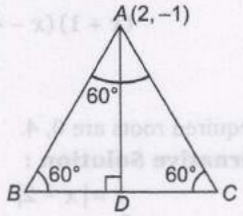
$$\Rightarrow \quad y = 0$$

$\Rightarrow$ x-axis.

5. **Key Idea :** *In equilateral triangle all sides of a triangle are equal.*

Given equation of BC is $x + 2y = 1$

The perpendicular distance from a point $A(2, -1)$ is



$$AD = \frac{|2 + 2(-1) - 1|}{\sqrt{1+4}} = \frac{|-1|}{\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}}$$

In ΔABD,

$$\sin 60° = \frac{AD}{AB}$$

$$\Rightarrow \quad AB = \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{15}}$$

Note : In equilateral triangle, median of a triangle is the mid point of base.

6. According to the given condition

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

and $R^2 = P^2 + Q^2 - 2PQ\cos\alpha$

$$\Rightarrow P^2 + Q^2 + 2PQ\cos\alpha = P^2 + Q^2 - 2PQ\cos\alpha$$

$$\Rightarrow \quad 4PQ\cos\alpha = 0$$

$$\Rightarrow \quad \alpha = 90°$$

7. Given, $s = t^3 + 2t^2 + t$

$\therefore$ Speed $v = \frac{ds}{dt} = 3t^2 + 4t + 1$

At $t = 1$,

$$v = 3(1)^2 + 4(1) + 1$$
$$= 8 \text{ cm/s}$$

8. **Case I :** When $x < 2$

$\therefore \quad (x-2)^2 - (x-2) - 6 = 0$

$\Rightarrow \quad x^2 + 4 - 4x - x - 4 = 0$

$\Rightarrow \quad x^2 - 5x = 0$

$\Rightarrow \quad x = 0, x = 5$

But we have $x < 2$

$\therefore$ We take $x = 0$

Case II : When $x \geq 2$

$\therefore \quad (x-2)^2 + (x-2) - 6 = 0$

$\Rightarrow \quad x^2 + 4 - 4x + x - 8 = 0$

$\Rightarrow \quad x^2 - 3x - 4 = 0$

$\Rightarrow \quad (x+1)(x-4) = 0$

$\Rightarrow \quad x = -1, 4$

$\Rightarrow \quad x = 4 \quad (\because x \geq 2)$

$\therefore$ Required roots are 0, 4.

Alternative Solution :

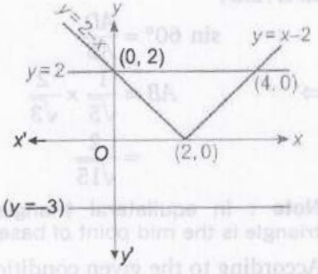
Let $y = |x - 2|$

$\therefore \quad y^2 + y - 6 = 0$

$\Rightarrow \quad y^2 + 3y - 2y - 6 = 0$

$\Rightarrow \quad (y+3)(y-2) = 0$

$\Rightarrow \quad y = -3, 2$

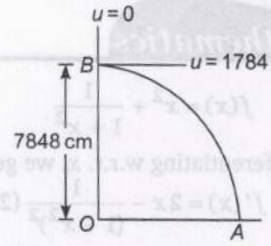


It is clear from the figure that line $y = 2$ intersect the curve $y = |x - 2|$ at two distinct points whose x-coordinates are 0, 4.

9. We know $h = ut + \frac{1}{2}gt^2$

Since, vertical velocity, $u = 0$

and $h = 7848$ cm



$\therefore \quad 7848 = 0 + \frac{1}{2} \times 981 \times t^2$

$\Rightarrow \quad t^2 = \frac{7848 \times 2}{981} = 8 \times 2 = 16$

$\Rightarrow \quad t = 4$ s

Note : The time taken by the particle to reach the ground is

$$t = \sqrt{\frac{2h}{g}}.$$

10. **Key Idea :** *If the equation of hyperbola is* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, *then equation of directrices are* $x = \pm \frac{a}{e}$.

Given equation of hyperbola be

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

Here, $a^2 = 9, b^2 = 4$

$\therefore \quad e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{4}{9}}$

$$= \frac{\sqrt{13}}{3}$$

$\therefore$ Equation of directrices, $x = \pm \frac{a}{e} = \pm \frac{9}{\sqrt{13}}$

11. $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\cos\frac{5\pi}{3}\right)$

$$= \frac{5\pi}{3} + \sin^{-1}\left[\sin\left(\frac{\pi}{2} - \frac{5\pi}{3}\right)\right]$$

$$= \frac{5\pi}{3} + \frac{\pi}{2} - \frac{5\pi}{3} = \frac{\pi}{2}$$

12. Given that, $f(x) = \log\left(\frac{1+x}{1-x}\right)$

$\therefore \quad f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1 + \frac{2x}{1+x^2}}{1 - \frac{2x}{1+x^2}}\right)$

$$= \log\left(\frac{1+x}{1-x}\right)^2$$

$$= 2\log\left(\frac{1+x}{1-x}\right) = 2f(x)$$

13. We have $(1 + x - 2x^2)^6 = 1 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_{12} x^{12}$ ...(i)

On putting $x = 1$ and $x = -1$ respectively in Eq. (i), we get

$(1 + 1 - 2)^6 = 1 + a_1 + a_2 + a_3 + \ldots + a_{12}$

$\Rightarrow \quad 0 = 1 + a_1 + a_2 + a_3 + \ldots + a_{12}$ ...(ii)

and $(1 - 1 - 2)^6 = 1 - a_1 + a_2 - a_3 + \ldots + a_{12}$

$\Rightarrow \quad 64 = 1 - a_1 + a_2 - a_3 + \ldots + a_{12}$ ...(iii)

On adding Eqs. (ii) and (iii), we get

$64 = 2(1 + a_2 + a_4 + \ldots + a_{12})$

$\Rightarrow \quad a_2 + a_4 + a_6 + \ldots + a_{12} = 31$

14. **Key Idea :** *For increasing function,* $f'(x) > 0$ *and decreasing function,* $f'(x) < 0$.

Let $f(x) = 2x^3 - 6x + 5$

On differentiating, we get

$f'(x) = 6x^2 - 6$

For increasing function, $f'(x) > 0$

$\Rightarrow \quad 6x^2 - 6 > 0 \Rightarrow x^2 > 1$

$\Rightarrow \quad x < -1$ or $x > 1$

Note : (i) For increasing function, if

$x_1 > x_2$ or $x_1 < x_2$

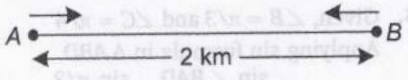
$\Rightarrow \quad f(x_1) > f(x_2)$ or $f(x_1) < f(x_2)$

(ii) For decreasing function, if

$x_1 > x_2$ or $x_1 < x_2$

$\Rightarrow \quad f(x_1) < f(x_2)$ or $f(x_1) > f(x_2)$

15. Total distance travelled by two trains to cross each other



$= 2000 + 300 + 200$

$= 2500$ m

The total speed $= 20 + 30 = 50$ m/s

$\therefore \quad$ Time, $t = \dfrac{\text{total distance travelled}}{\text{total speed}}$

$= \dfrac{2500}{50} = 50$ s

16. We have, $\dfrac{d^3y}{dx^3} + 2\left[1 + \dfrac{d^2y}{dx^2}\right] = 1$

$\therefore$ Degree $= 1$ and order $= 3$

17. We have,

$I = \int_0^1 x \left|x - \frac{1}{2}\right| dx$

$= \int_0^{1/2} x\left(\frac{1}{2} - x\right) dx + \int_{1/2}^1 x\left(x - \frac{1}{2}\right) dx$

$= \int_0^{1/2}\left(\frac{x}{2} - x^2\right) dx + \int_{1/2}^1 \left(x^2 - \frac{x}{2}\right) dx$

$= \left[\frac{x^2}{4} - \frac{x^3}{3}\right]_0^{1/2} + \left[\frac{x^3}{3} - \frac{x^2}{4}\right]_{1/2}^1$

$= \left[\frac{1}{16} - \frac{1}{24}\right] + \left[\frac{1}{3} - \frac{1}{4} - \left(\frac{1}{24} - \frac{1}{16}\right)\right]$

$= \frac{2}{16} - \frac{2}{24} + \frac{1}{12}$

$= \frac{6 - 4 + 4}{48} = \frac{6}{48}$

$= \frac{1}{8}$

18. **Key Idea :** *If* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, *then* $\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

We have,

$A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}$

Cofactors of A are

$C_{11} = 4, C_{12} = -3$

$C_{21} = -2, C_{22} = 4$

$\therefore \quad \text{adj } A = \begin{bmatrix} 4 & -2 \\ -3 & 4 \end{bmatrix}$

$\Rightarrow \quad \text{adj } A = \begin{vmatrix} 4 & -2 \\ -3 & 4 \end{vmatrix}$

$= 16 - 6 = 10$

Note : In any matrix we do not determine a value.

19. $\therefore \quad \vec{a} \cdot (\vec{b} + \vec{c}) \times (\vec{a} + \vec{b} + \vec{c})$

$= \vec{a} \cdot [\vec{b} \times \vec{a} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b} + \vec{c} \times \vec{c}]$

$= [\vec{a}\,\vec{b}\,\vec{a}] + [\vec{a}\,\vec{b}\,\vec{b}] + [\vec{a}\,\vec{b}\,\vec{c}] + [\vec{a}\,\vec{c}\,\vec{a}] + [\vec{a}\,\vec{c}\,\vec{b}] + [\vec{a}\,\vec{c}\,\vec{c}]$

$= 0 + 0 + [\vec{a}\,\vec{b}\,\vec{c}] + 0 - [\vec{a}\,\vec{b}\,\vec{c}] + 0$

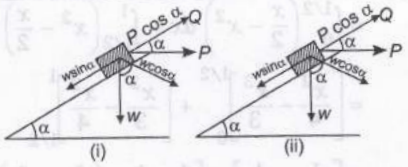
$= 0$

20. From figure (i) $P \cos\alpha = w \sin\alpha$

$\Rightarrow \quad P = \dfrac{w \sin\alpha}{\cos\alpha}$

And from figure (ii) $Q = w \sin\alpha$

$\therefore \quad \dfrac{1}{P^2} + \dfrac{1}{w^2} = \dfrac{\cos^2\alpha}{w^2 \sin^2\alpha} + \dfrac{1}{w^2}$



$$= \frac{\cos^2\alpha + \sin^2\alpha}{w^2 \sin^2\alpha} = \left(\frac{1}{w \sin\alpha}\right)^2$$

$$= \frac{1}{Q^2}$$

21. Let $I = \int_0^2 |x-1|\, dx$

$$= \int_0^1 -(x-1)\, dx + \int_1^2 (x-1)\, dx$$

$$= \left[-\frac{x^2}{2} + x\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^2$$

$$= \left[-\frac{1}{2} + 1\right] + \left[2 - 2 - \left(\frac{1}{2} - 1\right)\right]$$

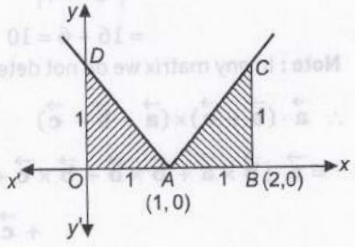
$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

Alternative Solution :

Let $I = \int_0^2 |x-1|\, dx$

It is clear from the figure,



Required area = Area of ΔOAD + Area of ΔABC

$$= \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 1 \times 1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

22. Let E_1 = Event of getting an ace card

$$\therefore \quad P(E_1) = \frac{4}{52}$$

And E_2 = Event of getting an coloured card

$$P(E_2) = \frac{15}{51}$$

$$\therefore \text{ Required probability} = \frac{4}{52} \times \frac{15}{51}$$

$$= \frac{5}{221}$$

23. $\therefore$ Displacement,

$$\vec{\mathbf{d}} = (6-5)\,\hat{\mathbf{i}} + (2+5)\,\hat{\mathbf{j}} + (-2+7)\,\hat{\mathbf{k}}$$

$$= \hat{\mathbf{i}} + 7\,\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$$

And total force $\vec{\mathbf{P}} = \vec{\mathbf{P}}_1 + \vec{\mathbf{P}}_2 + \vec{\mathbf{P}}_3$

$$= (10\,\hat{\mathbf{i}} - \hat{\mathbf{j}} + 11\hat{\mathbf{k}}) + (4\,\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) + (-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 9\hat{\mathbf{k}})$$

$$= 12\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}} + 8\,\hat{\mathbf{k}}$$

$\therefore$ Work done $W = \vec{\mathbf{P}} \cdot \vec{\mathbf{d}}$

$$= (12\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}} + 8\,\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + 7\,\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$$

$$= 12 + 35 + 40 = 87 \text{ unit}$$

24. Given that,

$$\sin x + \cos x = \frac{1}{5}$$

On squaring both sides, we get

$$\sin^2 x + \cos^2 x + 2 \sin x \cos x = \frac{1}{25}$$

$$\Rightarrow \quad \sin 2x = \frac{1}{25} - 1 = -\frac{24}{25}$$

Now, $\cos 2x = \sqrt{1 - \sin^2 2x} = \sqrt{1 - \frac{576}{625}}$

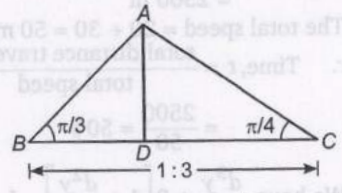
$$= \sqrt{\frac{49}{625}} = -\frac{7}{25}$$

$$\therefore \quad \tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{-24/25}{-7/25} = \frac{24}{7}$$

25. Given, $\angle B = \pi/3$ and $\angle C = \pi/4$

Applying sin formula in ΔABD

$$\frac{\sin \angle BAD}{BD} = \frac{\sin \pi/3}{AD}$$



$$\Rightarrow \quad \sin \angle BAD = \frac{BD}{AD} \times \frac{\sqrt{3}}{2} \qquad \ldots(i)$$

Again applying sin formula in ΔADC

$$\frac{\sin \angle CAD}{DC} = \frac{\sin \pi/4}{AD}$$

$$\Rightarrow \quad \sin \angle CAD = \frac{DC}{AD} \times \frac{1}{\sqrt{2}} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{\sin\angle BAD}{\sin\angle CAD}=\frac{BD}{AD}\times\frac{\sqrt{3}}{2}\times\frac{AD}{DC}\times\sqrt{2}$$

$$=\frac{BD}{DC}\times\frac{\sqrt{3}}{\sqrt{2}}$$

Given, $DC = 3BD$

$$\therefore\quad \frac{\sin\angle BAD}{\sin\angle CAD}=\frac{BD}{3BD}\times\frac{\sqrt{3}}{\sqrt{2}}=\frac{1}{\sqrt{6}}$$

26. Given, $|\vec{a}\times\vec{b}|=|\vec{a}\cdot\vec{b}|$

$$\therefore\quad ab\sin\theta|\hat{\mathbf{n}}|=ab\cos\theta$$

$$\Rightarrow\quad \tan\theta=1\qquad(\because|\hat{\mathbf{n}}|=1)$$

$$\Rightarrow\quad \theta=\frac{\pi}{4}$$

Note : If $\vec{a}$ and $\vec{b}$ are two vectors, then $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{\mathbf{n}}$ where $\hat{\mathbf{n}}$ is a unit vector which is perpendicular to the vectors $\vec{a}$ and $\vec{b}$.

27. Since, A, B and C are the angles of a triangle

$$\therefore\quad A+B+C=\pi$$

$$\Rightarrow\quad \frac{A}{2}+\frac{B}{2}=\frac{\pi}{2}-\frac{C}{2}$$

$$\Rightarrow\quad \tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)$$

$$\Rightarrow\quad \cot\frac{C}{2}=\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}$$

$$=\frac{\frac{1}{3}+\frac{2}{3}}{1-\frac{1}{3}\times\frac{2}{3}}=\frac{\frac{3}{3}}{\frac{7}{9}}=\frac{9}{7}$$

$$\Rightarrow\quad \tan\frac{C}{2}=\frac{7}{9}$$

28. The given limit can be rewritten as

$$\lim_{x\to1}\frac{(1-x)}{\cot\frac{\pi}{2}x}\qquad\left(\frac{0}{0}\right)$$

Using L'Hospital's rule

$$=\lim_{x\to1}\frac{-1}{-\operatorname{cosec}^2\left(\frac{\pi}{2}x\right)\frac{\pi}{2}}=\frac{2}{\pi}$$

29. We have, $f(x)=\left(\frac{1}{x}\right)^x$

On taking log on both sides, we get

$$\log f(x)=x\log\left(\frac{1}{x}\right)$$

$$\Rightarrow\quad \log f(x)=-x\log x$$

On differentiating both sides, we get

$$\frac{1}{f(x)}f'(x)=-(1+\log x)$$

$$\Rightarrow\quad f'(x)=-f(x)(1+\log x)\quad\ldots(i)$$

For maximum or minimum, put $f'(x)=0$

$$\Rightarrow\quad -(1+\log x)f(x)=0$$

$$\Rightarrow\quad 1+\log x=0$$

$$\Rightarrow\quad \log x=\log\left(\frac{1}{e}\right)$$

$$\Rightarrow\quad x=\frac{1}{e}$$

Again differentiating Eq. (i), we get

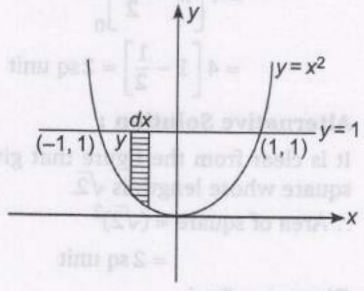
$$f''(x)=-\left[f'(x)(1+\log x)+\frac{f(x)}{x}\right]$$

At $x=\frac{1}{e}$, $f''(x)<0$, maximum

$\therefore$ Maximum value of $f(x)=\left(\frac{1}{1/e}\right)^{1/e}=e^{1/e}$

Note : If $f''(x)>0$, then the function is minimum.

30. $\therefore$ Required volume $=\int_{-1}^{1}\pi y^2dx$



$$=\pi\int_{-1}^{1}x^4dx=2\pi\left[\frac{x^5}{5}\right]_0^1$$

$$=\frac{2\pi}{5}\text{ cu unit}$$

31. **Key Idea :** *If integral is in the form of* $\int\frac{dx}{(ax+b)\sqrt{px+q}}$, *then put* $px+q=t^2$,

Let $\quad I=\int_8^{15}\frac{dx}{(x-3)\sqrt{x+1}}$

Put $\quad x+1=t^2\Rightarrow dx=2t\,dt$

$$\therefore\quad I=\int_3^4\frac{2t\,dt}{(t^2-4)t}=2\int_3^4\frac{dt}{t^2-4}$$

$$=2\times\frac{1}{2\times2}\left[\log\frac{t-2}{t+2}\right]_3^4$$

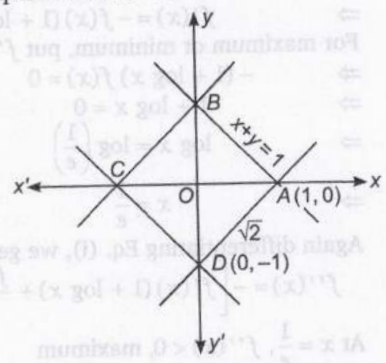
$$=\frac{1}{2}\left(\log\frac{2}{6}-\log\frac{1}{5}\right)$$

$$=\frac{1}{2}\log\frac{5}{3}$$

32. Given equation of curve is

$$|x| + |y| = 1$$

$\therefore$ Required area of curve



= Area of curve $ABCD$

= 4 area of curve OAB

$$= 4\int_0^1 (1-x)\,dx$$

$$= 4\left[x - \frac{x^2}{2}\right]_0^1$$

$$= 4\left[1 - \frac{1}{2}\right] = 2 \text{ sq unit}$$

Alternative Solution :

It is clear from the figure that given curve is a square whose length is $\sqrt{2}$.

$\therefore$ Area of square $= (\sqrt{2})^2$

$= 2$ sq unit

33. Given, $x = 3 + i$

$\therefore$ $x^3 - 3x^2 - 8x + 15$

$= (3+i)^3 - 3(3+i)^2 - 8(3+i) + 15$

$= 3^3 + 3 \times 3^2 \times i + 3 \times 3 \times i^2 + i^3$

$\quad - 3(9 + i^2 + 6i) - 24 - 8i + 15$

$= 27 + 27i - 9 - i - 27 + 3 - 18i$

$\quad - 24 - 8i + 15$

$= -15$

34. We have, $f(x) = \log(x + \sqrt{x^2+1})$

$$\therefore\ f(-x) = \log(-x + \sqrt{x^2+1}) \times \frac{(x + \sqrt{x^2+1})}{(x + \sqrt{x^2+1})}$$

$$= \log\left(\frac{-x^2 + x^2 + 1}{x + \sqrt{x^2+1}}\right)$$

$$= -\log(x + \sqrt{x^2+1})$$

$\Rightarrow f(-x) = -f(x)$

$\therefore$ $f(x)$ is an odd function.

Note : (i) If $f(-x) = f(x)$, is an even function.

(ii) If $f(x + T) = f(x)$, is a periodic function with period T.

35. Given equation of curves are

$$x\cos\alpha + y\sin\alpha = a \quad \ldots(i)$$

and $$x\sin\alpha - y\cos\alpha = b \quad \ldots(ii)$$

On squaring and adding Eqs. (i) and (ii), we get

$$x^2(\cos^2\alpha + \sin^2\alpha) + y^2(\sin^2\alpha + \cos^2\alpha) = a^2 + b^2$$

$$\Rightarrow \quad x^2 + y^2 = a^2 + b^2$$

$\therefore$ It is a equation of circle.

36. Given that, $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

and $$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\Rightarrow \quad (\vec{a} + \vec{b} + \vec{c})^2 = 0$$

$$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow \quad 1 + 1 + 1 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{3}{2}$$

37. Key Idea : *By the properties of logarithm* $\log_a b \times \log_b c = \log_a c$.

Given that,

$$x = \log_b a,\ y = \log_c b,\ z = \log_a c$$

$\therefore$ $xyz = \log_b a \times \log_c b \times \log_a c$

$= \log_b a \times \log_a c \times \log_c b$

$= \log_b c \times \log_c b$

$= \log_b b = 1$

38. Let $\Delta = \begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$

Applying $R_2 \to R_2 - \cos(\alpha-\beta)\,R_1$, $R_3 \to R_3 - \cos\alpha\, R_1$

$$= \begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ 0 & 1-\cos^2(\alpha-\beta) & \cos\beta - \cos\alpha\cos(\alpha-\beta) \\ 0 & \cos\beta - \cos\alpha\cos(\alpha-\beta) & 1-\cos^2\alpha \end{vmatrix}$$

$= [1 - \cos^2(\alpha-\beta)][1 - \cos^2\alpha] - [\cos\beta - \cos\alpha\cos(\alpha-\beta)]^2$

$$= 1 - \cos^2\alpha - \cos^2(\alpha - \beta) + \cos^2\alpha \cos^2(\alpha - \beta) - \cos^2\beta - \cos^2\alpha \cos^2(\alpha - \beta) + 2\cos\alpha \cos\beta \cos(\alpha - \beta)$$

$$= 1 - \cos^2\alpha - \cos^2\beta - \cos^2(\alpha - \beta) + 2\cos\alpha \cos\beta \cos(\alpha - \beta)$$

$$= 1 - \cos^2\alpha - \cos^2\beta - \cos(\alpha - \beta)[\cos(\alpha - \beta) - 2\cos\alpha \cos\beta]$$

$$= 1 - \cos^2\alpha - \cos^2\beta - \cos(\alpha - \beta)[\cos(\alpha - \beta) - \cos(\alpha + \beta) - \cos(\alpha - \beta)]$$

$$= 1 - \cos^2\alpha - \cos^2\beta - \cos(\alpha - \beta)[-\cos(\alpha + \beta)]$$

$$= 1 - \cos^2\alpha - \cos^2\beta + \cos(\alpha - \beta)\cos(\alpha + \beta)$$

$$= 1 - \cos^2\alpha - \cos^2\beta + \cos^2\alpha - \sin^2\beta$$

$$= 1 - (\cos^2\beta + \sin^2\beta) = 1 - 1$$

$$= 0$$

39. We know,

$$P(A' \cap B') = 1 - P(A \cup B)$$

$$\Rightarrow P(A \cup B) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{2}{3} = x + x - \frac{1}{3}$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

40. Since, p and q are the roots of the equation

$$x^2 + px + q = 0$$

$\therefore$ $p + q = -p$ and $pq = q$

$\Rightarrow$ $2p + q = 0$ and $q(p - 1) = 0$

If $q = 0$, then $p = 0$

If $p = 1$, then $q = -2$

$\therefore$ $p = 1$ or 0

41. The probability of getting 4, $p = \frac{1}{6}$

The probability of not getting 4, $q = \frac{5}{6}$

The probability of getting 4 at least one throw

$$= {}^2C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^1 + {}^2C_2\left(\frac{1}{6}\right)^2$$

$$= \frac{2 \cdot 5}{6^2} + \frac{1}{6^2} = \frac{11}{36}$$

Alternative Solution :

Let E_1 and E_2 be the events of getting 4 in first and second throw.

$\therefore$ Required probability

$$= P(E_1 \cap \bar{E}_2) + P(\bar{E}_1 \cap E_2) + P(E_1 \cap E_2)$$

$$= P(E_1)P(\bar{E}_2) + P(\bar{E}_1)P(E_2) + P(E_1)P(E_2)$$

$$= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{10}{36} + \frac{1}{36}$$

$$= \frac{11}{36}$$

42. Let $I = \int_0^8 |x - 5|\, dx$

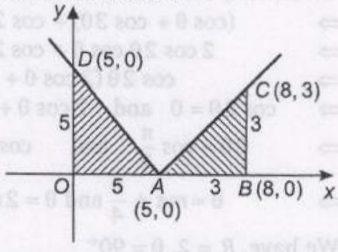
$$= \int_0^5 (5 - x)\, dx + \int_5^8 (x - 5)\, dx$$

$$= \left[5x - \frac{x^2}{2}\right]_0^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$$

$$= 25 - \frac{25}{2} + \left[\frac{64}{2} - 40 - \left(\frac{25}{2} - 25\right)\right]$$

$$= \frac{25}{2} - 8 + \frac{25}{2} = 17$$

Alternative Solution :



It is clear from the figure that required area = area of ΔOAD + area of ΔABC

$$= \frac{1}{2} \times 5 \times 5 + \frac{1}{2} \times 3 \times 3$$

$$= \frac{34}{2} = 17$$

43. Let $I = \int_0^{\pi} |\sin^3\theta|\, d\theta$

Since, $\sin\theta$ is positive in the interval 0 to π.

$\therefore$ $I = \int_0^{\pi} \sin^3\theta\, d\theta$

$$= \int_0^{\pi} (1 - \cos^2\theta)\sin\theta\, d\theta$$

Put, $\cos\theta = t \Rightarrow -\sin\theta\, d\theta = dt$

$$\therefore I = -\int_1^{-1} (1 - t^2)\, dt = -2\left[t - \frac{t^3}{3}\right]_0^{-1}$$

$$= -2\left[-1 + \frac{1}{3}\right] = \frac{4}{3}$$

Alternative Solution :

Let $I = \int_0^{\pi} |\sin^3\theta|\, d\theta$

$$= \int_0^{\pi} \sin^3\theta\, d\theta$$

$$= 2\int_0^{\pi/2} \sin^3\theta\, d\theta$$

Using Gamma function

$$= 2\left[\frac{2}{3 \cdot 1}\right] = \frac{4}{3}$$

44. By the law of conservation of momentum

$$2\times 6 - 4\times 3 = (2+4)v$$
$$\Rightarrow \quad 6v = 12 - 12$$
$$\Rightarrow \quad v = 0$$

45. Since, α, β, γ are the roots of the equation $x^3 + 4x + 1 = 0$, then

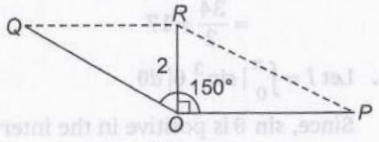
$$\alpha+\beta+\gamma = 0,\ \alpha\beta+\beta\gamma+\gamma\alpha = 4 \text{ and } \alpha\beta\gamma = -1$$
$$\therefore \frac{1}{\alpha+\beta}+\frac{1}{\beta+\gamma}+\frac{1}{\gamma+\alpha} = -\frac{1}{\gamma}-\frac{1}{\alpha}-\frac{1}{\beta}$$
$$(\because \alpha+\beta+\gamma = 0)$$
$$= -\frac{(\alpha\beta+\beta\gamma+\gamma\alpha)}{\alpha\beta\gamma}$$
$$= \frac{-4}{-1} = 4$$

46. Given that, $\cos\theta + \cos 2\theta + \cos 3\theta = 0$

$$\Rightarrow \quad (\cos\theta + \cos 3\theta) + \cos 2\theta = 0$$
$$\Rightarrow \quad 2\cos 2\theta\cos\theta + \cos 2\theta = 0$$
$$\Rightarrow \quad \cos 2\theta(2\cos\theta + 1) = 0$$
$$\Rightarrow \quad \cos 2\theta = 0 \text{ and } 2\cos\theta + 1 = 0$$
$$\Rightarrow \quad 2\theta = \cos\frac{\pi}{2} \text{ and } \cos\theta = -\frac{1}{2}$$
$$\Rightarrow \quad \theta = m\pi \pm \frac{\pi}{4} \text{ and } \theta = 2m\pi \pm \frac{2\pi}{3}$$

47. We have, $R = 2$, $\theta = 90°$

$$\Rightarrow \quad P + Q\cos\alpha = 0$$
$$\therefore \quad 2^2 = P^2 + Q^2 + 2PQ\cos\alpha$$
$$\Rightarrow \quad 4 = P^2 + Q^2 + 2P(-P)$$



$$\Rightarrow \quad 4 = Q^2 - P^2 \qquad \ldots(i)$$

Also, $P + Q\cos 150° = 0$

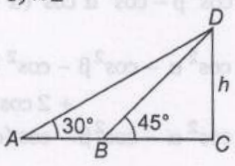
$$\Rightarrow \quad P = \frac{Q\sqrt{3}}{2}$$

From Eq. (i)

$$4 = Q^2 - \left(-\frac{Q\sqrt{3}}{2}\right)^2$$
$$4 = Q^2 - \frac{3Q^2}{4}$$
$$\Rightarrow \quad 4 = \frac{Q^2}{4} \Rightarrow Q = 4$$

and $P = 2\sqrt{3}$

48. The distance covered by a person from A to $B = 25(\sqrt{3}-1)\times 2$



$$= 50(\sqrt{3}-1)$$

In ΔDBC, $\tan 45° = \frac{DC}{BC}$

$$\Rightarrow \quad BC = h$$

In ΔDAC, $\tan 30° = \frac{h}{AB + BC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50(\sqrt{3}-1)+h}$$
$$\Rightarrow \quad 50(\sqrt{3}-1) + h = h\sqrt{3}$$
$$\Rightarrow \quad 50(\sqrt{3}-1) = h(\sqrt{3}-1)$$
$$\Rightarrow \quad h = 50 \text{ m}$$

49. $\lim\limits_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+2}$

$$= \lim_{x\to\infty}\left(1+\frac{2}{x+1}\right)^{\left(\frac{x+1}{2}\right)\times\frac{2}{(x+1)}\times(x+2)}$$
$$= \lim_{x\to\infty}\left(1+\frac{2}{x+1}\right)^{\left(\frac{x+1}{2}\right)\frac{2(x+2)}{x+1}}$$
$$= \lim_{x\to\infty}\left[\left(1+\frac{2}{x+1}\right)^{\frac{x+1}{2}}\right]^{\frac{2\left(1+\frac{2}{x}\right)}{\left(1+\frac{1}{x}\right)}}$$
$$= e^2$$

50. Since, $A + B + C = 270°$

$$\therefore \quad A = B = C = 90°$$
$$\cos 2A + \cos 2B + \cos 2C + 4\sin A\sin B\sin C$$
$$= \cos 180° + \cos 180° + \cos 180° + 4\sin 90°\sin 90°\sin 90°$$
$$= -1 - 1 - 1 + 4$$
$$= 1$$