

Forecasting

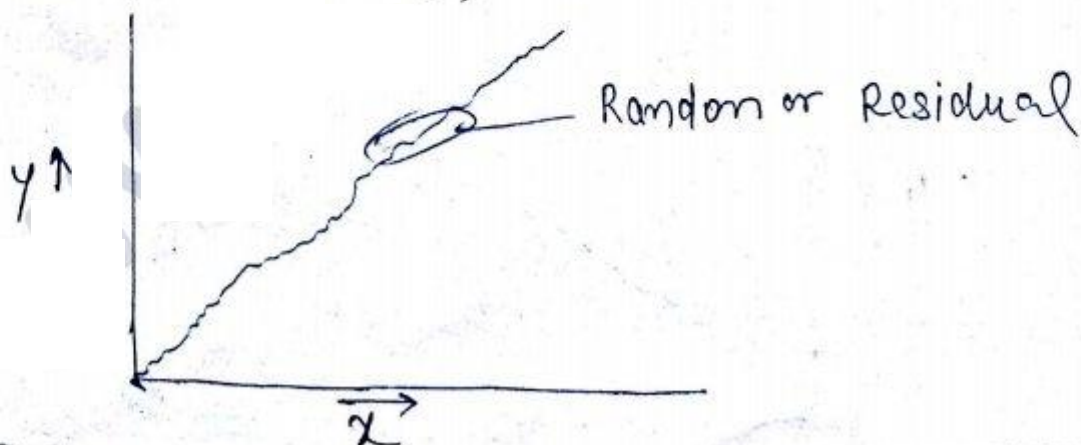
Forecasting can be termed as prediction of future sales or demand of a product it is a projection based upon past data and act of human judgement.

Need or benefits :-

- 1) it helps in determining the volume of production and production rate.
- 2) it forms the basis for production budget, material budget, labour budget etc.
- 3) it suggests the need for plant expansion.
- 4) it is essential for product design and development.
- 5) it helps in determining the price policy.
6. It helps in determining the extent of marketing advertising and distribution required.

Types of demand variation :-

1. Trend variation (T)

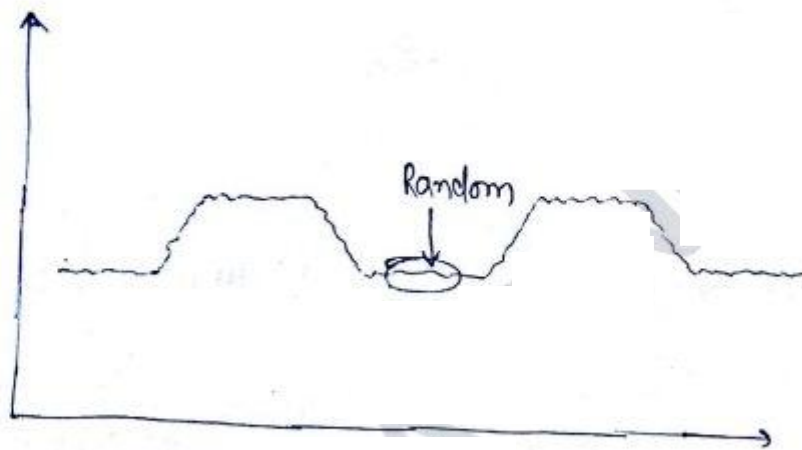


Forecasting

(5)

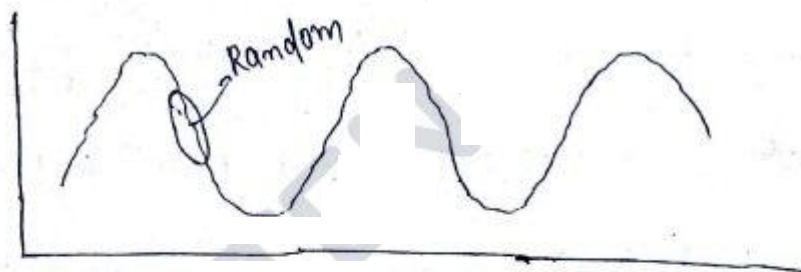
it shows a long term upward or downward movement in demand pattern of a particular product

2) Seasonal (S):



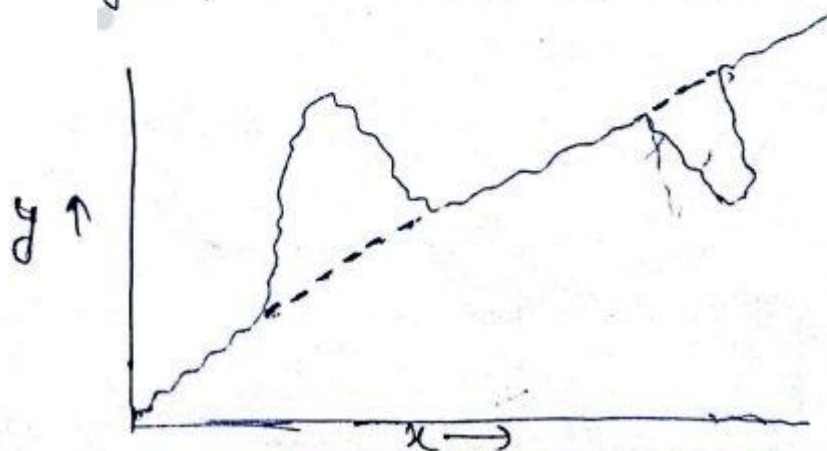
it shows a short term regular variation related to a particular time of day or day of a week.

3) Cyclic (C)



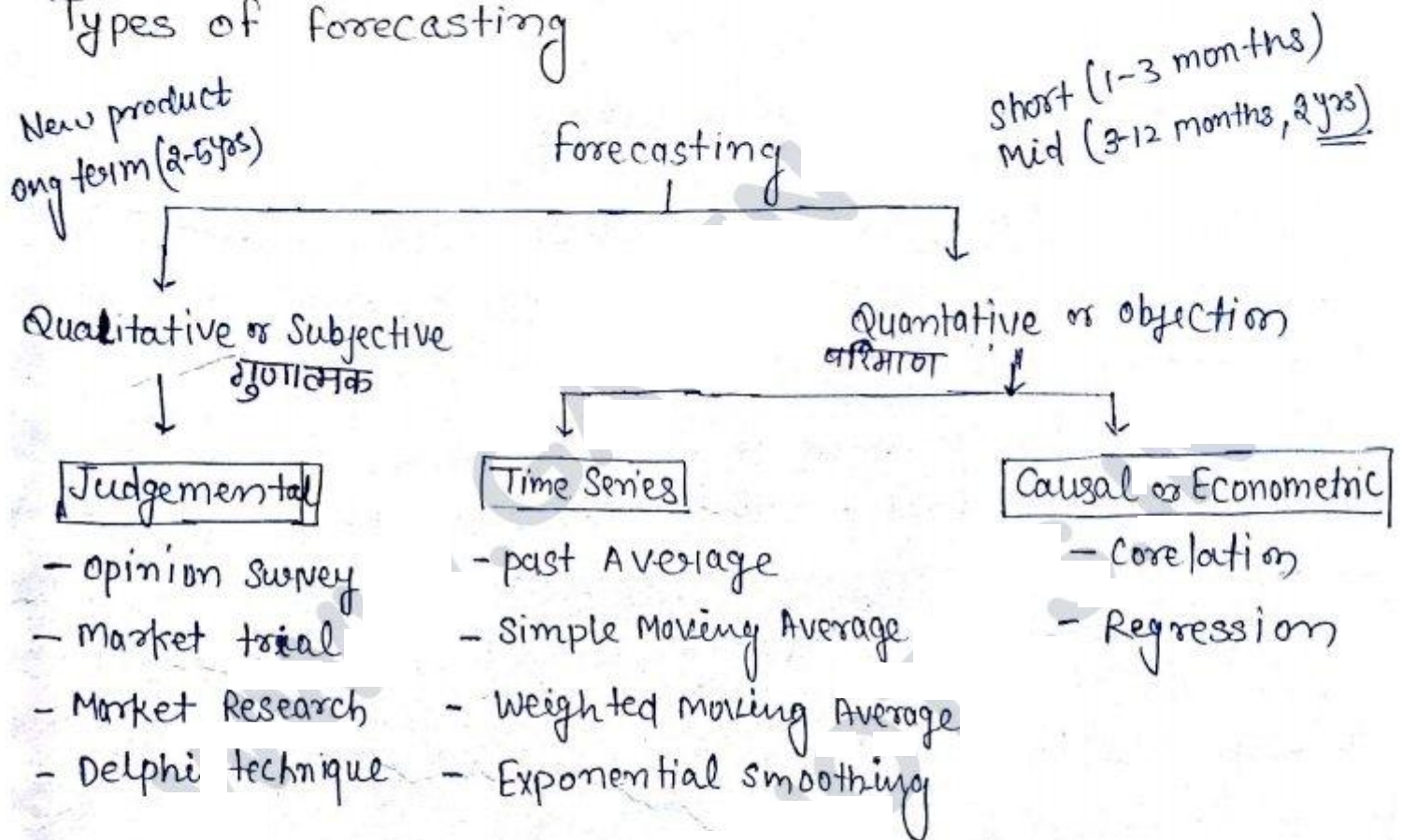
it shows a long term wave like demand variation normally for a year or more.

4. Irregular (I)



These are caused due to unusual circumstances which are not reflective of normal behaviour. These may be due to govt. policy change, price hike, strike, shut down etc.

Types of forecasting



Judgemental: This method is based upon the art of human judgement how well a human being can predict the demand of a product in future. This method does not require or past data or sale figure.

1) **Opinion Survey:** (रय, सलाह, अनुमान) - In this method opinions are collected from the customer, retailer and distributor regarding the demand pattern of a product these information are used while forecasting.

2. Market trial, it is applicable for new product and in that case product is introduced between a limited population in the form of free sample. The response from the limited population is used to project the demand for bigger population. It is applied for low cost consumable like toothpaste, cold drink, chocolate cosmetic items etc.
3. Market Research:- In this method work of survey is assigned to external marketing agencies and the purpose of research is to collect information regarding the demand of our product. The details about various factors which influence the demand like customer income, occupation, location, quantity, quality etc. are related to get the forecast.
4. Delphi technique:- In this method a panel of experts are asked sequential questions in which response to one question is used to produce next question. It is a step by step procedure in which information available to some expert is made available to others and final forecast is obtained by the common opinion of all the experts.

Time Series :- In this method past data are arranged in ~~proportional~~ chronological order as dependent variable and time as independent variable based upon these past data we need to project the demand in future.

1. Past average:- In this method forecast is given by average or mean of the actual demand data for the previous period.
2. Simple moving Average/rolling Average:-

n = no. of period for SMA

1st forecast = $(n+1)^{th}$ period

This method use past data and calculate a rolling average for a constant period. Fresh average is computed at the end of each period by adding the actual demand data for the most recent period and deleting the data for older period. In this method as data changes from period to period, it is termed as moving average method.

Year	2009	2010	2011	2012	2013	2014	2015
Demand	410	430	490	460	490	520	580

$$\text{if } n = 4 \text{ (SMA)} = F_{2012} = \frac{460 + 490 + 430 + 410}{4} = 0.25 \times 460 + 0.25 \times 490 + 0.25 \times 430 + 0.25 \times 410$$

3. Weighted moving average:- (WMA) :-

$$WMA(n=4) = F_{2013} = 0.4 \times 490 + 0.3 \times 460 + 0.2 \times 430 + 0.1 \times 410$$

↓

This Method gives unequal weight to each demand data in such a manner summation of all weight always equal to 1. the most recent data is given the highest weight and weight assigned to oldest data will be least.

⊗

Sum of digit Method:-

n = no. of period for WMA

1) Find the sum of n natural number

$$\sum n = \frac{n \cdot (n+1)}{2}$$

2) Arrange them in decreasing order of weight as

$$\frac{n}{\sum n}, \frac{n-1}{\sum n}, \frac{n-2}{\sum n} \dots \frac{1}{\sum n}$$

e.g. $n = 4$

$$\sum n = 10$$

$$\frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}$$

$n = 5$

$$\sum = 15$$

$$\frac{5}{15}, \frac{4}{15}, \frac{3}{15}, \frac{2}{15}, \frac{1}{15}$$

Problem 29 Project the demand for each of the time period using SMA for $n=3$ period and WMA for $n=4$ period. Also find the forecast for 9th, 10th & 11th period.

Period	Demand	SMA($n=3$)	WMA($n=4$)
1	460	—	—
2	340	—	—
3	520	—	—
4	730	440	—
5	400	530	562
6	430	550	517
7	610	520	490
8	760	480	526
9	—	600	613
10	—	600.00	613
11	—	600.00	613

WMA $n=4$

weight are = $\frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}$

we cannot forecast from forecast so forecast for 10th & 11th period will be equal to 9th period

4. Exponential Smoothing:-

This method requires only the current demand and forecasted value for the current period to give the next forecast. It is a modified form of WMA which gives weight to all previous data but the weights assigned are in exponentially decreasing order. The most recent data is given the highest weight and weight assigned to older data decreases exponentially.

General Form:-

$$F_t = \alpha \cdot D_{t-1} + \alpha(1-\alpha)D_{t-2} + \alpha(1-\alpha)^2D_{t-3} + \alpha(1-\alpha)^3D_{t-4} + \dots$$

$$F_t = \alpha \cdot D_{t-1} + (1-\alpha) \left[\alpha D_{t-2} + \alpha(1-\alpha)D_{t-3} + \dots \right]$$

$$F_t = \alpha D_{t-1} + (1-\alpha) F_{t-1}$$

for finite No. of data

$$0 < \alpha < 1$$

$$F_t = F_{t-1} + \alpha (D_{t-1} - F_{t-1})$$

$$\text{Error} = e_t = \Delta_t = D_t - F_t$$

$$F_t = F_{t-1} + \alpha e_{t-1}$$

Forecast horizon - length of time in future for which forecast to be done (exponential best method for long period)

α - is known is smoothing Constant/coefficient and is equal to n period of moving average and is given by

$$\alpha = \frac{2}{n+1}$$

Practically it is in the range of 0.1-0.2

Note

If For the initial period forecasted value is not given then.

i) Take the actual demand for the first period equal to forecast that is take $D_1 = F_1$ and proceed.

ii) Take the average of actual demand data as the forecast for the first period and proceed.

Problem 30:- The sales of car in a showroom in 4 consecutive months is 70, 68, 82, 95 respectively with $\alpha = 0.4$ Find the forecast for the next month.

Solⁿ

$$D_1 = 70, F_1 = 70$$

$$F_2 = 70 + 0.4(0) \quad F_2 = 70$$

$$F_3 = 70 + 0.4(68 - 70) = 69.2$$

$$F_4 = 69.2 + 0.4(82 - 69.2) = 74.32$$

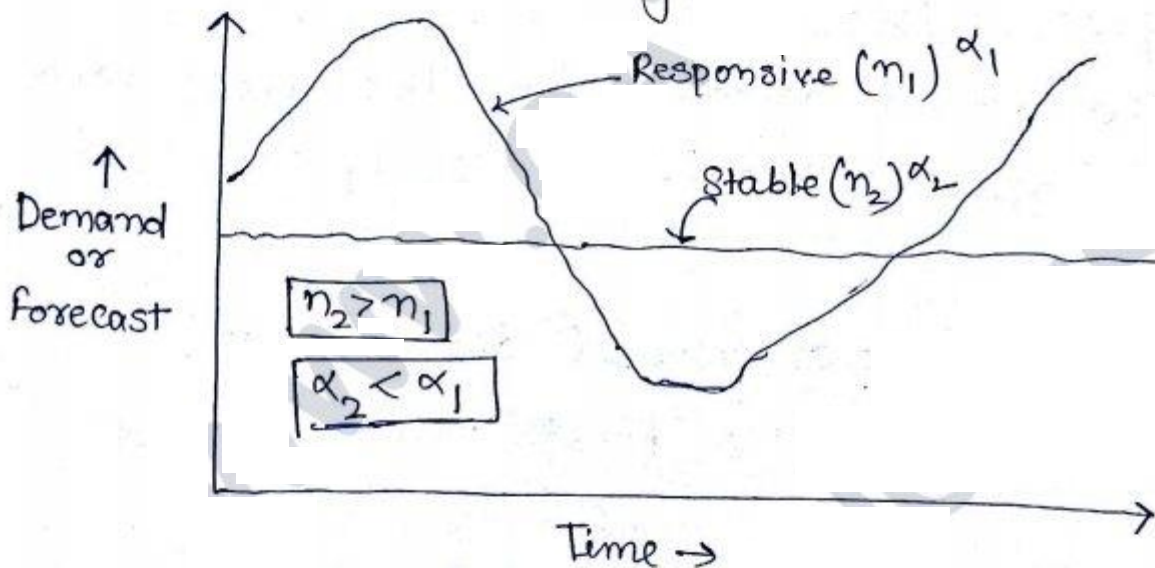
$$F_5 = 74.32 + 0.4(95 - 74.32) = 82.59$$

$$F_5 = 82.59 \approx 83$$

Month	D_i	f_i	e_i
1	70	70	0
2	68	70	-2
3	82	69.2	12.8
4	95	74.32	20.68

$$\boxed{82.59} \approx 83$$

Responsive & stability! -



$$\alpha = \frac{2}{n+1}$$

Responsive!:- Responsiveness indicate that the forecast have fluctuating or swing pattern. It is preferred for new product and for that the no. of period is kept small.

Stability!:- stability means that the forecast pattern is flat, smooth or has less fluctuation it is preferred for old existing period and for that no. of period is kept large.

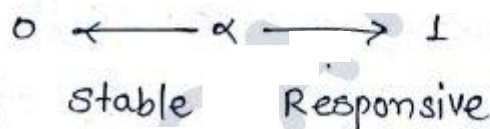
$$\alpha = \frac{2}{n+1} \quad F_t = F_{t-1} + \alpha (D_{t-1} - F_{t-1})$$

1. IF $\alpha = 0$, $n \rightarrow \infty$ [limit of stability]

$$F_t = F_{t-1}$$

2. IF $\alpha = 1$, $n = 1$ [limit of responsiveness]

$$F_t = D_{t-1}$$



Forecast error:-

$$e_i = D_i - F_i$$

when forecast error is studied for long duration it becomes helpful to find the particular pattern or trend which may regulate for future production. The most commonly used method to find forecast error are

1. Mean Absolute Deviation (MAD):-

$$MAD = \frac{\sum_{i=1}^n |D_i - F_i|}{n}$$

e.g.

S.No.	D_i	F_i	e_i
1	250	240	+10
2	250	280	-30
3	270	250	+20
4			

$$MAD = \frac{60}{3} = \underline{\underline{20}} \quad \checkmark$$

It indicate the average magnitude of error made in every period without considering sign that is in absolute terms.

2) Mean forecast error or Bias

$$\text{MFE or Bias} = \frac{\sum_{i=1}^n (D_i - F_i)}{n}$$

it measure the forecast error with regard to direction and shows any tendency of over ~~and~~ or under forecast. Positive Bias indicate under estimated forecasting and negative Bias indicate over estimate forecasting,

Running sum forecast error (RSFE)

$$\text{RSFE} = \sum_{i=1}^n (D_i - F_i)$$

$$\text{Bias} = \frac{\text{RSFE}}{n}$$

3) Mean Square Error :- (MSE)

$$\text{MSE} = \frac{\sum_{i=1}^n (D_i - F_i)^2}{n}$$

$$\sigma = \text{MSE}$$

MST is used to compute standard deviation for forecast error which is utilized to plot control chart for forecast error

4) Mean Absolute Percentage Error (MAPE)

$$\text{MAPE} = \frac{\sum_{i=1}^n \left| \frac{D_i - F_i}{D_i} \right| \times 100}{n}$$

$$\begin{array}{c} \textcircled{30\%} \\ \downarrow \\ \pm 15\% \end{array}$$

$$\begin{array}{c} \textcircled{40\%} \\ \downarrow \\ \pm 20\% \end{array}$$

It is the average of percentage error compare to actual demand it is used to put error in prospective because there is difference between 60 out of 100 and 60 out of 1000.

5. Tracking Signal (TS)

$$\text{TS} = \frac{\text{RSFE}}{\text{MAD}}$$

$$\textcircled{\pm 4} \text{ or } \textcircled{\pm 5}$$

assumed process under control.

it is a term used to represent how well the forecast is predicting the actual value, a value of zero would be ideal, but ± 4 or ± 5 is the acceptable range.

Problem 3! The demand for luxury car has been shown below the expert forecasted A sale of 100 car for the month of march with a smoothing constant $\alpha = 0.15$ find the forecast for the month of Aug. Also determine MAD, MSE, MAPE, Bias

Month	Demand	forecast
march	150	100
April	200	100 107.5
May	100	121.37 118.16
Jun	50	118.16
July	150	107.94
Aug.		114.25

$$F_{\text{march}} = 100, \alpha = 0.15$$

$$F_{\text{April}} = 100 + 0.15(150 - 100) = 107.5$$

$$F_{\text{may}} = 107.5 + 0.15(200 - 107.5) = 121.37$$

$$F_{\text{Jun}} = 121.37 + 0.15(100 - 121.37) = 118.16$$

$$F_{\text{July}} = 118.16 + 0.15(50 - 118.16) = 107.94$$

$$F_{\text{Aug}} = 107.94 + 0.15(150 - 107.94) = 114.25$$

month	Demand	forecast	e_i	e_i^2	$\left \frac{e_i}{D_i} \times 100 \right $
March	150	100	50	2500	33.33
April	200	107.5	92.5	8556.25	46.25
May	100	121.375	-21.375	456.81	21.375
June	50	118.16	-68.16	4645.78	136.12
July	150	107.94	42.06	1769.04	28.04
Aug	$n=5$	114.95	$\sum e_i = 95.02$	$\sum e_i^2 =$	$\sum \left \frac{e_i}{D_i} \times 100 \right $
			$\sum e_i = 274.1$	17928.5	= 265.3

$$MAD = \frac{274.1}{5} = 54.82$$

$$MSE = \frac{17928.5}{5} = 3585.7$$

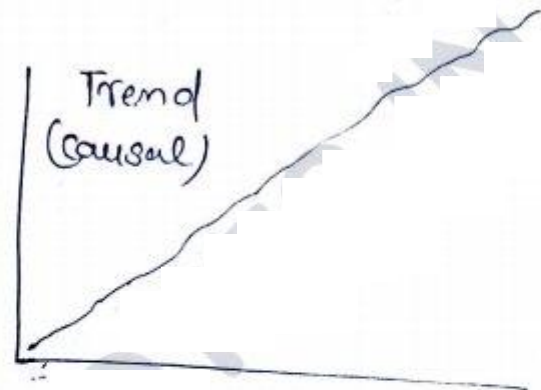
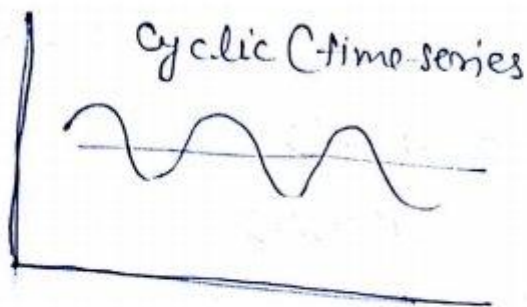
$$MAPE = \frac{265.3}{5} = 53.06$$

$$Bias = \frac{\sum e_i}{n} = \frac{95.02}{5} = 19.0$$

it is an underestimated forecasting

$$TS = \frac{95.02}{54.82} = 1.73$$

Causal or Econometric :—



In this method forecasted price to establish cause and effect relation between demand of a product and any other variable on which demand is dependent. The objective is to establish a relation such that changes in one variable become useful for prediction of other.

Correlation Analysis!—

$$\boxed{r \rightarrow +1 \text{ to } -1}$$

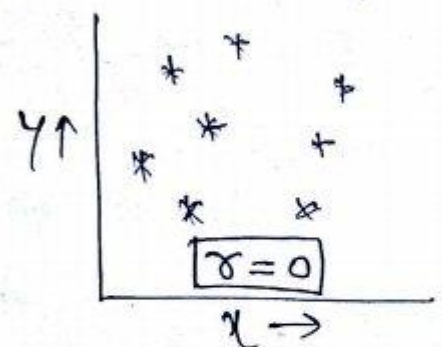
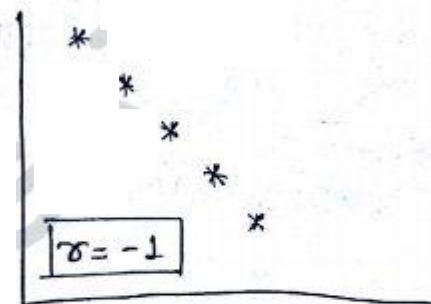
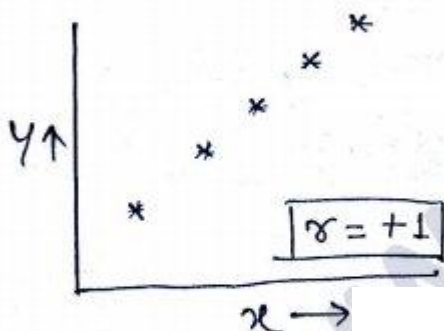
$x \& y$

$$r = 0.68$$

$$x \uparrow 100 \rightarrow y \uparrow 68$$

$$r = -0.47$$

$$x \uparrow 100 \rightarrow y \downarrow 47$$



it indicate the degree of closeness between two variable at its value range is +1 to -1

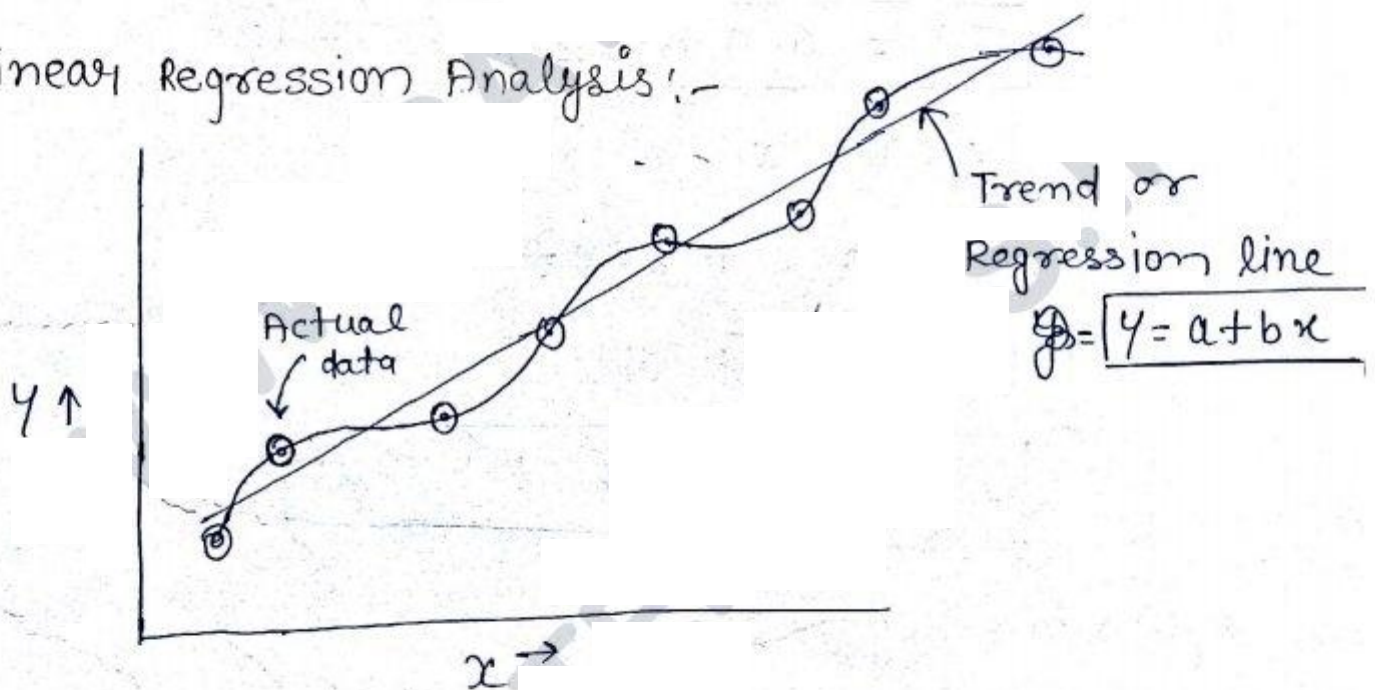
it is an indication by change one variable become helpful to predict changes in other

The correlation coefficient between two variable x and y is given by

$$r = \frac{\sum (x - \bar{x}) \cdot (y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \cdot \sum (y - \bar{y})^2}}$$

where $\bar{x}$ & $\bar{y}$ are the average value of individual x and y value

Linear Regression Analysis:-



it is a mathematical technique of obtaining the line of best fit between the dependent variable which is usually demand of a product and any other variable on which demand is dependent regression analysis relationship between some independent variable x and dependent variable y represent by a straight line

$$y = a + bx$$

trend line

independent

Dependent

$a \rightarrow$ Intercept on y -axis

$b \rightarrow$ slope of line

$$Y = a + bx \quad \text{--- (1)}$$

$n =$ no. of period of data

Taking Σ both side of equation (1)

$$\Sigma Y = a \cdot n + b \Sigma X \quad \text{--- (2)}$$

equ (1) $\times x$

$$x \cdot Y = a \cdot x + b \cdot x^2$$

taking Σ both side

$$\Sigma x \cdot Y = a \Sigma x + b \Sigma x^2 \quad \text{--- (3)}$$

eq (3) $\times n$ - eq (2) $\times \Sigma x$

$$n \cdot \Sigma x \cdot Y = a \cdot n \cdot \Sigma x + b \cdot n \cdot \Sigma x^2$$

$$-(\Sigma x \cdot \Sigma Y = a \cdot \Sigma x + b \cdot (\Sigma x)^2)$$

$$n \Sigma x \cdot Y - \Sigma x \cdot \Sigma Y = b [n \Sigma x^2 - (\Sigma x)^2]$$

$$b = \frac{n \cdot \Sigma x \cdot Y - \Sigma x \cdot \Sigma Y}{n \cdot \Sigma x^2 - (\Sigma x)^2}$$

$$a = \frac{\Sigma Y - b \cdot \Sigma x}{n}$$

So trend line $\rightarrow$

$$Y = a + bx$$

Special case

least square Method: -

When the independent variable is linear and uniform and is in such a form that it can be modified to make $\sum x = 0$ then the calculation become very simple and method is called least square method (LSM)

$$\sum x = 0$$

$$b = \frac{\sum x \cdot y}{\sum x^2}$$

$$a = \frac{\sum y}{n}$$

(a) $n = \text{odd}$

Year	Sales(Y)	x
2009		-2
2010		-1
2011	-----	0
2012		+1
2013		+2

(b) $n = \text{even}$

Year	Sales(Y)	x	<u>x^2</u>
2008		-2.5	-5
2009		-1.5	-3
2010		-0.5	-1
2011		+0.5	+1
2012		+1.5	+3
2013		+2.5	+5

Problem 32 A Car manufacturer has recently had road side car exhibition for new model of car to the no. of salesman exhibition and no. of car booked is given below fit a linear regression equation and estimate the no. of car booked if 10 salesman employed in exhibition

No. of cars booked (y)	No. of Salesman (x)	x^2	$x \cdot y$
132	5	25	660
160	8	64	1280
148	6	36	888
156	8	64	1248
168	9	81	1512
102	3	9	306
142	5	25	710
98	4	16	392
152	6	36	912
142	6	36	852
<u>1400</u>	<u>60</u>	<u>392</u>	<u>8760</u>
$\Sigma y = 1400$	$\Sigma x = 60$	$\Sigma x^2 = 392$	$\Sigma x \cdot y = 8760$

$$y = a + bx \quad \text{--- (1)}$$

$$\Sigma y = a \cdot n + b \cdot \Sigma x \quad \text{--- (2)}$$

$$1400 = 10 \cdot a + 60 \cdot b$$

$$\Sigma x \cdot y = a \cdot \Sigma x + b \Sigma x^2$$

$$8760 = 60 \cdot a + 392 \cdot b$$

$$a = 72.5 \quad b = 11.25$$

$$y = 72.5 + 11.25x$$

$$\Rightarrow x = 10$$

$$y = \underline{\underline{185}}$$

Problem 33 The sales of an automobile company is as given below forecast the demand for next 2 years using least square method

Year	Sales (cr)	x	x^2	$x \cdot y$
2005	30	-9	81	-270
2006	33	-7	49	-231
2007	37	-5	25	-185
2008	39	-3	9	-117
2009	42	-1	1	-42
2010	46	+1	1	46
2011	48	+3	9	144
2012	50	+5	25	250
2013	55	+7	49	385
2014	58	+9	81	522
	$\Sigma y = 438$	$\Sigma x = 0$	$\Sigma x^2 = 330$	$\Sigma x \cdot y = 502$

$$y = a + bx$$

$$b = \frac{\Sigma x \cdot y}{\Sigma x^2} \quad a = \frac{\Sigma y}{n}$$

$$b = \frac{502}{330} = 1.52 \quad a = \frac{438}{10} = 43.8$$

$$\boxed{y = 43.8 + 1.52x}$$

for $x = 11$ i.e. 2014 $y = 60.52$ cr,

$x = 12$ i.e. 2015 $y = 63.56$ cr.