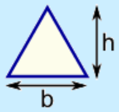
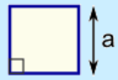
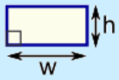
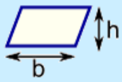
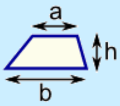
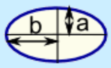
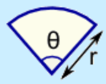
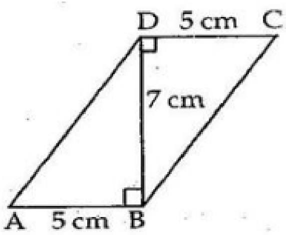
Area

Exercise 10A

	Triangle Area = ½ × b × h b = base h = vertical height		Square Area = a^2 a = length of side
	Rectangle Area = w × h w = width h = height		Parallelogram Area = b × h b = base h = vertical height
	Trapezoid (US) Trapezium (UK) Area = ½(a+b) × h h = vertical height		Circle Area = **π** × r^2 Circumference = 2 × π × r r = radius
	Ellipse Area = **π**ab		Sector Area = ½ × r^2 × θ r = radius θ = angle in **radians**

Question 1:

Area of $\Delta ABD = \frac{1}{2} \times \text{base} \times \text{height}$



$$= \left(\frac{1}{2} \times 5 \times 7\right) \text{cm}^2 = \frac{35}{2} \text{cm}^2$$

$$\text{Area of } \Delta CBD = \left(\frac{1}{2} \times 5 \times 7\right) \text{cm}^2 = \frac{35}{2} \text{cm}^2$$

Since the diagonal BD divides ABCD into two triangles of equal area.

$\therefore$ ABCD is a parallelogram.

$\therefore$ Area of parallelogram = Area of ΔABD + Area of ΔCBD

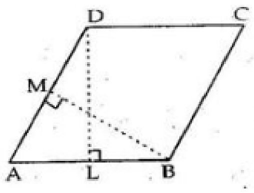
$$= \left(\frac{35}{2} + \frac{35}{2}\right) \text{cm}^2 = \frac{70}{2} \text{cm}^2$$

$$= 35 \text{ cm}^2$$

$\therefore$ Area of parallelogram = 35 cm^2

Question 2:

Since ABCD is a parallelogram and DL is perpendicular to AB.



So, its area $= AB \times DL$

$= (10 \times 6)\ cm^2$

$= 60\ cm^2$

Also, in parallelogram ABCD,

$BM \perp AD$

$\therefore$ Area of parallelogram $ABCD = AD \times BM$

$60 = AD \times 8 cm$

$\therefore \quad AD \times 8 = 60$

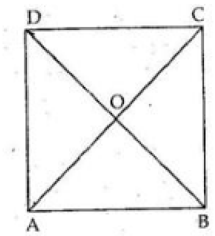
$\Rightarrow \quad AD = \frac{60}{8} = 7.5$ cm

$\therefore \quad AD = 7.5$ cm

Question 3:

ABCD is a rhombus in which diagonal AC=24 cm and BD = 16 cm.

These diagonals intersect at O.



Since diagonals of a rhombus are perpendicular to each other. So, in Δ ACD,

OD is its altitude and AC is its base.

So, area of $\Delta ACD = \frac{1}{2} \times AC \times OD$

$= \frac{1}{2} \times 24 \times \frac{BD}{2}$

$= \left(\frac{1}{2} \times 24 \times 8\right) cm^2 \quad [\because BD = 16 cm]$

$= 96\ cm^2$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2} \times AC \times OB$

$= \left(\frac{1}{2} \times 24 \times 8\right) cm^2 = 96\ cm^2$

Now, area of rhombus = Area of ΔACD + Area of ΔABC

$= (96 + 96)\ cm^2$

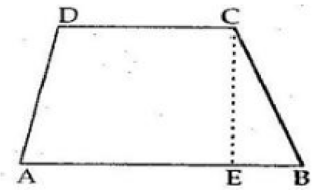
$= 192\ cm^2$

Question 4:

ABCD is a trapezium in which, AB || CD

AB = 9 cm and CD = 6 cm

CE is a perpendicular drawn to AB through C and CE = 8 cm



Area of trapezium = $\frac{1}{2}$(sum of parallel sides) × distance between them

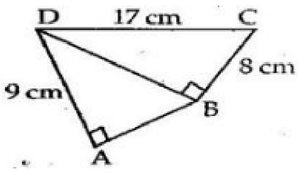
$$= \left[\frac{1}{2}(9+6)\times 8\right] \text{ cm}^2$$

$$= \left(\frac{1}{2}\times 15\times 8\right) \text{ cm}^2 = 60 \text{ cm}^2$$

$\therefore$ Area of trapezium = 60 cm^2

Question 5:

(i) ABCD is a quadrilateral.



Now in right angled Δ DBC,

$$DB^2 = DC^2 - CB^2$$

$$= 17^2 - 8^2$$

$$= 289 - 64 = 225 \text{ cm}^2$$

$\therefore$ $DB = \sqrt{225} = 15 \text{cm}$

So, area of ΔDBC $= \left(\frac{1}{2}\times 15\times 8\right) \text{cm}^2 = 60 \text{ cm}^2$

Again, in right angled ΔDAB,

$$AB^2 = DB^2 - AD^2$$

$$= 15^2 - 9^2$$

$$= 225 - 81 = 144 \text{cm}^2$$

$\therefore$ $AB = \sqrt{144} = 12 \text{ cm.}$

$\therefore$ area of ΔDAB $= \left(\frac{1}{2}\times 12\times 9\right)\text{cm}^2 = 54 \text{ cm}^2$

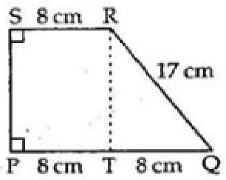
So, area of quadrilateral ABCD

= Area of ΔDBC + Area of ΔDAB

$= (60 + 54) \text{ cm}^2 = 114 \text{ cm}^2$

$\therefore$ area of quadrilateral ABCD = 114 cm^2

(ii)



$RT \perp PQ$

In right angled ΔRTQ

$RT^2 = RQ^2 - TQ^2$

$= 17^2 - 8^2$

$= 289 - 64 = 225\ cm^2$

$\therefore \quad RT = \sqrt{225} = 15\ cm$

$\therefore$ Area of trapezium $= \frac{1}{2}$(sum of parallel sides) x distance between them

$= \frac{1}{2} \times (PQ + SR) \times RT$

$= \frac{1}{2} \times (16 + 8) \times 15$

$= \left(\frac{1}{2} \times 24 \times 15\right) cm^2 = 180 cm^2$

$\therefore$ area of trapezium $= 180 cm^2$

Question 7:

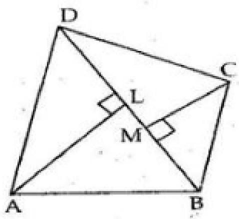
Given: ABCD is a quadrilateral and BD is one of its diagonals.

$AL \perp BD$ and $CM \perp BD$

To Prove: area (quad. ABCD)

$= \frac{1}{2} \times BD \times (AL + CM)$

Proof:



Area of Δ BAD $= \frac{1}{2} \times BD \times AL$

Area of Δ CBD $= \frac{1}{2} \times BD \times CM$

$\therefore$ Area of quard. ABCD = Area of ΔABD + Area of ΔCBD

$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$

$\therefore$ Area of quard. ABCD $= \frac{1}{2} \times BD[AL + CM]$

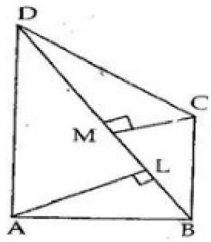
Question 8:

Area of $\Delta BAD = \frac{1}{2} \times BD \times AL$

$= \left(\frac{1}{2} \times 14 \times 8\right) cm^2 = 56\ cm^2$

Area of $\Delta CBD = \frac{1}{2} \times BD \times CM$

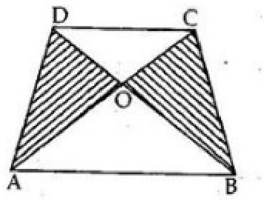
$= \left(\frac{1}{2} \times 14 \times 6\right) cm^2 = 42\ cm^2$



$\therefore$ area of quad. ABCD = Area of ΔABD + Area of ΔCBD

$= (56 + 42)\ cm^2 = 98\ cm^2$

Question 9:



Consider ΔADC and ΔDCB. We find they have the same base CD and lie between two parallel lines DC and AB.

Triangles on the same base and between the same parallels are equal in area.

So ΔCDA and ΔCDB are equal in area.

$\therefore$ area(ΔCDA) = area(ΔCDB)

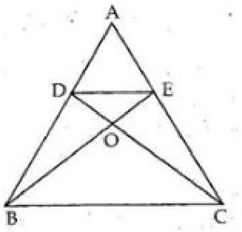
Now, area(ΔAOD) = area(ΔADC) – area(ΔOCD)

and area(ΔBOC) = area(ΔCDB) – area(ΔOCD)

= area(ΔADC) – area(ΔOCD)

$\Rightarrow$ area(ΔAOD) = area(ΔBOC)

Question 10:



(i) ΔDBE and ΔDCE have the same base DE and lie between parallel lines BC and DE.

So, area (ΔDBE) = area (ΔDCE).....(1)

Adding area (ΔADE) on both sides, we get

$ar(\Delta DBE) + ar(\Delta ADE) = ar(\Delta DCE) + ar(\Delta ADE)$

$\Rightarrow \quad ar(\Delta ABE) = ar(\Delta ACD)$

(ii) Since $\quad ar(\Delta DBE) = ar(\Delta DCE)$ [from (1)]

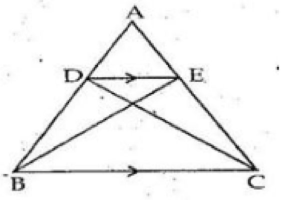
Subtracting $ar(\Delta ODE)$ from both sides we get

$ar(\Delta DBE) - ar(\Delta ODE) = ar(\Delta DCE) - ar(\Delta ODE)$

$\Rightarrow \quad ar(\Delta OBD) = ar(\Delta OCE)$

Question 11:

Given: A ΔABC in which points D and E lie on AB and AC, such that $ar(\Delta BCE) = ar(\Delta BCD)$



To Prove: DE $\parallel$ BC

Proof : As ΔBCE and ΔBCD have same base BC, and are equal in area, they have same altitudes.

This means that they lie between two parallel lines.

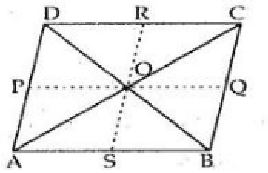
$\therefore$ DE $\parallel$ BC

Question 12:

Given : A parallelogram ABCD in which O is a point inside it

To Prove: (i) $ar(\Delta OAB) + ar(\Delta OCD) = \frac{1}{2} ar(\|gm\ ABCD)$

(ii) $ar(\Delta OAD) + ar(\Delta OBC) = \frac{1}{2} ar(\|gm\ ABCD)$



Construction: Through O draw PQ ∥ AB and RS ∥ AD

Proof : (i) Δ AOB and parallelogram ABQP have same base AB and lie between parallel lines AB and PQ.

If a triangle and a parallelogram are on the same base, and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

$\therefore \quad ar(\Delta AOB) = \frac{1}{2} ar(\|gm\ ABQP)$

Similarly, $ar(\Delta COD) = \frac{1}{2} ar(\|gm\ PQCD)$

So, $ar(\Delta AOB) + ar(\Delta COD)$

$= \frac{1}{2} ar(\|gm\ ABQP) + \frac{1}{2} ar(\|gm\ PQCD)$

$= \frac{1}{2} [ar(\|gm\ ABQP) + ar(\|gm\ PQCD)]$

$= \frac{1}{2} [ar \|gm\ ABCD]$

(ii) Δ AOD and ∥ gm ASRD have the same base AD and lie between same parallel lines AD and RS.

So, $ar(\Delta AOD) = \frac{1}{2} ar(\|gm\ ASRD)$

Similarly, $ar(\Delta BOC) = \frac{1}{2} ar(\|gm\ RSBC)$

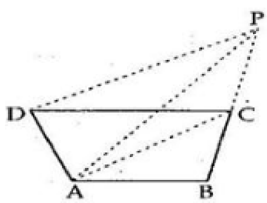
$\therefore \quad ar(\Delta AOD) + ar(\Delta BOC) = \frac{1}{2} [ar(\|gm ASRD) + ar(\|gm RSBC)]$

$= \frac{1}{2} [ar(\|gm ABCD)]$

Question 13:

Given: ABCD is a quadrilateral in which through D, a line is drawn parallel to AC which meets BC produced in P.

To Prove : $ar(\Delta ABP) = ar(quad. ABCD)$



Proof : Δ ACP and Δ ACD have same base AC and lie between parallel lines AC and DP.

$\therefore \quad ar(\Delta ACP) = ar(\Delta ACD)$

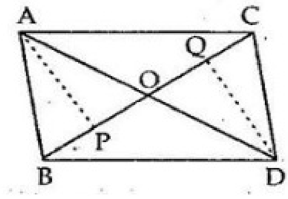
Adding $ar(\Delta ABC)$ on both sides, we get;

$ar(\Delta ACP) + ar(\Delta ABC) = ar(\Delta ACD) + ar(\Delta ABC)$

$\Rightarrow \quad ar(\Delta ABP) = ar(quad. ABCD)$

Question 14:

Given: Two triangles, i.e. ΔABC and ΔDBC which have same base BC and points A and D lie on opposite sides of BC and $ar(\Delta ABC) = ar(\Delta BDC)$



To Prove: $OA = OD$

Construction: Draw $AP \perp BC$ and $DQ \perp BC$

Proof: We have

$$ar(\Delta ABC) = \frac{1}{2} \times BC \times AP \text{ and}$$

$$ar(\Delta BCD) = \frac{1}{2} \times BC \times DQ$$

So, $\frac{1}{2} \times BC \times AP = \frac{1}{2} \times BC \times DQ$ [from (1)]

$\Rightarrow$ $AP = DQ$ ......(2)

Now, in ΔAOP and ΔQOD, we have

$\angle APO = \angle DQO = 90°$

and $\angle AOP = \angle DOQ$ [vertically opp. angles]

$AP = DQ$ [from (2)]

Thus, by Angle-Angle-Side criterion of congruence, we have

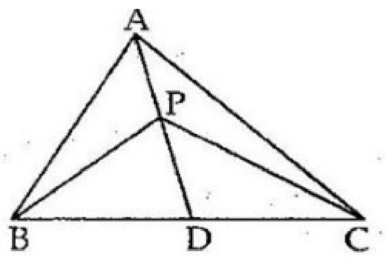
$\therefore$ $\Delta AOP \cong \Delta QOD$ [AAS]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $OA = OD$ [C.P.C.T.]

Question 15:

Given: A ΔABC in which AD is the median and P is a point on AD.



To Prove: (i) $ar(\Delta BDP) = ar(\Delta CDP)$

(ii) $ar(\Delta ABP) = ar(\Delta APC)$

Proof: (i) In Δ BPC, PD is the median. Since median of a triangle divides the triangle into two triangles of equal areas

So, $ar(\Delta BPD) = ar(\Delta CDP)$......(1)

(ii) In ΔABC, AD is the median

So, $ar(\Delta ABD) = ar(\Delta ADC)$

But, $ar(\Delta BPD) = ar(\Delta CDP)$ [from (1)]

Subtracting $ar(\Delta BPD)$ from both the sides of the equation, we have

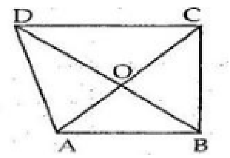
$\therefore ar(\Delta ABD) - ar(\Delta BPD) = ar(\Delta ADC) - ar(\Delta BPD)$

$= ar(\Delta ADC) - ar(\Delta CDP)$ from (1)

$\Rightarrow$ $ar(\Delta ABP) = ar(\Delta ACP)$.

Question 16:

Given : A quadrilateral ABCD in which diagonals AC and BD intersect at O and BO = OD



To Prove : ar(ΔABC) = ar(ΔADC)

Proof: Since OB = OD [Given]

So, AO is the median of ΔABD

$\therefore$ ar(ΔAOD) = ar(ΔAOB) ....(i)

As OC is the median of ΔCBD

ar(ΔDOC) = ar(ΔBOC) .....(ii)

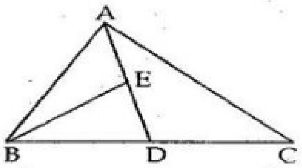
Adding both sides of (i) and (ii), we get

ar(ΔAOD) + ar(ΔDOC) = ar(ΔAOB) + ar(ΔBOC)

$\therefore$ ar(ΔADC) = ar(ΔABC)

Question 17:

Given : A ΔABC in which AD is a median and E is the mid – point of AD



To Prove : $ar(\Delta BED) = \frac{1}{4} ar(\Delta ABC)$

Proof : Since, ar(ΔABD) = ar(ΔACD) [$\because$ AD is the median]

i.e. $ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC)$ .....(1)

[$\because ar(\Delta ABC) = ar(\Delta ABD) + ar(\Delta ADC)$]

Now, as BE is the median of ΔABD

ar(ΔABE) = ar(ΔBED) .....(2)

Since ar(ΔABD) = ar(ΔABE) + ar(ΔBED) .....(3)

$\therefore$ ar(ΔBED) = ar(ΔABE) [from (2)]

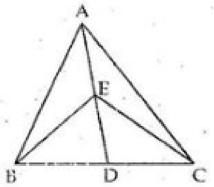
$= \frac{1}{2} ar(\Delta ABD)$ [from (2) and (3)]

$= \frac{1}{2}\left[\frac{1}{2} ar(\Delta ABC)\right]$ [from (1)]

$= \frac{1}{4} ar(\Delta ABC)$

Question 18:

Given: A Δ ABC in which E is the mid – point of line segment AD where D is a point on BC.



To Prove: $ar(\Delta BEC) = \frac{1}{2} ar(\Delta ABC)$

Proof: Since BE is the median of ΔABD

So, $ar(\Delta BDE) = ar(\Delta ABE)$

$\therefore \quad ar(\Delta BDE) = \frac{1}{2} ar(\Delta ABD) \quad ....(i)$

As, CE is median of ΔADC

So, $ar(\Delta CDE) = \frac{1}{2} ar(\Delta ACD) \quad ....(ii)$

Adding (i) and (ii), we get

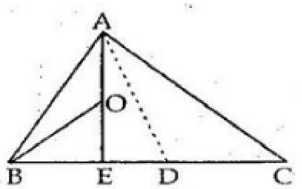
$$ar(\Delta BDE) + ar(\Delta CDE) = \frac{1}{2} ar(\Delta ABD) + \frac{1}{2} ar(\Delta ACD)$$

$$ar(\Delta BEC) = \frac{1}{2}[ar(\Delta ABD) + ar(\Delta ACD)]$$

$$= \frac{1}{2} ar(\Delta ABC).$$

Question 19:

Given: A Δ ABC in which AD is the median and E is the mid – point of BD. O is the mid – point of AE.



To Prove: $ar(\Delta BOE) = \frac{1}{8} ar(\Delta ABC)$

Proof: Since O is the midpoint of AE.

So, BO is the median of ΔBAE

$\therefore \quad ar(\Delta BOE) = \frac{1}{2} ar(\Delta ABE) .....(1)$

Now, E is the mid – point of BD

So AE divides ΔABD into two triangles of equal area.

$\therefore \quad ar(\Delta ABE) = \frac{1}{2} ar(\Delta ABD).....(2)$

As D is the mid point of BC

So $\quad ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC).....(3)$

$\Rightarrow \quad ar(\Delta BOE) = \frac{1}{2} ar(\Delta ABE) \quad$ [from (1)]

$$= \frac{1}{2}\left[\frac{1}{2} ar(\Delta ABD)\right] \quad \text{[from (2)]}$$

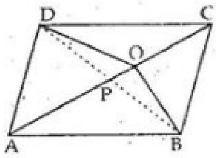
$$= \frac{1}{4} ar(\Delta ABD)$$

$$= \frac{1}{4} \times \frac{1}{2} ar(\Delta ABC) \quad \text{[from (3)]}$$

$$= \frac{1}{8} ar(\Delta ABC)$$

Question 20:

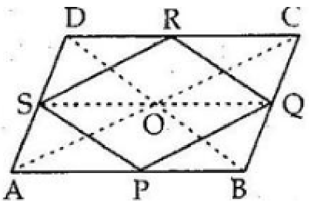
Given: A parallelogram ABCD in which O is any point on the diagonal AC.



To Prove: ar(ΔAOB) = ar(ΔAOD).
Construction: Join BD which intersects AC at P.
Proof: As diagonals of a parallelogram bisect each other,
so, OP is the median of ΔODB
$\therefore$ ar(ΔODP) = ar(ΔOBP).
Also, AP is the median of ΔABD
$\therefore$ ar(ΔADP) = ar(ΔABP)
Adding both sides, we get
ar(ΔODP) + ar(ΔADP) = ar(ΔOBP) + ar(ΔABP)
$\Rightarrow$ ar(ΔAOD) = ar(ΔAOB).

Question 21:

Given: ABCD is a parallelogram and P, Q, R and S are the midpoints of AB, BC, CD and DA respectively.

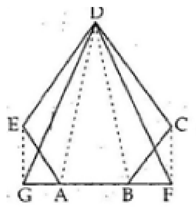


To Prove: PQRS is a parallelogram and ar(||gmPQRS) $= \frac{1}{2}$ ar(||gm ABCD)
Construction: Join AC, BD and SQ.
Proof: As S and R are the midpoints of AD and CD. So, in ΔADC,
SR || AC [By mid point theorem]
Also, as P and Q are the midpoints of AB and BC. So, in ΔABC,
PQ || AC
$\therefore$ PQ || AC || SR
$\therefore$ PQ || SR
Similarly, we can prove SP || RQ.
Thus PQRS is a parallelogram as its opposite sides are parallel
since diagonals of a parallelogram bisect each other.
So in ΔABD,
O is the midpoint of AC and S is the midpoint of AD.
$\therefore$ OS || AB [By midpoint theorem]
Similarly in ΔABC, we can prove that,
OQ || AB
i.e. SQ || AB

Thus, ABQS is a parallelogram.
Now, ar(ΔSPQ) $= \frac{1}{2}$ ar(||gm ABQS) ....(i)
[$\because$ ΔSPQ and ||gm ABQS have the same base and lie between same parallel lines]
Similarly, we can prove that;
ar(ΔSRQ) $= \frac{1}{2}$ ar(||gm SQCD) ....(ii)
Adding (i) and (ii) we get
ar(ΔSPQ) + ar(ΔSRQ) $= \frac{1}{2}$[ar(||gm ABQS) + ar(||gm SQCD)]
$\therefore$ ar(||gmPQRS) $= \frac{1}{2}$ ar(||gm ABCD)

Question 22:

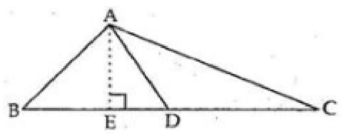
Given: ABCDE is a pentagon. EG, drawn parallel to DA, meets BA produced at G, and CF, drawn parallel to DB, meets AB produced at F.



To Prove: ar(Pentagon ABCDE) = ar (ΔDGF)
Proof:
Triangles on the same base and between the same parallels are equal in area.
Since ΔDGA and ΔAED have same base AD and lie between parallel lines AD and EG
$\therefore$ ar (ΔDGA) = ar (ΔAED).....(1)
Similarly, ΔDBC and ΔBFD have same base DB and lie between parallel lines BD and CF.
$\therefore$ ar(ΔDBF) = ar(ΔDBC).....(2)
Adding both the sides of the equations (1) and (2), we have
$\therefore$ ar(ΔDGA) + ar(ΔDBF) = ar(ΔAED) + ar(ΔBCD)
Adding ar(ΔABD) to both sides, we get,
ar(ΔDGA) + ar(ΔDBF) + ar(ΔABD)
= ar(ΔAED) + ar(ΔBCD) + ar(ΔABD)
$\therefore$ ar(ΔDGA) = ar(pentagon ABCDE)

Question 23:

Given : ABC is a triangle in which AD is the median.
To Prove: ar(ΔABD) = ar(ΔACD)
Construction: Draw AE $\perp$ BC



Proof: ar(ΔABD) $= \frac{1}{2} \times BD \times AE$

and, ar(ΔADC) $= \frac{1}{2} \times DC \times AE$

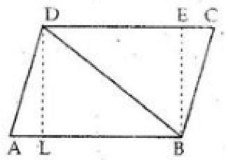
Since, BD = DC [Since D is the median]

So, ar(ΔABD) $= \frac{1}{2} \times BD \times AE$

$= \frac{1}{2} \times DC \times AE =$ ar(ΔADC)

$\therefore$ ar(ΔABD) = ar(ΔACD)

Question 24:



Given : ABCD is a parallelogram in which BD is its diagonal.

To Prove: $ar(\Delta ABD) = ar(\Delta BCD)$

Construction : Draw $DL \perp AB$ and $BE \perp CD$

Proof: $ar(\Delta ABD) = \frac{1}{2} \times AB \times DL$ .....(i)

and, $ar(\Delta CBD) = \frac{1}{2} \times CD \times BE$ .....(ii)

Now, since ABCD is a parallelogram.

$\therefore$ $AB \parallel CD$

and $AB = CD$ .....(iii)

Since distance between two parallel lines is constant,

$\Rightarrow$ $DL = BE$ .....(iv)

Form (i),(ii) , (iii), and (iv) we have

$$ar(\Delta ABD) = \frac{1}{2} \times AB \times DL$$

$$= \frac{1}{2} \times CD \times BE = ar(\Delta CBD)$$

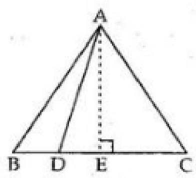
$\therefore$ $ar(\Delta ABD) = ar(\Delta CBD)$

Question 25:

Given : A ΔABC in which D is a point on BC such that;

$BD = \frac{1}{2}DC$

To Prove: $ar(\Delta ABD) = \frac{1}{3} ar(\Delta ABC)$



Consruction: Draw $AE \perp BC$

Proof: $ar(\Delta ABD) = \frac{1}{2} \times BD \times AE$ .....(1)

and, $ar(\Delta ABC) = \frac{1}{2} \times BC \times AE$ .....(2)

Given that $BD = \frac{1}{2}BC$

So, $BC = BD + DC = BD + 2BD = 3BD$

$\therefore$ $BD = \frac{1}{3} BC$ .....(3)

From (1),

$$ar(\Delta ABD) = \frac{1}{2} \times BD \times AE$$

$$= \frac{1}{2} \times \frac{BC}{3} \times AE \quad \text{[from (3)]}$$

$$\therefore ar(\Delta ABD) = \frac{1}{3} \times \left(\frac{1}{2} \times BC \times AE\right)$$

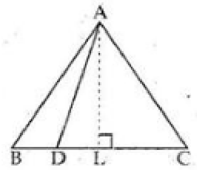
$$= \frac{1}{3} \times ar(\Delta ABC) \quad \text{[from (2)]}$$

$$\therefore ar(\Delta ABD) = \frac{1}{3} \times ar(\Delta ABC)$$

Question 26:

Given: ABC is a triangle in which D is a point on BC such that;

$$BD : DC = m : n$$



To Prove: $ar(\Delta ABD) : ar(\Delta ACD) = m : n$

Proof: $ar(\Delta ABD) = \frac{1}{2} \times BD \times AL$

and, $ar(\Delta ADC) = \frac{1}{2} \times DC \times AL$

Now, $BD : DC = m : n$

$\therefore \quad BD = DC \times \frac{m}{n}$

$$\begin{aligned} \therefore \quad ar(\Delta ABD) &= \frac{1}{2} \times BD \times AL \\ &= \frac{1}{2} \times \left(DC \times \frac{m}{n}\right) \times AL \\ &= \frac{m}{n} \times \left(\frac{1}{2} \times DC \times AL\right) \\ &= \frac{m}{n} \times ar(\Delta ADC) \end{aligned}$$

$\Rightarrow \quad \frac{ar(\Delta ABD)}{ar(\Delta ADC)} = \frac{m}{n}$

$\Rightarrow \quad ar(\Delta ABD) : ar(\Delta ADC) = m : n$