Class XII Session 2024-25
Subject - Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If the matrix A is both symmetric and skew symmetric, then **[1]**

a) A is a null matrix b) A is a zero matrix

c) A is a square matrix d) A is a diagonal matrix

2. For which value of x, are the determinants $\begin{vmatrix} 2\text{x} & -3 \\ 5 & \text{x} \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal? **[1]**

a) ± 3 b) 2

c) ± 2 d) -3

3. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ then A^{-1} = ? **[1]**

a) -adj A b) adj A

c) -A d) A

4. The function f(x) = cot x is discontinuous on the set **[1]**

a) $\left\{x = (2n+1)\frac{\pi}{2}; n \in \mathbf{Z}\right\}$ b) $\{x = 2n\pi : n \in \mathbf{Z}\}$

c) $\left\{x = \frac{n\pi}{2}; n \in \mathbf{Z}\right\}$ d) $\{x = n\pi : n \in \mathbf{Z}\}$

5. The Cartesian equations of a line are $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2}$. What is its vector equation? **[1]**

a) $\vec{r} = (2\hat{i} - 3\hat{j} - 2\hat{k})$ b) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

c) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2i + 3j - 2k)$ d) $\vec{r} = (2\hat{i} + 3\hat{j} - 2\hat{k})$

6. The degree of the differential equation $\left(1 + \frac{dy}{dx}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$ is **[1]**

a) 2

b) 1

c) 3

d) 4

7. The position of origin (0, 0) w.r.t. feasible region represented by x + y $\geq$ 1 is **[1]**

a) on the line x + y = 0

b) on the line x - y = 0

c) not in the region

d) in the region

8. Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5). **[1]**

a) $\frac{\sqrt{65}}{3}$

b) $\frac{\sqrt{65}}{2}$

c) $\frac{\sqrt{61}}{3}$

d) $\frac{\sqrt{61}}{2}$

9. If $\int\limits_{-2}^{5} f(x)dx = 4, \int\limits_{0}^{5} (1 + f(x))dx = 7,$ then the value of the integral $\int\limits_{-2}^{0} f(x)$ dx is equal to **[1]**

a) –3

b) 2

c) 3

d) 5

10. Let A = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$, then **[1]**

a) $A^2 = 0$

b) $A^2 = A$

c) $A^2 = I$

d) $A^2 = 4$

11. The value of objective function is maximum under linear constraints **[1]**

a) at (0, 0)

b) at any vertex of feasible region

c) the vertex which is maximum distance from (0, 0)

d) at the centre of feasible region

12. Consider the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and b = $4\hat{i} - 4\hat{j} + 7\hat{k}$. **[1]**
What is the scalar projection of $\vec{a}$ on $\vec{b}$?

a) $\frac{23}{9}$

b) $\frac{17}{9}$

c) 1

d) $\frac{19}{9}$

13. If the value of a third-order determinant is 12, then the value of the determinant formed by replacing each element by its cofactor will be **[1]**

a) -12

b) 12

c) 13

d) 144

14. Let A and B be independent events with P(A) = 0.3 and P(B) = 0.4. Find P (A|B) **[1]**

a) 0.27

b) 0.3

c) 0.2

d) 0.33

15. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is **[1]**

a) $y = 2x^2 - 4$

b) y = 2x

c) y = 2

d) y = 2x - 4

16. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$, then which one of the following is correct? **[1]**

a) $\vec{a}$ is parallel to $\vec{b}$

b) $\vec{a}$ = 0 or $\vec{b}$ = 0

c) $\vec{a}$ is perpendicular to $\vec{b}$

d) $\vec{a}$ and $\vec{b} \neq 0$

17. The value of p and q for which the function f(x) = $\begin{cases} \frac{\sin(p+1)x+\sin x}{x} & ,x<0 \\ q & ,x=0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{x^{\frac{3}{2}}} & ,x>0 \end{cases}$ is continuous for all x ∈ R, are **[1]**

a) p = $-\frac{3}{2}$, q = $\frac{1}{2}$

b) p = $-\frac{3}{2}$, q = $-\frac{1}{2}$

c) p = $\frac{5}{2}$, q = $\frac{7}{2}$

d) p = $\frac{1}{2}$, q = $\frac{3}{2}$

18. If the direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$, then: **[1]**

a) 0 < a < 1

b) a > 2

c) a = $\pm\sqrt{3}$

d) a > 0

19. **Assertion (A):** The function f(x) = x^2 - 4x + 6 is strictly increasing in the interval (2, ∞). **[1]**

Reason (R): The function f(x) = x^2 - 4x + 6 is strictly decreasing in the interval $(-\infty, 2)$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The modulus function f : R → R given by f(x) = |x| is neither one-one nor onto. **[1]**

Reason (R): The signum function f : R → R given by f(x) = $\begin{cases} 1, & x>0 \\ 0, & x=0 \\ -1, & x<0 \end{cases}$ is bijective.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the principal value of $\operatorname{cosec}^{-1}(-1)$. **[2]**

OR

Find the domain of f(x) = $\sin^{-1}(-x^2)$.

22. Find the least value of a such that the function f given by f(x) = x^2 + ax + 1 is strictly increasing on (1, 2). **[2]**

23. The volume of a cube is increasing at the rate of 9 cm^3 /sec. How fast is the surface area increasing when the length of an edge is 10 cm? **[2]**

OR

The total revenue in Rupees received from the sale of x units of a product is given by R(x) = $3x^2$ + 36x + 5. Find the marginal revenue, when x = 5, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at any instant.

24. Evaluate: $\int \frac{x^2 \tan^{-1} x}{(1+x^2)} dx$ **[2]**

25. Find the maximum and minimum values of f(x) = sin x in the interval [π, 2π]. **[2]**

Section C

26. Evaluate: $\int \frac{(2x+3)}{\sqrt{x^2+x+1}} dx$ **[3]**

27. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found **[3]**

to be both diamonds. Find the probability of the lost card being a diamond.

28. Evaluate: $\int x \sin^3 x \cos x\, dx$. **[3]**

OR

Evaluate the integral: $\int (x+1)\sqrt{x^2+x+1}dx$

29. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009? **[3]**

OR

Solve the differential equation $(1 + y^2)(1 + \log x)\, dx + x\, dy = 0$ given that when x = 1, y = 1

30. The feasible region for a LPP is shown in Figure. Evaluate Z = 4x + y at each of the corner points of this region. Find the minimum value of Z, if it exists. **[3]**

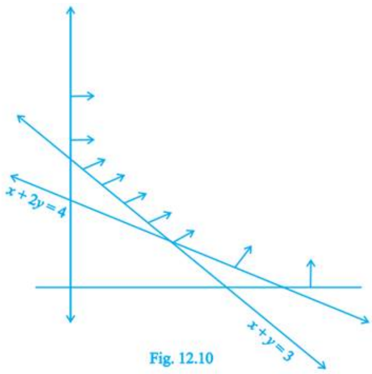


Fig. 12.10

OR

Find the maximum value of Z = 7x + 7y subject to the constraints $x \geq 0$, $y \geq 0$, $x + y \geq 2$ and $2x + 3y \leq 6$

31. If $x^x + y^x = 1$, prove that $\frac{dy}{dx} = -\left\{\frac{x^x(1+\log x)+y^x\cdot\log y}{x\cdot y^{(x-1)}}\right\}$ **[3]**

Section D

32. If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $\frac{a^2}{12}$ sq. units, then using integration, find the value of m. **[5]**

33. Let R be relation defined on the set of natural number N as follows: **[5]**

R = {(x, y): $x \in N$, $y \in N$, 2x + y = 41}. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

OR

Show that the function f : R - {3} $\rightarrow$ R - {1} given by $f(x) = \frac{x-2}{x-3}$ is a bijection.

34. Show that the matrix, $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ satisfies the equation, $A^3 - A^2 - 3A - I_3 = O$. Hence, find A^{-1} **[5]**

35. Find the distance of a point (2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. **[5]**

OR

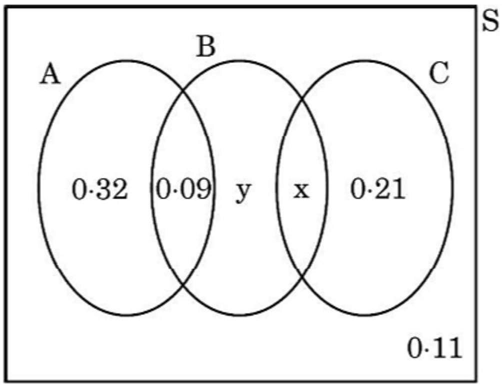
Show that the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$ intersect. Also, find their point intersection.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



i. Find the value of x. (1)
ii. Find the value of y. (1)
iii. Find $P\left(\frac{C}{B}\right)$. (2)

OR

Find the probability that a randomly selected person of the society does Yoga of type A or B but not C. (2)

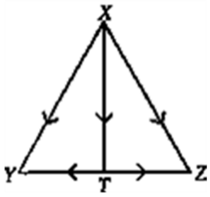
37. **Read the following text carefully and answer the questions that follow:** **[4]**

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

i. If $\vec{p}, \vec{q}, \vec{r}$ are the vectors represented by the sides of a triangle taken in order, then find $\vec{q} + \vec{r}$. (1)
ii. If ABCD is a parallelogram and AC and BD are its diagonals, then find the value of $\overrightarrow{AC} + \overrightarrow{BD}$. (1)
iii. If ABCD is a parallelogram, where $\overrightarrow{AB}$ = $2\vec{a}$ and $\overrightarrow{BC}$ = $2\vec{b}$, then find the value of $\overrightarrow{AC} - \overrightarrow{BD}$. (2)

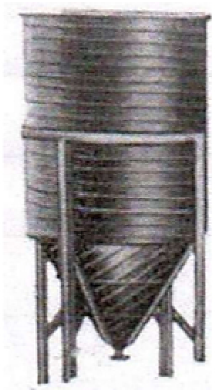
OR

If T is the mid point of side YZ of $\triangle$XYZ, then what is the value of $\overrightarrow{XY} + \overrightarrow{XZ}$. (2)



38. **Read the following text carefully and answer the questions that follow:** **[4]**

A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.



A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is 45^o.

i. Find the volume of water in the tank in terms of its radius r. (1)
ii. Find rate of change of radius at an instant when r = $2\sqrt{2}$ cm. (1)
iii. Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius r = $2\sqrt{2}$ cm. (2)

OR

Find the rate of change of height **h** at an instant when slant height is 4 cm. (2)

Solution

Section A

1.

(b) A is a zero matrix

Explanation: Only a null matrix can be symmetric as well as skew symmetric.

In Symmetric Matrix $A^T = A$,

Skew Symmetric Matrix $A^T = -A$,

Given that the matrix is satisfying both the properties.

Therefore, Equating the RHS we get A = -A i.e, 2A = 0.

Therefore A = 0, which is a null matrix.

2.

(c) ± 2

Explanation: ± 2

$\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$

$2x^2 + 15 = 20 + 3$

$2x^2 = 23 - 15$

$2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

3.

(b) adj A

Explanation: $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$|A| = \cos^2\theta - (-\sin^2\theta)$

$= \cos^2\theta + (\sin^2\theta)$

= 1 ...(i)

We know that $A^{-1} = \frac{1}{|A|}$ adj A

= adj A [From I]

4.

(d) $\{x = n\pi : n \in \mathbf{Z}\}$

Explanation: We have f(x) = cot x is continuous in $R - \{n\pi : n \in Z\}$

Since, $f(x) = \cot x = \frac{\cos x}{\sin x}$ [since, sin x = 0 at $n\pi, n \in Z$]

Hence, f(x) = cot x is discontinuous on the set $\{x = n\pi : n \in Z\}$

5.

(c) $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2i + 3j - 2k)$

Explanation: Fixed point is $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and the vector is $2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

Equation $(2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$

6. **(a)** 2

Explanation: We have $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} = \frac{d^2y}{dx^2}$

$\Rightarrow \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$

So, the degree of differential equation is 2.

7.

(c) not in the region

Explanation: Since (0, 0) does not satisfy $x + y \geq 1$

i.e., $0 + 0 \neq 1$

$\Rightarrow$ (0, 0) not lie in feasible region represented by $x + y \geq 1$.

8.

(d) $\frac{\sqrt{61}}{2}$

Explanation: Given position vector of A ,$\overrightarrow{OA} = \hat{i} + \hat{j} + 2\hat{k}$ position vector of B,$\overrightarrow{OB} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ and that of C, $\overrightarrow{OC} = \hat{i} + 5\hat{j} + 5\hat{k}$ therefore,$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) - \left(\hat{i} + \hat{j} + 2\hat{k}\right) = \hat{i} + 2\hat{j} + 3\hat{k}$ (by triangle law of vector addition) thus we may write

$\vec{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{AC} = 4\hat{j} + 3\hat{k}$,

$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 4\hat{k}$

$\Rightarrow |\overrightarrow{AB} \times \vec{AC}| = \sqrt{61}$

$\Rightarrow \frac{1}{2}|\overrightarrow{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{61}$

Therefore, the area of triangle ABC is $=\frac{1}{2}\sqrt{61}$

9.

(b) 2

Explanation: $\because \int\limits_{0}^{5} (1 + f(x))dx$ = 7

$\therefore \int\limits_{0}^{5} dx + \int\limits_{0}^{5} f(x)dx$ = 7

$\Rightarrow [x]_0^5 + \int\limits_{0}^{5} f(x)dx$ = 7

$\Rightarrow \int\limits_{0}^{5} f(x)dx$ = 7 - 5 = 2,

Also, $\int\limits_{-2}^{5} f(x)dx$ = 4

$\Rightarrow \int\limits_{-2}^{0} f(x)dx + \int\limits_{0}^{5} f(x)dx$ = 4

$\Rightarrow \int\limits_{-2}^{0} f(x)dx$ = 2

10.

(b) A^2 = A

Explanation: A = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$, then A^2 = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$ = A

11.

(b) at any vertex of feasible region

Explanation: In linear programming problem we substitute the coordinates of vertices of feasible region in the objective function and then we obtain the maximum or minimum value. Therefore, the value of objective function is maximum under linear constraints at any vertex of feasible region.

12.

(d) $\frac{19}{9}$

Explanation: Scalar projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

= $\frac{(\hat{i} - 2\hat{j} + \hat{k}) \cdot (4\hat{i} - 4\hat{j} + 7\hat{k})}{|4i - 4\hat{j} + 7\hat{k}|}$

$= \frac{(4+8+7)}{\sqrt{(4)^2 + (-4)^2 + (7)^2}} = \frac{19}{9}$

which is the required scalar projection of $\vec{a}$ and $\vec{b}$.

13.

(d) 144

Explanation: Let A is the determinant.

$\therefore$ |A| = 12

Also, we know that, if A is a square matrix of order n, then $|\text{adj } A| = |A|^{n-1}$.

For n = 3, $|\text{adj } A| = |A|^{3-1} = |A|^2$.

$\therefore$ $|\text{adj } A| = (12)^2 = 144$.

14.

(b) 0.3

Explanation: Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4 P(A/B) = P(A) = 0.3.

15.

(d) y = 2x - 4

Explanation: Let, $\frac{dy}{dx} = p$

$\therefore p^2 - xp + y = 0$

$y = xp - p^2$ .... (i)

$\Rightarrow \frac{dy}{dx} = (x - 2p)\frac{dy}{dx} + p$

$\Rightarrow p = (x - 2p)\frac{dp}{dx} + p$

$\therefore \frac{dp}{dx} = 0$

$\Rightarrow$ P is constant

from Eqn. (i), $y = x \cdot c - c^2$

$\therefore$ $y = 2x - 4$ is the correct option

16.

(b) $\vec{a} = 0$ or $\vec{b} = 0$

Explanation: Given that, $\vec{a} \cdot \vec{b} = 0$,

i.e. $\vec{a}$ and $\vec{b}$ are perpendicular to each other and $\vec{a} \times \vec{b} = 0$

i.e. $\vec{a}$ and $\vec{b}$ are parallel to each other. So, both conditions are possible iff $\vec{a} = 0$ and $\vec{b} = 0$

17. **(a)** p = $-\frac{3}{2}$, q = $\frac{1}{2}$

Explanation: p = $-\frac{3}{2}$, q = $\frac{1}{2}$

18.

(c) a = $\pm\sqrt{3}$

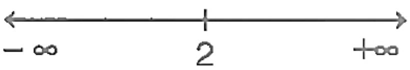
Explanation: a = $\pm\sqrt{3}$

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: We have, f(x) = x^2 - 4x + 6

or f'(x) = 2x - 4 = 2 (x - 2)



Therefore, f'(x) = 0 gives x = 2.

Now, the point x = 2 divides the real line into two disjoint intervals namely, $(-\infty, 2)$ and $(2, \infty)$.

In the interval $(-\infty, 2)$, f'(x) = 2x - 4 < 0.

Therefore, f is strictly decreasing in this interval.

Also, in the interval $(2, \infty)$, f'(x) > 0 and so the function f is strictly increasing in this interval.

Hence, both the statements are true but Reason is not the correct explanation of Assertion.

20.

(c) A is true but R is false.

Explanation: Assertion: Here, f : R $\rightarrow$ R is given by

f(x) = |x| = $\begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

It is seen that f(-1) = |-1| = 1, f(1) = |1| = 1

Therefore, f(-1) = f(1) but -1 $\neq$ 1

Therefore, f is not one-one.

Now, consider -1 $\in$ R

It is known that f(x) = |x| is always non-negative

Thus, there does not exist any element x in domain R such that f (x) = |x| = -1.

Therefore, f is not onto.

Hence, the modulus function is neither one-one nor onto.

Reason: f : R $\rightarrow$ R, f(x) = $\begin{cases} 1, & \text{if} \quad x > 0 \\ 0, & \text{if} \quad x = 0 \\ -1, & \text{if} \quad x < 0 \end{cases}$

It is seen that f(1) = f(2) = 1 but 1 $\neq$ 2.

Therefore, f is not one-one

Now, as f(x) takes only three values (1, 0 or -1), therefore for the element -2 in codomain R, there does not exist any x in domain R such that f(x) = -2

Therefore, f is not onto. Hence, the Signum function is neither one-one nor onto.

Section B

21. We know that the range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - [0]

Let cosec^{-1} (-1) = θ. Then we have, cosec θ = -1

cosec θ = -1 = -cosec $\frac{\pi}{2}$ = cosec $\left(\frac{-\pi}{2}\right)$

$\therefore \theta = \frac{-\pi}{2} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - [0]

Hence, the principal value of cosec^{-1} (-1) is equal to $\frac{-\pi}{2}$

OR

The domain of sin^{-1} x is [-1,1]. Therefore, f(x) = sin^{-1} (-x^2) is defined for all x satisfying -1 $\leq$ -x^2 $\leq$ 1

$\Rightarrow \quad 1 \geq x^2 \geq -1$

$\Rightarrow 0 \leq x^2 \leq 1$

$\Rightarrow x^2 \leq 1$

$\Rightarrow x^2 - 1 \leq 0$

$\Rightarrow$ (x - 1)(x + 1) $\leq$ 0

$\Rightarrow -1 \leq x \leq 1$

Hence, the domain of f(x) = sin^{-1} (-x^2) is [-1, 1].

22. f(x) = x^2 + ax + 1

$\Rightarrow$ f'(x) = 2x + a

Since f(x) is strictly increasing on (1, 2), therefore f'(x) = 2x + a > 0 for all x in (1, 2)

$\therefore$ On (1, 2) 1 < x < 2

$\Rightarrow$ 2 < 2x < 4

$\Rightarrow$ 2 + a < 2x + a < 4 + a

$\therefore$ Minimum value of f' (x) is 2 + a and maximum value is 4 + a.

Since f'(x) > 0 for all x in (1, 2)

$\therefore$ 2 + a > 0 and 4 + a > 0

$\Rightarrow$ a > -2 and a > -4

Therefore least value of a is - 2.

Which is the required solution.

23. Let x be the side and V be the volume of the cube at any time t Then,

$V = x^3$

Differentiating both sides with respect to t,

$\Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$\Rightarrow 9 = 3(10)^2 \frac{dx}{dt} \left[\because x = 10\text{cm and } \frac{dV}{dt} = 9cm^3/\text{sec}\right]$

$\Rightarrow \frac{dx}{dt} = 0.03\text{cm/sec}$

Let S be the surface area of the cube at any time then t ,

$S = 6x^2$

Differentiating both sides with respect to t,

$\Rightarrow \frac{dS}{dt} = 12x \frac{dx}{dt}$

$\Rightarrow \frac{dS}{dt} = 12 \times 10 \times 0.03 \left[\because x = 10\text{cm and } \frac{dx}{dt} = 0.03\text{cm/sec}\right]$

$\Rightarrow \frac{dS}{dt} = 3.6\text{cm}^2/\text{sec}$

OR

Since Marginal Revenue is the rate of change of total revenue with respect to the number of units sold, we have

Marginal Revenue (MR) = $\frac{dR}{dx}$ = 6x + 36

When x = 5, MR = 6(5) + 36 = 66

Hence, the required marginal revenue is ₹ 66.

24. Let I = $\int \frac{x^2 \tan^{-1} x}{(1+x^2)} dx$

Now let $\tan^{-1} x = t$ and x = tan t

Differentiating both sides, we get

$\frac{1}{1+x^2} dx = dt$

Now we have

$I = \int \frac{x^2 \tan^{-1} x}{(1+x^2)} dx = \int \tan^2 t \cdot t dt$ = $\int t(\sec^2 t - 1)\, dt$

= $\int t \sec^2 t dt - \int t dt$

Here t is the first function and $\sec^2 t$ as the second function.

$I = \int \text{tsec}^2\, \text{tdt} - \int \text{tdt} = t \int \sec^2 \text{tdt} - \int \left(\frac{dt}{dt} \cdot \int \sec^2 t dt\right) dt - \frac{t^2}{2}$

$= \text{t.} \tan \text{t} - \int \tan \text{tdt} - \frac{\text{t}^2}{2}$

$= t \cdot \tan t - \ln |\sec t| - \frac{t^2}{2} + c$

We know that sec t = $\sqrt{\tan^2 t + 1}$

$I = \tan^{-1} x \cdot x - \ln |\sqrt{\tan^2 t + 1}| - \frac{\tan^2 x}{2} + c$

$= x \tan^{-1} x - \ln |\sqrt{x^2 + 1}| - \frac{\tan^2 x}{2} + c$

25. The given function is, f(x) = sinx

Therefore, f'(x) = cos x

At stationary points, we must have

f'(x) = 0 ⇒ cos x = 0 ⇒ x = $\frac{3\pi}{2}$

Let us now compute the values of f(x) at x = $\pi, \frac{3\pi}{2}, 2\pi$.

Now, f(π) = sin π = 0, $f\left(\frac{3\pi}{2}\right)$ = sin $\frac{3\pi}{2}$ = -1 and f(2π) = sin 2π = 0.

The greatest and the least of these values are 0 and -1 respectively.

Hence, the maximum value of f(x) is 0 which it attains at x = π and 2π, and the minimum value is -1 which it attains at x = $\frac{3\pi}{2}$.

Section C

26. Formula to be used $- \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$\therefore \int \frac{(2x+3)}{\sqrt{x^2+x+1}} dx$

$= \int \frac{(2x+1)+2}{\sqrt{x^2+x+1}} dx$

$= \int \frac{(2x+1)}{\sqrt{x^2+x+1}} dx + \int \frac{2}{\sqrt{x^2+x+1}} dx$

Put, $x^2 + x + 1 = a^2$, (2x + 1) dx = 2ada

$\therefore \int \frac{(2x+1)}{\sqrt{x^2+x+1}} dx$

$= \int \frac{2ada}{a}$

$= \int 2\text{da}$

= 2a + c_1

$= 2\sqrt{x^2 + x + 1} + c_1$

For 2nd part of integral.

$\therefore \int \frac{2}{\sqrt{x^2+x+1}} dx$

$= 2 \int \frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}}$

$= 2 \log |\left(x + \frac{1}{2}\right) + \sqrt{\text{x}^2 + \text{x} + 1}| + \text{c}_2$

$\therefore \int \frac{(2x+1)}{\sqrt{x^2+x+1}}dx + \int \frac{2}{\sqrt{x^2+x+1}}dx$
$= 2\sqrt{x^2+x+1} + 2\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right| + c$, c is the integrating constant

27. E_1 : lost card is diamond

E_2 : lost card is not diamond

let A: two cards drawn from the remaining pack are diamonds.
$P(E_1) = \frac{13}{52} = \frac{1}{4}, P(E_2) = \frac{39}{52} = \frac{3}{4}$
$P\left(\frac{A}{E_1}\right) = \frac{12C_2}{51C_2} = \frac{12\times11}{51\times50}$
$P\left(\frac{A}{E_2}\right) = \frac{13C_2}{51C_2} = \frac{13\times12}{51\times50}$
$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$
$= \frac{\frac{13}{52}\times\frac{12\times11}{51\times50}}{\frac{13}{52}\times\frac{12\times11}{51\times50}+\frac{3}{4}\times\frac{13\times12}{51\times50}}$
$= \frac{11}{50}$

28. We can write it as $\int x \sin^2 x \sin x \cos x\, dx$
We also know that 2sinx.cosx = sin2x
$\int x \sin^2 x \sin x \cos x\, dx = \frac{1}{2}\int x \sin^2 x \sin 2x dx$
We also know that $\sin^2 x = \frac{1-\cos 2x}{2}$
$\frac{1}{2}\int x\sin^2 x\sin 2x dx = \frac{1}{2}\int x\cdot\left(\frac{1-\cos 2x}{2}\right)\sin 2x\, dx$
$= \frac{1}{2}\left[\left(\int \frac{x\sin 2x}{2}dx - \int \frac{x\cos 2x\sin 2x}{2}dx\right)\right]$
Here Sin4x = 2sin2x.cos2x
$= \frac{1}{2}\left[\left(\int \frac{x\sin 2x}{2}dx - \frac{1}{4}\int x\sin 4x\, dx\right)\right]$
Using BY PART METHOD.
Here x is first function and Sin2x and sin4x as the second function.
$\int$a.b.dx=a $\int$ b.dx-$\int\left[\frac{da}{dx}\cdot\int b\, dx\right]dx$
$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{x\int\sin 2x dx - \int\left(\frac{dx}{dx}\cdot\int\sin 2x dx\right)dx\right\}\right) - \left(\frac{1}{4}\left\{x\int\sin 4x - \int\left(\frac{dx}{dx}\cdot\int\sin 4x dx\right)dx\right\}\right)\right]$
$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2} + \int\frac{\cos 2x}{2}dx\right\}\right) - \left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4} + \int\frac{\cos 4x}{4}dx\right\}\right)\right]$
$= \frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2} + \frac{\sin 2x}{4}\right\}\right) - \left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4} + \frac{\sin 4x}{16}\right\}\right)\right] + c$
$= \frac{-x\cos 2x}{8} + \frac{\sin 2x}{16} + \frac{x\cos 4x}{32} - \frac{\sin 4x}{128} + c$

OR

Let the given integral be,
I = $\int(x+1)\sqrt{x^2+x+1}dx$
Also, x + 1 = $\lambda\frac{d}{dx}\left(x^2+x+1\right)+\mu$
$\Rightarrow$ x + 1 = λ(2x + 1) + μ
$\Rightarrow$ x + 1 = (2λ)x + λ + μ
Equating coefficient of like terms
2λ = 1
$\Rightarrow \lambda = \frac{1}{2}$
And
$\lambda + \mu$ = 1
$\Rightarrow \frac{1}{2} + \mu$ = 1
$\therefore \mu = \frac{1}{2}$
$\therefore$ I = $\frac{1}{2}\int(2x+1)\sqrt{x^2+x+1}dx$ + $\frac{1}{2}\int\sqrt{x^2+x+1}dx$
= $\frac{1}{2}\int(2x+1)\sqrt{x^2+x+1}dx$ + $\frac{1}{2}\int\sqrt{x^2+x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}dx$
= $\frac{1}{2}\int(2x+1)\sqrt{x^2+x+1}dx$ + $\frac{1}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$

Let x^2 + x + 1 = t

$\Rightarrow$ (2x + 1) dx = dt

Then,

I = $\frac{1}{2}\int\sqrt{t}dt$ + $\frac{1}{2}\left[\frac{x+\frac{1}{2}}{2}\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right.$ + $\left.\frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right|\right]$ + C

= $\frac{1}{2}\times\frac{2}{3}t^{\frac{3}{2}}$ + $\frac{1}{2}\left[\left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1}\right.$ + $\left.\frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|\right]$ + C

= $\frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}}$ + $\frac{1}{2}\left[\left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1}\right.$ + $\left.\frac{3}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+1}\right|\right]$ + C

29. Let the population at any instant (t) be y.

Now it is given that the rate of increase of population is proportional to the number of inhabitants at any instant.

$\therefore\frac{dy}{dt}\alpha y$

$\Rightarrow\frac{dy}{dt}=ky$ (k is constant)

$\Rightarrow\frac{dy}{y}=kdt$

Now, integrating both sides, we get,

log y = kt + C ....(i)

According to given conditions,

In the year 1999, t = 0 and y = 20000

$\Rightarrow$ log20000 = C .....(ii)

Also, in the year 2004, t = 5 and y = 25000

$\Rightarrow$ log 25000 = k.5 + C

$\Rightarrow$ log 25000 = 5k + log 20000

$\Rightarrow 5\mathrm{k}=\log\left(\frac{25000}{20000}\right)=\log\left(\frac{5}{4}\right)$

$\Rightarrow\mathrm{k}=\frac{1}{5}\log\left(\frac{5}{4}\right)$ ......(iii)

Also, in the year 2009, t = 10

Now, substituting the values of t, k and c in equation (i), we get

$\log y=10\times\frac{1}{5}\log\left(\frac{5}{4}\right)+\log(20000)$

$\Rightarrow\log y=\log\left[20000\times\left(\frac{5}{4}\right)^2\right]$

$\Rightarrow\mathrm{y}=20000\times\frac{5}{4}\times\frac{5}{4}$

$\Rightarrow$ y = 31250

Therefore, the population of the village in 2009 will be 31250.

OR

We have,

(1 + y^2) (1 + log x) dx + x dy = 0

$\Rightarrow$ (1 + log x) (1 + y^2) dx = -x dy

$\Rightarrow\frac{(1+\log x)}{x}dx=-\frac{1}{1+y^2}dy$

$\Rightarrow\int\frac{1+\log x}{x}dx=-\int\frac{1}{1+y^2}dy$ ...[Integrating both sides]

$\Rightarrow\int tdt=-\int\frac{1}{1+y^2}dy$, where 1 + log x = t

$\Rightarrow\frac{t^2}{2}$ = $-\tan^{-1}$ y + C

$\Rightarrow\frac{1}{2}$ (1 + log x)^2 = $-\tan^{-1}$ y + C

It is given that when x = 1, y = 1. So, putting x = 1, y = 1 in (i), we obtain

$\frac{1}{2}(1+\log 1)^2$ = $-\tan^{-1}$ 1 + C

$\Rightarrow\frac{1}{2}=-\frac{\pi}{4}+C\Rightarrow C=\frac{1}{2}+\frac{\pi}{4}$

Putting C = $\frac{1}{2}+\frac{\pi}{4}$ in (i), we obtain

$\frac{1}{2}$ (1 + log x)^2 = $-\tan^{-1}$ y + $\frac{1}{2}+\frac{\pi}{4}$

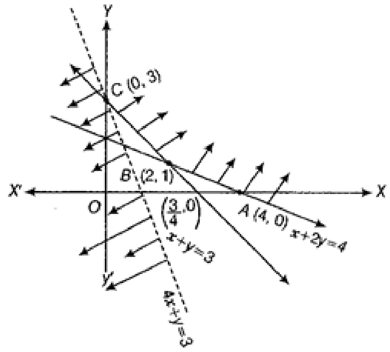
$\Rightarrow\tan^{-1}$ y = $\frac{\pi}{4}+\frac{1}{2}-\frac{1}{2}$ (1 + log x)^2

$\Rightarrow y=\tan\left\{\frac{\pi}{4}+\frac{1}{2}-\frac{1}{2}(1+\log x)^2\right\}$, which is the solution of the given differential equation.

30. From the shaded region, it is clear that feasible region is unbounded with the corner points A(4, 0), B(2, 1) and C(0, 3).

Also, we have Z = 4x + y.

[Since, x + 2y = 4 and x + y = 3 ⇒ y = 1 and x = 2]



Corner Points	Corresponding value of Z
(4, 0)	16
(2, 1)	9
(0, 3)	3 (minimum)

Now, we see that 3 is the smallest value of Z at the corner point (0, 3). Note that here we see that the region is unbounded, therefore 3 may or may not be the minimum value of Z.

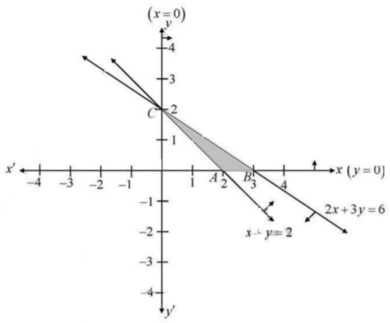
To decide this issue, we graph the inequality 4x + y < 3 and check whether the resulting open half plan has no point in common with feasible region otherwise, Z has no minimum value.

From the shown graph above, it is clear that there is no point in common with feasible region and hence Z has minimum value of 3 at (0, 3).

OR

Given Z = 7x + 7y subject to the constraints x $\geq$ 0, y $\geq$ 0, x + y $\geq$ 2 and 2x + 3y $\leq$ 6

Now, draw the line x + y = 2 and 2x + 3y = 6



And shaded region satisfied by above inequalities

here the feasible region is bounded.

The value of Z at A(3, 0), Z = 7 $\times$ 2 + 7 $\times$ 0 = 14 at B(3, 0), Z = 7 $\times$ 3 + 7 $\times$ 0 = 21 and at C(0, 2), Z = 7 $\times$ 0 + 7 $\times$ 2 = 14

Therefore,the maximum value of Z is 21, this is the required solution which occurs at B(3.0)

31. ATQ , $x^x + y^x = 1$

$\Rightarrow e^{\log x^x} + e^{\log y^x} = 1$ {As $e^{\text{Log } a} = a$}

$\Rightarrow e^{x \log x} + e^{x \log y} = 1$

Differentiating with respect to x using chain rule,

$\frac{d}{dx}(e^{x \log x}) + \frac{d}{dx}(e^{x \log y}) = \frac{d}{dx}(1)$

$\Rightarrow e^{x \log x} \frac{d}{dx}(x \log x) + e^{x \log y} \frac{d}{dx}(x \log y) = 0$

$\Rightarrow e^{x \log x} [x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)]e^{x \log x} + e^{\log y^x}\left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = 0$

$\Rightarrow x^x [x(\frac{1}{x}) + \log x(1)] + y^x[x(\frac{1}{x})\frac{dy}{dx} + \log y(1)] = 0$

$\Rightarrow x^x [1 + \log x] + y^x (\frac{dy}{dx} + \log y) = 0$

$\Rightarrow y^x \times \frac{x}{y}\frac{dy}{dx} = -[x^x(1 + \log x) + y^x \log y]$

$\Rightarrow (xy^{x-1}) \frac{dy}{dx} = -[x^x(1 + \log x) + y^x\log y]$

$\Rightarrow \frac{dy}{dx} = -\left[\frac{x^x(1+\log x)+y^x \log y}{xy^{x-1}}\right]$

LHS = RHS

Hence Proved.

Section D

32. The given equations are :

$y^2 = 16ax$ ...(1)

$y = 4mx$ ....(2)

Equation (1) represent a parabola having centre at the origin and vertex along positive x−axis.

Equation (2) represents a straight line passing through the origin and making an angle of 45 with x−axis.

POINTS OF INTERSECTION :

Put y = 4mx in (1), we get

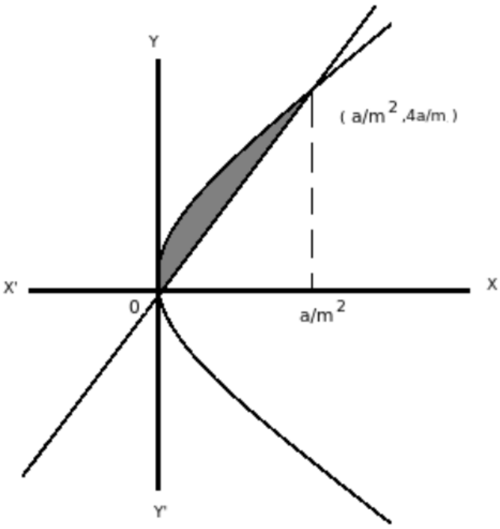
$16m^2x^2 - 16ax = 0$

$\Rightarrow 16x\,[m^2x - a] = 0$

$\Rightarrow x = 0;\ x = \frac{a}{m^2}$

When x = 0; y = 0

When $x = \frac{a}{m^2}$, then $y = \frac{4a}{m}$



Required area =Area under parabola - Area under line

$= 4\sqrt{a}\int_0^{a/m^2} \sqrt{x}dx - 4m\int_0^{a/m^2} xdx$

$= 4\sqrt{\mathrm{a}} \times \frac{2}{3}\left[\mathrm{x}^{\frac{3}{2}}\right]_0^{\frac{\mathrm{a}}{\mathrm{m}^2}} - \frac{4\mathrm{m}}{2}\left[\mathrm{x}^2\right]_0^{\frac{\mathrm{a}}{\mathrm{m}^2}}$

$= \frac{8}{3}\frac{a^2}{m^3} - \frac{2a^2}{m^3}$

$= \frac{8}{3}\frac{a^2}{m^3} - \frac{2a^2}{m^3} = \frac{2}{3}\frac{a^2}{m^3}$

Now, area = $\frac{a^2}{12}$

So, $\frac{2}{3}\frac{a^2}{m^3} = \frac{a^2}{12}$

$\Rightarrow \mathrm{m}^3 = 8$

$\Rightarrow$ m = 2

33. Given that,

R = {(1, 39), (2, 37), (3, 35) .... (19, 3), (20, 1)}

Domain = {1,2,3,.......,20}

Range = {1,3,5,7......,39}

R is not reflexive as $(2, 2) \notin R$ as

$2 \times 2 + 2 \neq 41$

R is not symmetric

as $(1, 39) \in R$ but $(39, 1) \notin R$

R is not transitive

as $(11, 19) \in R$, $(19, 3) \in R$

But $(11, 3) \notin R$

Hence, R is neither reflexive, nor symmetric and nor transitive.

OR

A = R - {3}, B = R - {1}

f : A $\rightarrow$ B is defined as $f(x)=\left(\frac{x-2}{x-3}\right)$.

Let x, y $\in$ A such that f(x) = f(y).

$\Rightarrow \frac{x-2}{x-3}=\frac{y-2}{y-3}$

$\Rightarrow$ (x - 2) (y - 3) = (y - 2) (x - 3)

$\Rightarrow$ xy - 3x - 2y + 6 = xy - 3y - 2x + 6

$\Rightarrow$ -3x - 2y = -3y - 2x

$\Rightarrow$ 3x - 2x = 3y - 2y

$\Rightarrow$ x = y

Therefore, f is one-one.

Let y $\in$ B = R - {1}

Then, y $\neq$ 1.

The function f is onto if there exists x $\in$ A such that f(x) = y.

Now, f (x) = y

$\Rightarrow \frac{x-2}{x-3}=y$

$\Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = -3y + 2

$\Rightarrow x=\frac{2-3y}{1-y}\in \mathbf{A} \qquad [y\neq 1]$

Thus, for any y $\in$ B, there exists $\frac{2-3y}{1-y}\in \mathrm{A}$ such that

$f\left(\frac{2-3y}{1-y}\right)=\frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3}=\frac{2-3y-2+2y}{2-3y-3+3y}=\frac{-y}{-1}=y$

$\therefore$ f is onto.

Hence, function f is one-one and onto.

34. Here, we have:

$A=\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}$

$A^3 = A^2.A$

$A^2=\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}$

= $\begin{bmatrix}1+0-6 & 0+0-8 & -2+0-2\\ -2+2+6 & 0+1+8 & 4-2+2\\ 3-8+3 & 0-4+4 & -6+8+1\end{bmatrix}=\begin{bmatrix}-5 & -8 & -4\\ 6 & 9 & 4\\ -2 & 0 & 3\end{bmatrix}$

$A^2.A$ = $\begin{bmatrix}-5 & -8 & -4\\ 6 & 9 & 4\\ -2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}$

= $\begin{bmatrix}-5^2+16-12 & 0-8+16 & 10-16-4\\ 6-18+12 & 0-9+16 & -12+18+4\\ -2-0+9 & 0-0-12 & 4+0+3\end{bmatrix}$

= $\begin{bmatrix}-1 & -8 & -10\\ 0 & 7 & 10\\ 7 & 12 & 7\end{bmatrix}$

Now, $A^3 - A^2 - 3A - I$

= $\begin{bmatrix}-1 & -8 & -10\\ 0 & 7 & 10\\ 7 & 12 & 7\end{bmatrix}-\begin{bmatrix}-5 & -8 & -4\\ 6 & 9 & 4\\ -2 & 0 & 3\end{bmatrix}-3\begin{bmatrix}1 & 0 & -2\\ -2 & -1 & 2\\ 3 & 4 & 1\end{bmatrix}-\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

= $\begin{bmatrix}-1+5 & -8+8 & -10+4\\ 0-6 & 7-9 & 10-4\\ 7+2 & 12-0 & 7-3\end{bmatrix}+\begin{bmatrix}-3-1 & -0-0 & 6-0\\ 6-0 & +3-1 & -6-0\\ -9-0 & -12+0 & -3-1\end{bmatrix}$

$= \begin{bmatrix} 4 & 0 & -6 \\ -6 & -2 & 6 \\ 9 & 12 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 6 \\ 6 & 2 & -6 \\ -9 & -12 & -4 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Thus, $A^3 - A^2 - 3A - I = 0$

Multiply both sides by A^{-1}, we get

$A^{-1}A^3 - A^{-1}A^2 - 3A^{-1}A - IA^{-1} = 0$

$A^2 - A - 3I = A^{-1}$ ...(since $A^{-1}A = I$)

$\Rightarrow A^{-1} = (A^2 - A - 3I)$

$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} -5-1-3 & -8-0-0 & -4+2-0 \\ 6+2-0 & 7+1-3 & 4-2-0 \\ -2-3-0 & 0-4-0 & 3-1-3 \end{bmatrix}$

$= \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

35. We have equation of the line as $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$.

$\Rightarrow x = \lambda - 5, y = 4\lambda - 3, z = 6 - 9\lambda$

Let the coordinates of L be $(\lambda - 5, 4\lambda - 3, 6 - 9\lambda)$, then Dr's of PL are $(\lambda - 7, 4\lambda - 7, 7 - 9\lambda)$.

Also, the direction ratios of given line are proportional to 1, 4, -9.

Since, P L is perpendicular to the given line.

$\therefore (\lambda - 7) \cdot 1 + (4\lambda - 7) \cdot 4 + (7 - 9\lambda) \cdot (-9) = 0$

$\Rightarrow \lambda - 7 + 16\lambda - 28 + 81\lambda - 63 = 0$

$\Rightarrow 98\lambda = 98 \Rightarrow \lambda = 1$

So, the coordinates of L are (-4, 1, -3).

$\therefore$ Required distance, PL$= \sqrt{(-4-2)^2 + (1-4)^2 + (-3+1)^2}$

$= \sqrt{36 + 9 + 4} = 7\ units$

OR

Here,it is given that

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

Here,

$\overrightarrow{a_1} = i + 2\hat{j} + 3\hat{k}$

$\overrightarrow{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{a_2} = 4\hat{i} + \hat{j}$

$\overrightarrow{b_2} = 5\hat{i} + 2\hat{j} + \hat{k}$

Thus,

$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$

$= \hat{i}(3-8) - \hat{j}(2-20) + \hat{k}(4-15)$

$\therefore \overrightarrow{b_1} \times \overrightarrow{b_2} = -5\hat{i} + 18\hat{j} - 11\hat{k}$

$\therefore |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$

$= \sqrt{25 + 324 + 121}$

$= \sqrt{470}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = (4-1)\hat{i} + (1-2)\hat{j} + (0-3)\hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 3\hat{i} - \hat{j} - 3\hat{k}$

Now,we have

$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (-5\hat{i} + 18\hat{j} - 11\hat{k}) \cdot (3\hat{i} - \hat{j} - 3\hat{k})$

$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$

= -15 - 18 + 33

= 0

Thus, the distance between the given lines is

$d = \left| \frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|} \right|$

$\therefore d = \left| \frac{0}{\sqrt{470}} \right|$

$\therefore$ d = 0 units

As d = 0

Thus, the given lines intersect each other.

Now, to find a point of intersection, let us convert given vector equations into Cartesian equations.

For that putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$\Rightarrow \vec{L}_1 : x\hat{i} + y\hat{j} + z\hat{k} = (i + 2j + 3\hat{k}) + \lambda(2i + 3\hat{j} + 4\hat{k})$

$\Rightarrow \vec{L}_2 : x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

$\Rightarrow \vec{L}_1 : (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$

$\Rightarrow \vec{L}_2 : (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$

$\Rightarrow \vec{L}_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$\therefore \vec{L}_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$

General point on L1 is

$x_1 = 2\lambda + 1$, $y_1 = 3\lambda + 2$, $z_1 = 4\lambda + 3$

Suppose, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Thus, point P satisfies the equation of line $\vec{L}_2$.

$\Rightarrow \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = \frac{4\lambda+3-0}{1}$

$\therefore \frac{2\lambda-3}{5} = \frac{3\lambda+1}{2}$

$\Rightarrow 4\lambda - 6 = 15\lambda + 5$

$\Rightarrow 11\lambda = -11$

$\Rightarrow \lambda = -1$

Thus, $x_1 = 2(-1) + 1$, $y_1 = 3(-1) + 2$, $z_1 = 4(-1) + 3$

$\Rightarrow x_1 = -1$, $y_1 = -1$, $z_1 = -1$

Therefore, point of intersection of given lines is (-1, -1, -1).

Section E

36. i. $x + 0.21 = 0.44 \Rightarrow x = 0.23$

ii. $0.41 + y + 0.44 + 0.11 = 1 \Rightarrow y = 0.04$

iii. $P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)}$

P(B) = 0.09 + 0.04 + 0.23 = 0.36

$P\left(\frac{C}{B}\right) = \frac{0.23}{0.36} = \frac{23}{36}$

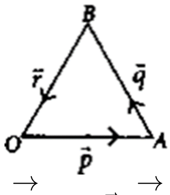
OR

P(A or B but not C)

= 0.32 + 0.09 + 0.04

= 0.45

37. i. Let OAB be a triangle such that



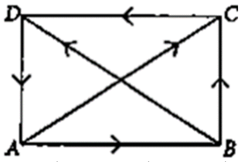
$\overrightarrow{AO} = -\vec{p}, \overrightarrow{AB} = \vec{q}, \overrightarrow{BO} = \vec{r}$

Now, $\vec{q} + \vec{r} = \overrightarrow{AB} + \overrightarrow{BO}$

$= \overrightarrow{AO} = -\vec{p}$

ii. From triangle law of vector addition,

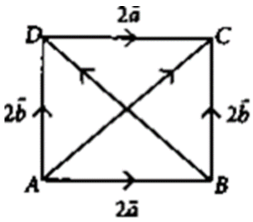
$\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$



= $\overrightarrow{AB} + 2\overrightarrow{BC} + \overrightarrow{CD}$

= $\overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB} = 2\overrightarrow{BC}$ $[\because \overrightarrow{AB} = -\overrightarrow{CD}]$

iii. In $\triangle$ABC, $\overrightarrow{AC}$ = $2\vec{a}$ + $2\vec{b}$ ...(i)



and in $\triangle$ABD, $2\vec{b}$ = $2\vec{a}$ + $\overrightarrow{BD}$ ...(ii) [By triangle law of addition]

Adding (i) and (ii), we have $\overrightarrow{AC} + 2\vec{b} = 4\vec{a} + \overrightarrow{BD} + 2\vec{b}$

$\Rightarrow \overrightarrow{AC} - \overrightarrow{BD} = 4\vec{a}$

OR

Since T is the mid point of YZ

So, $\overrightarrow{YT} = \overrightarrow{TZ}$

Now, $\overrightarrow{XY} + \overrightarrow{XZ} = (\overrightarrow{XT} + \overrightarrow{TY}) + (\overrightarrow{XT} + \overrightarrow{TZ})$ [By triangle law]

= $2\overrightarrow{XT} + \overrightarrow{TY} + \overrightarrow{TZ} = 2\overrightarrow{XT}$ $[\because \overrightarrow{TY} = -\overrightarrow{YT}]$

38. i. v = $\frac{1}{3}\pi r^2$ h = $\frac{1}{3}\pi r^3$ [as θ = 45^o gives r = h]

ii. $\frac{dv}{dt}$ = $\pi r^2 \frac{dr}{dt}$

$\Rightarrow \left(\frac{dr}{dt}\right)_{r=2\sqrt{2}}$ = $-\frac{1}{4\pi}$ cm/sec

iii. C = πrl = $\pi r\sqrt{2}$r = $\sqrt{2}\,\pi r^2$

$\frac{dC}{dt}$ = $\sqrt{2}\,\pi$ 2r $\frac{dr}{dt}$

$\left(\frac{dC}{dt}\right)_{r=2\sqrt{2}}$ = -2 cm^2/sec

OR

$l^2 = h^2 + r^2$

I = 4 $\Rightarrow$ r = h = $2\sqrt{2}$

h = r $\Rightarrow$ $\frac{dh}{dt} = \frac{dr}{dt}$ = $-\frac{1}{4\pi}$ cm/sec