

CBSE Test Paper 05
Chapter 13 Kinetic Theory

1. The number of degrees of freedom a monatomic molecule is **1**
 - a. 3.0
 - b. 4
 - c. 2
 - d. 1
2. Four liters of a diatomic ideal gas ($\gamma = 1.4$) confined to a cylinder is subject to a closed cycle. Initially, the gas is at 1.00 atm and at 300 K. First, its pressure is tripled under constant volume. Then, it expands adiabatically to its original pressure. Finally, the gas is compressed isobarically to its original volume. Find the temperature at the end of the cycle **1**
 - a. 285 K
 - b. 276 K
 - c. 300 K
 - d. 332 K
3. One mole of an ideal monatomic gas is at an initial temperature of 300 K. The gas undergoes an isovolumetric process, acquiring 500 J of energy by heat. It then undergoes an isobaric process, losing this same amount of energy by heat. Determine the new temperature of the gas **1**
 - a. 316 K
 - b. 343 K
 - c. 301 K
 - d. 333 K
4. Four liters of a diatomic ideal gas ($\gamma = 1.4$) confined to a cylinder is subject to a closed cycle. Initially, the gas is at 1.00 atm and at 300 K. First, its pressure is tripled under constant volume. Then, it expands adiabatically to its original pressure. Finally, the gas is compressed isobarically to its original volume. What was the net work done for this cycle? **1**
 - a. 356 J
 - b. 316 J

-
- c. 376 J
d. 327 J
5. In a 30.0-s interval, 500 hailstones strike a glass window with an area of 0.600 m^2 at an angle of 45.0° to the window surface. Each hailstone has a mass of 5.00 g and a speed of 8.00 m/s. If the collisions are elastic, what is the pressure on the window? **1**
- a. 1.57 Pa
b. 1.32 Pa
c. 1.97 Pa
d. 1.13 Pa
6. Although the velocity of air molecules is very fast but the fragrance of a perfume spreads at a much slower rate, explain? **1**
7. The volume of a given mass of a gas at 27°C , 1 atm is 100 cc. What will be its volume at 327°C ? **1**
8. If a molecule have N atoms and k number of constraints, how many degrees of freedom does the gas possess? **1**
9. Calculate the degree of freedom for monatomic, diatomic and triatomic gas. **2**
10. State Graham's law of diffusion and derive it. **2**
11. The molecules of a given mass of a gas have root mean square speeds of 100 ms^{-1} at 27°C and 1.00 atmospheric pressure. What will be the root mean square speeds of the molecules of the gas at 127°C and 2.0 atmospheric pressure? **2**
12. A cylinder of fixed capacity contains 44.8 L of helium gas at STP. Calculate the amount of heat required to raise the temperature of container by 15°C ? **3**
13. Two perfect gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. Find out the temperature of the mixture if the masses of molecules are m_1 and m_2 and number of molecules is n_1 and n_2 ? **3**
14. An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0 . If the container is suddenly stopped, find the change in temperature. **3**
15. Derive an expression for the pressure due to an ideal gas. **5**

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Answer

1. a. 3.0

Explanation: Degrees of freedom of a system refers to the possible independent motions a system can have.

Monoatomic gas molecule can have 3 independent motion and hence have 3 degrees of freedom (all are translational).

2. c. 300 K

Explanation: After Isochoric process

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$
$$\frac{1}{3} = \frac{300}{T_2}$$

$$T_2 = 900 \text{ K}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_4} = \frac{P_4 V_4}{T_4}$$
$$\frac{P_2 V_2}{T_2} = \frac{P_4 V_4}{T_4}$$
$$\frac{3V}{900} = \frac{1V}{T_4}$$

$$T_4 = 300 \text{ K}$$

3. a. 316 K

Explanation: for monoatomic gas $C_V = 1.5R$, $C_P = 2.5R$

At const volume,

$$Q = 500 \text{ J}$$

$$Q = nC_V \Delta T$$

$$500 = 1 \times 1.5 \times 8.31 (T_1 - 300)$$

$$T_1 = 340 \text{ K}$$

At const pressure $Q = 500 \text{ J}$

$$Q = nC_P \Delta T$$

$$500 = 1 \times 2.5 \times 8.31 (340 - T_2)$$

$$T_2 = 316 \text{ K}$$

4. d. 327 J

Explanation: Work done in isochoric process $W_1 = 0$

Work done in adiabatic process

$$W_2 = \frac{1}{\gamma-1} (P_2V_2 - P_3V_3)$$

$$P_2V_2^\gamma = P_3V_3^\gamma$$

$$V_3^\gamma = \frac{P_2}{P_3} V_2^\gamma$$

$$V_3 = \left(\frac{P_2}{P_3}\right)^{1/\gamma} V_2 = \left(\frac{3}{1}\right)^{1/1.4} \times 4 = 8.79 \text{ lit}$$

$$W_2 = \frac{1}{\gamma-1} (P_2V_2 - P_3V_3)$$

$$= \frac{1}{1.4-1} \times (12 - 8.79) \times 1.01 \times 10^5 \times 10^{-3}$$

$$= 810 \text{ J}$$

Work done in isochoric process

$$W_3 = P\Delta V = 1 \times 1.01 \times 10^5 \times (4 - 8.79) \times 10^{-3} = -483 \text{ J}$$

Net work done for cycle

$$W = W_1 + W_2 + W_3 = 0 + 810 - 483 = 327 \text{ J}$$

5. a. 1.57 Pa

Explanation: $P = \frac{F}{A}$

$$F = 500 \times \frac{\Delta p}{\Delta t} = 500 \times \frac{2mv \cos 45^\circ}{\Delta t}$$

$$= \frac{500 \times 2 \times 5 \times 10^{-3} \times 8 \times 1}{30 \times 1.41}$$

$$A = 0.6 \text{ m}^2$$

$$P = \frac{500 \times 2 \times 5 \times 10^{-3} \times 8 \times 1}{30 \times 1.41 \times 0.6} = 1.57 \text{ Pa}$$

6. Scent vapour molecules do not travel uninterrupted, they undergo their motion with a number of collisions and trace a zig-zag path. As a result their effective displacement per unit time becomes small. Which results the spreading at a much slower rate.

7. By gas equation of ideal gas

$$P_1 = 1 \text{ atm}, P_2 = 1 \text{ atm}, V_1 = 100 \text{ cc} \quad V_2 = ?$$

$$T_1 = 273 + 27 = 300 \text{ K} \quad T_2 = 327 + 273 = 600 \text{ K}$$

$$\therefore \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{1 \times 100}{300} = \frac{1 \times V_2}{600} \Rightarrow V_2 = \frac{100 \times 600}{300} = 200 \text{ cc}$$

Units of (P_1, P_2) and (V_1, V_2) must be same separately but unit of T must be in only on

Kelvin scale.

8. Degree of freedom, $f = 3N - k$. N and k are the number of atoms and number of constraints respectively.

9. The degrees of freedom of the system is given by:- $f = 3N - K$

Where, f = degrees of freedom

N = Number of Particles in the system.

K = Independent relation among the particles.

i. For a monatomic gas; $N = 1$ and $K = 0$

$$f = 3 \times 1 - 0 = 3$$

ii. For a diatomic gas ; $N = 2$ and $K = 1$

$$f = 3 \times 2 - 1 = 5$$

iii. For a triatomic gas; $N = 3$ and $K = 3$

$$f = 3 \times 3 - 3$$

$$f = 6$$

10. According to Graham's law of diffusion, the rates of diffusion of two gases are inversely proportional to the square roots of their densities.

Consider two gases A and B diffusing into each other at a Pressure P . Let S_A and S_B be their densities. The root Mean square velocities of the molecules of gases A and B will be:→

$$V_{r.m.s}^A = \sqrt{\frac{3P}{S_A}} \dots(i)$$

$$V_{r.m.s}^B = \sqrt{\frac{3P}{S_B}} \dots(ii)$$

Dividing equation (i) by (ii)

$$\frac{V_{r.m.s}^A}{V_{r.m.s}^B} = \sqrt{\frac{3P}{S_A}} \times \sqrt{\frac{S_B}{3P}} = \sqrt{\frac{S_B}{S_A}} \dots(i)$$

Now, the rate of diffusion of a gas is directly proportional to r.m.s. velocity of its molecules. If r_A and r_B are the rates of diffusion of gases A and B respectively then

$$\frac{r_A}{r_B} = \frac{V_{r.m.s}^A}{V_{r.m.s}^B} = \sqrt{\frac{S_B}{S_A}}$$

This is Graham's law.

$$\frac{r_A}{r_B} = \sqrt{\frac{S_B}{S_A}}$$

11. $v_{rms} = 100 \text{ m/s}$

$$T_1 = 27 + 273 = 300 \text{ K}$$

$$v_{rms} = ?$$

$$T_2 = 127 + 273 = 400 \text{ K}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

M = Molar mass of gas for a gas M is constant

$$\therefore v_{rms} \propto \sqrt{T}$$

$$\frac{v_{rms1}}{v_{rms2}} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{100}{v_{rms2}} = \sqrt{\frac{300}{400}}$$

$$v_{rms2} = \frac{100 \times \sqrt{400}}{\sqrt{300}} = \frac{100 \times 2 \times 10}{10\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{200 \times \sqrt{3}}{3} = \frac{200 \times 1.732}{3}$$

$$v_{rms2} = 115.4 \text{ ms}^{-1}.$$

12. At STP, 1 mole of a gas occupy 22.4 L of volume.

$$\text{Moles of helium in container, } \mu = \frac{44.8}{22.4} = 2 \text{ moles}$$

Now, helium is monoatomic, so, $C_v = \frac{3}{2} R$ (as, degrees of freedom for monoatomic gas=3)

Change in temperature,

$$\Delta T = T_2 - T_1 = 15^\circ \text{C} = 15 \text{ K}$$

Therefore, volume of gas remains constant

$$\therefore \Delta W = p\Delta V = 0$$

$$\text{and } \Delta Q = \Delta U + \Delta W$$

Amount of heat required,

$$\Delta Q = \Delta U = \mu C_v \Delta T$$

$$= 2 \times \frac{3}{2} R \times 15 = 45R$$

$$= 45 \times 8.31 = 374 \text{ J (since universal gas constant, } R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1})$$

13. In a perfect gas, there is no mutual interaction between the molecules.

$$\text{Now, kinetic energy of gas} = \frac{1}{2} m v^2$$

From the law of equi-partition of energy, kinetic energy of a single molecule with 3

degrees of freedom:

$$\frac{1}{2}mv^2 = \frac{3}{2}KT$$

Kinetic energy of n_1 molecules of one gas at temperature $T_1 = n_1 \times \left(\frac{3}{2}KT_1\right) \rightarrow (1)$

Kinetic energy of n_2 molecules of other gas at temperature T_2

$$= n_2 \times \left(\frac{3}{2}KT_2\right) \rightarrow (2)$$

n_1, n_2 = Number of molecules in the two gases

K = Boltzmann's Constant

$T_1, T_2 \rightarrow$ Temperatures of the two gases

Total K.E. = $\frac{3}{2}K(n_1T_1 + n_2T_2) \dots (3)$ (adding equation (1) & (2))

Let T be the absolute temperature of the mixture of gases

Then,

$$\text{Total Kinetic energy of the mixture} = n_1 \times \left(\frac{3}{2}KT\right) + n_2 \times \left(\frac{3}{2}KT\right)$$

$$\text{Total Kinetic energy} = \frac{3}{2}KT(n_1 + n_2) \rightarrow (4)$$

Since there is no loss of energy, hence on comparing equations (3) & (4) for total kinetic energy: $\rightarrow$

$$\frac{3}{2}KT(n_1 + n_2) = \frac{3}{2}K(n_1T_1 + n_2T_2)$$

$$\Rightarrow T(n_1 + n_2) = (n_1T_1 + n_2T_2)$$

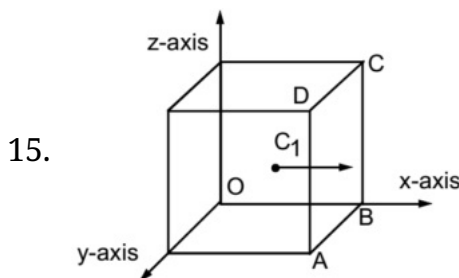
$$\Rightarrow T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2} \text{ This is required temperature of the mixture.}$$

14. Loss in K.E. of the gas = $\Delta E = \frac{1}{2}mnv_o^2$

where n = number of moles.

If its temperature changes by ΔT , then

$$\frac{3}{2}nR\Delta T = \frac{1}{2}mnv_o^2 \Rightarrow \Delta T = \frac{mv_o^2}{3R}$$



Consider an ideal gas contained in a cubical container shown in figure above, having each of side a and having volume V . Now, $V = a^3$ [$\because$ (Side) 3 = volume of cube]

Let n = number of molecules of the gas contained in the container

m = Mass of each molecule

M = mn = Total mass of the gas contained

Consider one molecule is moving with velocity C_1 such that,

$$\vec{C}_1 = v_{1x}\hat{i} + v_{1y}\hat{j} + v_{1z}\hat{k}$$

X-component of change in momentum of the molecule along +X axis = $mv_{1x} - (-v_{1x}) =$

$2mv_{1x}$ in time $t = 2a/v_{1x}$

Now rate of change of momentum (X component) = $(2mv_{1x})/t = \frac{mv_{1x}^2}{a}$, this is the X component of the force, F_{1x} by the molecule.

Now X component of pressure exerted by the molecule $P_{1x} = F_{1x}/a^2 = mv_{1x}^2/a^3$

Now taking all the molecules, net X component of pressure, $P_x =$

$$\frac{m}{a^3} \times (v_{1x}^2 + v_{2x}^2 + \dots + v_{nx}^2) \text{ (for n number of molecules)}$$

Similarly $P_y = \frac{m}{a^3} (v_{1y}^2 + v_{2y}^2 + \dots + v_{ny}^2)$ and

$$P_z = \frac{m}{a^3} (v_{1z}^2 + v_{2z}^2 + \dots + v_{nz}^2)$$

All three component of pressures are equal for all the gas molecules.

$$P = \text{Total pressure of a single molecule} = \frac{P_x + P_y + P_z}{3}$$

$$= \frac{1}{3} \left[\frac{m}{a^3} (v_{1x}^2 + v_{2x}^2 + \dots + v_{nx}^2) + \frac{m}{a^3} (v_{1y}^2 + v_{2y}^2 + \dots + v_{ny}^2) + \frac{m}{a^3} (v_{1z}^2 + v_{2z}^2 + \dots + v_{nz}^2) \right]$$

$$\therefore P = \frac{m}{3V} [C_1^2 + C_2^2 + \dots + C_n^2]$$

where $C_1^2 = v_{1x}^2 + v_{1y}^2 + v_{1z}^2$, $C_2^2 = v_{2x}^2 + v_{2y}^2 + v_{2z}^2$, ..., $C_n^2 = v_{nx}^2 + v_{ny}^2 + v_{nz}^2$

multiplying and dividing by n (total no of molecules of gas)

$$\therefore P = \frac{m \times n}{3V} \left[\frac{C_1^2 + C_2^2 + \dots + C_n^2}{n} \right]$$

$$\Rightarrow P = \frac{M}{3V} C^2 = \frac{1}{3} \rho C^2$$

$$\text{where } C^2 = \frac{C_1^2 + C_2^2 + \dots + C_n^2}{n} \text{ or } C = \sqrt{\frac{C_1^2 + C_2^2 + \dots + C_n^2}{n}}$$

C = r. m s. velocity of gas and ρ being the mass density of the gas.