Lecture - 10

Note :-

$R, L, G, C \rightarrow$ Primary constant — $\frac{V}{I}$ Waves
|
Transmission line

$\mu, \sigma, \epsilon \rightarrow$ Material constant — $E/_H$ Waves
|
Materials

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \qquad \gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \qquad \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

Note :- (1)

$$\mu = \frac{|\gamma\eta|}{\omega}$$

$$\sigma = \text{Real}\left[\gamma/\eta\right]$$

$$\epsilon = \frac{\text{Imag}\left[\gamma/\eta\right]}{\omega}$$

Note 2 :-

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \text{complex} = \frac{E_x}{H_y} = |\eta|\angle\theta$$

$$= \left|\frac{E_x}{H_y}\right|\angle\theta$$

→ In general material E_x and H_y are orthogonal in directions but have a phase shift or time delay which is η's phase

Example :- $E \rightarrow \sin(\omega t - \beta z)\, a_x$

$H \rightarrow \sin(\omega t - \beta z - \theta)\, a_y$

Note 3 :-

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\alpha^2 - \beta^2 + j2\alpha\beta = j\omega\mu\sigma - \omega^2\mu\epsilon$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)}$$

Loss Tangent or Dissipation Factor :-

$\frac{\sigma}{\omega\epsilon}$ decides the propogation aspects α or β of the EM Waves

If $\frac{\sigma}{\omega\epsilon} >> 1 \Rightarrow$ very good conductor

α is very large $\Rightarrow$ EM Wave propogation is very tough

→ EM waves cannot easily propogate in good conductors.

→ The propogation aspects depends on $\frac{\sigma}{\omega\epsilon}$ and hence on frequency - ω also

eg :- $\frac{\sigma}{\omega\epsilon} >> 1$ For humanbody upto light frequency

$\frac{\sigma}{\omega\epsilon} << 1$ For human body beyond X-ray frequen eg:- $\frac{\sigma}{\omega\epsilon} >> 1$ For earth upto MHz only

$\frac{\sigma}{\omega\epsilon} << 1$ for earth beyond microwaves

Summary:-

The term $\frac{\sigma}{\omega\epsilon} = \left|\frac{J_c}{J_d}\right|$ = loss tangent and hence decides the behaviour of the material

$J_c >> J_d \Rightarrow$ good conductor

$J_d >> J_c \Rightarrow$ good dielectric

$$\frac{J_c}{J_d} = \frac{\sigma E}{\epsilon \frac{\partial E}{\partial t}} = \frac{\sigma E}{j\omega\epsilon E} = \frac{\sigma}{\omega\epsilon}$$

Case-(III):-

EM Wave Propogation in very good conductors ($\sigma >> \omega\epsilon$)

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \simeq \sqrt{j\omega\mu\sigma}$$

$$= \sqrt{\omega\mu\sigma \angle 90} = \sqrt{\omega\mu\sigma} \angle 45$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \simeq \sqrt{\frac{j\omega\mu}{\sigma}} \simeq \sqrt{\frac{\omega\mu}{\sigma}\angle 90^\circ}$$

$$= \sqrt{\frac{\omega\mu}{\sigma}} \angle 45 = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

$$R = X = \sqrt{\frac{\omega\mu}{2\sigma}}$$

Note (1) :-

→ η's real part (R) = Cause of attenuation (α)

η's Imaginary part (X) = Cause of phase shift (β)

η's phase = 45° means E_x & H_y are delayed in time by 45°

Skin Depth or Depth of penetration :-

$$|E| = E_0 e^{-\alpha z}$$

$$E_0 = 100$$

$$\Big\downarrow \; z = \frac{1}{\alpha}, \; \frac{1}{e}$$

$$37$$

$$\Big\downarrow \; z = \frac{2}{\alpha}, \; \frac{1}{e^2}$$

$$13$$

$$\Big\downarrow \; \frac{3}{\alpha}, \; \frac{1}{e^3}$$

$$4$$

→ The first $\frac{1}{\alpha}$ distance travelled by the wave is where majority of the ~~wave~~ amplitude is attenuated. This distance is called as skin depth. Hence skin depth is the distance travelled by the wave where the wave leakage to $\frac{1}{e}$ times (37%) of the initial value.

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2\sigma}{\omega\mu}} \cdot \frac{1}{\sigma} = \frac{1}{\sigma R_s}$$

where R_s = η's real part = skin resistance

25. $V_p = \frac{\omega}{\beta} = \frac{\omega}{\alpha} = \omega\delta = 2\pi \times 2\times10^5 \times 4\times10^{-6}$

$= 1.6\pi \text{ m/s} \cong 5 \text{ m/s}$

26. $\omega' = 4\omega \qquad \delta = \sqrt{\frac{2}{\omega\mu\sigma}} \Rightarrow \delta' = \frac{\delta}{2}$

$V_p' = \omega'\delta' = 4\omega\frac{\delta}{2} = 2V_p = 10 \text{ m/s}$

Note:-

$\frac{\sigma}{\omega\epsilon} = 0$ — Free space $\longrightarrow 3\times10^8$ m/s

Ideal dielectric $\longrightarrow \frac{3\times10^8}{\sqrt{\epsilon_R}}$

$\frac{\sigma}{\omega\epsilon}\uparrow$ — General Material $\longrightarrow 10^5 - 10^6$ m/s

$\frac{\sigma}{\omega\epsilon} >> 1$ — Good conductor $\longrightarrow$ few m/s

$\frac{1}{\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)}}$

$$V_p = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)}}$$

27. $\frac{\sigma}{\omega\epsilon} = 1.73$

η's phase $= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{\omega\mu\angle 90}{K\angle\tan^{-1}\left(\frac{\omega\epsilon}{\sigma}\right)}}$

$= \sqrt{K'\angle 90 - \tan^{-1}\left(\frac{\omega\epsilon}{\sigma}\right)}$

$\frac{\tan^{-1}(\sigma/\omega\epsilon)}{2}$

η's phase $= \frac{90 - \tan^{-1}\left(\frac{\omega\epsilon}{\sigma}\right)}{2} = \frac{\tan^{-1}\sqrt{3}}{2} = \frac{60}{2}$

$= 30°$

Note :-

η's phase = E_x to H_y Phase

	Free space $\longrightarrow$	$\theta = 0$	
$\frac{\sigma}{\omega\epsilon} = 0$	Ideal dielectric $\longrightarrow$		
$\frac{\sigma}{\omega\epsilon} \uparrow$	General Material $\longrightarrow$	$20^\circ - 30^\circ$	$\frac{\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}{2}$
$\frac{\sigma}{\omega\epsilon} >> 1$	Good conductor $\longrightarrow$	45°	

28.

$$J_c = J_d$$

$$\frac{\sigma}{\omega\epsilon} = 1$$

$$\Rightarrow \frac{\sigma}{2\pi f\epsilon} = 1$$

$$f = \frac{10^{-2}}{2\pi \times 4 \times \frac{1}{36\pi \times 10^9}} = 45\,MHz$$

29.

$$\frac{I_c}{I_d} = \frac{J_c}{J_d} = \frac{\sigma}{\omega\epsilon}$$

$$\Rightarrow I_d = \frac{I_c\omega\epsilon}{\sigma} = \frac{1 \times 2\pi \times 50 \times 8.8 \times 10^{-12}}{58}$$

Ans - (B)

30.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

$$= R + jX$$

Ans - D

31. $\eta = 0.02 \angle 45^\circ$ Ans-(a)

32. lossy dielectric, $\frac{\sigma}{\omega\epsilon} << 1$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j\omega\mu \cdot j\omega\epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)}$$

$$= j\omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{j2\omega\epsilon}\right) = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} + j\omega\sqrt{\mu\epsilon}$$

Power, Power density and Poynting Vector :-

→ Energy that can be transformed into other formats is said to be a power format

$$\frac{\text{Joules}}{\text{second}} = \text{Watts}$$

Using Maxwells IV Equation

$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

Take $\cdot E$ on both sides

$$(\nabla \times H)\cdot E = \sigma E^2 + \epsilon \frac{\partial E}{\partial t}\cdot E$$

$$\Rightarrow \quad -\nabla\cdot(E \times H) + H\cdot(\nabla \times E) = \sigma E^2 + \epsilon\frac{\partial E}{\partial t}\cdot E$$

$$\Rightarrow \quad -\nabla\cdot(E\times H) - \sigma E^2 = \epsilon\frac{\partial E}{\partial t}\cdot E + \mu\frac{\partial H}{\partial t}\cdot H$$

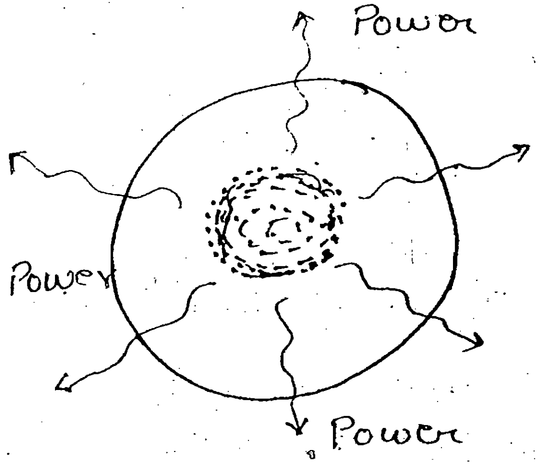
$$= \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H\right)^2$$

Take $\int dv$ on both sides

$$\int \nabla\cdot(E\times H)\,dv + \int \sigma E^2\,dv = -\frac{\partial}{\partial t}\int\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right)dv$$

Using Divergence theorem

$\oint (E \times H)\,ds + \int \sigma E^2 dV$ = Rate of decrease of E/H Energy inside a volume



The LHS is a EM wave power crossing the surface

Hence $E \times H$ = Power density of EM Wave
= Strength of power of EM Wave
= Poynting Vector of the EM Wave
= It has the direction of propogation

$E(z,t)_x \times H(z,t)_y = P(z,t)_z$

= Power density at a time at a point in space

= Instantaneous Poynting Vector

$P(z)_{avg.}$ = Time Average Poynting Vector

$= E_{RMS} \cdot H_{RMS} = \frac{E_0 e^{-\alpha z}}{\sqrt{2}} \cdot \frac{H_0 e^{-\alpha z}}{\sqrt{2}}$

$P(z)_{avg.} = \frac{1}{2} E_0 H_0 e^{-2\alpha z} = \frac{1}{2} \frac{E_0^2}{\eta} e^{-2\alpha z} = \frac{1}{2} \eta H_0^2 e^{-2\alpha z}$

Note 1 :-

The average power decays at 2α rate exponentially in the medium

→ This power lost in the wave is acquired by the medium as ohmic power

$$J \cdot E \quad \text{or} \quad \sigma E^2 \quad \text{Watts}/m^3$$

Note 2 :-

Alternatively

$$P(z)_{avg.} = \frac{1}{2}(E \times H^*)$$

* conjugate operation means average power is independent of harmonic and its phase and depends only on amplitude.

> In free space

$$P_{avg} = \frac{1}{2} E_o H_o = \frac{1}{2}\frac{E_o^2}{\eta} = \frac{1}{2}\eta H_o^2$$

Workbook :-

3. $P_{avg.} = \frac{1}{2}\eta H_o^2 \, (-a_y)$

$H_o = 0.1$

$= \frac{1}{2} \cdot 120\pi \cdot (0.1)^2 (-a_y) \longrightarrow \underline{B}$

34. $W = \int P_{avg.} \, ds$

$x = \text{constant}$

$= \int \frac{1}{2}\frac{E_o^2}{\eta} a_x \cdot ds \, a_x$

$= \frac{1}{2} \cdot \frac{60 \cdot 60}{120\pi} \pi \times 4^2 = 240W$

35 C

36 (1) $H = 50 \sin(\omega t - \beta z) a_x + 150 \sin(\omega t - \beta z) a_y$

$P_{avg} = \frac{1}{2}\eta H_o^2 \, (a_z)$

$H = 50 \sin(\omega t - \beta z) \left(\frac{a_x + 3a_y}{\sqrt{10}}\right) \sqrt{10}$

$= (50\sqrt{10})\ \sin(\omega t - \beta z) \left(\frac{a_x + 3a_y}{\sqrt{10}}\right)$

$P_{avg} = \frac{1}{2} \cdot 120\pi \cdot (50\sqrt{10})^2\ a_z$

$H_0 = 50\sqrt{10}$

(ii) $H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_x$

$= 50 \sin(\omega t - \beta z)\, a_x\ (1 + 3j)$

$= 50\ (1 + 3j)\ \sin(\omega t - \beta z)\, a_x$

Note:-

orthogonal field components in time ~~and~~ or space have same effect

(iii) $H = 50 \sin(\omega t - \beta z) a_x + 150 \cos(\omega t - \beta z) a_y$

$H = 50 \sin(\omega t - \beta z)\ (a_x + 3j a_y)$

$P_{avg} = \frac{1}{2} (E \times H^*)$

$E = (50\eta) \sin(\omega t - \beta z)(-a_y) + (150\eta) \cos(\omega t - \beta z)(a_x)$

$= (50\eta)\ \sin(\omega t - \beta z)\ (3j a_x - a_y)$

$P_{avg} = \frac{1}{2} \cdot 50 \cdot 120\pi\ (3j a_x - a_y) \times 50\ (a_x - 3j a_y)$

$= \frac{1}{2} \cdot 120\pi \cdot 50^2\ (a_z + 9a_z)$

$= \frac{1}{2} \cdot 120\pi\ (50\sqrt{10})^2\ a_z$

37.

$$E = (a_x + j a_y)\, e^{jkz - j\omega t}$$

$$H = \frac{k}{\omega\mu}(a_y + j a_x)\, e^{jkz - j\omega t}$$

$$P_{avg.} = \frac{1}{2}(a_x + j a_y) \times \frac{k}{\omega\mu}(a_y - j a_x)$$

$$= \frac{k}{2\omega\mu}(a_z - a_z) = 0 \longrightarrow \text{Null vector}$$

<u>Conventional</u> :–

$$E = 100\,(j a_x + 2a_y - j a_z)\, e^{j\omega t}$$

$$H = (-a_x + j a_y + a_z)\, e^{j\omega t}$$

$$P_{avg.} = \frac{1}{2}(E \times H^*) = \frac{1}{2}\, 100\,(j a_x + 2a_y - 2a_z) \times (-a_x - j a_y + a_z)$$

$$= 50 \begin{vmatrix} a_x & a_y & a_z \\ j & 2 & -j \\ -1 & -j & 1 \end{vmatrix}$$

$$= 50\,(3a_x + 3a_z)$$

$$|P_{avg.}| = 150\sqrt{2}$$

$$P_{avg.}\ \text{direction} = (a_x + a_z)$$

$$\text{Unit } P_{avg.}\ \text{direction} = \frac{a_x + a_z}{\sqrt{2}}$$

Wave Polarization :-

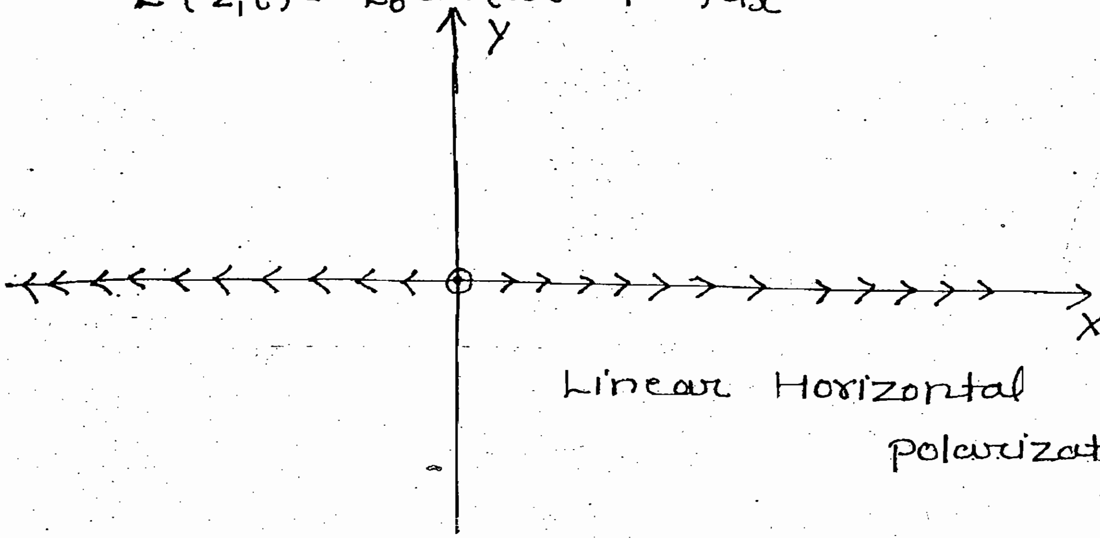
It is the electric field orientation & the possible planar components of the E field satisfying the basic transverse nature

EM wave is propogating along z-axis in the following cases :-

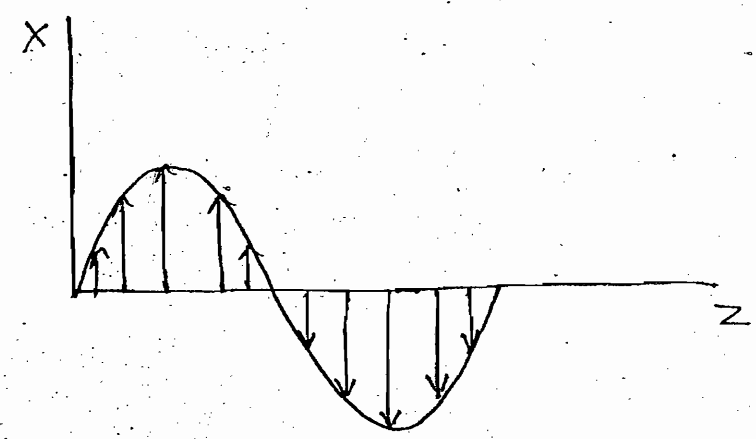
Case (I) :-

E field is oriented in x-direction

$$E(z,t) = E_0 \sin(\omega t - \beta z)\, a_x$$



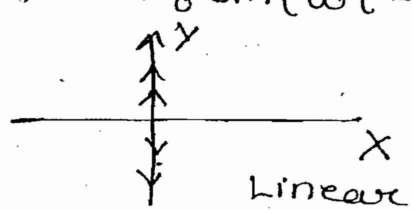
Linear Horizontal polarization



Case -(II) :-

E. field is oriented in y-direction

$$E(z,t) = E_0 \sin(\omega t - \beta z)\, a_y$$

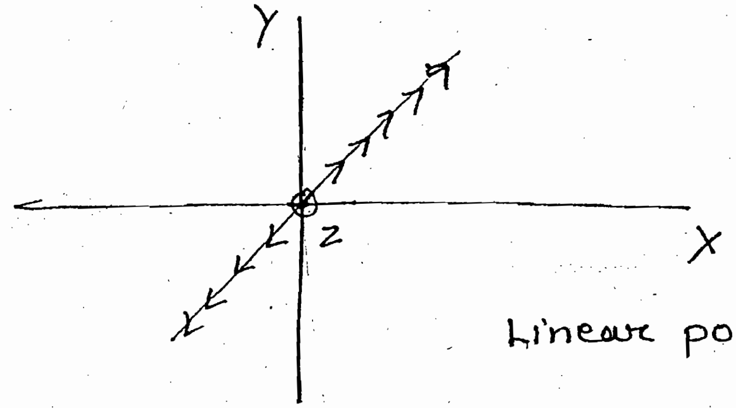


Linear vertical polarization

Case-(III) :-

$\bar{E}$ field is oriented in x & y directions

$$E(z,t) = E_1 \sin(\omega t - \beta z)\, a_x + E_2 \sin(\omega t - \beta z)\, a_y$$



Linear polarization

Note :-

If the EM waves has single E field components or two planar components both inphase the wave is said to be linearly polarized

Case-(IV) :-

E field is oriented in x & y directions

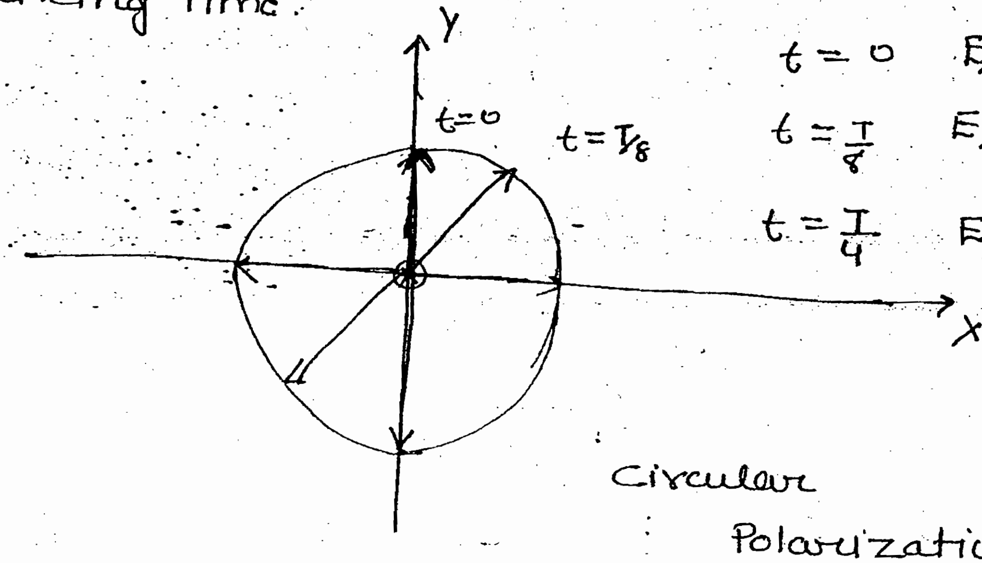
$$E(z,t) = E_0 \sin(\omega t - \beta z)\, a_x + E_0 \cos(\omega t - \beta z)\, a_y$$

→ Polarization is identified by studying the trace of the E field on the $z=0$ plane for various advancing time.

$t = 0$ $E_x = 0$, $E_y = E_0$

$t = \frac{T}{8}$ $E_x = \frac{E_0}{\sqrt{2}}$, $E_y = \frac{E_0}{\sqrt{2}}$

$t = \frac{T}{4}$ $E_x = E_0$, $E_y = 0$



Circular Polarization.

Summary:-

If the EM Waves has two planar components of E field both out of phase at 90° and equal amplitude, the wave is circularly polarised.

Note:-

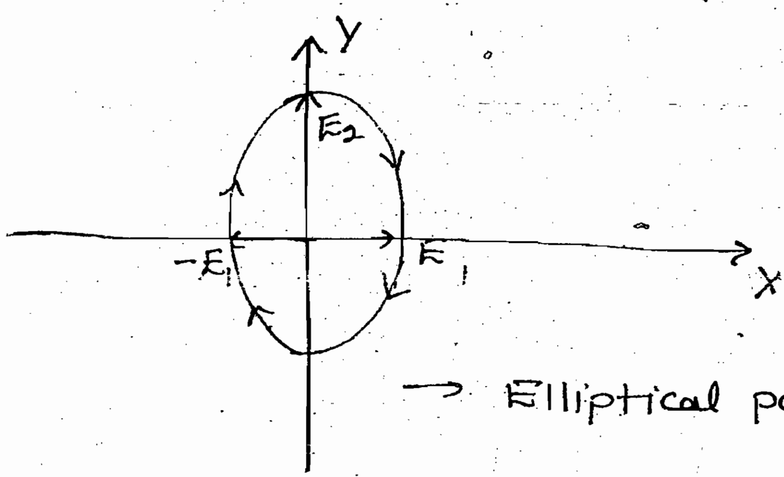
Sense of rotation left or right, if the left hand thumb points towards propagation direction and the closed finger along advancing time, the wave is left circularly polarised

eg:- out of paper - propogation clockwise time advancement - left circular

Case-(V):-

E field is oriented in x & y directions

$$E(z,t) = E_1 \sin(\omega t - \beta z)\, a_x + E_2 \cos(\omega t - \beta z)\, a_y$$

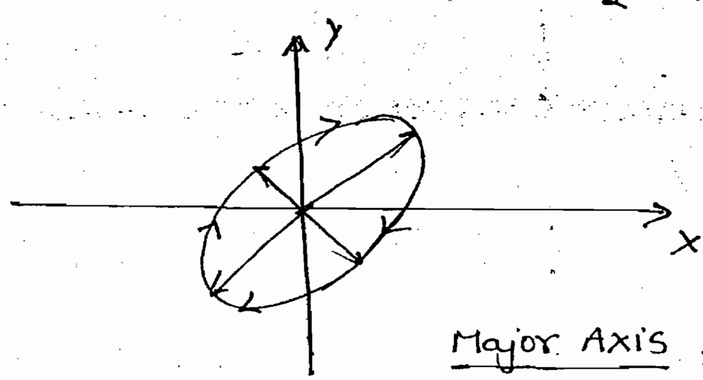


→ Elliptical polarization

Case-(VI):-

E field is oriented in x & y direction

$$E(z,t) = E_1 \sin(\omega t - \beta z)\, a_x + E_2 \sin(\omega t - \beta z + \theta)\, a_y$$



$$\frac{\text{Major Axis}}{\text{Minor Axis}} = AR = [1, \infty)$$

Circle ↙ (1), Linear ↓ (∞)

Workbook :—

3 (I) $\vec{E} = 25 \sin(\omega t + 4x)(a_y + 6a_z)$

$\longrightarrow$ Linear

II) $\vec{E} = 25 \sin(\omega t + 4x) a_y + 25 \cos(\omega t + 4x) a_z$

$\longrightarrow$ Circular

III) $\vec{E} = 25 \sin(\omega t + 4x + 60^\circ) a_y + 25 \cos(\omega t + 4x + 60^\circ) a_z$

$\longrightarrow$ Circular

(IV) $\vec{E} = 25 \sin(\omega t + 4x) a_y + (25\sqrt{2}) \sin(\omega t + 4x + 45^\circ) a_z$

$\longrightarrow$ Elliptical

V) Elliptical

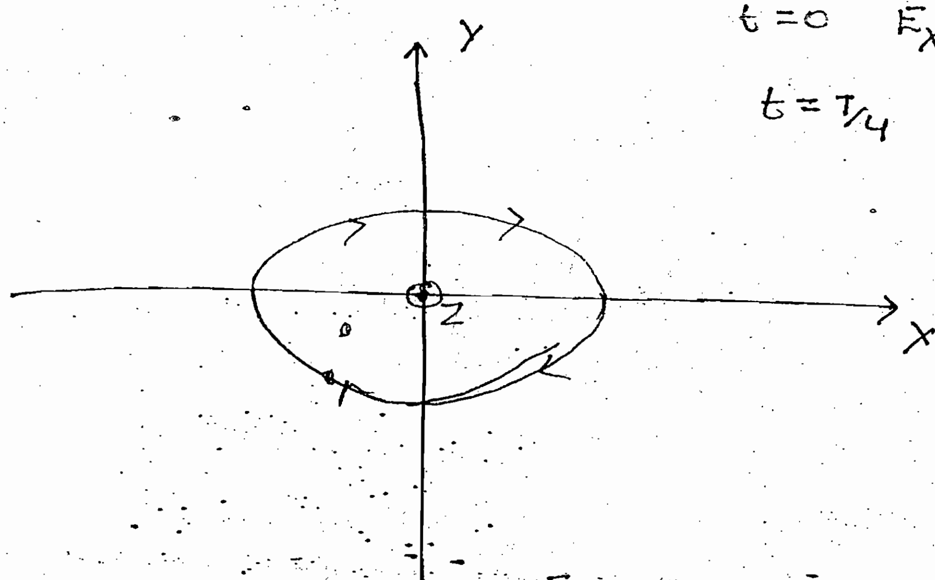
VI) Circular

9. $\theta = 60^\circ \rightarrow$ elliptical

10. $\theta = 90^\circ$

$t = 0 \quad E_x = E_1 \quad E_y = 0$

$t = T/4 \quad E_x = 0 \quad E_y = E_2$



Summary :—

$$\left.\begin{array}{l} E_x \longrightarrow E_1 \cos \omega t \\ E_y \longrightarrow E_2 \sin \omega t \end{array}\right\} e^{-jkz}$$

E_x	E_y	Prop.	Polarization
cos	−sin	z	Left
cos	sin	z	Right
cos	−sin	−z	Right
−sin	cos	z	Right

48

$(a_y + j a_z)\, e^{j\beta x}$

$a_y \downarrow \sin \downarrow \cos$

$a_z \downarrow \cos \downarrow -\sin$