MATHMETICS SAMPLE PAPER
CLASS – 12

SECTION-A

1. If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} - 3\hat{k}$ then find the projection of $\vec{a}$ and $\vec{b}$

2. Find λ, if the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = \lambda + 3\hat{k}$ are coplanar.

3. If a line makes angles 90^o, 60^o and θ with x, y and z-axis respectively, where θ is acute, then find θ.

4. Write the element a_{23} of a 3×3 matrix $A = (a_{ij})$ whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$

5. Find the differential equation representing the family of curves $\upsilon = \frac{A}{r} + B,$ where A and B are arbitrary constants.

6. Find the integrating factor of the differential equation:

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$$

SECTION-B

7. If A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$**,** find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.

OR

If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$

8. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants, find the value of f(2x) – f(x)

9. Find: $\int \frac{dx}{\sin x + \sin 2x}$

OR

Integrate the following w.r.t.x

$$\frac{x^2 - 3x + 1}{\sqrt{1-x^2}}$$

10. Evaluate: $\int_{-n}^{n} (\cos ax - \sin bx)^2 dx$

11. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

OR

An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained.

12. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r}\times\hat{i}).(\vec{r}\times\hat{j}) + xy$.

13. Find the distance between the point (-1, -5, -10) and the pint of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane x – y +z = 5.

14. If sin $\left[\cot^{-1}(x+1)\right] = \cos(\tan^{-1} x)$, then find x.

15. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), x^2 \leq$, then find $\frac{dy}{dx}$.

16. If x = a cos $\theta + b\ \sin\ \theta$, y = a sin θ - b cos θ, show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

17. The side of an equilateral triangle is increasing at the rate of 2cm/s. At what rate is its area increasing when the side of the triangle is 20 cm?

18. Find $\int (x+3)\sqrt{3-4x-x^2}dx$

19. Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand made fans, mats and plates from recycled material at a cost of Rs. 100 and Rs. 50 each. The number of articles sold are given below:

Article / School	A	B	C
Hand-fans	40	25	35
Mats	50	40	50
Pates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also find the total funds collected for the purpose.
Write one value generated by the above situation.

20. Let N denote the set of all natural numbers and R be relation on $N\times N$ defined by (a, b) r (c, d) if ad(b +c) = bc(a + d). Show that R is an equivalence relation.

21. Using integration find the area of the triangle formed by positive x-axis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$.

OR

Evaluate $\int_1^3 (e^{2-3x} + x^2 + 1)dx$ as a limit of a sum.

22. Solve the differential equation:

$$(\tan^{-1} y - x)dy = (1+y^2)dx$$

OR

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ given that y = 1, when x =x.

23. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z}{1}$ intersect, then find the value of k and hence find the equation of the plane containing these lines.

24. If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$, then find P(A) and P(B).

25. Find the local maxima and local minima, of the function f(x) = sin x – cos x, $0 < x < 2\pi$. Also find the local maximum and local minimum values.

26. Find graphically, the maximum value of Z = 2+ 5y, , subject to constraints given below: $2x + 4y \le 8$, $3x + 4y \le 6$, $x + y \le 4$, $x \ge 0$, $y \ge 0$

MATHEMATICS SAMPLE PAPER

SOLUTIONS

SECTION–A

1. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$= \frac{7\times 2 + 1\times 6 + (-4)\times 3}{\sqrt{2+6^2+3^2}} = \frac{14+6-12}{\sqrt{49}} = \frac{8}{7}$

2. Since $\vec{a}, \vec{b}$ and $\vec{c}$ vectors are coplanar.

$\therefore \quad \left[\vec{a}\,\vec{b}\,\vec{c}\right] = 0$

$\Rightarrow \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \qquad \Rightarrow \quad 1(-3+\lambda) - 3(6+0) + 1(2\lambda+0) = 0$

$\Rightarrow \quad -3+\lambda-18+2\lambda = 0 \qquad \Rightarrow \quad 3\lambda - 21 = 0$

$\Rightarrow \quad \lambda = 7$

3. Let l, m, n, be direction cosine of given line.

$\therefore \quad \boldsymbol{l} = \cos 90^o = 0; \qquad m = \cos 60^o = \frac{1}{2}$ and n = cos θ and n = cos θ

$\because \boldsymbol{l}^2 + m^2 + n^2 = 1 \qquad \Rightarrow \qquad 0 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = \frac{3}{4} \qquad \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ ($\because$ θ is acute angle)

$\Rightarrow \qquad \theta = \frac{\pi}{6}$

4. $a_{23} = \frac{|2-3|}{2} = \frac{|-1|}{2} = \frac{1}{2}$

5. Given family of curve is $\quad v = \frac{A}{r} + B$

Differentiating with respect to r, we get

$$\frac{dv}{dr} = \frac{-A}{r^2}$$

Again differentiating with respect to r, we get

$$\frac{d^2v}{dr^2} = \frac{2A}{r^3} \qquad \Rightarrow \frac{d^2v}{dr^2} = \frac{2}{r}.\frac{A}{r^2}$$

$$\Rightarrow \quad \frac{d^2v}{dr^2} = \frac{2}{r}.\left(-\frac{dv}{dr}\right) \qquad \Rightarrow r\frac{d^2v}{dr^2} = -2\frac{dv}{dr}$$

$$\Rightarrow r\frac{d^2v}{dr^2} = +2\frac{dv}{dr} = 0$$

6. Give differential equation is

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1 \qquad \Rightarrow \left(\frac{e^{-2\sqrt{x}}-y}{\sqrt{x}}\right).\frac{dx}{dy}=1$$

$$\Rightarrow \frac{dx}{dy}=\frac{\sqrt{x}}{e^{-2\sqrt{x}}-y} \qquad \Rightarrow \frac{dx}{dx}=\frac{e^{-2\sqrt{x}}-y}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}} \qquad \Rightarrow \frac{dy}{dx}+\frac{1}{\sqrt{x}}.y=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$$IF=e\int\frac{1}{\sqrt{x}}dx=e\int x^{\frac{-1}{2}}dx=e^{2\sqrt{x}}$$

SECTION-B

7.
$$A=\begin{vmatrix}2&0&1\\2&1&3\\1&-1&0\end{vmatrix}$$

$$\therefore A^2=A\times A=\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}4+0+1&0+0-1&2+0+0\\4+2+3&0+1-3&2+3+0\\2-2-+0&0-1+0&1-3+0\end{bmatrix}=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}$$

Now, $A^2-5A+4I=\begin{vmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{vmatrix}-5\begin{vmatrix}2&0&1\\2&1&3\\1&-1&0\end{vmatrix}+4\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$$=\begin{bmatrix}-1&-1&-3\\-1&-3&-10\\-5&4&2\end{bmatrix}$$

Now given $A^2-5A+4I+X=0$

$$\Rightarrow \begin{bmatrix}-1&-1&-3\\-1&-3&-10\\-5&4&2\end{bmatrix}+X=0$$

$$\Rightarrow X=-\begin{bmatrix}-1&-1&-3\\-1&-3&-10\\-5&4&2\end{bmatrix} \qquad \Rightarrow X=\begin{bmatrix}1&1&3\\1&3&10\\5&-4&-2\end{bmatrix}$$

OR

Given

$$A=\begin{bmatrix}1&-2&3\\0&-1&4\\-2&2&1\end{bmatrix}$$

$$A'=\begin{bmatrix}1&0&-2\\-2&-1&2\\3&4&1\end{bmatrix}$$

$$|A'|=1(-1-8)-0-2(-8+3)=-9+10=1\neq 0$$

Hence, $(A')^{-1}$ will exist.

$A_{11}=\begin{vmatrix}-1&2\\4&1\end{vmatrix}=-1-8=-9;$ $A_{12}=-\begin{vmatrix}-2&2\\3&1\end{vmatrix}=-(-2-6)=8$

$A_{13}=-\begin{vmatrix}-2&-1\\3&4\end{vmatrix}=-8+3=-5;$ $A_{21}=-\begin{vmatrix}0&-2\\4&1\end{vmatrix}=-(0+8)=-8$

$A_{22}=\begin{vmatrix}1&-2\\3&1\end{vmatrix}=1+6=7;$ $A_{23}=-\begin{vmatrix}1&0\\3&4\end{vmatrix}=-(4-0)=-4$

$A_{31}=\begin{vmatrix}0&-2\\-1&2\end{vmatrix}=0-2=-2;$ $A_{32}=-\begin{vmatrix}1&-2\\-2&2\end{vmatrix}=-(2-4)=2$

$A_{33}=\begin{vmatrix}1&0\\-2&-1\end{vmatrix}=-10=-1$

Adj(A’) $=\begin{bmatrix}-9&8&-5\\-8&7&-4\\-2&2&-1\end{bmatrix}^7=\begin{bmatrix}-9&-8&-2\\8&7&2\\-5&-4&-1\end{bmatrix}$

$$(A')^{-1}\frac{1}{1}\begin{bmatrix}-9&-8&-2\\8&7&2\\-5&-4&-1\end{bmatrix}=\begin{bmatrix}-9&-8&-2\\8&7&2\\-5&-4&-1\end{bmatrix}$$

8. Here f(x) = $\begin{vmatrix}a&-1&0\\ax&a&-1\\ax^2&ax&a\end{vmatrix}$

Taking a common from C_1,we get

f(x) = $\begin{vmatrix}a&-1&0\\ax&a&-1\\ax^2&ax&a\end{vmatrix}$

applying $C_2\to C_2+C_{1,}$ we get

f(x) =a$\begin{vmatrix}1&0&0\\x&a+x&-1\\x^2&ax+x^2&a\end{vmatrix}$

Expanding along R_1, we get

$f(x) = a[1(a^2 + ax + ax + x^2) - 0 + 0\}$

$\Rightarrow f(x) = a(a^2 + 2ax + x^2)$

$\Rightarrow f(x) = a(a+x)^2$

Now, $f(2x)-f(x)=a(a+2x)^2-a(a+x)^2$

$=a\{(a+2x)^2-(a+x)^2\}=a(a+2x+a+x)(a+2x-a-x)$

$=ax(2a+3x)$

9. Here

$I = \int \frac{1}{\sin x + \sin 2x} dx$

$\Rightarrow I = \int \frac{1}{\sin x + 2\sin x \cos x} dx \quad \Rightarrow I = \int \frac{1}{\sin x(1+2\cos x)} dx$

$\Rightarrow I = \int \frac{\sin x}{\sin^2 x(1+2\cos x)} dx \quad \Rightarrow I = \int \frac{\sin x}{(1-\cos^2 x)(1+2\cos x)} dx$

Let $\cos x = z \quad \Rightarrow - \sin x \, dx = dz$

$\Rightarrow \quad I = \int \frac{-dz}{(1-z^2)(1+2z)} \quad \Rightarrow \quad I = -\int \frac{dz}{(1+z)(1-z)(1+2z)}$

Here , integrand is proper rational function. Therefore by the form of partial function , we can write

$\frac{1}{(1+z)(1-z)(1+2z)} = \frac{A}{1+z} + \frac{B}{1-z} + \frac{C}{1+2z}$ ....(i)

$\Rightarrow \quad \frac{1}{(1+z)(1-z)(1+2z)} = \frac{A(1-z)(1+2z) + B(1+z)(1+2z) + C(1+z)(1-z)}{(1+z)(1-z)(1-2z)}$

$1= A(1-z)(1+2z) +B(1+z)(1+2z)+C(1+z)(1 - z)$ ....(ii)

Putting the value of z=-1 in (ii) we get

$\Rightarrow 1 = -2A + 0 + 0 \quad \Rightarrow \quad A = -\frac{1}{2}$

Again, putting the value of z=1 in (ii),we get

$\Rightarrow 1=0+B.2.(1+2)+0 \quad \Rightarrow \quad 1=6B \quad \Rightarrow$ **B=** $\frac{1}{6}$

Similarly, putting the value of z= $-\frac{1}{2}$ **in(ii) ,we get**

$\Rightarrow 1 = 0 + 0 + C\left(\frac{1}{2}\right)\left(\frac{3}{2}\right) \qquad \Rightarrow 1=\frac{3}{4}C \qquad \Rightarrow c=\frac{4}{3}$

Putting the value of A,B,C in (i) we get

$\frac{1}{(1+z)(1-z)(1+2z)} = -\frac{1}{2(1+z)} + \frac{1}{6(1-z)} + \frac{4}{3(1+2Z)}$

$I = -\int \left[-\frac{1}{2(1+z)} + \frac{1}{6(1-z)} + \frac{4}{3(1+2Z)} \right] dz$

$$I=\int\left[\frac{1}{2(1+z)}-\frac{1}{6(1-z)}-\frac{4}{3(1+2Z)}\right]dz$$

$$I=\frac{1}{2}\log|1+z|+\frac{1}{6}\log|1-z|-\frac{4}{3\times 2}\log|1+2z|+C$$

putting the value of z , we get

$$\Rightarrow \quad I=\frac{1}{2}\log|1+\cos x|+\frac{1}{6}\log|1-\cos x|-\frac{2}{3}\log|1+2\cos x|+C$$

OR

$$I=\int\frac{x^2-3x+1}{\sqrt{1-x^2}}dx=\int\frac{x^2-1+2-3x}{\sqrt{1-x^2}}dx$$

$$=\int\frac{-(1-x^{2)}}{\sqrt{1-x^2}}dx+2\int\frac{dx}{\sqrt{1-x^2}}-3\int\frac{xdx}{\sqrt{1-x^2}}$$

Let $=-\int\sqrt{1-x^2}dx+2\int\frac{dx}{\sqrt{1-x^2}}-3\int\frac{xdx}{\sqrt{1-x^2}}$

$$=-\frac{1}{2}x\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}x+2\sin^{-1}x+3\sqrt{1-x^2}+C$$

$$=\frac{3}{2}\sin^{-1}x-\frac{1}{2}x\sqrt{1-x^2}+3\sqrt{1-x^2}+C$$

10. Here $I=\int_{-\pi}^{\pi}(\cos\ ax-\sin bx)^2\ dx$

$$I=\int_{-\pi}^{\pi}\cos^2 ax+\sin^{2bx}-2\cos ax\sin bx)dx$$

$$I=\int_{-\pi}^{\pi}\cos^2 ax\,dx+\int_{-\pi}^{\pi}\sin^2 ax\,dx-0$$

[First two integranda are even function while third is odd function]

$$I=2\int_0^{\pi}2\cos^2 ax\,dx+\int_0^{\pi}2\sin^2 bx\,dx$$

$$I=\int_0^{\pi}(1+\cos 2ax)\,dx+\int_0^{\pi}(1-\cos 2bx)\,dx$$

$$I=\int_0^{\pi}dx+\int_0^{\pi}\cos 2ax\,dx+\int_0^{\pi}dx-\int_0^{\pi}\cos 2bx\,dx$$

$$I=2[x]_0^{\pi}+\left[\frac{\sin 2ax}{2a}\right]_0^{\pi}-\left[\frac{\sin 2bx}{2b}\right]_0^{\pi}$$

$$I=2\pi+\frac{\sin 2a\pi}{2a}-\frac{\sin 2b\pi}{2b}$$

11. Let E,F and A three events such that

E = selection of Bag A and F=selection of bag B

A= getting one red and one black ball of two

Here ,p(E)=P(getting 1 or 2 in a throw of die)= $\frac{2}{6}=\frac{1}{3}$

$\therefore \ p(F)=1-\frac{1}{3}=\frac{2}{3}$

Also, P(A/E)=P (getting one red and one black if bag A is selected)= $\frac{^6C_1\times {}^4C_1}{^{10}C_2}=\frac{24}{45}$

and P(A/F)=P(getting one red and one black if bag Black if bag B is selected)= $\frac{^3C_1\times 7C_1}{^{10}C_2}=\frac{21}{45}$

Now, by theorem of total probability,
p(A)=P(E).P(A/E)+P(F).P(A/F)

$\Rightarrow p(A)=\frac{1}{3}\times\frac{24}{45}+\frac{2}{3}\times\frac{21}{45}=\frac{8+14}{45}=\frac{22}{45}$

OR

Let number of head be random variable X in four tosses of a coin .X may have values 0,1,2,3 or 4 obviously repeated tosses of a coin are Bernoulli trials and thus X has binomial distribution with n=4 and p= probability of getting head in one toss= $\frac{1}{2}$

q=probability of getting tail (not head) in one toss= $1-\frac{1}{2}=\frac{1}{2}$

since, we know that P(X=r)= $^nC_rP^rq^{n-r}$, $r=0,1,2.........n$

therefore,

P(X=0)= $^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{4-0}=1\times1\times\left(\frac{1}{2}\right)^4=\frac{1}{16}$

$P(X=1)={}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{4-1}=4\times\left(\frac{1}{2}\right)^1\times\left(\frac{1}{2}\right)^4=\frac{4}{16}=\frac{1}{4}$

$P(X=2)={}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-2}=6\times\left(\frac{1}{2}\right)^2\times\left(\frac{1}{2}\right)^2=\frac{6}{16}=\frac{3}{8}$

$P(X=3)={}^4C_3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-3}=4\times\left(\frac{1}{2}\right)^3\times\left(\frac{1}{2}\right)^1=\frac{4}{16}=\frac{1}{4}$

$P(X=4)={}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{4-4}=1\times\left(\frac{1}{2}\right)^4\times\left(\frac{1}{2}\right)^0=\frac{1}{16}$

Now required probability distribution of X is

x	0	1	2	3	4
4P(x)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Required mean = $\mu=\sum x_ip_i$

$$=0\times\frac{1}{16}+1\times\frac{1}{4}+2\times\frac{3}{8}+3\times\frac{1}{4}+4\times\frac{1}{16}$$
$$=\frac{1}{4}+\frac{3}{4}+\frac{3}{4}+\frac{1}{4}=\frac{8}{4}=2$$

variance = $\sigma^2_x = \sum x_i p_i - \left(\sum x_i p_i\right)^2 = \sum X^2_i p_i - \mu^2$

$= \left(0\times\frac{1}{16}+1^2\times\frac{1}{4}+2^2\times\frac{3}{8}+3^2\times\frac{1}{4}+4^2\times\frac{1}{16}\right)-2^2$

$= \frac{1}{4}+\frac{3}{4}+\frac{9}{4}+1-4$

$= \frac{1}{4}+\frac{3}{4}+\frac{9}{4}-3$

$= \frac{1+6+9-12}{4} = \frac{4}{4} = 1$

12. Here

Now

$\left(\vec{r}\times\hat{i}\right)\left(\vec{r}\times\hat{j}\right)+xy = \{(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{i}\}.\{(x\hat{i}+y\hat{j}+z\hat{k})\times\hat{j}\}+xy$

$= (-y\hat{k}+z\hat{j}).(x\hat{k}-z\hat{i})+xy = (0\hat{i}+z\hat{j}-y\hat{k})\cdot(-z\hat{i}+0\hat{j}+x\hat{k})+xy$

$= 0+0-xy+xy = 0$

13. Let P (α,β,γ) be the point of intersection of the given line (i) and plane (ii)

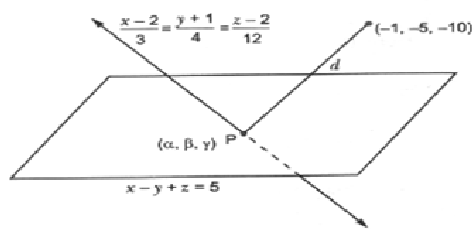
$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ ....(i)

and x – y + z = 5 ....(ii)

since ,point P (α,β,γ) lies on line (i)(therefore it satisfy(i)

$\Rightarrow \frac{\alpha-2}{3} = \frac{\beta+1}{4} = \frac{\gamma-2}{12} = \lambda$

$\Rightarrow \alpha = 3\lambda+2; \beta = 4\lambda-1; \gamma = 12\lambda+2$



Also point P (α,β,γ) lie on plane (ii)

$\Rightarrow \alpha-\beta+\gamma = 5$ ......(iii)

putting the value of α,β,γ in (iii) we get

$\Rightarrow 3\lambda+2-4\lambda+1+12\lambda+2 = 5$

$\Rightarrow 11\lambda+5 = 5 \Rightarrow \lambda = 0$

$\Rightarrow \alpha = 2; \quad \beta = -1; \quad \gamma = 2$

hence the coordinate of the point of intersect ion p is (-2,-1,2)

therefore ,required distance= d $= \sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$\sqrt{9+16+144} = \sqrt{169} = 13$units

14. Here sin$\left[\cot^{-1}(x+1)\right] = \cos(\tan^{-1}x)$

let $\cot^{-1}(x+1) = \theta \quad \Rightarrow \cot\theta = x+1$

$\Rightarrow \quad \operatorname{cosec}\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+(x+1)^2} = \sqrt{x^2+2x+2}$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{x^2+2x+2}} \qquad \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{x^2+2x+2}}\right)$

$\Rightarrow \cot^{-1}(x+1) = \sin^{-1}\left(\frac{1}{\sqrt{x^2+2x+2}}\right)$

again $\tan^{-1} x = \alpha \Rightarrow \tan\alpha = x$

$\therefore \sec\alpha = \sqrt{1+\tan^2\alpha} = \sqrt{1+x^2}$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{1+x^2}} \qquad \Rightarrow \quad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

$\Rightarrow \quad \tan^{-1} = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

now equation (i) becomes

$\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{x^2+2x+2}}\right)\right) = \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)$

$\frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{1+x^2}} \Rightarrow \sqrt{x^2+2x+2} = \sqrt{1+x^2}$

$x^2+2x+2 = 1+x^2 \qquad \Rightarrow \quad 2x+2 = 1$

$\Rightarrow x = -\frac{1}{2}$

Or

Here

$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$

$\Rightarrow \quad (\tan^{-1}x)^2 + (\pi - \tan^{-1}x)^2 = \frac{5\pi^2}{8}$

$\Rightarrow \quad (\tan^{-1}x)^2 + (\tan^{-1}x)^2 + \frac{\pi^2}{4} - \pi\tan^{-1}x = \frac{5\pi^2}{8}$

$\Rightarrow \quad 2(\tan^{-1}x)^2 - \pi\tan^{-1}x + \frac{\pi^2}{4} - \frac{5\pi^2}{8} = 0$

$\Rightarrow \quad 2(\tan^{-1}x)^2 - \pi\tan^{-1}x - \frac{3\pi^2}{8} = 8$

let $\tan^{-1}x = y$

$2y^2 - \pi y - \frac{3\pi^2}{8} = 0 \qquad \Rightarrow \quad 16y^2 - 8\pi y - 3\pi^2 = 0$

$16y^2 - 12\pi y + 4\pi y - 3\pi^2 = 0 \quad \Rightarrow 4y(4y-3\pi) + \pi(4y-3\pi) = 0$

$\Rightarrow (4y-3\pi)(4y+\pi) = 0 \quad \Rightarrow y = -\frac{\pi}{4}$ or $\quad y = \frac{3\pi}{4}$

$\Rightarrow \tan^{-1}x = -\frac{\pi}{4} \qquad [\because \frac{3\pi}{4}$ does not belongs to domain of $\tan^{-1}x$ i.e, $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)]$

$\Rightarrow x = \tan\left(-\frac{\pi}{4}\right) = -1$

15.

$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}} \times \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$$

$$= \tan^{-1}\left(\frac{2+2\sqrt{1-x^4}}{1+x^2-1+x^2}\right) = \tan^{-1}\left(\frac{2+2\sqrt{1-x^4}}{2x^2}\right)$$

$$= \tan^{-1}\left(\frac{1+\sqrt{1-x^4}}{x^2}\right)$$

let $x^2 = \sin\theta$ $\Rightarrow \sin^{-1}(x^2) = \theta$

putting the value of x^2, we get

$$= \tan^{-1}\left\{\frac{1+\sqrt{1-\sin^2\theta}}{\sin\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{1+\cos\theta}{\sin\theta}\right\} = \tan^{-1}\left\{\frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\}$$

$$= \tan^{-1}\left\{\cot\frac{\theta}{2}\right\} = \tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right\}$$

$$= \frac{\pi}{2}-\frac{\theta}{2} = \frac{\pi}{2}-\sin^{-1}x^2$$

$$\left(\begin{array}{l} \because 0 \le x^2 \le 1 \\ \Rightarrow \sin 0 < \sin\theta < \sin\frac{\pi}{2} \\ \Rightarrow 0 < \theta < \frac{\pi}{2} \Rightarrow 0 < \frac{\theta}{2} < \frac{\pi}{2} \\ \Rightarrow 0 < -\frac{\theta}{2} < -\frac{\pi}{4} \\ \Rightarrow \frac{\pi}{2} > \frac{\pi}{2}-\frac{\theta}{2} > \frac{\pi}{2}-\frac{\pi}{4} \\ \Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2}-\frac{\theta}{2}\right) > \frac{\pi}{4} \\ \left(\frac{\pi}{2}-\frac{\theta}{2}\right) \in \left(\frac{\pi}{4},\frac{\pi}{2}\right) \subset \left(-\frac{\pi}{2},\frac{\pi}{2}\right) \end{array}\right)$$

differentiating both sides with respect to x, we get

$$\Rightarrow \frac{dy}{dx} = -\frac{2x}{2\sqrt{1-x^4}} \quad \Rightarrow \quad \Rightarrow \frac{dy}{dx} = -\frac{x}{\sqrt{1-x^4}}$$

16. Given x= a cos θ +bsin θ

$$\Rightarrow \frac{dx}{d\theta} = -a\sin\theta + b\cos\theta$$

Also, $y = a\sin\theta - b\cos\theta$

$$\Rightarrow \frac{dy}{d\theta} = a\cos\theta + b\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\cos\theta + b\sin\theta}{-a\sin\theta + b\cos\theta}$$

$$\frac{dy}{dx} = -\frac{x}{y} \qquad \Rightarrow \frac{d^2y}{dx^2} = -\left(\frac{y - x.\frac{dy}{dx}}{y^2}\right)$$

$$\Rightarrow y^2\frac{d^2y}{dx^2} = -y + x\frac{dy}{dx} \qquad \Rightarrow \qquad y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$

17. Let ‘A’ be the area and ‘a’ be the side of an equilateral triangle.

$$A = \frac{\sqrt{3}}{4}a^2$$

Differentiating with respect to t we get

$\Rightarrow$ $$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a\frac{da}{dt}$$

$\Rightarrow$ $$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \times 2$$ [Given $\frac{da}{dt}$ = 2cm/sec.]

$\Rightarrow$ $$\frac{dA}{dt} = \sqrt{3a}$$ $\Rightarrow \left.\frac{da}{dt}\right]_{a=20cm} = 20\sqrt{3}\text{sq cm / s}$

18. Let $I = \int (x+3)\sqrt{3-4x-x^2}\,dx$

Let x + 3 = $A\frac{d}{dx}(3-4x-x^2) + B$

$\Rightarrow$ x + 3 = A (-4 – 2x) +B $\Rightarrow$ x + 3 =- 4A =-2Ax + B

$\Rightarrow$ x + 3= (-4A + B) =- 2Ax [By comparing coefficients]

$\because$ -2A = 1 $\Rightarrow$ A = $= \frac{1}{-2}$

Again, $\because$ -4A + B = 3

$\Rightarrow$ $-4 \times \frac{1}{2} + B = 3$ $\Rightarrow$ 2 +B = 3 $\Rightarrow$ B = 1

Here, $x + 3 = -\frac{1}{2}(-2-4) + 1$

$$I = \int \left\{ -\frac{1}{2}(-2x-4) + 1 \right\} \sqrt{3-4x-x^2}\,dx$$

$$I = -\frac{1}{2}\int (-2x-4)\sqrt{3-4x-x^2}\,dx + \int \sqrt{3-4x-x^2}\,dx$$

$$I = -\frac{1}{2}I_1 + I_2$$ …(i), where……………..

……………

Now $I_1 \int (-2x-4)\sqrt{3-4x-x^2}\,dx$

Let 3-4x – x² = z $\Rightarrow$ (-2x – 4) dx = dz

$I_1 \int \sqrt{z}dz = \frac{2}{3}(z)^{\frac{2}{3}} + C_1$ $\Rightarrow$ $I_2 = \frac{2}{3}(3-4x-x^2)^{\frac{3}{2}+} C_1$

Again $I_2 \int \sqrt{3-4x-x^2}\,dx$

$\Rightarrow$ $I_2 = \int \sqrt{-(x^2+4x-3)}\,dx$ $\Rightarrow$ $I_2 = \int \sqrt{-\{x+2)^2 - 7\}}\,dx$

$$I_2 = \int \sqrt{(\sqrt{7})^2 - (x+2)^2}\,dx$$

$$I_2 = \frac{1}{2}(x+2)\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}} + C_2$$

Putting the value of I_1 and I_2 in (i), we get

$\because \qquad I=-\frac{1}{2}\times\frac{2}{3}(3-4x-x^2)^{\frac{3}{2}}-\frac{C_1}{C_2}(x+2)\sqrt{3-4x-x^2}+\frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}}+C_2$

$\Rightarrow \qquad I=-\frac{1}{3}(3-4x-x^2)^{\frac{3}{2}}+\frac{1}{2}(x+2)\sqrt{3-4x-x^2}+\frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}}+C_2$

$$=\sqrt{3-4x-x^2}\left[-\frac{1}{3}(3-4x-x^2)+\frac{1}{2}(x+2)\right]+\frac{7}{2}\sin^{-1}\frac{x+2}{7}+C$$

$$=\frac{x}{6}\sqrt{3-4x-x^2}(2x+11)+\frac{7}{2}\sin^{-1}\frac{x+2}{7}+C,\text{ where } C=C_2-\frac{C_1}{2}$$

19. The number of handmade fans, mats and plates sold by three school A, B and can be represented by 3×3 matrix as

$$X=\begin{matrix}A\\B\\C\end{matrix}\begin{bmatrix}40 & 50 & 20\\25 & 40 & 30\\35 & 50 & 40\end{bmatrix}$$

And their selling price can be represented by 3×1 matrix as

$$Y=\begin{bmatrix}25\\100\\50\end{bmatrix}\begin{matrix}\rightarrow \text{Handmade fans}\\ \rightarrow \text{Mats}\\ \rightarrow \text{Plates}\end{matrix}$$

Now, the total funds collected by each school is given by the matrix multiplication as

$$XY=\begin{matrix}A\\B\\C\end{matrix}\begin{bmatrix}40 & 50 & 20\\25 & 40 & 30\\35 & 50 & 40\end{bmatrix}\begin{bmatrix}25\\100\\50\end{bmatrix}$$

$$\Rightarrow \qquad XY=\begin{bmatrix}40\times25+50\times100+20\times50\\25\times25+40\times100+30\times50\\35\times25+50\times100+40\times50\end{bmatrix}\Rightarrow \quad XY=\begin{bmatrix}7000\\6125\\7875\end{bmatrix}$$

Hence, total funds collected by school A = Rs.7000
Total funds collected by school B = Rs.6125
Total funds collected by school C = Rs.7875
Total funds collected for the purpose = Rs.(7000+6125+7875)
= Rs. 21000
Value: Students are motivated for social service.

SECTION-C

20 Reflexivity: By commutative law under addition and multiplication

B + a = a +b $\quad \forall a,b\in N$
Ab=ba $\quad \forall a,b\in N$
Ab(b+a)=ba(a+b) $\quad \forall a,b\in N$
(a,b)R(a,b) $\quad$ Hence, R is reflexive

Symmetry: Let(a,b)R(c,d)
(a,b)R(c,d) $\Rightarrow$ ad(b + c) = bc (a + d)
$\Rightarrow$ bc(a + d) = ab(b + c)

$\Rightarrow$ cb(d + a) da(c + b)

[By commutative law under addition and multiplication]

$\Rightarrow$ (c + d) R(a, b)

Hence, R is symmetric.

Transitivity: Let(a, b) R (c, d) and (c, d) R (e, f)

Now, (a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow$ ad(b +c) = bc(a + d) and cf(d + e) de(c + f)

$$\frac{b+c}{bc}=\frac{a+d}{ad} \text{ and } \frac{d+e}{de}=\frac{c+f}{cf}$$

$$\Rightarrow \quad \frac{1}{c}+\frac{1}{b}=\frac{1}{d}+\frac{1}{a} \text{ and } \frac{1}{e}+\frac{1}{d}=\frac{1}{f}+\frac{1}{c}$$

Adding both, we get

$$\Rightarrow \quad \frac{1}{c}+\frac{1}{v}+\frac{1}{e}+\frac{1}{d}=\frac{1}{d}+\frac{1}{a}+\frac{1}{f}+\frac{1}{c}$$

$$\Rightarrow \quad \frac{1}{b}+\frac{1}{e}=\frac{1}{a}+\frac{1}{f} \quad \Rightarrow \quad \frac{e+b}{be}=\frac{f+a}{af}$$

$\Rightarrow$ af(b+e) = be(a +f) $\Rightarrow$ (a, b) R (e, f) [c,d $\neq 0$]

Hence, r is transitive.

In this way, r is reflexive symmetric and transitive

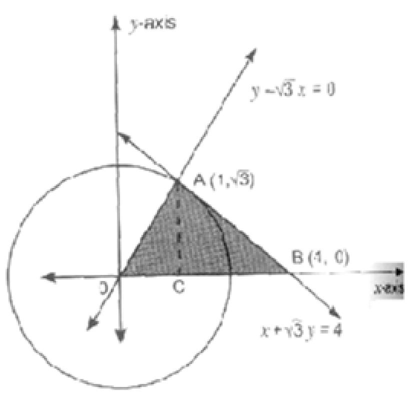
Therefore, r is an equivalence relation.

21. Given circle is $x^2+y^2=4$

$$\Rightarrow \quad 2x+2y\frac{dy}{dx}=0 \qquad \text{[By differentiating]}$$

$$\Rightarrow \quad \frac{dy}{dx}=-\frac{x}{y}$$

Now, slope of tangent at $(1,\sqrt{3})=\left.\frac{dy}{dx}\right]_{(1,\sqrt{3})}=-\frac{1}{\sqrt{3}}.$



$\therefore$ Slope of normal at $(1,\sqrt{3})=\sqrt{3}$

Therefore, equation of tangent is

$$\frac{y-\sqrt{3}}{x-1}=-\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x+\sqrt{3}y=4$$

Again, equation of normal is

$$\frac{y-\sqrt{3}}{x-1}=\sqrt{3} \quad \Rightarrow \quad y-\sqrt{3}x=0$$

To draw the graph of the triangle formed by the lines x-axis, (i) and (ii), we find the intersecting of these three lines which give vertices of required triangle. Let O, A, B be the intersecting of these lines.

Obviously, the coordinate of O, A, B are (0, 0), $(1,\sqrt{3})$ and (4, 0) respectively.

Required area = area of triangle OAB

= area of region OAC + area of region CAB

$=\int_0^1 y\,dx+\int_1^4 y\,dx$ [Where in 1st integrand $y=\sqrt{3x}$ and in 2nd $y=\frac{4-x}{\sqrt{3}}$]

$$=\int_0^1 \sqrt{3}x dx+\int_0^1 \frac{4-x}{\sqrt{3}}dx=\sqrt{3}\left[\frac{x^2}{2}\right]_0^1-\frac{1}{\sqrt{3}}\left[\frac{(4-x)^2}{2}\right]_1^4$$

$$=\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{3}}\left[0-\frac{9}{2}\right]$$

$$=\frac{\sqrt{3}}{2}+\frac{9}{2\sqrt{3}}=\frac{12}{2\sqrt{3}}=2\sqrt{3}\text{ sq units.}$$

Or

$$\int_1^3 (e^{2-3x}+x^2+1)dx=e^2\int_1^3 e^{-3x}dx+\int_1^3 (x^2+1)dx$$

$$=e^2.I_1+I_2 \qquad \text{.... (i), } I_2=\int_1^3 e^{-3x}dx; I_2=\int_1^3 (x^2+1)dx$$

We have, $\int_a^b f(x)dx=\lim_{h\to 0} h\{(a+b)+f(a+2h)+\ldots+f(a+nh)\}$

For I_1

$f(x)=e^{-3x}$, a = 1, b = 3

$$h=\frac{b-a}{n}\Rightarrow h=\frac{2}{n}\Rightarrow nh=2$$

Now, $\int_1^3 e^{-3x}dx=\lim_{h\to 0} h\{f(1+h)+f(1+2h)+\ldots f(1+nh)\}$

$$=\lim_{h\to 0} h\left\{e^{-3(1+h)}+e^{-3(1+2h)}+\ldots+e^{-3(1+nh)}\right\}$$

$$=\lim_{h\to 0} h\left\{e^{-3}.e^{-3h}+e^{-3}.e^{-6(1+2h)}+\ldots+e^{-3nh}\right\}$$

$$=\lim_{h\to 0} h\left\{e^{-3h}+(e^{-3h})^2+\ldots+(e^{-3h})^n\right\}$$

$$=e^{-3}.\lim_{h\to 0} h\left\{\frac{e^{-3h}(-1-(e^{-3h})^n}{1-e^{-3h}}\right\}$$

$$=e^{-3}.\lim_{h\to 0} h\left\{\frac{e^{-3h}(1-e^{-3nh})}{1-e^{-3h}}\right\}$$ [Applying formula for sum of GP]

$$= e^{-3}.\lim_{h\to 0}\left\{\frac{e^{-3h}((1-e^{-6})}{1-e^{-3h}}\right\} = e^{-3}(1-e^{-6}).e^{0}.\frac{1}{\lim\limits_{3h\to 0}\frac{e^{-3h}-1}{-3h}\times 3}$$

$$= \frac{e^{-3}(1-e^{-6})}{3}$$

For I_2 $F(x) = x^2 + 1$, a = 1, b = 3 $\Rightarrow$ $h = \frac{b-a}{n}$ $\Rightarrow$ $h = \frac{2}{n} \Rightarrow b$

nh=2

Now, $\int_1^3 (x^2+1)dx = \lim_{h\to 0} h\{f(1+h)+f(1+2h)+....+f(1+nh)\}$

$$= \lim_{h\to 0} h\left[\left\{(1+h)^2+1\right\}+1\left\{(1+2h)^2+1\right\}+...+\left\{(1+nh)^2+1\right\}\right]$$

$$= \lim_{h\to 0} h\left[n+(1+h^2+2h)+(1+4h^2+4h)+(1+9h^2+6h)+...+(1+n^2h^2+2nh)\right]$$

$$= \lim_{h\to 0} h\left[n+n+h^2(1^2+2^2+3^2+...n^2)+2h(1+2+3...+n)\right]$$

$$= \lim_{h\to 0} h\left[2n+\frac{h^2n(n+1)(2+1)}{6}+\frac{2h\,n(n+1)}{2}\right]$$

$$= \lim_{h\to 0}\left[2nh++\frac{h\,nh(nh+h)(2nh+h)}{6}+nh(nh+h)\right]$$

$$= \lim_{h\to 0}\left[4+\frac{2(2+h)(2\times 2+h)}{6}+nh(nh+h)\right] = 4+\frac{2(2+0)(4+0)}{6}+2(2+0)$$

$$= 4+\frac{16}{6}+4 = 8+\frac{8}{3} = \frac{32}{3}$$

Putting the value of I_1 and I_2 in (i), we ge,

$$I = \frac{e^2.e^{-3}(1-e^{-6})}{3}+\frac{32}{3} = \frac{e^{-1}(1-e^{-6})}{3}+\frac{32}{3} = \frac{32+(e^{-1}-e^{-7})}{3}$$

22. The given differential equation can be written as

$$\frac{dx}{dy}+\frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

Now (i) is linear differential equation of the form $\frac{dx}{dy}+P_1x = Q_1$,

where , $p_1 = \frac{1}{1+y^2}$ and $Q_1 = \frac{\tan^{-1}y}{1+y^2}$

Therefore, I.F = $e^{\int\frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

Thus , the solution of the given differential equation is

$$xe^{\tan^{-1}y} = \int\left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy + C$$

Let $I = \int\left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy$

substituting $\tan^{-1}y = t$ so that $\left(\frac{1}{1+y^2}\right)dy = dt$, we get

$$I=\int t\,e^t dt = t\,e^t-\int 1.e^t dt = t\,e^t - e^t \equiv e^t(t-1)$$

$$I=e^{\tan^{-1}y}(\tan^{-1}y-1)$$

substituting the valiue of I in the equation (ii) , we get

$x.e^{\tan^{-1}y}=e^{\tan^{-1}y}(\tan^{-1}y-1)+C$ or $x=(\tan^{-1}y-1)+C\,e^{-\tan^{-1}y}$

which is the gernal solution of the given differential equation

OR

Given differential equation is

$$\frac{dy}{dx}=\frac{xy}{x^2+y^2} \qquad .....(i)$$

Let y= νx $\Rightarrow$ $\frac{dy}{dx}=\nu+x\frac{d\nu}{dx}$

Now (i) becomes

$$\nu+x\frac{d\nu}{dx}=\frac{\nu x^2}{x^2+\nu^2x^2} \quad\Rightarrow\quad \nu+x\frac{d\nu}{dx}=\frac{\nu x^2}{x^2(1+\nu^2)}$$

$$\Rightarrow \nu+x\frac{d\nu}{dx}=\frac{\nu}{1+x^2} \quad\Rightarrow\quad x\frac{d\nu}{dx}=\frac{\nu}{1+\nu^2}-\nu$$

$$\Rightarrow \quad x\frac{d\nu}{dx}=\frac{\nu-\nu-\nu^3}{1+\nu^2} \quad\Rightarrow\quad x\frac{d\nu}{dx}=\frac{-\nu^3}{1+\nu^2}$$

$$\Rightarrow -\frac{dx}{x}=\frac{1+\nu^2}{\nu^3}d\nu$$

Integrating both sides, we get

$$\Rightarrow \int\frac{1+\nu^2}{\nu^3}d\nu=-\int\frac{dx}{x} \quad\Rightarrow\quad \int\frac{d\nu}{\nu^3}+\int\frac{d\nu}{\nu}=-\int\frac{dx}{x}$$

$$\Rightarrow \quad -\frac{1}{2\nu^2}+\log|\nu|=-\log|x|+C$$

putting the value of $\nu=\frac{y}{x}$,we get

$$\Rightarrow \quad -\frac{x^2}{2y^2}+\log\left|\frac{y}{x}\right|+\log|x|=C \qquad \Rightarrow \quad -\frac{x^2}{2y^2}+\log|y|-\log|x|+\log|x|=C$$

$$\Rightarrow \quad -\frac{x^2}{2y^2}+\log|y|=C$$

put y=1 and x=0 in(ii) 0+log$|1|$=C $\Rightarrow$ C = 0

Therefore required particular solution is $-\frac{x^2}{2y^2}+\log|y|=0$

23. Let the given lines

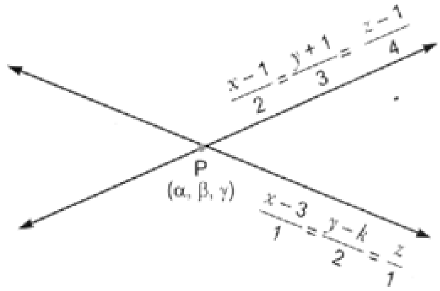
$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4} \qquad ......(i)$$

and $\frac{x-3}{1}=\frac{y-k}{3}=\frac{z}{1}$ ......(ii)intersect at P (α,β,γ)

$\because$ P lie in (i)

$$\Rightarrow \frac{\alpha-1}{2}=\frac{\beta+1}{3}=\frac{\gamma-1}{2}=\lambda \text{ (say)}$$

$\Rightarrow \; \alpha = 2\lambda+1, \beta = 3\lambda-1, \gamma = 4\lambda+1$



again, $\because$ P lie on (ii)also

$\Rightarrow \frac{\alpha-3}{1} = \frac{\beta-k}{2} = \gamma$

$\Rightarrow \quad \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1}$

$\Rightarrow \quad \frac{2\lambda-2}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1}$

I II III

from I and II

$\Rightarrow \quad 2\lambda - 2 = \frac{3\lambda-1-k}{2} \quad \Rightarrow \quad 4\lambda-4 = 3\lambda-1-k$

$\Rightarrow \quad k=3\lambda-1-4\lambda+4 \quad \Rightarrow k=-\lambda+3$

$\Rightarrow \quad k = \frac{3}{2}+3 = \frac{9}{2}$

Now, we know that equation of plane containing lines.

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Therefore , required equation is $\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow$ (x-1)(3-8)-(y+1)(2-4)+(z-1)(4-3)=0

$\Rightarrow$ -5(x-1)+2(y+1)+(z-1)=0

$\Rightarrow$ -5x+2y+z+6=0 $\quad \Rightarrow$ 5x-2y-z-6=0

24. we know that "if A and B are two independent events" then

$P(A \cap B) = P(A).P(B)$

Also, since A and B are two independent events $\bar{A}$,B and A, $\bar{B}$ are also independent events .

$\therefore$ $P(\bar{A} \cap B) = P(\bar{A}).P(B)$

$P(A \cap \bar{B}) = P(\bar{A}).P(\bar{B})$

Now, let P(A) = x and P(B) = y

$\Rightarrow P(\bar{A}) = 1 - x$ and $P(\bar{B}) = 1 - y$

Given $P(\overline{A}\cap B)=\frac{2}{15}$ and $P(A\cap\overline{B})=\frac{1}{6}$

$\Rightarrow P(\overline{A}).P(B)=\frac{2}{15}$ and $P(A).P(\overline{B})=\frac{1}{6}$

$\Rightarrow (1-x).y=\frac{2}{15}$ and $x.(1-y)=\frac{1}{6}$

$\Rightarrow y-xy=\frac{2}{15}$ .....(i) and $x-xy=\frac{1}{6}$ ......(ii)

From(i) y. (1-x) $=\frac{2}{15}$ $\Rightarrow$ $y=\frac{2}{15(1-x)}$

Putting the value of y in (ii) ,we get

$x - x\times\frac{2}{15(1-x)}=\frac{1}{6}$ $\Rightarrow$ $\frac{15x-15x^2-2x}{15-15x}=\frac{1}{6}$

$\Rightarrow 6(-15x^2+13x)=15-15x$ $\Rightarrow$ $-90x^2+78x=15-15x$

$\Rightarrow -90x^2+93x-15=0$ $\Rightarrow$ $30x^2-31x+5=0$

$\Rightarrow 30x^2-25x-6x+5=0$ $\Rightarrow$ $5x(6x-5)-1(6x-5)=0$

$\Rightarrow (6x-5)(5x-1)=0$ $\Rightarrow$ $x=\frac{5}{6}$ or $x=\frac{1}{5}$

Now, $x=\frac{5}{6}$ $\Rightarrow$ $y=\frac{2}{15\left(1-\frac{5}{6}\right)}=\frac{12}{15}=\frac{4}{5}$

and $x=\frac{1}{5}$ $\Rightarrow$ $y=\frac{2}{15\left(1-\frac{1}{5}\right)}=\frac{1}{6}$

Hence $P(A)=\frac{5}{6}$ and $P(B)=\frac{4}{5}$

or $P(A)=\frac{1}{5}$ $P(B)=\frac{1}{6}$

25. Given $f(x)=\sin x-\cos x$ $\Rightarrow f'(x)=\cos x+\sin x$

for critical points

$f'(x)=0$ $\Rightarrow$ $\cos x+\sin x=0$

$\Rightarrow \sin x=-\cos x$ $\Rightarrow$ $\tan x=-1$

$\Rightarrow \tan x=\tan\frac{3\pi}{4}$ $\Rightarrow$ $x=n\pi+\frac{3\pi}{4}$,$n\in Z$

$\Rightarrow$ $x=\frac{3\pi}{4},\frac{7\pi}{4}$ [other value does not belong to $(0,2\pi)$]

$\Rightarrow$ Now $f''(x)=-\sin x+\cos x$

$f''(x)_{x=\frac{3\pi}{4}}=-\sin\frac{3\pi}{4}+\cos\frac{3\pi}{4}=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=-\frac{2}{\sqrt{2}}=-\sqrt{2}<0$

i.e., $f(x)$ is maximum at $x=\frac{3\pi}{4}$

$\Rightarrow$ Local maximum value of $f(x)=f\left(\frac{3\pi}{4}\right)=\sin\frac{3\pi}{4}-\cos\frac{3\pi}{4}$

$$-\frac{1}{\sqrt{2}}-\left(-\frac{1}{\sqrt{2}}\right)=\frac{2}{\sqrt{2}}=\sqrt{2}$$

Again $f''(x)_{x=\frac{7\pi}{4}}$ = - sin $\frac{7\pi}{4}$ + cos $\frac{7\pi}{4}$ $=-\left(-\frac{1}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}>0$

i.e., f(x) is minimum at x= $\frac{7\pi}{4}$

⇒ Local minimum value of f(x) =f($\frac{7\pi}{4}$)=sin $\frac{7\pi}{4}$ - cos $\frac{7\pi}{4}$ $=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=-\frac{2}{\sqrt{2}}=-\sqrt{2}$

Therefore, local maximum and local minimum values are $\sqrt{2}$ and - $\sqrt{2}$ respectively.

26. Given constraints are

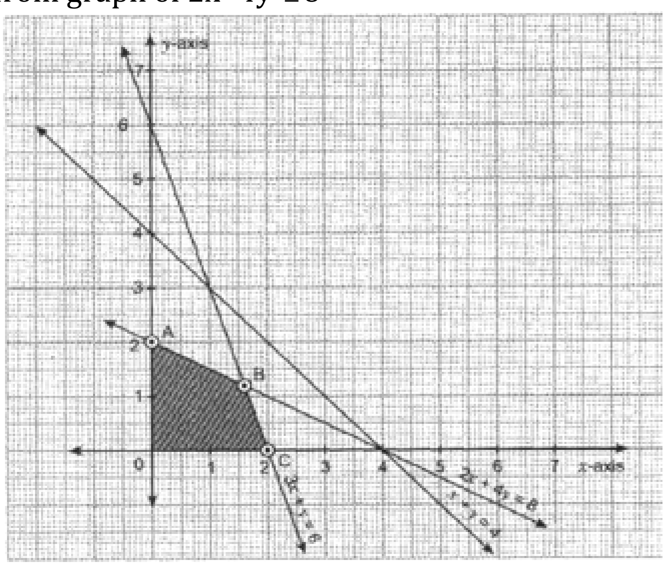
$2x+4y \le 8$ ....(i)

$3x+y \le 4$ ....(ii)

$x+y \le 4$ ....(iii)

$x \ge 0, y \ge 0$ ....(iv)

from graph of $2x+4y \le 8$



we draw the graph of 2x+ 4y = 8 as

X	0	4
y	2	0

∵ $2\times0+4\times0\le8$

⇒ (0,0) origin satisfy the constraints.

Hence , fesible region lie origin side of line 2x + 4y = 8

For graph $3x+y \le 4$

we draw the graph of line 3x+ y =6

X	0	2
y	6	0

∵ $3\times0+0\le6$

$\Rightarrow$ origin (0,0) satisfy $3x+y \le 6$
hence , fesible region lie origin side of line 3x + y =6
for graph of $x+y \le 4$
we draw the graph of line x+ y=4

X	0	4
y	4	0

$\because \quad 0+0 \le 4 \Rightarrow$ origin (0,0) satisfy $x+y \le 4$
hence feasible region lie origin side of line x+4= 4
also $\quad x \ge 0, y \ge 0$ says feasible region is in ist quadrant.
therefore, OABC is required feasible region.
Having corner point O(0,0) , (0,2) , B $(\frac{8}{5},\frac{6}{5})$, C(2,0)

here feasible region is bounded.
NOW the value of objective function Z = 2x + 5y is obtained as

Corner point	Z=2x+5y
O(0,0)	0
(0,2)	$2\times0\times5\times2$ =10
B $(\frac{8}{5},\frac{6}{5})$	$2\times\frac{8}{5}\times5\times\frac{6}{5}$ =9.2
C(2,0)	$2\times2\times5\times0$ =4

Hence, maximum value of Z is 10 at x =0, y =2