CHAPTER 6
Ratios, Rates, and Proportions

6-1 Ratios and Rates

A **ratio** is a comparison of two quantities by division.

The ratio of a to b can be written in three different ways: a to b, $a:b$, and $\dfrac{a}{b}$.

If two quantities are in the ratio of a to b, then the two numbers can be expressed as $\boldsymbol{ax}$ and $\boldsymbol{bx}$, in which x is a positive integer.

A **rate** is a ratio of two measurements having different units of measure. For example, a price of $2.59 per gallon of gasoline, an income of $750 in 3 days, and a speed of 60 miles per hour are all rates.

A **unit rate** is a rate that has a denominator of 1. Some examples of unit rates are defined as follows.

$$\textbf{Unit Price} = \frac{\text{Price of Package}}{\text{Number of Units in the Package}}$$

$$\textbf{Gas Mileage} = \frac{\text{Number of Miles Traveled}}{\text{Number of Gallons of Gas Used}}$$

$$\textbf{Speed (Miles per Hour)} = \frac{\text{Number of Miles Traveled}}{\text{Number of Hours}}$$

$$\textbf{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Example 1 □ Express each ratio as a unit rate.
 a. 1360 grams of coffee cost $17.68. What is the unit price of the coffee?
 b. A car travels 322 miles on 11.5 gallons of gas. What is the car's gas mileage?
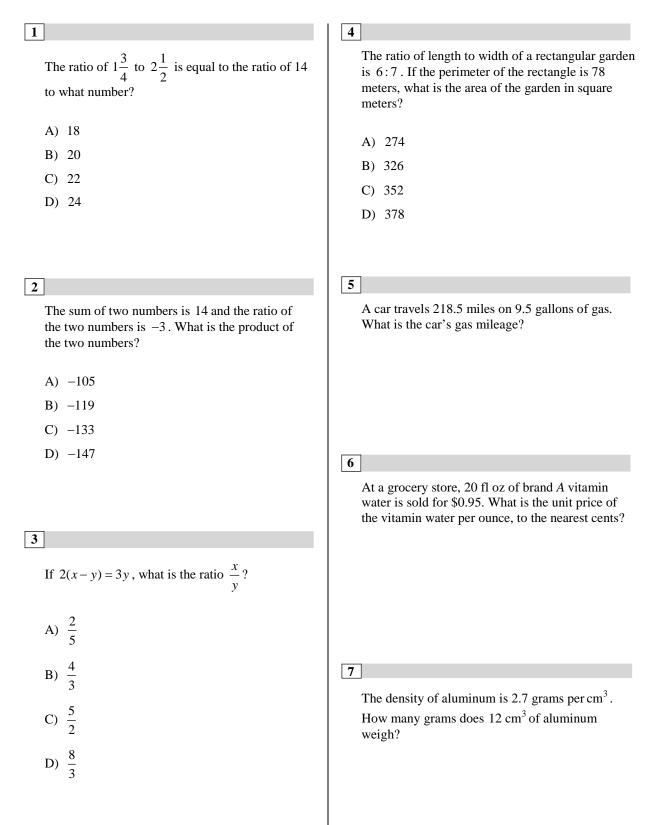 c. A driver traveled $485\frac{1}{3}$ miles in $8\frac{2}{3}$ hours. What is his speed?
 d. A volume of 46 cm^3 of silver has a mass of 483 grams. What is the density of silver?

Solution □ a. $\dfrac{\$17.68}{1360 \text{ grams}} = \dfrac{1768 \text{ cents}}{1360 \text{ grams}} = 1.3 \text{ cents / gram}$

 b. $\dfrac{322 \text{ mi}}{11.5 \text{ gal}} = 28 \text{ miles / gallon}$.

 c. $\dfrac{485\frac{1}{3} \text{ mi}}{8\frac{2}{3} \text{ hr}} = \dfrac{(485\frac{1}{3} \text{ mi}) \cdot 3}{(8\frac{2}{3} \text{ hr}) \cdot 3} = \dfrac{1456 \text{ mi}}{26 \text{ hr}} = 56 \text{ mph}$

 d. $\text{density} = \dfrac{\text{mass}}{\text{volume}} = \dfrac{483 \text{ grams}}{46 \text{ cm}^3} = 10.5 \text{ grams/cm}^3$

Example 2 □ 3 angles of a triangle are in the ratio of $3:5:7$. What is the measure of each angle?

Solution □ The measure of each angle of the triangle can be represented as $3x$, $5x$, and $7x$.
 $3x + 5x + 7x = 180$ The angle sum in a triangle is 180.
 $15x = 180$ Simplify.
 $x = 12$ Simplify.
 The measure of the 3 angles are $3x = 3 \cdot 12 = 36$, $5x = 5 \cdot 12 = 60$, and $7x = 7 \cdot 12 = 84$.

Exercises - Ratios and Rates

1

The ratio of $1\dfrac{3}{4}$ to $2\dfrac{1}{2}$ is equal to the ratio of 14 to what number?

A) 18

B) 20

C) 22

D) 24

2

The sum of two numbers is 14 and the ratio of the two numbers is -3. What is the product of the two numbers?

A) -105

B) -119

C) -133

D) -147

3

If $2(x - y) = 3y$, what is the ratio $\dfrac{x}{y}$?

A) $\dfrac{2}{5}$

B) $\dfrac{4}{3}$

C) $\dfrac{5}{2}$

D) $\dfrac{8}{3}$

4

The ratio of length to width of a rectangular garden is $6 : 7$. If the perimeter of the rectangle is 78 meters, what is the area of the garden in square meters?

A) 274

B) 326

C) 352

D) 378

5

A car travels 218.5 miles on 9.5 gallons of gas. What is the car's gas mileage?

6

At a grocery store, 20 fl oz of brand A vitamin water is sold for $0.95. What is the unit price of the vitamin water per ounce, to the nearest cents?

7

The density of aluminum is 2.7 grams per cm^3. How many grams does 12 cm^3 of aluminum weigh?

6-2 Proportions

A **proportion** is an equation stating that two ratios are equal.

The proportions $\dfrac{a}{b} = \dfrac{c}{d}$ is read "a is to b as c is to d." The numbers a and d are called the **extremes** of the proportion. The numbers b and c are called the **means** of the proportion.

In a proportion, the product of the extremes is equal to the product of the means.

If $\dfrac{a}{b} = \dfrac{c}{d}$, then $ad = bc$. The products ad and bc are called the **cross products** of the proportion $\dfrac{a}{b} = \dfrac{c}{d}$.

Example 1 □ Determine whether each pair of ratios forms a proportion.

a. $\dfrac{0.4}{1.5}, \dfrac{1.6}{6}$

b. $\dfrac{12}{25}, \dfrac{7}{15}$

Solution □ a. $\dfrac{0.4}{1.5} \overset{?}{=} \dfrac{1.6}{6}$ Write a proportion.

$0.4 \times 6 \overset{?}{=} 1.5 \times 1.6$ Find the cross products.

$2.4 = 2.4$ Simplify.

The cross products are equal, so the ratios form a proportion.

b. $\dfrac{12}{25} \overset{?}{=} \dfrac{7}{15}$ Write a proportion.

$12 \times 15 \overset{?}{=} 25 \times 7$ Find the cross products.

$180 \neq 175$ Simplify.

The cross products are not equal, so the ratios do not form a proportion.

Example 2 □ Solve the proportion $\dfrac{3}{7} = \dfrac{6}{x-4}$.

Solution □ $\dfrac{3}{7} = \dfrac{6}{x-4}$

$3(x-4) = 7(6)$ Find the cross products.

$3x - 12 = 42$ Distributive Property

$3x = 54$ Add 12 to each side.

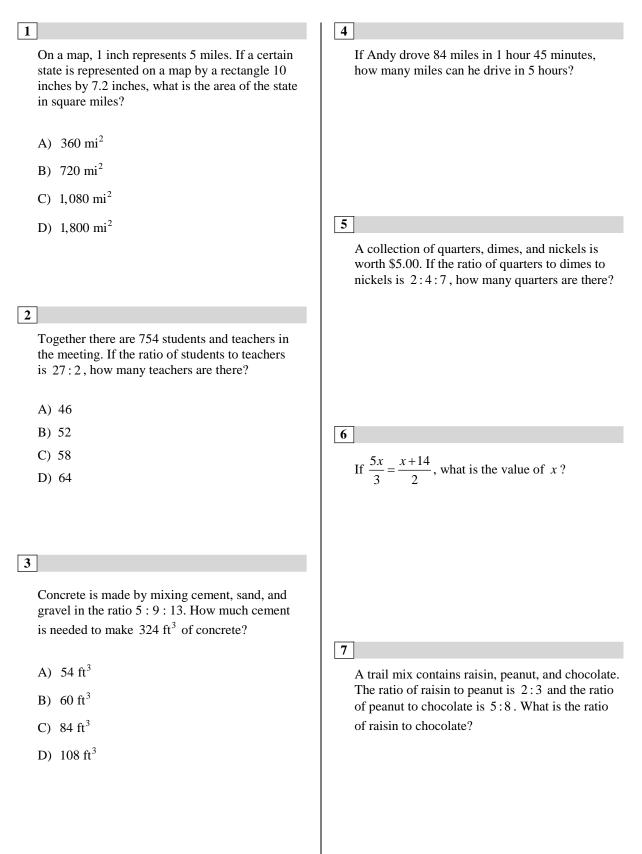$x = 18$ Divide each side by 3.

Example 3 □ Carter's SUV requires 8 gallons of gasoline to travel 148 miles. How much gasoline, to the nearest gallon, will he need for a 500 mile trip?

Solution □ a. Let g = the number of gallons of gas needed for a 500 mile trip.

$\begin{array}{l} \text{miles} \rightarrow \\ \text{gallons} \rightarrow \end{array} \dfrac{148}{8} = \dfrac{500}{g} \begin{array}{l} \leftarrow \text{miles} \\ \leftarrow \text{gallons} \end{array}$ Write a proportion.

$148g = 8 \times 500$ Find the cross products.

$g = \dfrac{8 \times 500}{148} \approx 27.03$ Divide each side by 148.

Carter's needs 27 gallons of gas for a 500 mile trip.

Exercises - Proportions

1

On a map, 1 inch represents 5 miles. If a certain state is represented on a map by a rectangle 10 inches by 7.2 inches, what is the area of the state in square miles?

A) 360 mi^2

B) 720 mi^2

C) $1{,}080 \text{ mi}^2$

D) $1{,}800 \text{ mi}^2$

2

Together there are 754 students and teachers in the meeting. If the ratio of students to teachers is $27:2$, how many teachers are there?

A) 46

B) 52

C) 58

D) 64

3

Concrete is made by mixing cement, sand, and gravel in the ratio 5 : 9 : 13. How much cement is needed to make 324 ft^3 of concrete?

A) 54 ft^3

B) 60 ft^3

C) 84 ft^3

D) 108 ft^3

4

If Andy drove 84 miles in 1 hour 45 minutes, how many miles can he drive in 5 hours?

5

A collection of quarters, dimes, and nickels is worth $5.00. If the ratio of quarters to dimes to nickels is $2:4:7$, how many quarters are there?

6

If $\dfrac{5x}{3} = \dfrac{x+14}{2}$, what is the value of x?

7

A trail mix contains raisin, peanut, and chocolate. The ratio of raisin to peanut is $2:3$ and the ratio of peanut to chocolate is $5:8$. What is the ratio of raisin to chocolate?

6-3 Ratios, Rates, and Proportions Word Problems

You can use conversion factors to convert a unit of measure from one system to another.
Sometimes you may need to use two or more conversion factors.

Example 1 □ A model car is scaled so that 1 inch of the model equals 6 feet of the actual

car. If the model is $1\frac{2}{3}$ inch long, how long is the actual car?

Solution □ Let $x =$ the length of actual car.

$$\begin{array}{l}\text{model} \to \\ \text{actual} \to\end{array} \quad \frac{1 \text{ in}}{6 \text{ ft}} = \frac{1\frac{2}{3} \text{ in}}{x \text{ ft}} \quad \begin{array}{l}\leftarrow \text{ model} \\ \leftarrow \text{ actual}\end{array}$$ Set up a proportion.

$$x = 6 \times 1\frac{2}{3} = 10$$ Cross product

The length of the actual car is 10 feet.

Example 2 □ A car is traveling at a constant rate of 54 miles per hour. How many kilometers
will the car travel in 5 minutes? (1 mile $= 1.6$ kilometers)

Solution □ $54 \text{ miles} = 54 \text{ mi} \times \dfrac{1.6 \text{ km}}{1 \text{ mi}} = 86.4 \text{ km}$

$$\frac{54 \text{ mi}}{1 \text{ hr}} = \frac{86.4 \text{ km}}{60 \text{ min}}$$ $54 \text{ miles} = 86.4 \text{ km}$ and $1 \text{ hour} = 60 \text{ min}$

$$\frac{86.4 \text{ km}}{60 \text{ min}} = \frac{x \text{ km}}{5 \text{ min}}$$ Set up a proportion.

$$86.4 \times 5 = 60x$$ Cross Products

$$x = \frac{86.4 \times 5}{60}$$ Divide.

$$= 7.2 \text{ km}$$

Example 3 □ The ratio of males to females in an office is $6 : 7$. If there are 42 males in the office,
what is the total number of people in the office?

Solution □ Let $f =$ the number of females in the office.

$$\begin{array}{l}\text{male} \to \\ \text{female} \to\end{array} \quad \frac{6}{7} = \frac{42}{f}$$ The ratio of males to females is 6 to 7.

$$6f = 7 \cdot 42 = 294$$ Cross products

$$f = \frac{294}{6} = 49$$ Divide.

The total number of people in the office is
$42 + 49$, or 91.

Exercises - Ratios, Rates, and Proportions Word Problems

1

If 20 machines produce 1,240 printers in a day, how many more machines are needed to produce 1,984 printers in a day?

A) 12

B) 20

C) 24

D) 32

2

If $\frac{3}{4}$ quart of lemonade concentrate is mixed with $6\frac{2}{3}$ quarts of water to make lemonade for 40 people, how many quarts of lemonade concentrate are needed to make the lemonade for 24 people?

A) $\frac{3}{10}$

B) $\frac{7}{20}$

C) $\frac{2}{5}$

D) $\frac{9}{20}$

3

A machine produced 735 tapes in $5\frac{1}{4}$ hours. What fraction of the 735 tapes was produced in one hour?

A) $\frac{1}{7}$

B) $\frac{4}{21}$

C) $\frac{5}{21}$

D) $\frac{2}{7}$

4

A 32-acre field yields 768 bushels of corn each year. How many more acres are needed to yield 960 bushels of corn each year?

A) 6

B) 8

C) 10

D) 12

5

The length of a rectangle is 8 inches longer than the width. If the ratio of the length to perimeter is $5:16$, what is the area of the rectangle?

A) 160 in^2

B) 180 in^2

C) 240 in^2

D) 280 in^2

6

If 12 grams of coffee costs x dollars and each gram makes y cups of coffee, what is the cost of one cup of coffee in terms of x and y?

A) $\frac{12y}{x}$

B) $\frac{y}{12x}$

C) $\frac{12x}{y}$

D) $\frac{x}{12y}$

Chapter 6 Practice Test

1

The density of an object is equal to the mass of the object divided by the volume of the object. What is the mass, in grams, of an object with a volume of 0.01 m^3 and a density of 4.54 grams per cubic centimeters? ($1 \text{ m} = 100 \text{ cm}$)

A) 454

B) 4,540

C) 45,400

D) 454,000

2

Jason and Donny painted a house and received $1,200. To complete the painting job Jason painted 4 hours 25 minutes and Donny spent 2 hours and 15 minutes. If they split the $1,200 in proportion to the amount of time each spent painting, how much did Donny receive?

A) $405.00

B) $443.00

C) $472.00

D) $492.00

3

The tennis balls in a bag are either white or yellow. If the ratio of white balls to yellow balls is $\dfrac{3}{10}$, which of the following could not be the number of balls in the bag?

A) 26

B) 39

C) 42

D) 52

4

A car is traveling at a constant rate of x miles per hour. How many miles will the car travel in y minutes?

A) $60xy$

B) $\dfrac{60x}{y}$

C) $\dfrac{xy}{60}$

D) $\dfrac{y}{60x}$

5

A tree is 8 feet tall and grows 8 inches each year. In how many years will the tree reach a height of 30 feet?

A) 27

B) 33

C) 45

D) 52

6

Aaron reads x pages of a science fiction book in m minutes. If he continues reading at this rate, what will be the number of pages he reads in $20\,m$ seconds?

A) $\dfrac{1}{3}x$

B) $\dfrac{1}{2}x$

C) $\dfrac{2}{3}x$

D) $2x$

7

If $\dfrac{x}{y} = 1$, what is the value of $x - y - 1$?

A) −1

B) 0

C) 1

D) The value cannot be determined from the information given.

8

In a certain room the ratio of males to females is 4 to 5. After 8 males enter the room, the ratio of males to females is 6 to 5. What is the total number of people in the room before the additional males enter the room?

A) 27

B) 36

C) 45

D) 54

9

A person is born every 5 seconds and a person dies every 12 seconds. How many seconds does it take for the population to grow by one person?
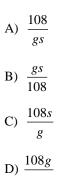
A) 7 sec

B) $8\dfrac{4}{7}$ sec

C) 10.5 sec

D) $10\dfrac{5}{7}$ sec

10

Steve is going to paint a wall that measures 9 feet by 12 feet. If one gallon of paint is needed for each s square foot of wall and each gallon costs g dollars, in terms of s and g how much does it cost to paint the entire wall?

A) $\dfrac{108}{gs}$

B) $\dfrac{gs}{108}$

C) $\dfrac{108s}{g}$

D) $\dfrac{108g}{s}$

11

If 2 inches are equivalent to 5 centimeters, how many square centimeters are in one square inch?

12

A large painting has a length of 18 inches and a width of 12 inches. If each dimension is reduced by x inches to make the ratio of length to width 5 to 3, what is the value of x?

Answer Key

Section 6-1

1. B   2. D   3. C   4. D   5. 23
6. 5   7. 32.4

Section 6-2

1. D   2. B   3. B   4. 240   5. 8
6. 6   7. $\dfrac{5}{12}$

Section 6-3

1. A   2. D   3. B   4. B   5. C
6. D

Chapter 6 Practice Test

1. C   2. A   3. C   4. C   5. B
6. A   7. A   8. B   9. B   10. D
11. $\dfrac{25}{4}$ or 6.25   12. 3

Answers and Explanations

Section 6-1

1. **B**

$$\dfrac{1\frac{3}{4}}{2\frac{1}{2}} = \dfrac{14}{x} \qquad \text{The ratio of } 1\frac{3}{4} \text{ to } 2\frac{1}{2} \text{ is equal to the ratio of 14 to } x.$$

$1\frac{3}{4} \cdot x = 14 \cdot 2\frac{1}{2}$ Cross Products

$\dfrac{7}{4} x = 14 \cdot \dfrac{5}{2}$ Simplify.

$\dfrac{7}{4} x = 35$ Simplify.

$\dfrac{4}{7} \cdot \dfrac{7}{4} x = \dfrac{4}{7} \cdot 35$ Multiply each side by $\dfrac{4}{7}$.

$x = 20$ Simplify.

2. **D**

Let x and y be the two numbers.

$x + y = 14$ The sum of two numbers is 14.

$\dfrac{x}{y} = -3$.   The ratio of the two numbers is -3.

$\dfrac{x}{y} = -3 \;\Rightarrow\; x = -3y$

$x + y = 14$ First equation
$(-3y) + y = 14$ Substitute $-3y$ for x.
$-2y = 14$ Simplify.
$y = -7$

Substitute $y = -7$ in the first equation.

$x + (-7) = 14 \;\Rightarrow\; x = 21$

Therefore the product of the two numbers is $x \cdot y = 21 \cdot (-7) = -147$.

3. **C**

$2(x - y) = 3y$
$2x - 2y = 3y$ Distributive property
$2x = 5y$ Add $2y$ to each side.
$\dfrac{2x}{2} = \dfrac{5y}{2}$ Divide each side by 2.
$x = \dfrac{5}{2} y$ Simplify.
$\dfrac{x}{y} = \dfrac{\frac{5}{2} y}{y}$ Divide each side by y.
$\dfrac{x}{y} = \dfrac{5}{2}$ Simplify.

4. **D**

Let $6x =$ the length and $7x =$ the width of the rectangle.
$P = 2\ell + 2w$ Perimeter of a rectangle.
$78 = 2(6x) + 2(7x)$ $P = 78$, $\ell = 6x$, and $w = 7x$
$78 = 26x$ Simplify.
$3 = x$ Divide each side by 26.
Therefore, the length of the rectangle is $6 \cdot 3$ or 18, and the width of the rectangle is $7 \cdot 3$ or 21. The area of the rectangle is $18 \cdot 21$ or 378.

5. **23**

$$\text{Gas Mileage} = \dfrac{\text{Number of Miles Traveled}}{\text{Number of Gallons of Gas Used}}$$

$= \dfrac{218.5}{9.5} = 23$

The car's gas mileage is 23 miles per gallon.

6. **5**

$$\text{Unit Price} = \dfrac{\text{Price of Package}}{\text{Number of Units in the Package}}$$

$= \dfrac{0.95}{20} = 0.0475$

The unit price of the vitamin water to the nearest cent is 5.

7. 32.4

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\frac{2.7 \text{ grams}}{1 \text{ cm}^3} = \frac{x \text{ grams}}{12 \text{ cm}^3}$$

$$x = 2.7 \times 12 = 32.4 \text{ grams}$$

Section 6-2

1. D

Set up a proportion.

$$\frac{1 \text{ inch}}{5 \text{ miles}} = \frac{10 \text{ inches}}{x \text{ miles}} \implies x = 50 \text{ miles}$$

$$\frac{1 \text{ inch}}{5 \text{ miles}} = \frac{7.2 \text{ inches}}{y \text{ miles}} \implies y = 7.2 \times 5 = 36 \text{ miles}$$

The area of the state is 50×36, or $1,800 \text{ mi}^2$.

2. B

Let the number of students $= 27x$ and let the number of teachers $= 2x$. Then, there will be $27x + 2x$, or $29x$ students and teachers who are in the meeting. Now set up a proportion.

$$\frac{\text{total in the meeting}}{\text{number of teachers}} = \frac{29x}{2x} \implies$$

$$\frac{754}{\text{number of teachers}} = \frac{29\cancel{x}}{2\cancel{x}} \implies$$

$$\text{number of teachers} = \frac{754 \times 2}{29} = 52$$

3. B

Let $5x =$ the volume of cement, $9x =$ the volume of sand, and $13x =$ the volume of gravel. Thus the total volume of concrete is $5x + 9x + 13x$, or $27x$, which is equal to 324 ft^3.

$$27x = 324 \implies x = 12$$

Therefore, the amount of cement is $5x = 5 \cdot 12 = 60$.

4. 240

$$1 \text{ hour } 45 \text{ minutes} = 1\frac{3}{4} \text{ hours}$$

Set up a proportion.

$$\frac{84 \text{ miles}}{1\frac{3}{4} \text{ hours}} = \frac{x \text{ miles}}{5 \text{ hours}}$$

$$1\frac{3}{4}x = 84 \cdot 5 \qquad \text{Cross Products}$$

$$\frac{7}{4}x = 420 \qquad \text{Simplify.}$$

$$\frac{4}{7} \cdot \frac{7}{4}x = \frac{4}{7} \cdot 420 \qquad \text{Multiply each side by } \frac{4}{7}.$$

$$x = 240$$

He can drive 240 miles in 5 hours.

5. 8

Let $2x =$ the number of quarters, $4x =$ the number of dimes, and $7x =$ the number of nickels. Then the total amount in terms of x, $2x(0.25) + 4x(0.1) + 7x(0.05)$, is equal to $5.00.

$$2x(0.25) + 4x(0.1) + 7x(0.05) = 5.00$$

$$0.5x + 0.4x + 0.35x = 5$$

$$1.25x = 5$$

$$x = 4$$

There are $2x = 2 \cdot 4$, or 8 quarters.

6. 6

$$\frac{5x}{3} = \frac{x+14}{2}$$

$$2(5x) = 3(x+14) \qquad \text{Cross Products}$$

$$10x = 3x + 42$$

$$7x = 42$$

$$x = 6$$

7. $\dfrac{5}{12}$

Let $r =$ the amount of raisin, $p =$ the amount of peanut, and $c =$ the amount of chocolate. Then

$$\frac{r}{p} = \frac{2}{3} \qquad \text{The ratio of raisin to peanut is } 2:3.$$

$$3r = 2p \qquad \text{Cross Products}$$

$$p = \frac{3}{2}r \qquad \text{Solve for } p.$$

$$\frac{p}{c} = \frac{5}{8} \qquad \text{The ratio of peanut to chocolate is } 5:8.$$

$$8p = 5c \qquad \text{Cross Products}$$

$$p = \frac{5}{8}c \qquad \text{Solve for } p.$$

Equate the two equations solved for p.

$$\frac{3}{2}r = \frac{5}{8}c \implies \frac{2}{3} \cdot \frac{3}{2}r = \frac{2}{3} \cdot \frac{5}{8}c$$

$$\implies r = \frac{5}{12}c \implies \frac{r}{c} = \frac{5}{12}$$

Section 6-3

1. A

Set up a proportion.

$$\frac{20}{1240} = \frac{x}{1984} \quad \begin{array}{l} \leftarrow \text{ number of machines} \\ \leftarrow \text{ number of printers} \end{array}$$

$1240x = 20 \cdot 1984$ Cross Products

$x = \dfrac{20 \cdot 1984}{1240} = 32$

Altogether we need 32 machines, therefore we need $32 - 20$, or 12, more machines.

2. D

Let $x =$ the number of quarts of lemonade concentrate needed for 24 people.

In this question "$6\dfrac{2}{3}$ quarts of water" was unnecessary information.

$$\frac{\frac{3}{4}}{40} = \frac{x}{24} \quad \begin{array}{l} \leftarrow \text{ quarts} \\ \leftarrow \text{ people} \end{array}$$

$40x = 24 \cdot \dfrac{3}{4}$ Cross products

$x = 24 \cdot \dfrac{3}{4} \cdot \dfrac{1}{40} = \dfrac{9}{20}$

3. B

The number of tapes produced in one hour is equal to $735 \div 5\dfrac{1}{4}$, or 140.

The fraction of 735 tapes produced in one hour is $\dfrac{140}{735}$, or $\dfrac{4}{21}$.

4. B

Set up a proportion.

$$\frac{32}{768} = \frac{x}{960} \quad \begin{array}{l} \leftarrow \text{ number of acres} \\ \leftarrow \text{ number of bushels} \end{array}$$

$768x = 32 \cdot 960$ Cross products

$x = \dfrac{32 \cdot 960}{768} = 40$

Altogether we need 40 acres, therefore we need $40 - 32$, or 8, more acres.

5. C

Let $x =$ the width of the rectangle,
then $x + 8 =$ the length of the rectangle.

$\begin{array}{ll} P = 2\ell + 2w & \text{Perimeter of a rectangle.} \\ P = 2(x+8) + 2(x) & \ell = x+8, \text{ and } w = x \\ P = 4x + 16 & \text{Simplify.} \end{array}$

$\dfrac{\text{length}}{\text{perimeter}} = \dfrac{x+8}{4x+16} = \dfrac{5}{16}$

$16(x+8) = 5(4x+16)$ Cross Products

$16x + 128 = 20x + 80$

$48 = 4x$

$12 = x$

The length of the rectangle is $12 + 8$, or 20 and the width of the rectangle is 12.
The area of the rectangle is $20 \cdot 12$, or 240.

6. D

If 12 grams of coffee cost x dollars, the cost of each gram of coffee is $\dfrac{x}{12}$ dollars. Let one cup of coffee cost d dollars, and set up a proportion to find the cost of one cup of coffee.

$$\frac{\frac{x}{12}}{y} = \frac{d}{1} \quad \begin{array}{l} \leftarrow \text{ cost in dollars} \\ \leftarrow \text{ number of cups} \end{array}$$

$y \cdot d = \dfrac{x}{12}$ Cross Products

$d = \dfrac{x}{12y}$

Chapter 6 Practice Test

1. C

$1 \text{ m} = 100 \text{ cm}$

$1 \text{ m}^3 = (100 \text{ cm})^3 = 1{,}000{,}000 \text{ cm}^3$

$0.01 \text{ m}^3 = 0.01 \times 1{,}000{,}000 \text{ cm}^3 = 10{,}000 \text{ cm}^3$

$\text{Density} = \dfrac{\text{Mass}}{\text{Volume}}$

$4.54 \text{ grams}/\text{cm}^3 = \dfrac{\text{Mass}}{0.01 \text{ m}^3} = \dfrac{\text{Mass}}{10{,}000 \text{ cm}^3}$

$\begin{aligned} \text{Mass} &= 4.54 \, \dfrac{\text{grams}}{\text{cm}^3} \cdot 10{,}000 \text{ cm}^3 \\ &= 45{,}400 \text{ grams} \end{aligned}$

2. A

$\begin{aligned} \text{Total time} &= \text{Jason's time} + \text{Donny's time} \\ &= 4 \text{ hour } 25 \text{ min} + 2 \text{ hour } 15 \text{ min} \\ &= 4\dfrac{5}{12} \text{ hour} + 2\dfrac{1}{4} \text{ hour} = 6\dfrac{2}{3} \text{ hour} \end{aligned}$

The amount Donny received

$$= 1,200 \times \frac{2\frac{1}{4} \text{ hour}}{6\frac{2}{3} \text{ hour}} = 1,200 \cdot \frac{\frac{9}{4}}{\frac{20}{3}} = 1,200 \cdot \frac{9}{4} \cdot \frac{3}{20}$$

$$= 405$$

3. **C**

If the ratio of white balls to yellow balls is $\frac{3}{10}$,

$3n$ represents the number of white balls and $10n$ represents the number of yellow balls (n is a positive integer).
Since the total number of balls in the bag is $3n + 10n$, or $13n$, and n is a positive integer, the number of balls will be a multiple of 13.

Choice C is correct, because 42 is not a multiple of 13.

4. **C**

Let m = the number of miles traveled in y minutes. Substitute 60 minutes for 1 hour and set up a proportion.

$$\frac{x}{60} = \frac{m}{y} \quad \begin{array}{l} \leftarrow \text{ number of miles} \\ \leftarrow \text{ number of minutes} \end{array}$$

$60m = xy$ Cross Products

$$m = \frac{xy}{60}$$

5. **B**

Let x = the number of years it will take the tree to reach a height of 30 feet.

Also, 8 inches $= \frac{8}{12}$ feet .

$8 + \frac{8}{12}x = 30$ The tree is 8 feet tall and will

grow $\frac{8}{12}x$ feet in x years.

$\frac{8}{12}x = 22$

$x = 22 \cdot \frac{12}{8} = 33$

6. **A**

m minutes $= 60m$ seconds
Let p = the number of pages he reads in $20m$ seconds.
Set up a proportion.

$$\frac{x}{60m} = \frac{p}{20m} \quad \begin{array}{l} \leftarrow \text{ number of pages} \\ \leftarrow \text{ number of seconds} \end{array}$$

$60m \cdot p = 20m \cdot x$ Cross Products

$$p = \frac{20m \cdot x}{60m} = \frac{1}{3}x$$

7. **A**

$\frac{x}{y} = 1$

$y \cdot \frac{x}{y} = y \cdot 1$ Multiply each side by y.

$x = y$ Simplify.

$x - y = y - y$ Subtract y from each side.

$x - y = 0$ Simplify.

$x - y - 1 = 0 - 1$ Subtract 1 from each side.

$x - y - 1 = -1$ Simplify.

8. **B**

Let m = the number of males in the room and let f = the number of females in the room.

$\frac{m}{f} = \frac{4}{5}$ The ratio of males to females is 4 to 5.

$5m = 4f$ Cross Products

$\frac{m+8}{f} = \frac{6}{5}$ After 8 males enter the room, the ratio of males to females is 6 to 5.

$5(m + 8) = 6f$ Cross Products

$5m + 40 = 6f$ Simplify.

$4f + 40 = 6f$ Substitute $4f$ for $5m$.

$40 = 2f$ Subtract $2f$ from each side.

$20 = f$ Divide each side by 2.

Substituting 20 for f in the equation $5m = 4f$ gives $5m = 4 \cdot 20$. Solving for m yields $m = 16$.

The total number of people in the room before the additional males enter the room is $m + f = 16 + 20 = 36$.

9. **B**

If a person is born every 5 seconds, 12 people are born per minute. If a person dies every 12 seconds, 5 people die per minute. Every minute the population grows by $12 - 5$, or 7, people.

Therefore, it takes $\frac{60}{7}$ seconds, or $8\frac{4}{7}$ seconds, for the population to grow by one person.

10. D

Total area of the wall $= 9 \times 12 = 108 \text{ ft}^2$.

Let it take p gallons of paint to paint 108 ft^2.

Set up a proportion.

$\dfrac{1}{s} = \dfrac{p}{108} \quad \begin{array}{l} \leftarrow \text{ number of gallons} \\ \hline \leftarrow \text{number of square feet} \end{array}$

$sp = 108 \qquad\qquad$ Cross Products

$p = \dfrac{108}{s}$

It takes $\dfrac{108}{s}$ gallons of paint to paint 108 ft^2.

Since each gallon costs g dollars, the total cost

will be $\dfrac{108}{s} \cdot g$ dollars.

11. $\dfrac{25}{4}$ or 6.25

$2 \text{ in} = 5 \text{ cm}$

$1 \text{ in} = \dfrac{5}{2} \text{ cm} \qquad$ Divide each side by 2.

$(1 \text{ in})^2 = (\dfrac{5}{2} \text{ cm})^2 \qquad$ Square both sides.

$1 \text{ in}^2 = \dfrac{25}{4} \text{ cm}^2 \qquad$ Simplify.

There are $\dfrac{25}{4}$ square centimeters in 1 square inch.

12. 3

The reduced length of the painting is $18 - x$ and
the reduced width of the painting is $12 - x$.

$\dfrac{18 - x}{12 - x} = \dfrac{5}{3} \qquad$ The new ratio is 5 to 3.

$3(18 - x) = 5(12 - x) \quad$ Cross Products

$54 - 3x = 60 - 5x \qquad$ Distributive Property

$54 + 2x = 60 \qquad\qquad$ Add $5x$ to each side.

$2x = 6 \qquad\qquad\qquad$ Subtract 54 from each side.

$x = 3 \qquad\qquad\qquad$ Divide each side by 2.