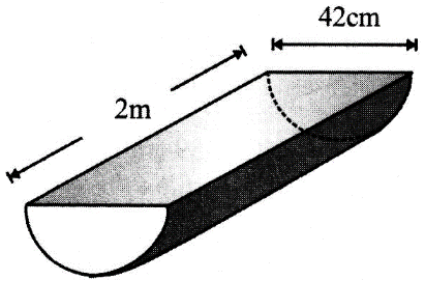
Surface Areas and Volumes

OLYMPIAD EXCELLENCE BOOK

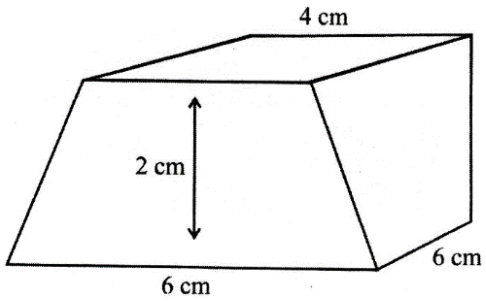
QUESTIONS

1. **A horsed drinking through has the dimensions shown. How long will it take to fill the through if water flows it in 22 litters per minute?**



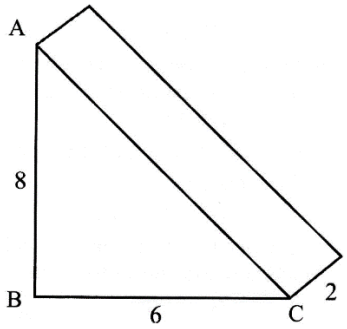
(a) 6 m (b) 6 m 18 sec (c) 6 m 42 sec (d) 6 m 22 sec

2. **An ice block with uniform cross section has the dimension shown alongside what is the volume of the ice block?**



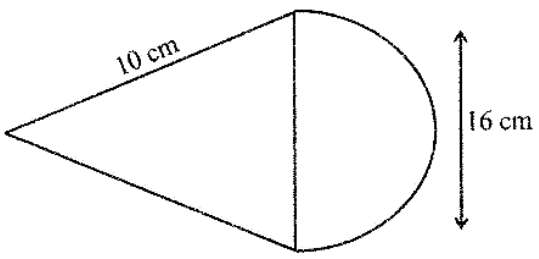
(a) 36 cm^2 (b) 40 cm^2 (c) 44 cm^2 (d) 50 cm^2

3. **What is the surface area of solid given alongside?**



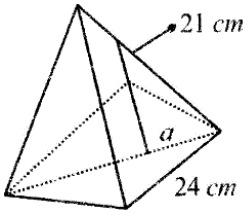
(a) 96 cm^2 (b) 108 cm^2 (c) 100 cm^2 (d) 92 cm^2

4. **What is the volume of the solid shown alongside?**



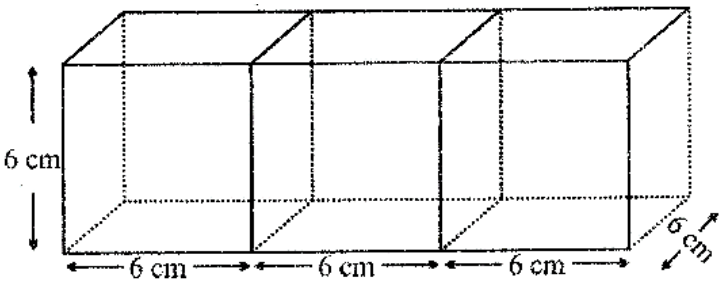
(a) 265 cm^3 (b) 255 cm^3 (c) 359 cm^3 (d) 569 cm^3

5. What is the volume of the solid shown alongside?



(a) 4032 cm^3 (b) 4023 cm^3 (c) 3210 cm^3 (d) 3040 cm^3

6. Three cubes each of side 6 cm are joined end-to-end as shown in the figure. Then what **is the surface area of the resulting cuboid?**



(a) 252 cm^2 (b) 504 cm^2 (c) 540 cm^2 (d) 450 cm^2

7. 10 cylindrical pillars of a building have to be painted. The radius of each pillar is 35 cm and the height is 2 m. what is the cost of painting at the rate of Rs 5 per sq m?

(a) Rs 200 (b) Rs 220 (c) Rs 280 (d) Rs 500

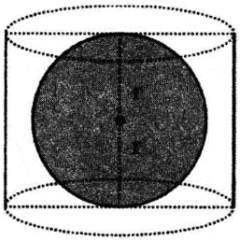
8. A right triangle ABC with its sides 5 cm, 12 cm, and 13 cm is revolved about the side 12 cm. Find the volume of the solid so formed.

(a) $59\ \pi cm^2$ (b) $5\pi cm^2$ (c) $100\pi cm^2$ (d) $120\pi cm^2$

9. A cardboard sheet in the form of a circular sector of radius 20 cm and central angle 108° is folded to make a cone. What is the radius of the cone?

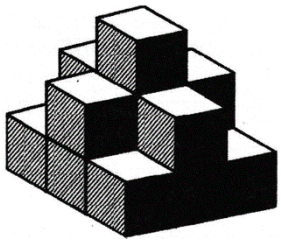
(a) 6 cm (b) 18 cm (c) 21 cm (d) 4 cm

10. A right cylinder just encloses a sphere of radius r as shown in the given figure. What is the ratio of surface area of sphere and curved surface area of the cylinder?



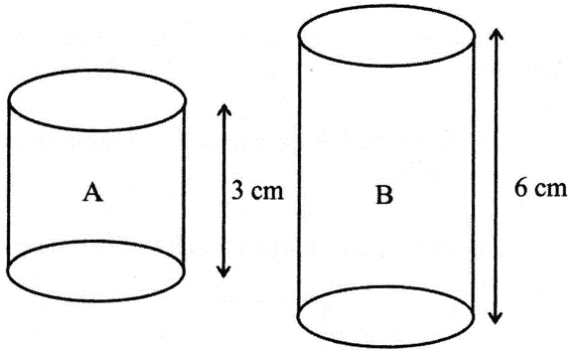
(a) 2 : 1 (b) 1 : 3 (c) $1:\sqrt{2}$ (d) 1 : 1

11. A child playing with building blocks which are of the shape of the cubes has build a structure as shown in the figure given below. If the each of the cube is 2 cm. what is the structure built by the cube?



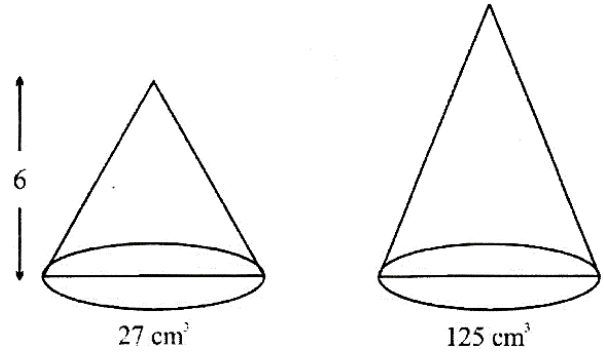
(a) 96 cm^3 (b) 120 cm^3 (c) 114 cm^3 (d) 112 cm^3

12. Cylinder A and B are with heights 2 cm and 4 cm respectively. If cylinder 4 has volume 20 cm^3, then the volume of cylinder B is



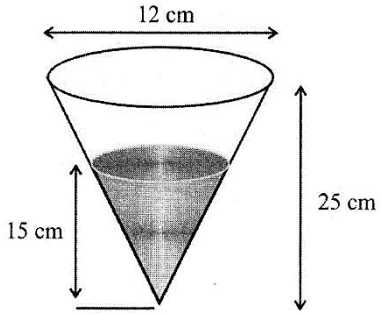
(a) 60 cm^2 (b) 70 cm^3 (c) 80 cm^3 (d) 160 cm^3

13. A and B are similar cones with volumes 27 cm^3 and 125 cm^2 respectively. If cone A is 6 cm high, then the height of cone B will be



(a) 10 cm (b) 5 cm (c) 15 cm (d) 20 cm

14. A conical flask has height 25 cm. and base diameter 12 cm. Water is poured into the flask in a depth of 15 cm. The diameter of the surface of the water is



(a) 6 cm (b) 7.2 cm (c) 8 cm (d) 9 cm

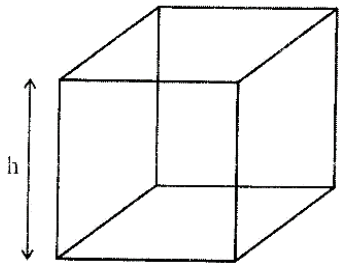
15. What is the length of the uniform wire of radius 0.2 cm that can be drawn from a solid sphere of radius 3 cm?

(a) 9 m (b) 24 m (c) 12 m (d) 60 m

16. What will be the cost to plaster the inner surface of a well 28 m deep and 2 m in radius at the rate of 25 **per sq m?**

(a) Rs 4000 (b) Rs 2200 (c) Rs 8800 (d) Rs 4400

17. In the given figure, volume of a square based box is 500 cm^2 and base length is 10 cm. The height h of the box is

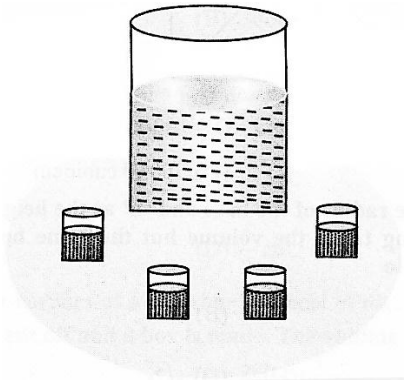


(a) 7 .5 cm (b) 10 cm (c) 5 cm (d) 2.5 cm

18. The length of a cold storage is double its breadth. Its height is 3 metres. The area of its four walls (including doors) is 180 in.'". What is its volume

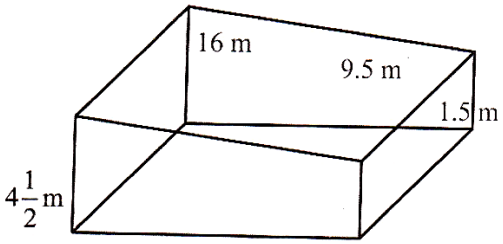
(a) 210 cm^3 (b) 220 cm^3 (c) 216 cm^3 (d) 250 cm^3

19. At a Dusherra Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 25 cm filled lap to height of 24 cm with orange juice. The juice is filled in small cylindrical glasses (See figure) of radius 5 cm up to a height of 6 cm, and sold for Rs. 10 each. How much money does the stall keeper receive by selling the juice completely?



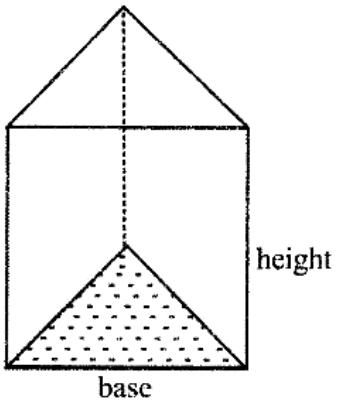
(a) Rs 1000 (b) Rs 800 (c) Rs 1200 (d) Rs 900

20. The Level of water of a swimming pool is rectangular of which length is 32 metres and width is 9.5 metres and the depth of water at one end is $1\frac{1}{2}$ metres which Increases to $4\frac{1}{2}$ metres at other end. Find the volume of water in the swimming pool.



(a) 900 *metre*^{3} (b) 456 *metre*^{3} (c) 942 *metre*^{3} (d) 924 *metre*^{3}

21. The base of a right prism is an equilateral triangle of side 6 cm and height of the prism is 20 cm. then the volume of the prism is



(a) $320\sqrt{3}$ cubic cm (b) $180\sqrt{3}$ cubic cm (c) $150\sqrt{3}$ cubic cm (d) $300\sqrt{3}$ cubic cm

22. A cylinder has 'r' as the radius of the base and 'V as the height. The radius of base of another cylinder, having triple the volume but the same height as that of the first cylinder must be equal to

(a) $\frac{r}{\sqrt{2}}$ (b) 2r (c) $r\sqrt{3}$ (d) $\sqrt{2r}$

23. The height of the cone is 40 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume is $\frac{1}{64}$ of the volume of the cone. At what height, above the base, is the section made?

(a) 6 cm (b) 8 cm (c) 10 cm (d) 20 cm

24. If the ratio of the diameters of two spheres is 4:5, then what is the ratio of their surface areas?

(a) 16 : 25 (b) 9 : 10 (c) 3 : 5 (d) 16 : 25

25. The radius of base of a right circular cone is 3.5 cm and slant height is 10 cm, then what is its lateral surface area?

(a) 110 sq cm (b) 100 sq cm (c) 70 sq cm (d) 49 sq cm

26. The base of a right pyramid is a square of side 30 cm long. If the volume of the pyramid is 4500 cm^3 then its height is

(a) 5 cm (b) 10cm (c) 15 cm (d) 20 cm

27. The respective height and volume of a cylinder and a right circular Hemisphere are equal, then the ratio of their radii is

(a) $\sqrt{2}:\sqrt{3}$ (b) $\sqrt{3}:1$ (c) $\sqrt{3}:\sqrt{2}$ (d) $2:\sqrt{3}$

28. **The radii of the base of Cone and a Cylinder are in the ratio $\sqrt{3}:\sqrt{2}$ are in the ratio $\sqrt{2}:\sqrt{3}$. Their volume are in the ratio of**

(a) $\sqrt{3}:\sqrt{2}$ (b) $3\sqrt{3}:\sqrt{2}$ (c) $\sqrt{3}:2\sqrt{2}$ (d) $\sqrt{2}:\sqrt{6}$

29. **A rectangular water tank is $160\text{m}\times 20\text{m}$. Water flows at the opening at a speed of 10 km/hr. The water level will rise in the tank in half an hour by**

(a) $\frac{3}{2}$ cm. (b) $\frac{4}{9}$ cm, (c) $\frac{5}{9}$ cm. (d) $\frac{5}{8}$ cm.

30. **30. From each of the four corners of a rectangular sheet of diamensions $20\text{cm}\times 16\text{cm}$, a square of side 3 cm is cut off and a box is made. The volume of the box is**

(a) 828 cm^3 (b) 280 cm^3 (c) 500 cm^3 (d) 1000 cm^3

ANSWER KEY & HINTS

1. (b): Volume $= \frac{1}{2} \times \frac{22}{7} \times \frac{21}{100} \times 2 \times 1000$

$$\text{Time} = \frac{\frac{1}{2} \times \frac{22}{7} \times \frac{21}{100} \times 2 \times 1000}{22} = 6m18\sec .$$

2. (b): Volume $= 4 \times 2 \times \frac{(6+4)}{2} = 40\ cm^3$

3. (a): $AC = \sqrt{6^2 + 8^2} = 10$

Surface Area $= 2 \times \frac{1}{2} \times 6 \times 8 + 2 \times 6 + 2 \times 8 + 2 \times 10 = 96\,cm^2$

4. (a): Volume $= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3 + \frac{2}{3} \times \frac{22}{7} \times 4 \times 4 \times 4 = 265\,cm^2$

5. (a):

6. (b): Surface Area $= 2\ [6 \times 18 + 18 \times 6 + 6 \times 6] = 504 cm^2$

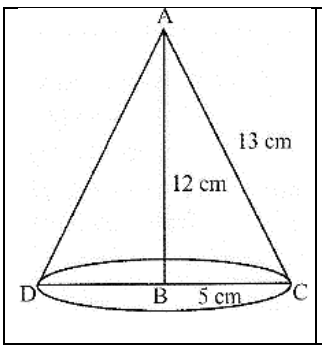
7. (b): Given, r = 35 cm, h = 2m

$\therefore$ Surface area of cylinder $10(2\pi rh)$

$$= 10\left(2 \times \frac{22}{7} \times 0.35 \times 2\right) = 44\text{ m}$$

$\therefore$ Total cost of painting $= 44 \times 5 = Rs220$

8. (c);



Let ABC be a right triangle with AB=12cm, BC=5 cm and AC= 13cm

When this triangle is revolved about AB, it forms a right circular cone of radius =BC=5cm and height AB= 12 cm.

$\therefore V_1$ = Volume of the solid formed= Volume of the cone of radius 5 cm and height 12 cm

$= \frac{1}{3} \times \pi \times 5 \times 5 \times 12 cm^3 = 100\pi cm^3$

9. (a): Here, area of sector, $2\pi r\left(\frac{\theta}{360^\circ}\right) = l$

Also, length of one $l = 2\pi r$

$$2\pi r = 2\pi \times 20 \times \frac{108}{360}$$

$$\Rightarrow r = \frac{20 \times 144}{360} = 6 \text{ cm}$$

10. (d) Clearly, the radius of the cylinder is r and its height is 2r

= Surface area of the sphere $= 4\pi r^2$

Radius of the cylinder = r, Height of the cylinder = 2r

$\therefore$ Curved surface area of the cylinder $= 2\pi r \times 2r = 4\pi r^2$

Required ratio $= 4\pi r^2 : 4\pi r^2 = 1:1$

11. (b): Not Available

12. (d): Suppose cylinder A is enlarged with scale factor k to give cylinder B

$$\therefore K = \frac{6}{3} = 2$$

Volume of $B = k^3 x$

Volume of $A = 8 \times 20 - 160\ cm^3$

13. (c): Suppose A is enlarged with scale factor k to give cone B.

Volume of $B = K^3 x$ Volume of A

$125 = K^3 \times 27$

$\frac{125}{125} = K^3 \qquad \therefore K = \frac{5}{3}$

So, height of the cone $B = \frac{5}{3} \times$ height of the cone $A = \frac{5}{3} \times 9 = 15 \text{ cm}$

14. (b): $\frac{25}{15} = \frac{12}{?}$

$? = \frac{36}{5} = 7.2$ cm

15. (a): Given that,

Radius of sphere $(r) = 3cm = 0.03$ m

Radius of wire $= 0.2$

$\Rightarrow R = 0.2cm = 0.002 \text{m}$

Given condition,

Volume of sphere=Volume of wire

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi R^2 h$$

$$\therefore h = \frac{4}{3}.\frac{r^3}{R^2}$$

$$= \frac{4}{3} \times \frac{0.03 \times 0.03 \times 0.03}{0.002 \times 0.002} = 9\,\text{cm}$$

16. (c): Curved surface area of the well $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 2 \times 28 = 352\ m^2$$

$\therefore$ Expenses of $352 \times 25 = 8800$

17. (c): $10 \times 10 \times h = 500$

$h = 5$ cm

18. (c): Let length, breadth and height of the cold storage be l metres, b metres and h metres respectively. Then, $l = 2b$ (given) and h = 3 metres.

Now, Area of four walls = 108

$\Rightarrow 2(l + b)h = 108$

$\Rightarrow 2(2b + b) \times 3 = 108 \Rightarrow 18b = 108 \Rightarrow b = 6$ metres

$\therefore l = 2b \Rightarrow l = 12$ metres

Hence, Volume of the cold storage $= (lbh)m^3 = (12 \times 6 \times 3)m^3 = 216$ m^3

19. (a): The volume of juice in the vessel $= \pi \times 25 \times 25 \times 24$

The volume of one glass $= \pi \times 5 \times 5 \times 6 cm^3$

So, number of glasses of juice that are sold

$$= \frac{Volume\ of\ the\ vessel}{Volume\ of\ each\ glass}$$

$$= \frac{\pi \times 25 \times 25 \times 24}{\pi \times 5 \times 5 \times 6}$$

$=100$

Therefore, amount received by the stall keeper $= Rs\,10 \times 100$

=Rs 1000

20. (b): The cross section of swimming pool is like a trapezium, of which parallel sides are $4\frac{1}{2}$ metres and $1\frac{1}{2}$ metres and vertical perpendicular is 32 metres

$\therefore$ Area of cross section

$= \frac{1}{2} \times 16 \times \left(4\frac{1}{2} + 1\frac{1}{2}\right) = 48$ sq. m.

$\therefore$ Volume of water in swimming pool

= Area of cross section $\times$ Length

$= 48 \times 9.5 = 456$ cubic metres.

21. (b): Area of the base $= \frac{\sqrt{3}}{4} \times (side)^2$

$= \frac{\sqrt{3}}{4} \times 6 \times 6 = 9\sqrt{3}$ sq.cm

$\therefore$ Volume of prism =Area of base $\times$ height

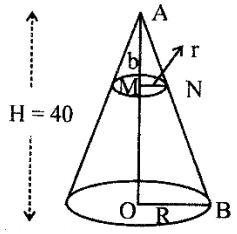
$= 9\sqrt{3} \times 20 = 180\sqrt{3}$ cu. cm

22. (c): Let the radius of the new cylinder be R then.

$3\pi r^2 h = \pi R^2 h$

$\Rightarrow R^2 = 3r^2 \Rightarrow R = \sqrt{3r} = r\sqrt{3}$

23. (d): Let H and R be the height and radius of bigger cone respectively and h and r that of smaller cone.



From triangles AOB and AMN.

$\angle A$ is common and $MN \| OB$.

$\therefore$ Triangles AOB and AMN are similar,

$\therefore \frac{AO}{MA} = \frac{BO}{MN}$

$\Rightarrow \frac{40}{h} = \frac{R}{r}$ .....(i)

Volume of smaller cone $= \frac{1}{3}\pi r^2 h$

Volume of bigger cone $= \frac{1}{3}\pi R^2 h$

According to the question

$$\frac{1}{3}\pi r^2 h = \left(\frac{1}{3}\pi R^2 h\right) \times \frac{1}{64}$$

$$\Rightarrow r^2 h = \frac{R^2 H}{64} \Rightarrow 64r^2 h = R^2 H$$

$$\Rightarrow \frac{64h}{H} = \left(\frac{40}{h}\right)^2 \quad \text{[form (i)]}$$

$$\Rightarrow \frac{64h}{H} = \frac{1600}{h^2}$$

$$\Rightarrow 64h^3 = 1600H = 1600 \times 40$$

$$\Rightarrow h^3 = \frac{1600 \times 40}{64} = 1000$$

$$\Rightarrow h^3 = \sqrt[3]{1000} = 10cm$$

$\therefore$ Required height $= 30 - 10 = 20cm$

24. (a): Given that, let the diameters of two sphere are d_1 and d_2 respectively.

$\therefore d_1 : d_2 = 4 : 5$

$\therefore$ Ratio of their surface areas

$$= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{(2r_1)^2}{(2r_2)^2} = \frac{d_1^2}{d_2^2}$$

$$= \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

$$= 16 : 25$$

25. (a): Given that,

$\therefore$ Radius of a right circular cone $= 3.5$ cm

and slant height of a right circular cone $(l) = 10$ cm

$\therefore$ lateral surface area of a cone = πrl

$$= \frac{22}{7} \times 3.5 \times 10 - 110cm^2$$

26. (c): Area of the base $= 30 \times 30 = 900$ sq.cm

We know, Volume of pyramid

$$= \frac{1}{3} \times area\ of\ base \times height$$

$$\Rightarrow 4500 = \frac{1}{3} \times 900 \times h$$

$$\Rightarrow h = \frac{4500 \times 3}{900} = 15cm$$

27. (c): Let the Radius of hemisphere = Height of cylinder =r units

$$\therefore \frac{Volume\ of\ hemisphere}{Volume\ of\ cylinder} = 1$$

$$\Rightarrow \frac{\frac{2}{3}\pi r^3}{\pi r_1^{\ 2}} = 1 \Rightarrow \frac{r^2}{r_1^{\ 2}} = \frac{3}{2}$$

$$\Rightarrow \frac{r}{r_1} = \frac{\sqrt{3}}{\sqrt{2}} or \sqrt{3} : \sqrt{2}$$

28. (b): $\frac{Volume\ of\ cone}{Volume\ of\ cylinder}$

$$= \frac{\pi r_1^{\ 2} h_1}{\pi r_2^{\ 2} h_2} = 3.\left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right)$$

$$= 3 \times \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2 \times \frac{\sqrt{2}}{\sqrt{3}} = 3 \times \frac{\sqrt{3}}{\sqrt{2}} = 3\sqrt{3} : \sqrt{2}$$

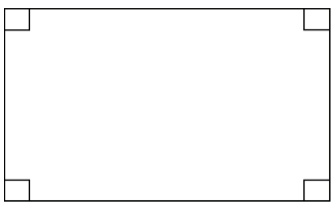
29. (d): Volume of water filled by pipe in 30 minutes

$$= \left(\frac{40 \times 1000000}{2}\right) \text{ cu. cm.}$$

=20000000 cu.cm

$$\therefore \text{ Height of water level} = \frac{20000000}{8000 \times 4000} = \frac{5}{8} \text{ cm.}$$

30. (b):



Length of box $= 20 - 2 \times 3 = 14$ cm

Width of box $= 16 - 2 \times 3 = 10$ cm

Height of box = 3 cm.

$\therefore$ Volume of box $= (14 \times 10 \times 2)$ cu.cm. = 280 cu.cm.