Class-X Session 2022-23

Subject - Mathematics (Standard)

Sample Question Paper - 37

With Solution

BLUE PRINT

Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Real Number	6	3(Q1, 5, 8)	1(Q19)	1(Q21)				6
2	Polynomials	20	2(Q2, 3)						2
3	Pair of Linear Equations in Two Variables		2(Q4, 6)				1(Q34)		7
4	Quadratic Equations		1(Q7)				1(Q32)		6
5	Arithmetic Progression		2(Q9, 10)			1(Q26)			5
6	Triangles	15	1(Q11)		1(Q22)				3
7	Circles		1(Q17)		1(Q25)		1(Q35)	1(Q36)	12
8	Coordinate Geometry	6	3(Q12, 13, 14)	1(Q20)	1(Q23)				6
9	Introduction to Trigonometry	12	2(Q15, 16)		1(Q24)	1(Q27)			7
10	Some Applications of Trigonometry						1(Q33)		5
11	Areas Related to Circles	10				1(Q28)		1(Q37)	7
12	Surface Areas and Volumes					1(Q29)			3
13	Statistics	11				1(Q30)			8
14	Probability		1(Q18)			1(Q31)		1(Q38)	8
Total Marks (Total Questions)		80	18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 case based integrated units of assessment (4 marks each) with sub parts of values of 1, 1 and 2 marks each respectively.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. The sum of three non-zero prime numbers is 100. One of them exceeds the other by 36. Then, the largest number is

(a) 73 (b) 91 (c) 67 (d) 57

2. Let $p(y) = y^4 - 3y^2 + 2y + 5$, then the remainder when $p(y)$ is divided by $(y - 1)$.

(a) 2 (b) 3 (c) -5 (d) 5

3. If $x^2 - 4$ is the factor of $2x^3 + k_1x^2 + k_2x + 12$, where k_1, k_2 are constant, then the value of $k_1 + k_2$ is

(a) 11 (b) 5 (c) -11 (d) -5

4. The difference between two numbers is 26 and one number is three times the other. Find them.

(a) 39, 13 (b) 41, 67 (c) 96, 70 (d) 52, 26

5. The value of $0.\overline{235}$ is :

(a) $\frac{233}{900}$ (b) $\frac{233}{990}$ (c) $\frac{235}{999}$ (d) $\frac{235}{990}$

6. ₹49 was divided among 150 children. Each girl got 50 paise and each boy got 25 paise. How many boys were there?

(a) 100 (b) 102 (c) 104 (d) 105

7. If $x^2 + y^2 = 25$, $xy = 12$, then $x =$

(a) $\{3, 4\}$ (b) $\{3, -3\}$ (c) $\{3, 4, -3, -4\}$ (d) $\{-3, -3\}$

8. The unit digit in the expression $55^{725} + 73^{5810} + 22^{853}$ is

(a) 0 (b) 4 (c) 5 (d) 6

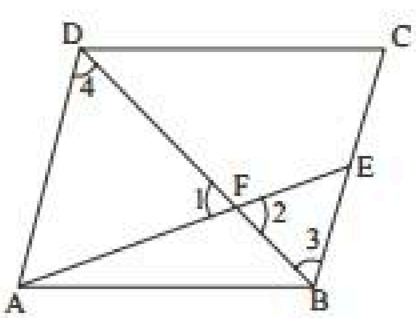
9. Sum of n terms of the series $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + ...$ is

(a) $\frac{n(n+1)}{2}$ (b) $2n((n+1)$ (c) $\frac{n(n+1)}{\sqrt{2}}$ (d) 1

10. If eight times the 8^{th} term of an A.P. is equal to 12 times the 12^{th} term of the A.P. then its 20^{th} term will be

(a) -1 (b) 1 (c) 0 (d) 2

11. The diagonal BD of a parallelogram ABCD intersects the segment AE at the point F, where E is any point on the side BC. Then



(a) $\frac{EF}{FA}=\frac{FB}{AB}$ (b) $DF \times EF = FB \times FA$ (c) $DF \times EF = (FB)^2$ (d) None of these

12. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(a) 5 (b) 12 (c) 11 (d) $7+\sqrt{5}$

13. If the point P(6, 2) divides the line segment joining A(6, 5) and B(4, y) in the ratio 3 : 1, then the value of y is

(a) 4 (b) 3 (c) 2 (d) 1

14. The points $A(-4,-1)$, $B(-2,-4)$, $C(4,0)$ and $D(2,3)$ are the vertices of a

(a) Parallelogram (b) Rectangle (c) Rhombus (d) Square

15. If $\sin(A+B)=\frac{\sqrt{3}}{2}$ and $\sin 2B=\frac{1}{2}$, then

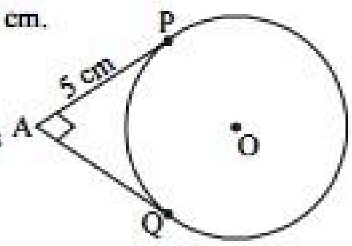
(a) $\tan B = 1$ (b) $B = 30°$ (c) $B = 45°$ (d) $\cos A = \frac{1}{2}$

16. If $\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=4$, then

(a) $\cos\theta=\frac{\sqrt{3}}{2}$ (b) $\sin\theta=\frac{1}{2}$ (c) $\theta=60°$ (d) $\tan\theta=\frac{1}{\sqrt{3}}$

17. In fig. the pair of tangents AP and AQ drawn from an external point A to a circle with centre O are perpendicular to each other and length of each tangent is 5 cm.

Then, the radius of the circle is



(a) 2 (b) 3 (c) 5 (d) 1

18. A bag contains 40 coins, consisting of '₹2, '₹5 and ₹10 denominations. If a coin is drawn at random, the probability of drawing a '₹2 coin is $\frac{5}{8}$. If x number of ₹2 coins are removed from the bag and then a coin is drawn at random, the probability of drawing a ₹2 coin is $\frac{1}{2}$. Find the value of x.

(a) 5 (b) 2 (c) 10 (d) 8

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***
(b) ***Both A and R are true but R is not the correct explanation of A.***
(c) ***A is true but R is false.***
(d) ***A is false but R is true.***

19. **Assertion :** If L.C.M. $\{p, q\} = 30$ and H.C.F $\{p, q\} = 5$, then $p.q = 150$.
Reason : L.C.M. of (a, b) × H.C.F of $(a, b) = a.b$.

20. **Assertion :** Mid-point of a line segment divides line in the ratio 1 : 1.
Reason : If area of triangle is zero that means points are collinear.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Prove that $\frac{1}{3+\sqrt{11}}$ is irrational.

22. AD is the median of ΔABC. The bisector of $\angle ADB$ and $\angle ADC$ meet AB and AC at points E and F. Prove that $EF \parallel BC$.

23. If (3,0), (2, a) and (b, 6) are the vertices of a Δ ABC, whose centroid is (2, 5). Find the values of a and b.

OR

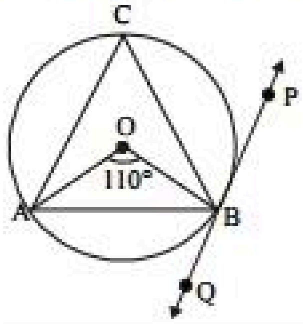
Using distance formula show that the points A(8,1), B(3, –4) and C(2, –5) are collinear.

24. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sqrt{mn}$

OR

Solve : $\dfrac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$; $(\theta < 90^\circ)$

25. In figure, AB is a chord of circle, and PQ is a tangent at point B of the circle. If $\angle AOB = 110^\circ$, then find $\angle ABQ$.



SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

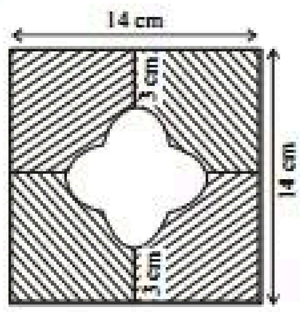
26. Find the sum of first 24 terms of the sequence whose n^{th} term is $a_n = 3 + \dfrac{2n}{3}$

OR

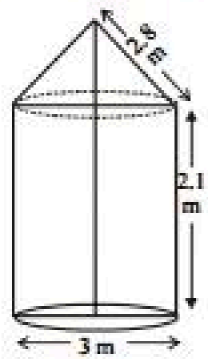
The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

27. If $4\tan\theta = 3$, then $\left(\dfrac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

28. In figure find the area of the shaded region [Use $\pi = 3.14$]



29. In figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of ₹ 500/sq. metre. (Use $\pi = \dfrac{22}{7}$]



OR

Show that the points A (1, 0), B (5, 3), C (2, 7) and D (–2, 4) are the vertices of a parallelogram.

30. The data regarding marks obtained by 48 students of a class in a class test is given below. Calculate the modal marks of students.

Marks Obtained	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50
Number of students	1	0	2	0	0	10	25	7	2	1

31. Find the chance that a non-leap year contains 53 Saturdays.

SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Sove for x : $\frac{1}{x+1}+\frac{3}{5x+1}=\frac{5}{x+4}, x \neq -1, -\frac{1}{5}, -4$

33. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr.

34. Solve the following pair of linear equations graphically :

$$x+3y=6,\ 2x-3y=12$$

Also shade the region bounded by the line $2x-3y=12$ and both the co-ordinate axes

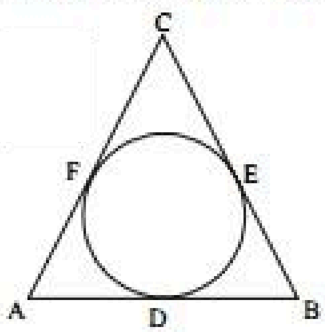
OR

Determine graphically whether the following pair of linear equations has :

$$3x-y=7$$
$$2x+5y+1=0$$ has :

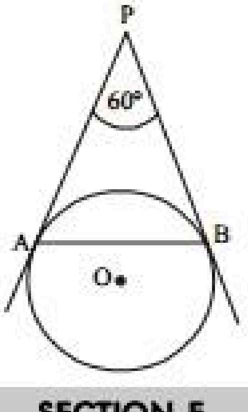
(i) a unique solution (ii) infinitely many solutions or (iii) no solution

35. In figure, a circle is inscribed in a Δ ABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



OR

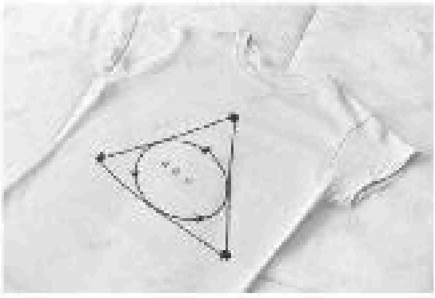
In figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and ∠APB = 60°. Find the length of chord AB.



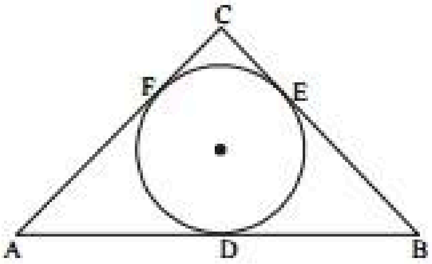
SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36. **Case - Study 1:** Read the following passage and answer the questions given below.
Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff . The logo design is as given in the figure and he is working on the fonts and different colours according to the theme.



In given figure, a circle with centre O is inscribed in a "ABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.



(i) Find the length of AD (ii) Find the Length of BE (iii) Find the length of CF

OR

If radius of the circle is 4cm, Find the area of "OAB

37. Case - Study 2: Read the following passage and answer the questions given below.
On school sport day, a sport teacher make a racing track whose left and right ends are semicircular shown in figure.

The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide then answer the following questions.

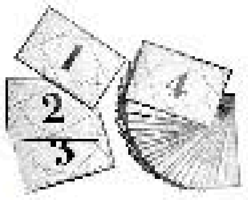
(i) Find the radius of inner semicircular end. (ii) Find the radius of outer semicircular end

(iii) The distance around the track along its inner edge is:

OR

The distance around the track along its outer edge is:

38. Case - Study 3: Read the following passage and answer the questions given below.
Rakesh and Mohit playing a card game. Rakesh picked up a card from properly mixed cards numbered from 1 to 25. Then answer the following questions :



(i) The probability of getting prime numbers is :
(ii) The probability of getting multiple of 3 is :
(iii) The probability of getting multiple of 2 is :

OR

The probability of getting multiple of 2 and 3 is :

Solution

SAMPLE PAPER-3

1. (c) Since, the sum of all the three prime numbrs is 100. Then, there are two cases

Case 1: All the three numbers should be even because 100 is an even number. But this case is not possible as there is only one even prime.

Case 2 : One prime is even and other two primes are odd. Since, 2 is only even prime, so it must be one of three primes. Let p and $p+36$ be the other two primes.

Then, according to question

$2+p+(p+36)=100; 2p+38=100$

$2p=100-38=62; p=\frac{62}{2}=31$

So, all the three primes are 2, 31 and 67.

Hence, largest prime number is 67.

2. (d) Substitute $y=1$

3. (c) $x^2-4=(x-2)(x+2)$ are the factors

$\therefore x=2,-2$ are roots of polynomial

$\therefore$ at $x=2$; $P(2)=2(2)^3+k_1(2)^2+k_2(2)+12=0$

$\Rightarrow 16+4k_1+2k_2+12=0 \Rightarrow 2k_1+k^2=-14$ ...(i)

at $x=2$; $P(-2)=2(-2)^3+k_1(-2)^2+k_2(-2)+12=0$

$\Rightarrow -16+4k_1-2k_2+12=0$

$\Rightarrow 2k_1-k_2=2$ ...(ii)

From (i) & (ii), $k_1=-3 \therefore k_1+k_2=-11$

4. (a) Let the two numbers be x and y $(x>y)$. Then,

$x-y=26$ ... (i)

$x=3y$ ... (ii)

Substituting value of x from equation (ii) in (i)

$3y-y=26 \Rightarrow 2y=26 \Rightarrow y=13$

Substituting value of y in equation (ii), $x=3\times 13=39$

Thus, two numbers are 13 and 39.

5. (c) Let $x=0.\overline{235}$ ...(i)

$1000x=235.\overline{235}$ ...(ii)

Subtract (i) from (ii), $999x=235 \Rightarrow x=\frac{235}{999}$

6. (c) Let the no. of girls be 'x' and the no. of boys be 'y'.

Given, $0.50x+0.25y=49$

and $x+y=150$ ...(i)

$\Rightarrow \frac{x}{2}+\frac{x}{4}=49$ ...(ii)

From (i) & (ii), $x=46, y=104$

Hence, number of boys $(y)=104$

7. (c) $x^2+y^2=25,\ xy=12$

$\Rightarrow x^2+\left(\frac{12}{x}\right)^2=25 \Rightarrow x^4+144-25x^2=0$

$\Rightarrow (x^2-16)(x^2-9) \Rightarrow x^2=16$ and $x^2=9$

$\Rightarrow x=\pm 4$ and $x=\pm 3$

8. (d) For given numbers,

$(55)^{725}$, unit digit $=5$; $(73)^{5810}$, unit digit $=9$

$(22)^{853}$, unit digit $=2$

Unit digit in the expression

$55^{725}+735^{810}+22^{853}$ is 6

9. (c) Here, $a_1=\sqrt{2}, a_2=\sqrt{8}=2\sqrt{2}$

$\therefore \quad d=2\sqrt{2}-\sqrt{2}=\sqrt{2}, a=\sqrt{2}$

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$=\frac{n}{2}\left[2\times\sqrt{2}+(n-1)\sqrt{2}\right]=\frac{n(n+1)}{\sqrt{2}}$

10. (c) $t_8=a+7d, t_{12}=a+11d$

According to question, $8t_8=12t_{12}$ (given)

$\Rightarrow 8(a+7d)=12(a+11d)$

$\Rightarrow 8a+56d=12a+132d$

$\Rightarrow 8a-12a+56d-132d=0$

$\Rightarrow -4a-76d=0$

$\Rightarrow a+19d=0$ ...(i)

$\therefore \quad t_{20}=a+19d=0$ using (i)

$\therefore \quad t_{20}=0$

11. (a) In ΔAFD & ΔFEB,

$\angle 1=\angle 2$ (V.O.A)

$\angle 3=\angle 4$ (Alternate angle)

$\therefore$ Δ FBE ~ Δ FDA

So, $\frac{EF}{FA}=\frac{FB}{DF}$

12. (b) A(0, 4), B(0, 0), C(3, 0)

$AB=\sqrt{(0-0)^2+(0-4)^2}=4$

$BC=\sqrt{(3-0)^2+(0-0)^2}=3$

$CA=\sqrt{(0-3)^2+(4-0)^2}=5$

$AB+BC+CA=12$

13. (d) $P(6,2)=\left(\frac{4\times 3+1\times 6}{3+1},\frac{3\times y+1\times 5}{3+1}\right)$

$\because \quad 6\neq\frac{18}{4}$ (Question is wrong)

$2=\frac{3y+5}{4}\Rightarrow 3y+5=8$

$3y=3 \Rightarrow y=1$

14. (b) Hint: Using distance formula

15. (c) We have, $\sin\ (A+B)=\frac{\sqrt{3}}{2}$

$\Rightarrow A+B=60°$ ...(i)

and $2B=30° \therefore B=15°$

Putting B in (i), we get

$A + 15° = 60° \Rightarrow A = 45°$

16. **(c)** We have, $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

$\Rightarrow \cos\theta\left(\frac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}\right) = 4$

$\Rightarrow \frac{2\cos\theta}{\cos^2\theta} = 4 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

17. **(c)**

18. **(c)** $n(S) = 40$, let $n(C) = C$

$P(C) = \frac{5}{8} \Rightarrow \frac{C}{40} = \frac{5}{8}$ or $C = 25$

Now, $\frac{25-x}{40-x} = \frac{1}{2} \Rightarrow x = 10$

19. **(a)** Since, $a \times b = HCF \times LCM$ of (a, b).

20. **(b)** Both statements are individually correct.

21. $\frac{1}{3+\sqrt{11}} = \frac{1}{3+\sqrt{11}} \times \frac{(3-\sqrt{11})}{(3-\sqrt{11})} = \frac{3-\sqrt{11}}{3^2-11}$

[1 Mark]

$= \frac{3-\sqrt{11}}{-2} = \frac{\sqrt{11}-3}{2} = \frac{\sqrt{11}}{2} - \frac{3}{2}$

[1 Mark]

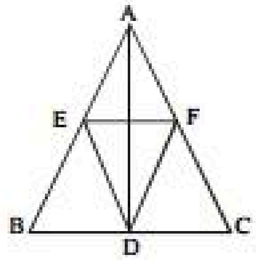
($\because$ Irrational number – Rational number = Irrational number)

Hence $\frac{1}{3+\sqrt{11}}$ is irrational.

22. In ΔADB, since DE is the bisector of $\angle ADB$

$\therefore \frac{BE}{EA} = \frac{BD}{AD}$ [Angle bisector theorem] ... (i)

In ΔADC, since DF is the bisector of $\angle ADC$



$\therefore \frac{CF}{FA} = \frac{DC}{AD} = \frac{BD}{AD}$ ......(ii) [1 Mark]

[$\because$ D is the mid-point of BC $\therefore$ BD = DC]

From (i) and (ii), we get $\frac{BE}{EA} = \frac{CF}{FA}$

In ΔABC, since $\frac{BE}{EA} = \frac{CF}{FA}$

$\therefore$ EF $\parallel$ BC [1 Mark]

(By the converse of Thale's theorem)

23. Given coordinates of vertices are (3, 0), (2, a) and (b, 6)

Centroid is (2, 5)

$\therefore \frac{x_1+x_2+x_3}{3} = 2 \Rightarrow \frac{3+2+b}{3} = 2 \Rightarrow 5+b = 6 \Rightarrow b = 6 - 5 \Rightarrow b = 1$ [1 Mark]

and $\frac{y_1+y_2+y_3}{3} = 5 \Rightarrow \frac{0+a+6}{3} = 5$

$\Rightarrow a+6 = 15 \Rightarrow a = 15-6 \Rightarrow a = 9$ [1 Mark]

OR

$AB = \sqrt{(3-8)^2+(-4-1)^2} = \sqrt{50} = 5\sqrt{2}$ [½ Mark]

$BC = \sqrt{(2-3)^2+(-5+4)^2} = \sqrt{2}$ [½ Mark]

$AC = \sqrt{(2-8)^2+(-5-1)^2} = \sqrt{72} = 6\sqrt{2}$ [½ Mark]

Since, $AB + BC = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = AC$ [½ Mark]

$\therefore$ given points are collinear

24. $m^2 - n^2 = (m+n) \times (m-n)$

$= \{(\tan\theta + \sin\theta) + (\tan\theta - \sin\theta)\} \times \{(\tan\theta + \sin\theta) - (\tan\theta - \sin\theta)\}$ [½ Mark]

$= \{2\tan\theta\} \times \{2\sin\theta\} = 4\tan\theta\sin\theta$

$= 4\sqrt{\tan^2\theta\sin^2\theta} = 4\sqrt{(\sec^2\theta-1)\sin^2\theta}$

$= 4\sqrt{\sec^2\theta\sin^2\theta - \sin^2\theta}$ $\left[\because \sec\theta = \frac{1}{\cos\theta}\right]$ [½ Mark]

$= 4\sqrt{\tan^2\theta - \sin^2\theta}$

$= 4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)} = 4\sqrt{mn}$ [1 Mark]

OR

$\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3 \Rightarrow \frac{\cos^2\theta}{\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta} = 3$ [½ Mark]

$\Rightarrow \frac{\cos^2\theta \times \sin^2\theta}{\cos^2\theta - \sin^2\theta\cos^2\theta} = 3$

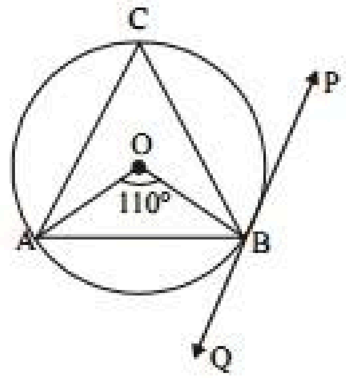
$\Rightarrow \frac{\sin^2\theta\cos^2\theta}{\cos^2\theta(1-\sin^2\theta)} = 3$ [1Mark]

$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = 3 \Rightarrow \tan^2\theta = 3 \Rightarrow \tan\theta = \sqrt{3}$

$\tan\theta = \tan 60° \Rightarrow \theta = 60°$ (acute angle) [½ Mark]

25. In ΔOAB, OA = OB (radii of the circle)

$\therefore \angle OAB = \angle OBA$



In ΔOAB, $\angle AOB + \angle OAB + \angle OBA = 180°$

$2\angle OBA = 180° - 110°$

$\Rightarrow \angle OBA = 35°$ [1 Mark]

Since BQ is a tangent at B

$\therefore \angle OBQ = 90°$

$\Rightarrow \angle OBA + \angle ABQ = 90° \Rightarrow 35° + \angle ABQ = 90°$

$\Rightarrow \angle ABQ = 90° - 35° = 55°$ [1 Mark]

26. $a_n = 3 + \frac{2n}{3}$. Here $T_n = a_n = 3 + \frac{2n}{3}$

$\Rightarrow T_1 = 3 + \frac{2(1)}{3} = \frac{11}{3}$ and $T_2 = 3 + \frac{2(2)}{3} = \frac{13}{3}$ [1 Mark]

$T_3 = 3 + \frac{2(3)}{3} = \frac{15}{3}$ ....

$\therefore \frac{11}{3}, \frac{13}{3}, \frac{15}{3}$, ... is an A.P.

Here $a = \frac{11}{3}$, $d = \frac{13}{3} - \frac{11}{3} = \frac{2}{3}$ [1 Mark]

Use, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{24} = \frac{24}{2}\left[2 \times \frac{11}{3} + (24-1)\left(\frac{2}{3}\right)\right]$

$= 12\left(\frac{22}{3} + \frac{46}{3}\right) = 12 \times \frac{68}{3} = 272$ [1 Mark]

OR

$a = 5, l = 45$

Let d = common difference

$\therefore \quad l = a + (n-1)d$

$(n-1)d = 45 - 5 = 40$ ...(i)

[½ Mark]

also, $S_n = 400$

$\therefore \frac{n}{2}[2a + (n-1)d] = 400$ [1 Mark]

$n[10 + 40] = 800$

$n = 16$ [½ Mark]

$d = \frac{40}{n-1} = \frac{40}{15} = \frac{8}{3}$ [1 Mark]

27. (c) Given, $4\tan\theta = 3$

$\Rightarrow \tan\theta = \frac{3}{4}$ [1 Mark]

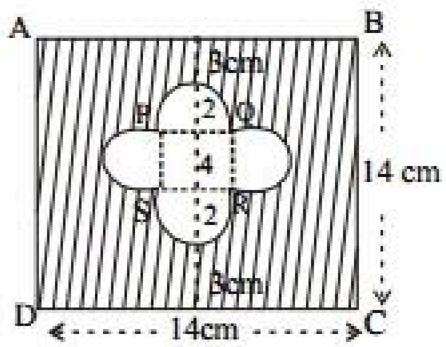
[divide by cos θ in both numerator and denominator]

$\therefore \frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1}$ [1 Mark]

$= \frac{4\tan - 1}{4\tan\theta + 1}\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3-1}{3+1} = \frac{1}{2}$ [1 Mark]

28.



Area of sq. ABCD = $(\text{side})^2 = 196 \text{ cm}^2$ [1 Mark]

Area of small sq. = $(\text{side})^2 = 4^2 = 16 \text{ cm}^2$

Area of 4 semi-circles = $4 \times \frac{1}{2}\pi r^2 = \left[4.\frac{1}{2}(3.14)(2)^2\right]\text{cm}^2$

$= 25.12 \text{ cm}^2$ [1 Mark]

$\therefore$ Area of shaded region

$= (196 - 16 - 25.12)\text{ cm}^2 = 154.88 \text{ cm}^2$ [1 Mark]

29. Canvas needed to make the tent = C.S.A of the conical part + C.S.A of the cylindrical part

Given that

Radius of the conical part = Radius of the cylindrical part

$= r = \frac{3}{2}\text{m}$

Slant height of the conical part = $l = 2.8$ m [1 Mark]

Height of the cylindrical part = h = 2.1m

C.S.A of the conical part $= \pi r l = \frac{22}{7} \times \frac{3}{2} \times 2.8 \text{ m}^2$

C.S.A of the cylindrical part $= 2\pi rh = 2 \times \frac{22}{7} \times \frac{3}{2} \times 2.1 \text{ m}^2$

$\therefore$ Total area of the canvas needed to make the tent

[1 Mark]

$= \frac{22}{7} \times \frac{3}{2} \times 2.8 + 2 \times \frac{22}{7} \times \frac{3}{2} \times 2.1$

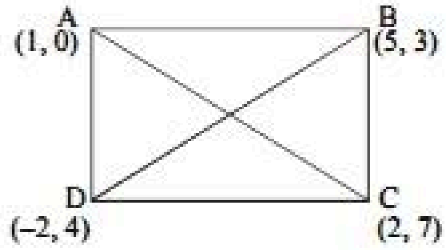
$= \frac{22}{7} \times \frac{3}{2} \times (2.8 + 4.2) = \frac{22}{7} \times \frac{3}{2} \times 7 = 33\text{m}^2$

Cost of the canvas = ₹ 500/m^2

So, total cost of the canvas needed to make the tent

= 500 × 33 = ₹ 16, 500 [1 Mark]

OR



Coordinates of the mid-point of diagonal AC =

$\left(\frac{1+2}{2}, \frac{0+7}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$ [1 Mark]

Coordinates of the mid-point of diagonal BD =

$\left(\frac{5-2}{2}, \frac{3+4}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$ [1 Mark]

Since, the coordinates of the mid-points of diagonals AC and BD are same.

∴ They bisect each other.

Hence, ABCD is a parallelogram. [1 Mark]

30. Modal class is 30 – 35, $l = 30$, $f_1 = 25$, $f_0 = 10$, $f_2 = 7$, $h = 5$ [1 Mark]

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ [1 Mark]

$\Rightarrow \text{Mode} = 30 + \left(\frac{25-10}{50-10-7}\right) \times 5 = 32.27$ approx.

[1 Mark]

31. S = {S, M, T, W, Th, F, Sa} ⇒ n(S) = 7 [½ Mark]

A non-leap year contains 365 days, i.e., 52 weeks + 1 day.

E = {Sa}, n(E) = 1 [1½ Marks]

$\therefore \; P(E) = \frac{n(E)}{n(S)} = \frac{1}{7}$ [1 Mark]

32. $\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, x \neq -1, -\frac{1}{5}, -4$

$\Rightarrow \frac{5x+1+3(x+1)}{(x+1)(5x+1)} = \frac{5}{x+4} \Rightarrow \frac{5x+1+3x+3}{5x^2+6x+1} = \frac{5}{x+4}$

[1 Mark]

$\Rightarrow (8x+4)(x+4) = 25x^2 + 30x + 5$

$\Rightarrow 8x^2 + 36x + 16 = 25x^2 + 30x + 5$ [1½ Marks]

$\Rightarrow 17x^2 - 6x - 11 = 0 \Rightarrow 17x^2 - 17x + 11x - 11 = 0$

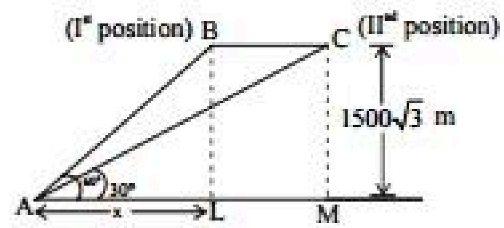
$\Rightarrow 17x(x-1) + 11(x-1) = 0$

$\Rightarrow (x-1)(17x+11) = 0$ [1 Mark]

$\Rightarrow x = 1, \frac{-11}{17}$ [1½ Marks]

Hence, the value of x is 1 or $\frac{-11}{17}$.

33.



[1 Mark]

Suppose, AL = x m

$\therefore \; \frac{BL}{x} = \tan 60°$

So, $\frac{1500\sqrt{3}}{x} = \sqrt{3}$ [2 Mark]

$\therefore x = 1500$ m

Now, $\frac{CM}{AL+LM} = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore \; 1500 + LM = 1500(3) = 4500$

Therefore, LM = 3000 m.

Hence, speed $= \frac{3000}{15}$

$= 200 \text{ m/s} = 720 \text{ km/hr}$ [2 Mark]

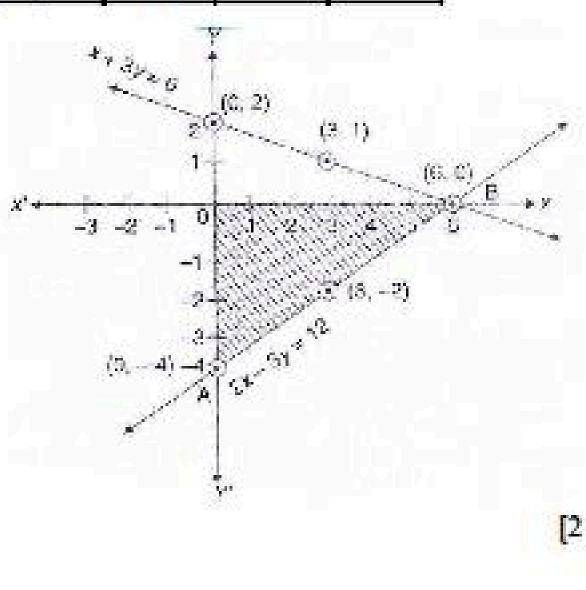
34. $x + 3y = 6 \Rightarrow y = \frac{6-x}{3}$ ...(i)

x	3	6	0
y	1	0	2

$2x - 3y = 12$ ...(ii)

$\Rightarrow y = \frac{2x-12}{3}$ [1½ Marks]

x	0	6	3
y	– 4	0	– 2



[2 Mark]

Clearly, the two lines intersect at point $B(6, 0)$. Hence, $x = 6$ and $y = 0$ is the solution of the system.

ΔOAB is the region bounded by the line

$2x - 3y = 12$ and both the co-ordinate axes. [1½ Marks]

OR

$$3x - y = 7$$

$$\Rightarrow \quad 3x - y - 7 = 0$$

$a_1 = 3, b_1 = -1, c_1 = -7$

$$2x + 5y + 1 = 0$$

$a_2 = 2, b_2 = 5, c_2 = 1$

$$\frac{a_1}{a_2} = \frac{3}{2}$$

$$\frac{b_1}{b_2} = \frac{-1}{5}$$

$$\frac{3}{2} \neq \frac{-1}{5}$$ [1 Mark]

Thus $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, given linear equations has a unique solution.

Now, we have

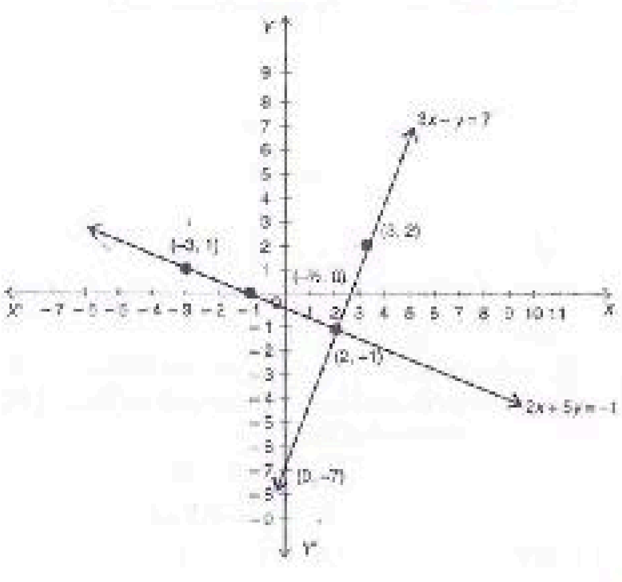
$3x - y - 7 = 0 \Rightarrow y = 3x - 7$

x	0	2	3
y	−7	−1	2

and $2x + 5y + 1 = 0$ [2 Marks]

$$\Rightarrow \quad y = \frac{-1 - 2x}{5}$$

x	− 1/2	2	−3
y	0	−1	1



[2 Marks]

Hence, $x = 2$ and $y = -1$ is the solution.

35. $\because$ AB = 12 cm

So, AD + BD = 12 cm ...(i)

$\because$ BC = 8 cm

$\therefore$ BE + CE = 8 cm ... (ii) [1 Mark]

$\because$ CA = 10 cm

Therefore, AF + CF = 10 cm ...(iii)

Here CF and CE act as tangents to the circle from the external point C. [1 Mark]

Since, the lengths of tangents drawn from an external point to a circle are equal.

$\Rightarrow$ CF = CE ... (iv)

Now, AF and AD act as tangents to the circle from the external point A.

$\Rightarrow$ AF = AD ...(v)

As, BD and BE act as tangents to the circle from the external point B.

$\Rightarrow$ BD = BE ... (vi) [1 Mark]

By (iv) and (ii),

BE + CF = 8 cm ... (vii)

By (v) and (iii),

AD + CF = 10 cm ... (viii)

By (vi) and (i),

AD + BE = 12 cm ...(ix)

After adding (vii), (viii) and (ix),

BE + CF + AD + CF + AD + BE

= 8cm + 10 cm + 12cm

$\therefore$ 2AD + 2BE + 2CF = 30 cm

So, AD + BE + CF = 15 cm ... (x)

After subtracting (vii) from (x),

AD + BE + CF − BE − CF = 15 cm − 8 cm

$\therefore$ AD = 7 cm [1 Mark]

After subtracting (viii) from (x),

AD + BE + CF − AD − CF = 15 cm − 10 cm

$\Rightarrow$ BE = 5 cm

After subtracting (ix) from (x),

AD + BE + CF − AD − BE = 15 cm − 12 cm

$\therefore$ CF = 3 cm [1 Mark]

Hence, the lengths of AD, BE and CF are 7 cm, 5 cm and 3 cm respectively.

OR

Since, AP and PB are tangents drawn to the given circle from an external point P. [1 Mark]

Since, the lengths of the tangents drawn from an external point to a circle are equal.

$\Rightarrow$ AP = PB [2 Marks]

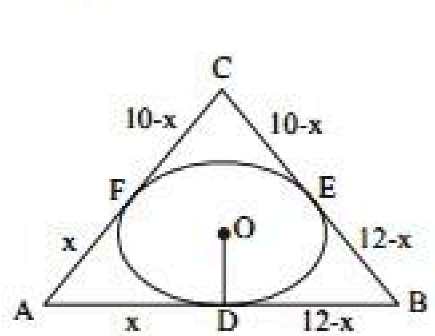
In ΔPAB, sides AP and PB are of the equal length.

So, ΔPAB is isosceles, with AP = PB and

∠PAB = ∠PBA = x (say).

Now, ∠APB = 60° [2 Marks]

36.



BC = 10 − x + 12 − x = 8 $\Rightarrow$ x = 7.

(i) AD = 7 cm [1 mark]

(ii) BE = 12 – x = 12 – 7 = 5 cm [1 mark]

(iii) CF = 10 – x = 10 – 7 = 3 cm [2 marks]

OR

$$\text{Ar } \Delta OAB = \frac{1}{2} \times AB \times OD$$

$$= \frac{1}{2} \times 12 \times 4 = 24 \text{ cm}^2$$ [2 marks]

37. (i) Radius of inner semicircular end [1 mark]

$$= \frac{60}{2} = 30 \text{ m}$$

(ii) Radius of outer semicircular end [1 mark]

= 30 + 10 = 40 m

(iii) The distance arounded the track along its inner edge

$= 106 \times 2 + 2 \times \pi r$ [1 mark]

$$= 212 + 2 \times \frac{22}{7} \times 30 = 212 + 188.57$$

= 400.57 m [1 mark]

OR

The distance arounded the track along its outer edge

$= 106 \times 2 + 2 \times \pi r$

$$= 212 + 2 \times \frac{22}{7} \times 40 = 212 + 251.43$$

= 463.43 m [2 marks]

38. (i) Let A be prime numbers from 1 to 25

A = 2, 3, 5, 7, 11, 13, 17, 19, 23

$$P(A) = \frac{9}{25}$$ [1 mark]

(ii) $P(C) = \frac{12}{25}$ [1 mark]

(iii) Let B is a multiply of 3 from 1 to 25.

B = 3, 6, 9, 12, 15, 18, 21, 24 [1 mark]

$$= \frac{8}{25}$$ [1 mark]

OR

$$P(d) = \frac{4}{25}$$ [2 marks]